Unbalanced beam splitters enabling enhanced phase sensitivity of a Mach-Zehnder interferometer using coherent and squeezed vacuum states

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An optical interferometer seeded by coherent and squeezed vacuum states seems to be the most promising platform for gravitational wave detection. Prior studies regarding this estimation protocol focus on the scenario in which the transmittances of two beam splitters are $\frac{1}{2}$. In this paper, with respect to the same inputs, we analyze the phase sensitivity of an unbalanced Mach-Zehnder interferometer (MZI) followed by balanced homodyne measurement. We give the optimal transmittance and demonstrate the advantage of our scheme over a balanced MZI scheme. Additionally, when the average photon number of the coherent state is dominant, the phase sensitivity of our scheme can nearly saturate the single-parameter quantum Cramér-Rao bound. Our results may contribute to the development of practical quantum sensing.

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I. INTRODUCTION

Optical interferometers play an essential role in the field of precision measurements in that they can be used to observe slight variations of many physical quantities, such as concentration [1,2], temperature [3–5], and angular displacement [6,7]. In general, a classical interferometer utilizes a solely coherent state as input and the corresponding phase sensitivity is governed by the shot-noise limit. Caves pointed out that this bound originates from vacuum fluctuation of the unseeded port and demonstrated that the shot-noise limit can be broken by injecting a quantum state into the unseeded port [8]. Since then, quantum interferometers have become a focus of research. Related to this, various exotic two-mode inputs have received a great deal of attention, such as NOON states [9], two-mode squeezed vacuum states [10], and entangled coherent states [11]. In theory, the phase sensitivity of an interferometer using these states can reach or even surpass the Heisenberg limit. However, preparing these states with high photon numbers remains a challenge, which causes difficulty when it comes to practical measurements. Therefore, using coherent and squeezed vacuum states is a feasible way due to the fact that a high-photon coherent state is effortless to obtain.

Over the past few years, phase estimation using a Mach-Zehnder interferometer (MZI) fed by coherent and squeezed vacuum states has been widely discussed. Pezzé and Smerzi proposed that phase sensitivity can reach the Heisenberg limit through the use of photon-number-resolving measurement [12]. Schäfermeier *et al.* demonstrated an MZI-based proof-of-principle experiment, in which phase sensitivity surpasses the shot-noise limit by a factor of 1.7 [13]. Lang and Caves proved that a squeezed vacuum state is the optimal choice for the second input port when a coherent state is seeded into the first port [14]. By calculating the quantum Fisher information, Ono and Hofmann studied the optimal weights of coherent and squeezed vacuum states in the presence of photon loss [15]. Through the same approach, phase sensitivity limits in lossless and lossy scenarios were given by Gao [16]. Gard *et al.* analyzed the effects of realistic factors on phase sensitivity using several measurement strategies and found that balanced homodyne measurement is highly robust against various factors [17].

It should be noted that all of these schemes are discussed in terms of a balanced MZI. Until recently, from the perspective of the quantum Fisher information, Zhong *et al.* calculated the sensitivity limit of an unbalanced MZI using coherent and squeezed vacuum states [18]. They found that an unbalanced MZI may provide improved performance when compared with a balanced one in some cases. Ataman showed that similar situations also exist with respect to other Gaussian and non-Gaussian inputs [19,20]. Mishra and Ataman reported a paradigmatic method for calculating the optimal transmittance using several measurement strategies [21]. As an example, they analyzed coherent and squeezed vacuum states with a specific intensity ratio and found the optimal transmittance, which can improve the sensitivity by using balanced homodyne measurement. In this paper, with balanced homodyne measurement, we comprehensively analyze an unbalanced MZI using coherent and squeezed vacuum states with arbitrary intensity ratios. We address the optimal transmittance and compare the phase sensitivity of our scheme with that of a balanced MZI. The results indicate that it is feasible to improve phase sensitivity through the use of unbalanced beam splitters. The optimal phase sensitivity of our scheme can nearly saturate the single-parameter quantum Cramér-Rao bound when the average photon number of the coherent state is dominant.

The remainder of this paper is organized as follows. Section II introduces our scheme and gives the analysis of phase sensitivity. In Sec. III we discuss the optimal transmittance

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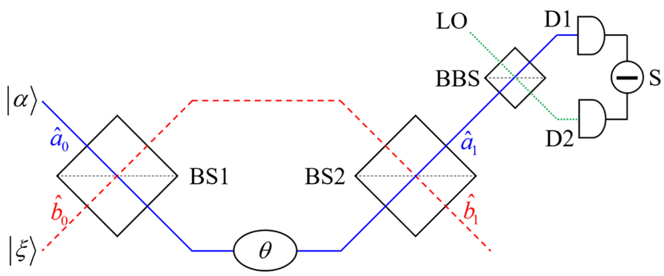

FIG. 1. Schematic of the phase estimation scheme with an MZI fed by coherent and squeezed vacuum states. The blue solid line and red dashed line represent modes A and B, respectively. The mode operators for the input and output are shown in the figure. The following denotations are used: BS, beam splitter; BBS, balanced beam splitter; LO, local oscillator; D, detector; and S, subtracter.

with respect to the phase sensitivity and show the advantage of our scheme over a balanced MZI. In Sec. IV the phase sensitivity with the optimal transmittance is compared with the quantum Cramér-Rao bound. We summarize our work in Sec. V.

II. MODEL AND SENSITIVITY ANALYSIS

In this section we introduce our scheme and analyze its phase sensitivity. Let us start with an unbalanced MZI, as depicted in Fig. 1, where $\hat{a}_0$ ($\hat{b}_0$) and $\hat{a}_1$ ($\hat{b}_1$) are annihilation operators of the input and output with respect to mode A (B). The transmittances of the two beam splitters in our scheme are arbitrary. A linear phase shift of θ occurring in arm A is the parameter we would like to estimate. We send a coherent state $|\alpha\rangle$ combined with a squeezed vacuum state $|\xi\rangle$ into the interferometer and perform balanced homodyne measurement at the output port A.

According to the theory of quantum parameter estimation, all operations before and after the estimated phase can be regarded as probe preparation and measurement. Hence, by adjusting the transmittances of two beam splitters, it is possible to construct a better probe and measurement than the use of a balanced MZI. For this reason, we consider an unbalanced MZI in this paper. In particular, when the transmittances of two beam splitters are $\frac{1}{2}$, our interferometer is simplified as a conventional interferometer.

Although the interferometer is unbalanced, the total mean photon number inside the interferometer remains $\mathcal{N} = \mathcal{N}_\mathrm{c} + \mathcal{N}_\mathrm{s}$, with $\mathcal{N}_\mathrm{c} = |\alpha|^2$ and $\mathcal{N}_\mathrm{s} = \sinh^2 r$. Throughout this paper, we assume that $\alpha = |\alpha|e^{i\delta}$ and $\xi = re^{i\varphi}$, where δ and φ are initial phases of the coherent and squeezed vacuum states, respectively.

For simplicity, we consider a particular homodyne measurement in the x direction of phase space (not real space), which takes the operator form

$$\hat{X} = \hat{a}_1 + \hat{a}_1^\dagger. \tag{1}$$

The transformation between the output and input modes of the interferometer is given by

$$\begin{aligned}\hat{a}_1 = {}&[e^{i\theta}\sqrt{\eta_1\eta_2} - \sqrt{(1-\eta_1)(1-\eta_2)}]\hat{a}_0 \\ &+ i[e^{i\theta}\sqrt{(1-\eta_1)\eta_2} + \sqrt{\eta_1(1-\eta_2)}]\hat{b}_0,\end{aligned} \tag{2}$$

where η_1 and η_2 represent the transmittance of BS1 and BS2, respectively. Further, we can calculate the expectation value of the measurement operator

$$\langle\hat{X}\rangle = 2|\alpha|[\sqrt{\eta_1\eta_2}\cos(\theta+\delta) - \sqrt{(1-\eta_1)(1-\eta_2)}\cos\delta] \tag{3}$$

and

$$\langle\hat{X}^2\rangle = 2C_1\sinh^2 r + C_2\sinh(2r) + 2C_3|\alpha|^2 + 1, \tag{4}$$

with

$$\begin{aligned}C_1 = {}&(1-\eta_1)\eta_2 + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)}\cos\theta \\ &+ \eta_1(1-\eta_2),\end{aligned} \tag{5}$$

$$\begin{aligned}C_2 = {}&(1-\eta_1)\eta_2\cos(2\theta+\varphi) + \eta_1(1-\eta_2)\cos\varphi \\ &+ 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)}\cos(\theta+\varphi),\end{aligned} \tag{6}$$

$$\begin{aligned}C_3 = {}&-2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)}[\cos(\theta+2\delta)+\cos\theta] \\ &+ (1-\eta_1)(1-\eta_2)[\cos(2\delta)+1] \\ &+ \eta_1\eta_2[\cos(2\theta+2\delta)+1].\end{aligned} \tag{7}$$

Based on Eqs. (3)–(7), the variance of the measurement results can be expressed as

$$\langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2 = 2C_1\sinh^2 r + C_2\sinh(2r) + 1, \tag{8}$$

irrespective of the initial phase δ and the amplitude $|\alpha|$ of the coherent state.

Through the use of the error propagation formula [22,23]

$$\Delta\theta = \frac{\sqrt{\langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2}}{|\partial\langle\hat{X}\rangle/\partial\theta|}, \tag{9}$$

the phase sensitivity of our scheme is calculated to be

$$\Delta\theta = \frac{\sqrt{2C_1\sinh^2 r + C_2\sinh(2r) + 1}}{2\sqrt{\eta_1\eta_2}|\alpha\sin(\theta+\delta)|}. \tag{10}$$

The smaller the value of phase sensitivity is, the closer the estimated value is to the true value.

One can verify that the phase sensitivity sits at its minimum $\Delta\theta_\mathrm{opt}^{\boldsymbol{\eta}}$ when the estimated phase is 0. Meanwhile, the optimal phase-matching conditions are found to be $\delta = \pi/2$ and $\varphi = \pi$. Since $\Delta\theta_\mathrm{opt}^{\boldsymbol{\eta}}$ is optimal for a transmittance combination $\boldsymbol{\eta} = (\eta_1, \eta_2)$, we define it as local optimal sensitivity.

In Figs. 2(a)–2(c) we show the dependence of local optimal sensitivity on the transmittances of two beam splitters with different weights of coherent and squeezed vacuum states. One can find that the local optimal sensitivity is symmetrically distributed with respect to the diagonal line $\eta_1 = \eta_2$, that is, the phase sensitivity stays the same when the transmittances of the two beam splitters are reversed.

In order to observe the effects of transmittance intuitively, we introduce a ratio which describes how close the sensitivity with transmittance combination $\boldsymbol{\eta}$ is to the sensitivity with the optimal transmittance combination. Its definition is $\mathcal{R} = \Delta\theta_\mathrm{min}/\Delta\theta_\mathrm{opt}^{\boldsymbol{\eta}}$, where $\Delta\theta_\mathrm{min}$ is the minimum of Eq. (10) over all

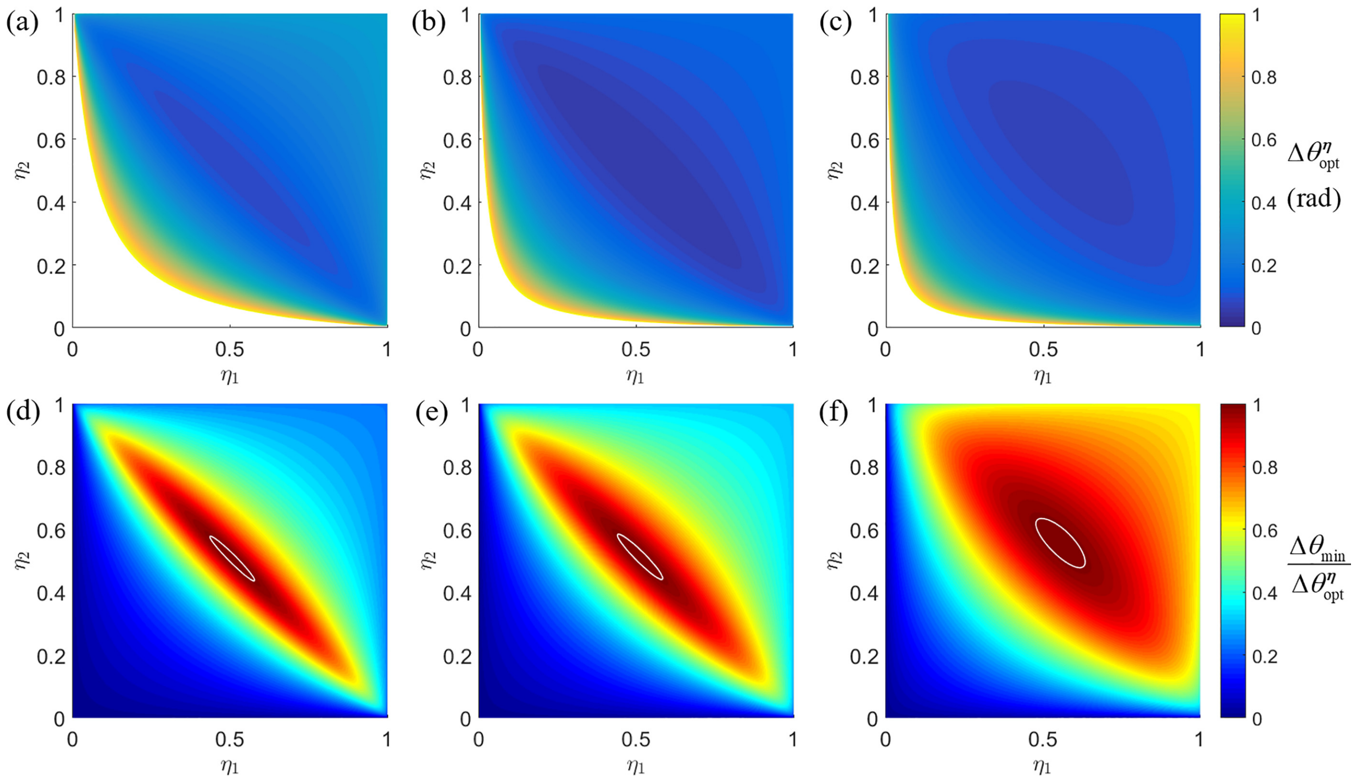

FIG. 2. The photon numbers are (a) and (d) $\mathcal{N}_{\rm c} = 2$ and $\mathcal{N}_{\rm s} = 18$, (b) and (e) $\mathcal{N}_{\rm c} = 10$ and $\mathcal{N}_{\rm s} = 10$, and (c) and (f) and $\mathcal{N}_{\rm c} = 18$ and $\mathcal{N}_{\rm s} = 2$. (a)–(c) Locally optimal sensitivity as a function of transmittances of two beam splitters, where the locally optimal sensitivity greater than 1 is not shown (white area). (d)–(f) Ratio $\Delta\theta_{\min}/\Delta\theta_{\rm opt}^{\eta}$ as a function of transmittances of two beam splitters, where ratio within the white solid line is greater than 0.99.

transmittance combination, i.e., $\Delta\theta_{\min} = \min_{\boldsymbol{\eta}}(\Delta\theta_{\rm opt}^{\boldsymbol{\eta}})$. Similar definitions are also used in other schemes [24,25]. The values of 1 and 0 correspond to the optimal transmittance combination and the worst one, respectively.

In Figs. 2(d)–2(f) we give the ratio corresponding to Figs. 2(a)–2(c). An evident phenomenon is that only when the transmittances of two beam splitters are the same does the ratio sit at 1. It also can be seen from Figs. 2(d) and 2(e) that when the photon number of the squeezed vacuum state is not less than that of the coherent state, the optimal transmittance is around $\frac{1}{2}$. However, when the number of photons in the coherent state is dominant, the optimal transmittance is no longer $\frac{1}{2}$.

These results suggest that unbalanced beam splitters can improve the phase sensitivity when compared to balanced ones. In addition, when the total number of photons is fixed, the area in which the ratio exceeds 0.99 increases with increasing weight of the coherent state. This indicates that more transmittance combinations can be used in our scheme to achieve the nearly optimal sensitivity.

III. PHASE SENSITIVITY WITH OPTIMAL TRANSMITTANCE

In Sec. II we showed that the optimal phase sensitivity is obtained when the transmittances of two beam splitters are the same. For this reason, in this section we consider the scheme using two beam splitters with the same transmittance η, i.e., $\eta_1 = \eta_2 = \eta$ and $\boldsymbol{\eta} = (\eta, \eta)$. At this point, the locally optimal sensitivity can be expressed as

$$\Delta\theta_{\rm opt}^{\boldsymbol{\eta}} = \frac{\mathcal{H}}{|\alpha|}, \tag{11}$$

with the squeezing-induced enhancement factor

$$\mathcal{H} = \frac{\sqrt{4\eta(1-\eta)(e^{-2r}-1)+1}}{2\eta}. \tag{12}$$

This result suggests that the sensitivity varies with the squeezing parameter and transmittance for a fixed coherent state. In particular, for $\eta = \frac{1}{2}$, the enhancement factor reduces to e^{-r}, i.e., $\Delta\theta_{\rm opt}^{\boldsymbol{\eta}} = e^{-r}/|\alpha|$, which is consistent with the optimal sensitivity of the scheme using a balanced MZI [17]. This also indirectly verifies the correctness of the result given by Eq. (10).

In order to determine the optimal transmittance, one can take the derivative of locally optimal sensitivity with respect to transmittance

$$\frac{\partial(\Delta\theta_{\rm opt}^{\boldsymbol{\eta}})}{\partial\eta} = \frac{2\eta(1-e^{-2r})-1}{2|\alpha|\eta^2\sqrt{1-4\eta(1-\eta)(1-e^{-2r})}} \tag{13}$$

and set this derivative to 0. The optimal transmittance minimizing the locally optimal sensitivity is found to be

$$\eta_{\rm opt}' = \frac{1}{2(1-e^{-2r})}. \tag{14}$$

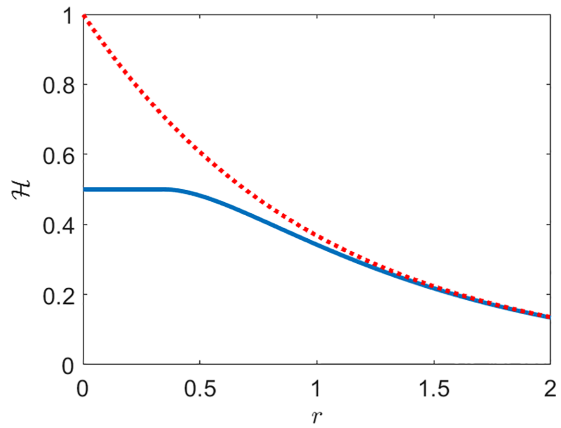

FIG. 3. Squeezing-induced enhancement factor as a function of the squeezing parameter. The red dotted line and blue solid line are the squeezing-induced enhancement factors of a balanced MZI and an unbalanced MZI with the optimal transmittance, respectively.

Here we use the prime to emphasize that this result is correct in mathematics instead of physics, since the value of transmittance should be between 0 and 1 in a realistic scenario.

With physical limitation considered, the optimal transmittance is rewritten as

$$\eta_{\text{opt}} = \begin{cases} 1, & r \leqslant \ln\sqrt{2} \\ \frac{1}{2(1-e^{-2r})}, & r > \ln\sqrt{2}. \end{cases} \tag{15}$$

In Fig. 3 we plot the squeezing-induced enhancement factors with η_{opt} (blue solid line) and $\eta = \frac{1}{2}$ (red dotted line) against the squeezing parameter, respectively. For $r = 0$, the phase sensitivity of our scheme using the optimal transmittance is twice that of a balanced MZI. This advantage is downplayed with an increase of the squeezing parameter. When the squeezing parameter is greater than 1.5, a balanced MZI becomes approximately the best choice.

This conclusion is not difficult to understand from the perspective of physics. For a small squeezing parameter, the squeezing effect will bring about the sensitivity improvement but the result is inapparent. Therefore, the sensitivity improvement is more significant when all photons of the coherent state are used through a beam splitter with a transmittance of 1. For a large squeezing parameter, the squeezing effect becomes the main thrust of sensitivity improvement. In contrast, when the transmittances are greater than $\frac{1}{2}$, the number of photons in the squeezed vacuum state passing through the estimated phase will decrease as most of the photons in the squeezed vacuum state after passing through the estimated phase are not output from the measured port. These two conditions are not conducive to improving the phase sensitivity; as a consequence, the best choice is a balanced MZI.

It is worth noting that $r = 2$ is almost the best squeezing parameter for the current experimental techniques. In this regard, adjusting the transmittances of two beam splitters is a beneficial choice for phase sensitivity. As a consequence, an unbalanced MZI is useful for a practical scheme using coherent and squeezed vacuum states.

To test the correctness of the above results, we compare them with the specific case given in the previous studies. In Ref. [21] the authors showed the optimal sensitivity with $\mathcal{N}_c = 10\,000$ and $r = 1.2$ (see Fig. 5 for details). Our phase-matching condition is $2\delta - \varphi = 0$ and the optimal transmittance calculated from Eq. (15) is 0.55. These results are the same as those in Ref. [21]. The optimal phase sensitivity in Ref. [21] is obtained with two equal transmittances, which is also consistent with our conclusion.[1]

IV. COMPARISON WITH THE QUANTUM CRAMÉR-RAO BOUND

Now we move on to compare the optimal sensitivity of our scheme with the corresponding quantum Cramér-Rao bound. Based on parameter estimation theory, the quantum Fisher

[1]The measured output port in Ref. [21] is different from our scheme. As a consequence, there is a π translation between the optimal phase points in the two schemes; meanwhile, the transmittance of BS2 in Ref. [21] is equal to the reflectivity of BS2 in our scheme.

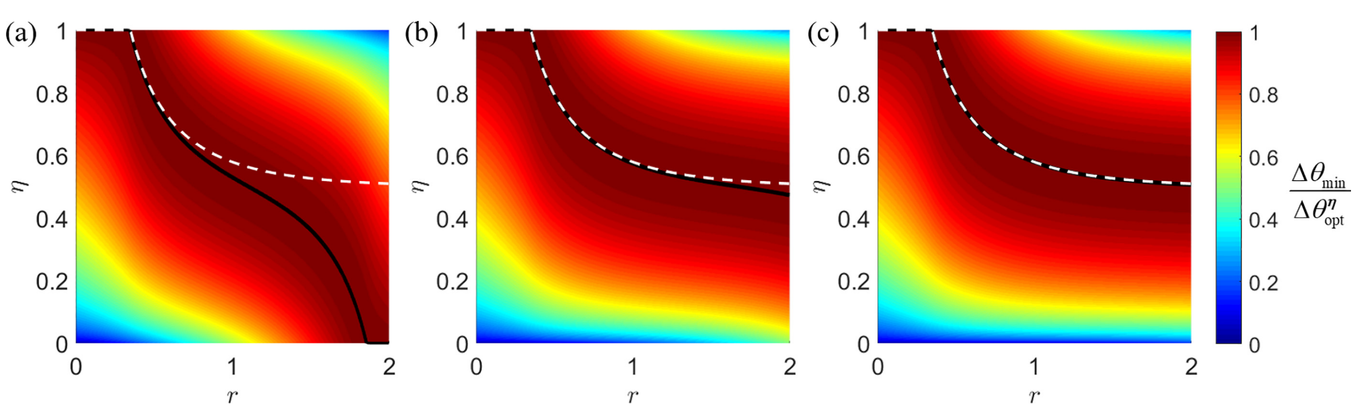

FIG. 4. Ratio $\Delta\theta_{\text{min}}/\Delta\theta^{\eta}_{\text{opt}}$ as a function of transmittance of two beam splitters and the squeezing parameter, where the white dashed line is the optimal transmittance of our scheme and the black solid line is the transmittance corresponding to the optimal QCRB, for (a) $\mathcal{N}_c = 10$, (b) $\mathcal{N}_c = 100$, and (c) $\mathcal{N}_c = 1000$.

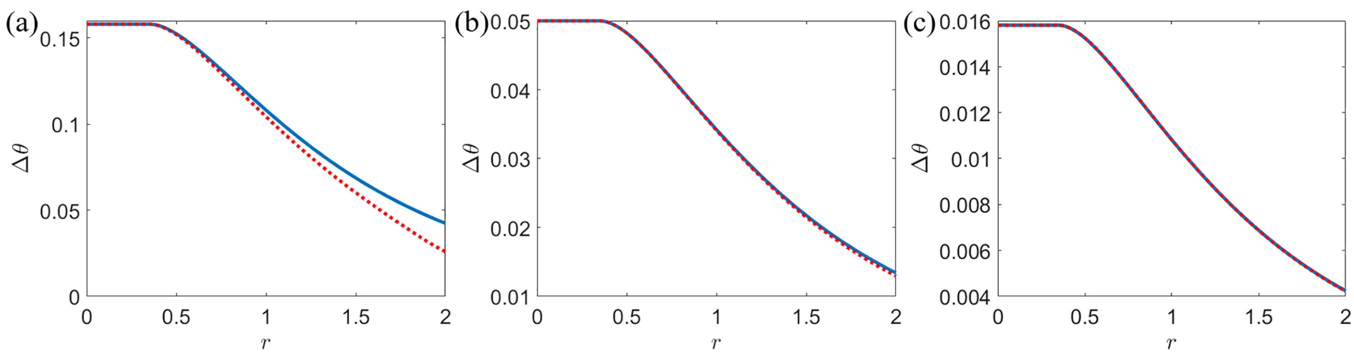

FIG. 5. Phase sensitivity of our scheme using the optimal transmittance (blue solid line) and the QCRB (red dashed line) versus squeezing parameter for (a) $\mathcal{N}_c = 10$, (b) $\mathcal{N}_c = 100$, and (c) $\mathcal{N}_c = 1000$.

information is given by[2] [26]

$$\mathcal{F} = 4\eta_1(1-\eta_1)(|\alpha|^2 e^{2r} + \sinh^2 r) + 4\eta_1^2|\alpha|^2 + 2(1-\eta_1)^2\sinh^2(2r). \quad (16)$$

Further, the quantum Cramér-Rao bound (QCRB) is found to be $\Delta\theta_{\text{QCRB}} = 1/\sqrt{\mathcal{F}}$. To each transmittance there corresponds a distinctive QCRB. In the following, the QCRB refers to the inverse of the square root of the quantum Fisher information calculated in terms of the optimal transmittance. In addition, it should be noted that the QCRB only contains the transmittance of BS1, as BS2 is a measuring device and will not increase the information of the parameter to be estimated.

For coherent states with different average photon numbers, Fig. 4 gives the ratio $\mathcal{R}$ versus different squeezing parameters and transmittances. The results show that when the average photon number of the coherent state is small, the transmittance corresponding to the optimal QCRB gradually tends to 0 as the squeezing parameter increases. For example, the optimal transmittance for $\mathcal{N}_c = 10$ and $r = 2$ is approximately 0. At this point, our scheme is equivalent to a single-mode phase estimation protocol based on a squeezed vacuum. For a coherent state with a high average photon number, the transmittance corresponding to the optimal QCRB gradually approaches $\frac{1}{2}$ with an increase of the squeezing parameter, which is completely consistent with our scheme.

So far, we have proved that our scheme has the same optimal transmittance as the optimal QCRB when a coherent state with a high average photon number is used. Finally, we further compare the QCRB and phase sensitivity of our scheme using the optimal transmittance. Figure 5 shows phase sensitivity and the QCRB as a function of the squeezing parameter with the optimal transmittance. It can be seen from the figure that, with a small average photon number in the coherent state, the phase sensitivity of our scheme can saturate and deviate from the QCRB with a low and a high squeezing parameter, respectively. When the average photon number of the coherent state increases, the QCRB can be saturated with the phase sensitivity of our scheme for any squeezing parameter. The results show that our scheme is the optimal candidate for phase estimation using coherent and squeezed vacuum states.

Here we provide a comparison of our results with the previous results in Refs. [19,20]. Figure 6 in Ref. [19] showed the QCRB with $\mathcal{N}_c = 100$ and $r = 0.5$ and that with $\mathcal{N}_c = 100$ and $r = 1.2$. The transmittances corresponding to the optimal QCRBs are about 0.8 and 0.55, respectively. These results are completely consistent with our results shown in Fig. 4(c). In addition, this work pointed out that the transmittance corresponding to the optimal QCRB is $\eta \approx 1/2(1 - e^{-2r})$ for $\mathcal{N}_c \gg \mathcal{N}_s$, which is in keeping with Eq. (15). This result proves, from a mathematical perspective, that our scheme is optimal when the average photon number of the coherent state is dominant. In Ref. [20] the QCRB with $\mathcal{N}_c \approx 249$ and $r = 1.9$ and that with $\mathcal{N}_c = 1000$ and $r = 0.88$ were studied (see Figs. 4 and 12 therein for details). The transmittances corresponding to the optimal QCRBs are about 0.5 and 0.6, respectively. We calculate the transmittances in terms of Eq. (15) since the photon number of the coherent state is dominant in the two cases. It turns out that our results are the same as those in Ref. [20]; meanwhile, the transmittance in the second case is also consistent with our result shown in Fig. 4(c).

The above results provide confirmation of the correctness of our work. Although this work is inspired by previous studies, there is a more detailed analysis showing three novel results. First, we proved that the optimal transmittance is only related to the squeezing parameter and provided the analytical expression for the optimal transmittance against squeezing parameter. Furthermore, we determined that equal transmittances of two beam splitters are the best configuration. Finally, the optimal sensitivity of our scheme can approach the QCRB when the average photon number of the coherent state is dominant.

V. CONCLUSION

We have proposed a phase estimation scheme which utilizes an MZI with two unbalanced beam splitters. We used coherent and squeezed vacuum states as inputs and performed balanced homodyne measurement at the output. The phase

[2]The phase sensitivity limit calculated from the two-parameter approach is suitable for schemes without any reference source, whereas that calculated from the single-parameter approach holds true for schemes using reference sources [26]. In our scheme, the phase of the local oscillator is a phase reference for the inputs. Therefore, we utilize the single-parameter approach to analyze the phase sensitivity limit of our scheme [Eq. (4) in Ref. [26]].

sensitivity of our scheme was analyzed and its advantage over a balanced MZI was demonstrated. We showed that using unbalanced beam splitters can improve phase sensitivity and the optimal phase sensitivity is reachable when the transmittances of two beam splitters are the same. A balanced MZI is approximately the best choice when the squeezing parameter is large. In addition, we compared the optimal phase sensitivity of our scheme with the quantum Cramér-Rao bound. For a high-photon-number coherent state, the quantum Cramér-Rao bound can be nearly saturated with the optimal phase sensitivity of our scheme, suggesting that our scheme is the optimal measurement strategy. The results in this paper may contribute to the development of practical quantum sensors, particularly next-generation gravitational wave detectors.

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