Radiative recombination of highly charged ions with polarized electrons

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We present a theoretical study of the radiative recombination (RR) of heavy highly charged ions with arbitrary (longitudinally and transversely) polarized electrons. In order to investigate how the spin state of incident electrons affects the linear polarization of emitted photons, we apply the density matrix theory and solutions of the relativistic Dirac equation. Explicit analytical expressions are obtained for the dependencies of the differential cross section and the polarization Stokes parameters of RR photons on the components of the incident electrons polarization vector. Detailed calculations of the degree of linear polarization and the tilt angle that characterizes the orientation of the polarization axis are carried out for radiative recombination with bare U^{92+} and Xe^{54+} ions for different polarization states of the incoming electrons. Based on the results of these calculations, we argue that the linear polarization of RR photons is very sensitive to the spin state of an electron target, and this sensitivity can be easily observed with the help of modern Compton polarization detectors.

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I. INTRODUCTION

Spin-polarized heavy ion beams are highly relevant for future investigations of the relativistic, QED, and parity-violation phenomena in a high-Z domain as well as for the search of new physics beyond the Standard Model [1–4]. The present challenge of storage-ring physics is the production and preservation of such beams. Various methods for the production of spin-polarized ion beams at the storage rings are currently discussed [5,6]. Apart from the optical pumping of magnetic ionic substates, the radiative recombination (RR) of free (or quasi-free) electrons from *polarized* targets may also result in a polarization of the residual ions [6]. The latter method attracts particular interest since RR is one of the dominant processes in ion-atom (or ion-electron) collisions, whose cross sections can reach the kilo- or even mega-Barn domain depending on collision energy [7,8]. Moreover, spin-polarized electronic (and atomic) targets are available today and can be installed at storage ring facilities [9,10]. For example, a spin-polarized electron target is planned to be installed at the GSI storage ring in Darmstadt. This will be the first step towards future investigations of the feasibility of RR with polarized electrons for the production of polarized ion beams.

Yet another advantage of radiative recombination as a tool for production of spin-polarized ions in storage rings is that this process can also be simultaneously used to *control* the polarization transfer to ions [11]. In particular, it was shown that the capture of longitudinally polarized electrons into the ground $1s$ state of initially bare ions leads to the remarkable modification of the linear polarization of emitted photons [1]. Namely, the direction of the linear polarization of K–RR photons is tilted by an angle proportional to the degree of longitudinal polarization of incident electrons. This tilt can be easily observed with the help of segmented Compton polarimeters [12,13] and can be used for the diagnostics of the polarization transfer in RR. Until the present, however, not much has been known about whether and how the polarization tilt angle (as well as the degree of electron polarization) depends on the *transverse* polarization components of incident electrons. The sensitivity of the linear polarization of RR photons on both the longitudinal and transversal components of the electron spin vector can open up a route for the "spin tomography" of electron targets and for the control of production of polarized ion beams.

While studies of the polarization transfer in RR were mainly restricted in the past to the case of longitudinally polarized electrons, a more general analysis was performed for the atomic bremsstrahlung. This process is very similar to RR [14–18], with the only difference that an electron after the photon emission is still in the continuum and not in a bound ionic state. For the bremsstrahlung process and for the case when the final electron remains unobserved, a detailed analysis of the polarization transfer was reported in Ref. [19]. In particular, theoretical calculations of the linear polarization of the emitted photons were performed for both longitudinally and transversely polarized incident electrons and their results were compared with experimental findings [14].

In this work we present the theoretical study of the radiative recombination of arbitrary (longitudinally and transversely) polarized electrons into a bound state of initially bare ions. Emphasis is given to the linear polarization of recombination x-ray radiation and to its sensitivity to the incoming electron polarization. Simple analytical expressions, which allow to analyze the polarization transfer between incident electrons and the emitted x-rays, are obtained in the framework of the relativistic Dirac theory and the density matrix approach. These expressions enable one to understand how the degree and direction of linear polarization of RR photons depend on the spin state of captured electrons. The developed theory opens up opportunities for the spin diagnostics of ion beams by means of the analysis of the linear polarization of recombination photons.

Relativistic units ($\hbar = c = m_e = 1$) are used throughout the paper unless stated otherwise.

II. THEORY

A. Geometry and setup

In order to describe the spin-polarization transfer in the radiative recombination, we need first to agree about the geometry under which electron-ion collisions and the emission of recombination photons are considered. In the present work we consider the capture of a polarized electron by an initially bare heavy ion. From the theoretical viewpoint it is more convenient to analyze the electron capture in the rest frame of the ion, i.e., in the projectile frame. In such a frame we can choose the quantization axis (z axis) along the incoming electron momentum $\mathbf{p}_i$. Together with the wave vector $\mathbf{k}_f$ of emitted photon this axis forms the reaction plane (xz plane). As one can see in Fig. 1, the direction of photon emission with respect to the quantization axis is determined by the polar angle θ_{RR}.

The polarization of incoming electrons can be determined by the vector $\mathbf{P} = (P_x, P_y, P_z)$. Since we have chosen the quantization axis along the incoming electron momentum, parameter P_z corresponds to the longitudinal polarization, while two parameters P_x and P_y describe the transversal polarization of electrons.

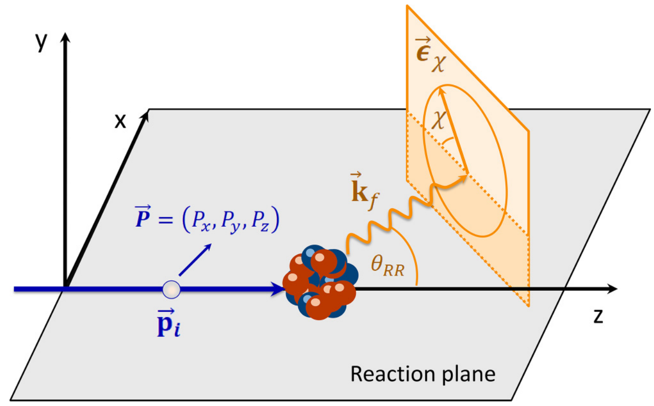

FIG. 1. The geometry of the radiative recombination of polarized electrons with heavy bare ions in the ion rest frame.

In order to describe the polarization of emitted photons the so-called Stokes parameters P_1, P_2, and P_3 are usually used [20]. The Stokes parameter P_3, which reflects the degree of circular polarization, cannot be measured by means of existing x-rays detectors. The Stokes parameters P_1 and P_2 indicate the degree and direction of the linear polarization of the light in the plane perpendicular to the photon momentum $\mathbf{k}_f$ (see Fig. 1). Experimentally, these parameters are determined by measuring the intensities of the light I_χ, whose polarization vector is oriented at different angles χ with respect to the reaction plane:

$$P_1 = \frac{I_{0^\circ} - I_{90^\circ}}{I_{0^\circ} + I_{90^\circ}}, \quad P_2 = \frac{I_{45^\circ} - I_{135^\circ}}{I_{45^\circ} + I_{135^\circ}}. \tag{1}$$

The Stokes parameters are very convenient for theoretical analysis. From the experimental viewpoint, however, it is more practical to visualize the linear polarization of light in terms of the polarization ellipse. The relative length of the principal axis of the ellipse defines the degree of linear polarization of light:

$$P_L = \sqrt{P_1^2 + P_2^2}, \tag{2}$$

while the orientation of this axis with respect to the reaction plane is described by the tilt angle χ:

$$\cos 2\chi = P_1/P_L, \quad \sin 2\chi = P_2/P_L. \tag{3}$$

In the present work we would like to investigate the dependence of Stokes parameters P_1 and P_2, and hence of parameters P_L and χ, on three projections of the electron polarization vector: $P_1(P_x, P_y, P_z)$, $P_2(P_x, P_y, P_z)$.

B. Radiative recombination: Basics

The theory of the radiative recombination of electrons with highly charged ions was considered in many previous works [21–23]. In the present study, therefore, we restrict ourselves to just a short review of basic formulas. Within the perturbative approach all the properties of emitted photons can be expressed in terms of the freebound transition amplitude $\langle n_b \kappa_b \mu_b | \hat{R}_\lambda^\dagger | p_i \mu_i \rangle$. Here $\hat{R}_\lambda = e\,\alpha^\nu A_\nu^* = -e\,\boldsymbol{\alpha} \cdot \mathbf{A}_\lambda{}^*$ is the interaction operator, where the wave function of emitted photons [24],

$$\mathbf{A}_\lambda(\mathbf{r}) = \frac{\boldsymbol{\epsilon}_{\mathbf{k}_f\lambda} \exp\left(i\,\mathbf{k}_f \cdot \mathbf{r} \right)}{\sqrt{2\,k_f^0\,(2\pi)^3}}, \tag{4}$$

is characterized by the energy $k_f^0 = |\mathbf{k}_f| = \omega$ and circular polarization vector $\boldsymbol{\epsilon}_{\mathbf{k}_f\lambda}$, with $\lambda = \pm 1$ being the photon's helicity.

The wave function of the incoming electron with well-defined asymptotic four-momentum $p_i = (p_i^0, \mathbf{p}_i)$ and the spin projection μ_i on the propagation direction is the continuum solution of the Dirac equation, which can be represented by the partial wave expansion

$$|p_i\,\mu_i\rangle = \frac{1}{\sqrt{4\pi}}\,\frac{1}{\sqrt{p_i\varepsilon_i}}\sum_\kappa \mathrm{i}^l \exp(i\Delta_\kappa)\sqrt{2l+1} \times C^{j\,\mu_i}_{l\,0,\,1/2\,\mu_i}|\varepsilon_i\,\kappa\,\mu_i\rangle, \tag{5}$$

where $|\varepsilon_i\,\kappa\,\mu_i\rangle$ is the partial electron wave with the energy $\varepsilon_i = p_i^0$ and the Dirac quantum number κ, and where $C^{j\,\mu_i}_{l\,0,\,1/2\,\mu_i}$ denotes the Clebsch-Gordan coefficients.

The differential cross section of the radiative recombination is given by [23]

$$\frac{d\sigma}{d\Omega} = \frac{(2\pi)^4}{v_i}\mathbf{k}_f^2\sum_{\mu_b}|\langle n_b\,\kappa_b\,\mu_b|\hat{R}^\dagger_\lambda|p_i\,\mu_i\rangle|^2, \tag{6}$$

where v_i is the incident electron velocity in the projectile frame.

C. Density matrix theory

After discussing the geometry and the amplitude for the radiative recombination, we are ready to start the investigation of the polarization transfer between incident electrons and emitted RR photons. The most natural and effective tool for studying the polarization transfer in atomic collisions [20,25,26] is the density matrix theory (DMT). In this theory, the initial and final states of the system are described by means of statistical operators $\hat{\rho}_i$ and $\hat{\rho}_f$, which are related to each other as

$$\hat{\rho}_f = \hat{R}_\lambda\,\hat{\rho}_i\,\hat{R}_\lambda^{\,\dagger}, \tag{7}$$

where for the case of RR, $\hat{R}_\lambda$ is the radiative transition operator, defined in Sec. II B. This equation contains the complete information about the system and can be used to obtain the observable properties of the radiative recombination. For example, the differential cross section of the RR can be written as

$$\frac{d\sigma}{d\Omega} = \frac{(2\pi)^4}{v_i}\mathbf{k}_f^2\sum_{\mu_b\lambda}\langle n_b\,\kappa_b\,\mu_b,\ \mathbf{k}_f\lambda|\hat{\rho}_f|n_b\,\kappa_b\,\mu_b,\ \mathbf{k}_f\lambda\rangle. \tag{8}$$

The density matrix of recombination photons can be obtained from the final-state density matrix simply by taking the trace over the total angular momentum projection of the bound electron μ_b:

$$\langle\mathbf{k}_f\lambda|\hat{\rho}_\gamma|\mathbf{k}_f\lambda'\rangle = \mathrm{Tr}_{\mu_b}(\hat{\rho}_f) = \sum_{\mu_b}\langle n_b\,\kappa_b\,\mu_b,\ \mathbf{k}_f\lambda|\hat{\rho}_f|n_b\,\kappa_b\,\mu_b,\ \mathbf{k}_f\lambda'\rangle. \tag{9}$$

By using Eq. (7), we can rewrite the DMT of photons in terms of the incident electron density matrix:

$$\langle\mathbf{k}_f\lambda|\hat{\rho}_\gamma|\mathbf{k}_f\lambda'\rangle = \sum_{\mu_b\mu_i\mu_i'}\langle n_b\,\kappa_b\,\mu_b|\hat{R}_\lambda|p_i\,\mu_i\rangle\langle p_i\,\mu_i| \times \hat{\rho}_i|p_i\,\mu_i'\rangle\langle p_i\,\mu_i'|\hat{R}^\dagger_{\lambda'}|n_b\,\kappa_b\,\mu_b\rangle. \tag{10}$$

Here $\hat{\rho}_i$ is the statistical operator of a free electron. In order to further evaluate Eq. (10), we note that the initial-state density matrix, which describes the spin state of incident electrons, can be parametrized by three real parameters:

$$\langle p_i\,\mu_i|\hat{\rho}_i|p_i\,\mu_i'\rangle = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - i\,P_y \\ P_x + i\,P_y & 1-P_z \end{pmatrix}, \tag{11}$$

which are the cartesian coordinates of the polarization vector $\mathbf{P} = (P_x, P_y, P_z)$ of an electron beam. Instead of the density matrix of the free electron, it is often more convenient to apply the (so-called) statistical tensors, which are mathematically equivalent to the DMT and can be represented as a linear combination of its elements [26]:

$$\langle p_i\,\mu_i|\hat{\rho}_i|p_i\,\mu_i'\rangle = \sum_{kq}(-1)^{1/2-\mu_i'}\,C^{k\,q}_{1/2\,\mu_i,\,1/2\,-\mu_i'}\,\rho^{(e)}_{kq}. \tag{12}$$

By making use of this expression and of Eq. (11), one can express the statistical tensors $\rho^{(e)}_{kq}$ in terms of the components of the polarization vector:

$$\rho^{(e)}_{00} = \frac{1}{\sqrt{2}} \qquad \rho^{(e)}_{11} = -\frac{1}{2}(P_x + \mathrm{i}\,P_y)$$
$$\rho^{(e)}_{10} = \frac{P_z}{\sqrt{2}}, \qquad \rho^{(e)}_{1-1} = \frac{1}{2}(P_x - \mathrm{i}\,P_y). \tag{13}$$

For exploring the transfer polarization in the radiative recombination it is also convenient to rewrite the photon density matrix in terms of experimentally observable parameters. Since the helicity of the photon takes only two values $\lambda = \pm 1$, the spin density matrix is a 2×2 matrix which can be represented in the form

$$\langle\mathbf{k}_f\lambda|\hat{\rho}_\gamma|\mathbf{k}_f\lambda'\rangle = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - i\,P_2 \\ P_1 + i\,P_2 & 1-P_3 \end{pmatrix}. \tag{14}$$

Here P_1, P_2, and P_3 are Stokes parameters which were introduced in Sec. II A.

D. Spin-polarization transfer

We can apply the concepts from the last section to analyze how the polarization of incoming electrons affects the differential cross section and linear polarization of emitted photons. Using expression (12), which relates the electron density matrix to the statistical tensors, we can rewrite the density matrix of emitted recombination photons as

$$\langle\mathbf{k}_f\,\lambda|\hat{\rho}_\gamma|\mathbf{k}_f\,\lambda'\rangle = \sum_{kq}\rho^{(e)}_{kq}\,\mathfrak{R}_{kq}(\lambda,\lambda',\mathbf{k}_f). \tag{15}$$

Here we introduced the tensors

$$\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f) = \sum_{\mu_b \mu_i \mu_i'} (-1)^{1/2-\mu_i'} C^{k\,q}_{1/2\,\mu_i,\,1/2\,-\mu_i'} \times \langle n_b\, \kappa_b\, \mu_b | \hat{R}_\lambda | p_i\, \mu_i \rangle \, \langle p_i\, \mu_i' | \hat{R}^\dagger_{\lambda'} | n_b\, \kappa_b\, \mu_b \rangle, \tag{16}$$

which are constructed from the (products of) transition matrix elements and whose properties are discussed in the Appendix.

By using Eq. (15) and symmetry properties (A1) and (A2) of the tensors $\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f)$, we can recast the RR differential cross section into the simple form

$$\frac{d\sigma}{d\Omega} = \sqrt{2}\,\mathfrak{R}_{00}(1, 1) + 2P_y \,\mathrm{Im}\,\mathfrak{R}_{11}(1, 1), \tag{17}$$

whereas the Stokes parameters of emitted photons are given by

$$P_1 = \frac{\sqrt{2}\,\mathfrak{R}_{00}(1, -1) - \mathrm{i}P_y[\mathfrak{R}_{1-1}(1, -1) + \mathfrak{R}_{11}(1, -1)]}{\sqrt{2}\,\mathfrak{R}_{00}(1, 1) + 2P_y \,\mathrm{Im}\,\mathfrak{R}_{11}(1, 1)}, \tag{18}$$

$$P_2 = i\frac{\sqrt{2}P_z\, \mathfrak{R}_{10}(1, -1) + P_x[\mathfrak{R}_{1-1}(1, -1) - \mathfrak{R}_{11}(1, -1)]}{\sqrt{2}\,\mathfrak{R}_{00}(1, 1) + 2P_y \,\mathrm{Im}\,\mathfrak{R}_{11}(1, 1)}, \tag{19}$$

where the short-hand notation $\mathfrak{R}_{kq}(\lambda, \lambda') \equiv \mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f)$ is used. The analytical expressions (17)–(19) show explicitly how the differential cross section and Stokes parameters P_1 and P_2 depend on the incoming electron polarization (P_x, P_y, P_z). Moreover, it immediately follows from Eqs. (A1)–(A3) that both the differential cross section and the Stokes parameters are purely real.

E. Relation to bremsstrahlung results

As mentioned above, radiative recombination of a free electron into a bound ionic state is closely related to another fundamental atomic process, atomic bremsstrahlung. In Ref. [14] the connection between these two processes was studied. The differential cross section and polarization correlations in electron-atom bremsstrahlung were investigated for the case when scattered electron remains unobserved. In particular, a series of symmetry relations for the dependence of the differential cross section and Stokes parameters on the polarization of the incoming electron had been presented [14]:

$$P_1(0, 0, 0) = P_1(1, 0, 0) = P_1(0, 0, 1), \tag{20}$$

$$P_2(0, 0, 0) = P_2(0, 1, 0) = 0, \tag{21}$$

$$d\sigma(0, 0, 0) = d\sigma(1, 0, 0) = d\sigma(0, 0, 1), \tag{22}$$

where

$$d\sigma(P_x, P_y, P_z) \equiv \frac{k_f}{Z^2}\,\frac{d\sigma(P_x, P_y, P_z)}{dk_f\, d\Omega_{k_f}}. \tag{23}$$

The same relations can be easily obtained from Eqs. (18) and (19) for radiative recombination. Thus, we can confirm that our formulas are in excellent agreement with the previous theoretical results.

III. RESULTS AND DISCUSSION

In the previous section we have derived simple analytical expressions which allow to analyze the polarization transfer from electrons to photons in the most general case. Though these expression can be applied to the electron capture into any hydrogenic state, we will consider the RR into the ground $1s_{1/2}$ state.

Let us start our analysis by summarizing the results of previous works and comparing them with the predictions obtained from Eqs. (17)–(19). In Ref. [1], for example, the RR into the K-shell of bare uranium was investigated for the case of longitudinally polarized electrons. It has been shown analytically that the P_2 parameter depends on the polarization of the incident electrons and that this parameter is proportional to the P_z component of the polarization vector. As can be seen from Eqs. (18) and (19), our analytical formulas confirm this prediction. Indeed, if incident electrons are longitudinally polarized and, hence, their polarization vector reads as $\mathbf{P} = (0, 0, P_z)$, the first Stokes parameter reads as $P_1 = \mathfrak{R}_{00}(1, -1)/\mathfrak{R}_{00}(1, 1)$ and does not depend on the polarization of the initial electron, while the second Stokes parameter $P_2 = \mathrm{i}P_z\mathfrak{R}_{10}(1, -1)/\mathfrak{R}_{00}(1, 1)$ is directly proportional to P_z. It is also easy to see that for the case of the capture of unpolarized electrons, $\boldsymbol{P} = (0, 0, 0)$, only the first Stokes parameter P_1 is nonzero. It implies that the linear polarization of the emitted photons will be oriented within or perpendicular to the reaction plane.

Having recalled the previous predictions about the recombination of longitudinally polarized electrons, we are ready now to discuss the general case. For example, Fig. 2 presents the numerical results for the radiative recombination of polarized electron into the $1s_{1/2}$ state of H-like uranium U^{91+} (the top panels) and xenon Xe^{53+} (the bottom panels). The calculations have been performed for the projectile energy $T_p = 400$ MeV/u, which corresponds to the electron kinetic energy $T_e = 219.4$ keV in the ion rest frame. We display the differential cross section (left column), the degree of linear polarization P_L (middle column), and the tilt polarization angle χ (right column) as functions of photon emission angle $\theta_{\rm lab}$ in the laboratory frame, i.e., in the rest frame of the electron target. The results are presented for four different electron polarizations: (i) unpolarized electrons $P_x = P_y = P_z = 0$ (black solid line), (ii) longitudinally polarized electrons $P_x = P_y = 0, P_z = 1$ (red dashed line), as well as electrons whose polarization vector has both longitudinal and transverse components: (iii) $P_x = 1/\sqrt{2}, P_y = 0, P_z = 1/\sqrt{2}$ (blue dash-dotted line) and (iv) $P_x = P_y = P_z = 1/\sqrt{3}$ (green dotted line). As can be seen, the differential cross section depends only weakly on the polarization of the initial electrons. In contrast, the degree of polarization P_L and, especially, the tilt angle χ, which determines the direction of the principal axis of the polarization ellipse with respect to the reaction plane, are strongly influenced by the polarization of incident electrons. In the case of unpolarized incoming electrons (black line) the tilt angle χ vanishes, while for the case of polarized electrons a significant rotation of the polarization axis can be observed. The most pronounced effect occurs for the forward photon emission, where the value of the tilt angle χ can reach 60°–70°. Such a remarkable rotation can be easily observed by means of segmented Compton polarimeters. One can also

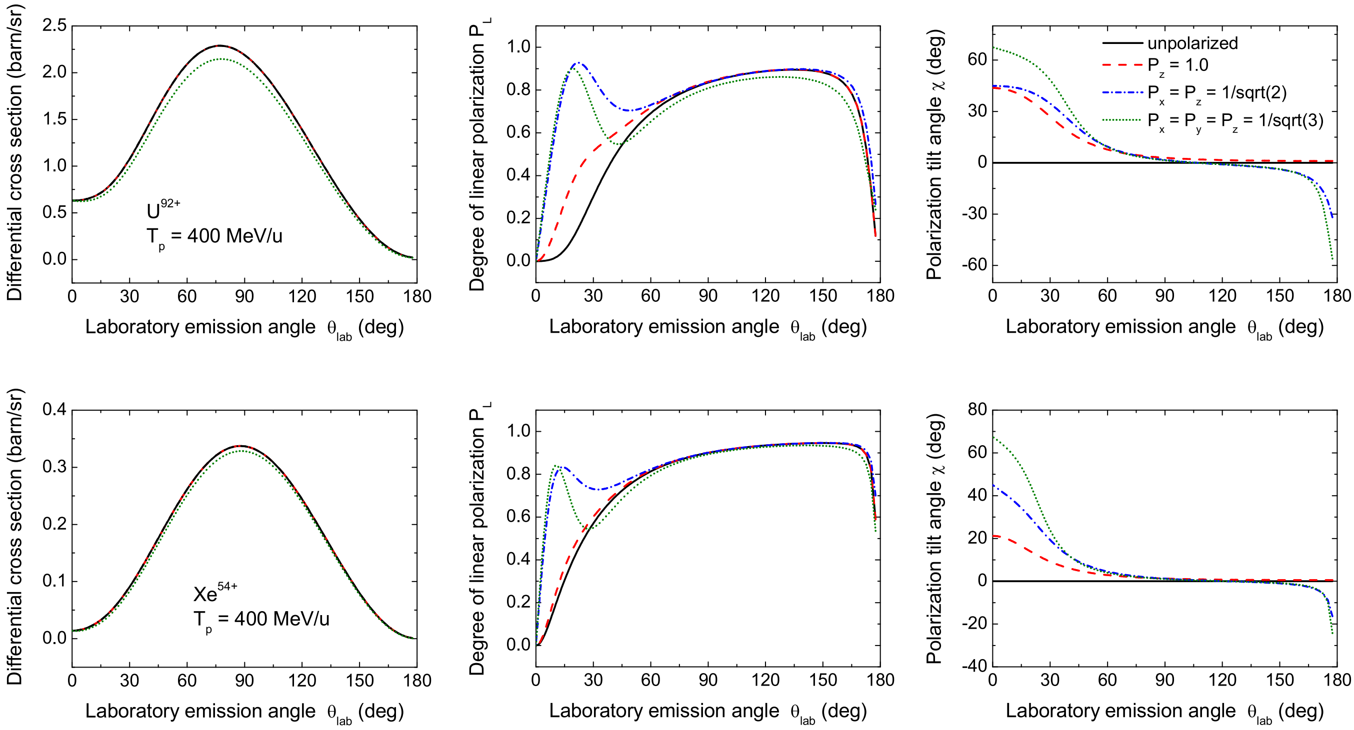

FIG. 2. The differential cross section (left column), the degree of linear polarization P_L (middle column), and the tilt polarization angle χ (right column) as functions of photon emission angle $\theta_{\rm lab}$ in the laboratory frame. Calculations have been performed for the radiative capture of polarized electrons into the ground state of (initially) bare uranium projectile U^{92+} (upper row) and bare xenon projectile Xe^{54+} (bottom row) with energy $T_p = 400$ MeV/u. The different polarization states of incident electrons have been considered: unpolarized electrons, $P_x = P_y = P_z = 0$ (black solid line); longitudinally polarized electrons, $P_x = P_y = 0, P_z = 1$ (red dashed line); as well as electrons whose polarization vector has both longitudinal and transverse components: $P_x = 1/\sqrt{2}, P_y = 0, P_z = 1/\sqrt{2}$ (blue dash-dotted line) and $P_x = P_y = P_z = 1/\sqrt{3}$ (green dotted line).

see that with a decrease in the nuclear charge, the tilt angle for the case of longitudinal polarization of incoming electrons decreases, while in the case of transversal polarization the tilt angle is kept approximately at the same level. This effect can be explained based on symmetry considerations. Indeed, for the case of longitudinally polarized electrons the entire system "ion + electron" still possesses azimuthal symmetry so that the rotation of the polarization ellipse is caused by the relativistic contributions to the electron-photon coupling. In contrast, the transverse polarization of incident electrons breaks the azimuthal symmetry of the system independent of the nuclear charge (as well as collision energy) and leads to a remarkable tilt of the RR linear polarization.

In order to analyze how the polarization transfer between incoming electrons and emitted photons depends on collisions energy, Fig. 3 displays the results for the projectile energy $T_p = 50$ MeV/u which corresponds to the electron kinetic energy $T_e = 27.4$ keV. As can be seen from the figure, the energy dependence of the polarization transfer is rather different for the cases of longitudinal and transverse polarization of incident electrons. In particular, the capture of longitudinally polarized electrons (blue line) at low energy leads to almost vanishing tilt angle χ. In contrast, when the incident electrons have a transverse polarization, there is still a significant rotation of the polarization vector out of the reaction plane. Again, such an energy behavior of the polarization transfer can be explained by the symmetry reasons from above.

As seen from Figs. 2 and 3 and the discussion above, the tilt angle of the linear polarization of RR photons is very sensitive to the polarization of incident photons. In order to better understand this dependence, we investigate how χ varies when the *direction* of the electron polarization vector $\mathbf{P}$ changes. For this reason we introduce an angle β which characterizes the rotation of the electron polarization vector from the z axis to the x axis: $P_z = |\mathbf{P}| \cos\beta$, $P_x = |\mathbf{P}| \sin\beta$. In Fig. 4 we display polarization tilt angle χ as a function of β for the electron recombination into the $1s$ state of initially bare uranium U^{92+} ions with energy $T_p = 400$ MeV/u. The results are presented for the laboratory photon emission angle $\theta_{\rm lab} = 30^\circ$ as well as for completely $|\mathbf{P}| = 1$ (red solid line) and partially $|\mathbf{P}| = 0.5$ (black dashed line) polarized electrons. As seen from the figure, the tilt angle χ is sensitive to both the degree and direction of electron polarization. For example, χ is enhanced approximately by a factor of 1.5 for all β's, if the degree of electron polarization is increased from $|\mathbf{P}| = 0.5$ to $|\mathbf{P}| = 1.0$. The tilt angle χ also significantly varies with the polarization orientation angle β, reaching its maximum for $\beta = 60^\circ$. Such remarkable sensitivity to the β and $|\mathbf{P}|$ makes the RR linear polarization a valuable tool for the diagnostics of electron polarization.

In the above analysis, we have assumed that an incident electron beam, as seen from the rest frame of an ion, is monochromatic and unidirectional. This is indeed not the case for realistic experimental scenarios. For example, in a pilot

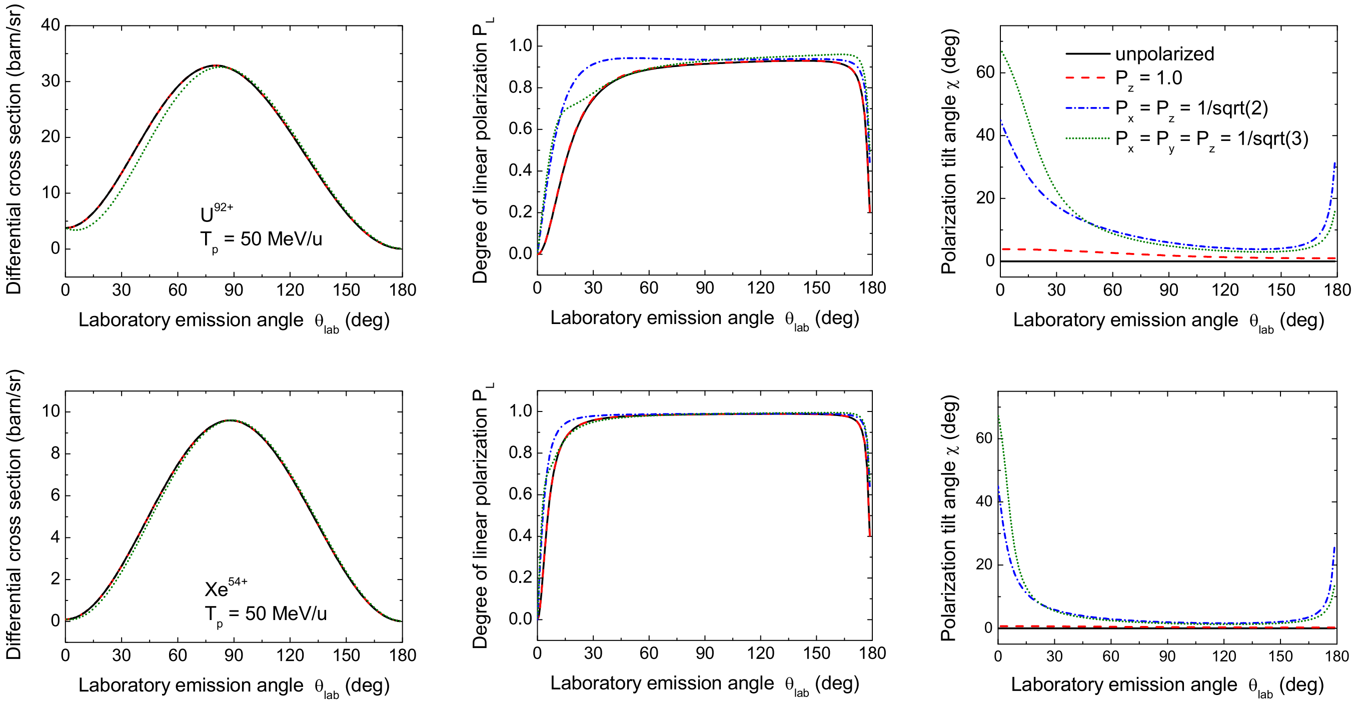

FIG. 3. The same as in Fig. 2 but for ions with projectile energy $T_p = 50$ MeV/u.

series of experiments, planned at the GSI facility, electrons will be radiatively captured from spin-polarized hydrogen atoms. It is natural to address the question, therefore, of how the momentum distribution of (loosely bound) target electrons will affect the polarization of RR photons. The influence of the momentum distribution was investigated by some of us within the framework of the impulse approximation in Ref. [27]. We have shown for relativistic collisions of heavy projectiles with low-Z atoms (such as hydrogen, for example) that the variation of angular distribution and polarization of RR photons caused by the electron momentum spread does not exceed 1%. This also complies with the agreement between experiment and theory, which was recently reported at the level of 1% and is limited by the overall experimental accuracy [28]. Based on these observations, we argue that contributions from the electron momentum distribution can be neglected in the present study.

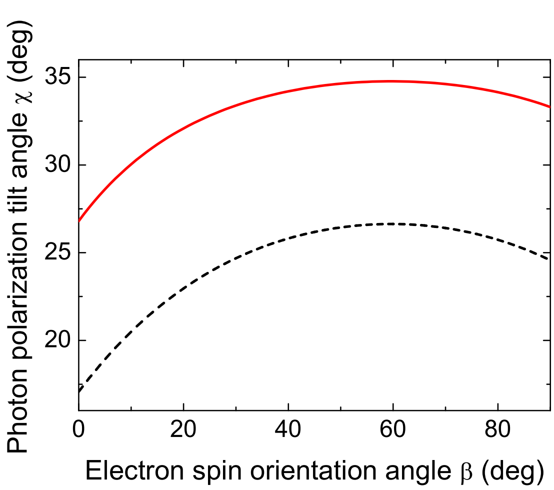

FIG. 4. The polarization tilt angle χ of K–RR photons as a function of the orientation angle β of the spin vector of incident electrons. Red solid and black dashed lines correspond to incident electron spin polarization $|\mathbf{P}| = 1$ and $|\mathbf{P}| = 0.5$, respectively. Calculations have been performed for the bare uranium projectile U^{92+} with energy $T_p = 400$ MeV/u and for the photon emission angle $\theta_{\rm lab} = 30^\circ$ in the laboratory frame.

IV. SUMMARY AND OUTLOOK

In this work we reinvestigate the radiative recombination of free (or quasi-free) electrons with heavy bare ions. Special attention was paid to the question of how the spin polarization of incident electrons may affect the linear polarization of emitted photons. In contrast to previous studies, we developed here a general theory which accounts for arbitrary (longitudinal and/or transversal) polarization of an electron beam. In order to investigate the spin-polarization transfer for this general case, we have employed Dirac's relativistic equation and the density matrix approach. Based on this theory, analytical and numerical calculations have been performed. In particular, we derived general analytical expressions which show how the differential cross section of the radiative recombination and Stokes parameters P_1 and P_2 of emitted x-rays depend on the incoming electron polarization (P_x, P_y, P_z). Based on these general formulas, detailed numerical calculations have been performed for electron recombination into the ground $1s_{1/2}$ state of initially bare uranium ($Z = 92$) and xenon ($Z = 54$). The calculations clearly indicate that the tilt angle χ of the linear polarization of emitted photons is very sensitive to the spin state of incident electrons. Even rather small variation of the degree and direction of electron spin polarization can result in remarkable change of χ, which can be easily observed with the help of currently available Compton polarimeters. We argue, therefore, that the radiative recombination of heavy bare ions can serve as a valuable tool for electron target spin diagnostics.

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APPENDIX: SYMMETRY PROPERTIES OF $\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f)$

We have shown in Sec. II D that the angle-differential cross section and the Stokes parameters of recombination photons can be expressed in terms of the tensors $\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f)$. The latter are constructed from the recombination matrix elements as follows from Eq. (16). By employing the well-known properties of these matrix elements, discussed, for example, in Refs. [21,22], we obtain

$$\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f) = (-1)^{k+q}\, \mathfrak{R}_{k-q}(-\lambda, -\lambda', \mathbf{k}_f), \quad \text{(A1)}$$

$$\mathfrak{R}^*_{kq}(\lambda, \lambda', \mathbf{k}_f) = (-1)^{q}\, \mathfrak{R}_{k-q}(\lambda', \lambda, \mathbf{k}_f). \quad \text{(A2)}$$

Based on these two symmetry relations, one can further analyze the basic properties of $\mathfrak{R}_{kq}(\lambda, \lambda', \mathbf{k}_f)$. For example, by setting $\lambda = \lambda'$ in Eq. (A2), we conclude that $\mathfrak{R}_{00}(\lambda, \lambda, \mathbf{k}_f)$ is purely real, thus implying the real-valued cross section (17). Moreover, the additional relation

$$\mathfrak{R}^*_{kq}(\lambda, -\lambda, \mathbf{k}_f) = (-1)^{k}\, \mathfrak{R}_{kq}(\lambda, -\lambda, \mathbf{k}_f) \quad \text{(A3)}$$

is derived by combining Eqs. (A1) and (A2), and indicates that $\mathfrak{R}_{1q}(\lambda, -\lambda, \mathbf{k}_f)$ is purely imaginary, while $\mathfrak{R}_{00}(\lambda, -\lambda, \mathbf{k}_f)$ is purely real. Again, this proves the realness of the Stokes parameters (18)–(19), as it should indeed be for physical observables.

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