Effects of reservoir squeezing on trace-distance correlations and emergence of the pointer basis

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We investigate theoretically the dynamics of 1-norm geometric quantum correlations and their classical counterparts in a two-qubit system. Both qubits are initially prepared in Bell-diagonal states and locally coupled to separated thermal squeezed baths or a common squeezed thermal bath via energy-preserving interactions. We then unveil the effects of reservoir squeezing on the abrupt changes in the evolution of geometric correlations. It is found that adequately tuning the squeezing phase can efficiently suppress the dephasing rate and delay the appearance of sudden transitions in geometric correlations. Further, we show that the squeezing phase of the bath renders a different avenue to enhance the finite time interval for frozen quantum correlation. On the other hand, in this context, we show that the squeezing strength of the reservoir exhibits a negative role. In addition, in the common bath case, we observe the steady-state correlations and decoherence-free subspace, which can be governed via squeezing parameters. Moreover, the abrupt change from a decaying regime to a constant nonzero value in classical correlation signals the emergence of a pointer-state basis. We show that the emergence of a pointer-state basis can be delayed by suitably adjusting the bath squeezing parameters. Remarkably, we find the optimal value of the squeezing phase, which introduces maximum retardation in the appearance of a pointer-state basis.

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I. INTRODUCTION

One of the foremost traits of quantum information theory is the existence of quantum correlations between distant quantum systems. The best-known measure of the genuine nonclassical correlations is quantum discord (QD), which was first introduced by Ollivier and Zurek [1]. Notably, this entropic-based measure can capture quantum correlations not only in the entangled states but also in mixed separable states [2,3]. Quantum discord has vital applications in the numerous fields of quantum optics [4–14]. Nevertheless, from an analytical viewpoint, the generic dynamics of QD is challenging due to the complex optimization methods involved; therefore, exact expressions are available only for certain states [15–19]. To overcome this hurdle, a geometric approach has been exploited to quantify the quantum correlations via various distance-based formulations, which are typically easier to evaluate [20–27]. Among them, the trace-norm (Schatten 1-norm) geometric qualifier has gained much attention, as it characterizes a rigorous and physically motivated measure for the quantum and classical correlations [26,27].

Despite the quantification of correlations, a central issue and the key prerequisite for modern quantum technologies is the understanding of how the quantum and classical correlations behave in the presence of various sources of decoherence. In this framework, it is widely known that quantum correlations beyond entanglement are more robust against decoherence and do not exhibit the phenomenon of sudden death [28–34]. It was demonstrated both theoretically and experimentally that for a specific class of initial states undergoing local Markovian dephasing noise, 1-norm geometric quantum discord (GQD-1) manifested some intriguing phenomena, e.g., freezing and double sudden transitions [35–37]. Furthermore, the dynamical decoupling (DD) protocol has been used to enhance the finite time interval over which GDQ-1 (QD) exhibits constant magnitude and control the occurrence of the double (single) sudden transitions phenomenon [38–41]. However, the DD method has some limitations, e.g., the simultaneous application of DD and non-Markovianity, deleterious for coherence preservation [42]. Similarly, the efficiency of the DD technique is highly sensitive to the pulse timing [40–42].

Moreover, from the viewpoint of applications, abrupt changes in the correlations dynamics are used to spotlight certain remarkable features. For example, the sudden transition behavior in the GQD-1 has been employed as an alternative method to precisely indicate the critical point associated with the phase transition in spin-chain models [37,43,44]. In contrast, the abrupt change from a decaying regime to a nonvanishing level in the classical correlation characterizes the emergence of a pointer-state basis (i.e., quantum-to-classical transition) [45,46]. However, these phenomena are strongly dependent on the detailed configuration of the environment being considered. For instance, due to non-Markovianity, multiple sudden changes in classical correlation evolution

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have been observed, revealing the emergence of a metastable pointer-state basis [47–49]. Recently, we reported that the nonequilibrium nature of the environment can control sudden transitions in the correlations and suppress the metastable pointer-state basis [50]. In addition, we also showed that the nonequilibrium feature of the dephasing environment can be exploited as an alternative and effective way to extend the time interval over which both entropic and geometric quantum correlations remain unchanged [34,50].

In previous studies, the environment (reservoir) was usually assumed to be initially in the thermal or vacuum state. However, current technologies allow us to create a nonthermal state for the open quantum systems; for instance, quantum coherence or squeezing could also exist in the reservoir and makes it a nonthermal environment. Interestingly, the utilization of nonthermal baths has manifested remarkable results. For example, a quantum heat engine exploiting the squeezed thermal reservoir as a working source could surpass the Carnot limit [51–55]. Furthermore, squeezed thermal and vacuum reservoirs play a crucial role in various phenomena such as entanglement sudden death [56–59], violation of Leggett-Garg-type inequalities [60], enhancing the lifetime of the cat state [61], resonance fluorescence [62,63], and several others [64–72]. Recently, in Ref. [73] the authors studied the entropy dynamics of the dephasing model, where a single qubit is coupled with a squeezed thermal bath via a nondemolition interaction. Remarkably, they reported that the dephasing rate of the system relies on the squeezing phase of the bath. This phase dependence cannot be precisely obtained from the Born-Markovian approximation, which is broadly exploited in open quantum systems. Moreover, we recently showed that the abrupt change from classical to quantum decoherence in a two-qubit system can be efficiently harnessed via bath-squeezing parameters [74]. These studies have inspired us to show that the squeezing parameters of the reservoir render a different avenue to control the phenomena of double sudden transitions in GQD-1 and quantum-to-classical transition (characterized by the emergence of the pointer-state basis).

In our previous work [74] we investigated the dynamics of quantum correlations (measured by an entropic quantifier) in two qubits, locally coupled to their own squeezed thermal baths. It was revealed that the quantum correlations exhibit only a single sudden change from a nonvanishing constant magnitude to a decaying regime. However, in this paper we examine the dynamics of quantum and classical correlations in a two-qubit system measured by trace-norm geometric quantifiers. Both qubits are initially considered in Bell-diagonal states and locally coupled to spatially separated squeezed thermal reservoirs or a common squeezed thermal reservoir via nondemolition interactions. We then explore the effects of reservoir squeezing on the abrupt changes in the evolution of 1-norm geometric correlations. Strikingly, we find that by properly tuning the squeezing phase of the reservoir, one can significantly suppress the dephasing rate that leads to a delay in the appearance of double sudden transitions in GQD-1. Furthermore, unlike the dynamical decoupling protocol, we show that the squeezing phase of the bath provides a promising tool to efficiently prolong the finite time interval for the frozen GQD-1, despite the application of any external operation on the system of interest. On the other hand, we show that the squeezing strength of the reservoir displays a negative role in the whole process. Moreover, we observe the generation of steady-state geometric correlations and decoherence-free subspace in the common bath scenario. Interestingly, their emergence can be controlled through the squeezing parameters of the bath. Controlling the time interval for frozen GQD-1 and decoherence-free subspace via squeezing parameters can be a useful strategy for overcoming the challenges of decoherence in quantum computing and communication.

Nevertheless, the classical correlations between the system $\mathcal{S}$ and quantum apparatus $\mathcal{A}$ can be employed to precisely define the exact time τ_E for the emergence of the pointer-state basis [45,75,76]. More specifically, τ_E is the instant of time when the classical correlation exhibits an abrupt change from a decaying regime to a certain nonvanishing stationary value [36,45,46]. The emergence of the pointer-state basis strongly depends on the nature of the decoherent environment. For example, for the Markovian noise models, classical correlation exhibits a single sudden change along the dynamics, which is associated with the emergence of a stable pointer-state basis [36,45]. However, in the case of non-Markovian dynamics, the classical correlation experiences successive abrupt changes before reaching a permanent constant value, implying the appearance of the metastable pointer-state basis [47–49]. Furthermore, we recently demonstrated that the environmental nonequilibrium feature delays the emergence of the pointer-state basis and suppresses the number of metastable pointer-state basis [50]. Now it is natural to wonder how the exact pure dephasing process in the squeezed thermal environments influences the emergence of the pointer-state basis associated with the abrupt change in geometric classical correlation. In this paper we show that the emergence of a pointer-state basis can be delayed by suitably adjusting the bath squeezing parameters in both local baths and common bath setups. Remarkably, we find the optimal value of the squeezing phase of the reservoir, which introduces maximum retardation in the appearance of a pointer-state basis. Furthermore, we find that the pointer-state basis emerges earlier in the common bath case. Our study provides insights to control the phenomenon of the quantum-to-classical transition.

The paper is arranged as follows. In Sec. II the physical dephasing model with the exact solution is introduced. Section III is devoted to the dynamics of 1-norm geometric quantum and classical correlations in a two-qubit system, locally subjected to spatially separated squeezed thermal baths or a common bath. The effects of reservoir squeezing parameters on the phenomena of double sudden transitions in GQD-1 and the emergence of a pointer-state basis are also discussed here in detail. A summary of our findings is presented in Sec. IV.

II. DEPHASING MODEL IN THE SQUEEZED RESERVOIR AND ITS SOLUTION

The pure dephasing model under consideration is composed of a two-qubit system, where each qubit is locally and independently coupled to a squeezed thermal bath or both qubits interact with a common squeezed thermal bath. Below, these different scenarios are discussed in detail.

A. Two qubits in local baths

We begin with the case when two qubits are coupled to independent local but identical squeezed thermal baths via the nondemolition interactions. To solve this model, first we consider a two-level system ($H_S = \frac{1}{2}\omega_0\hat{\sigma}_z$) interacting with a bosonic reservoir ($H_R = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k$) [77–80]. The system-reservoir interaction is characterized by the nondemolition Hamiltonian

$$H_{SR} = \hat{\sigma}_z\left(\sum_k (\lambda_k \hat{a}_k + \lambda_k^* \hat{a}_k^\dagger)\right), \tag{1}$$

where $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ such that $|e\rangle$ ($|g\rangle$) shows the excited (ground) state of the qubit with transition frequency ω_0. Further, ω_k represents the kth-mode frequency of the bath with creation (annihilation) operator $\hat{a}_k^\dagger$ ($\hat{a}_k$) and λ_k characterizes the coupling strength between the qubit and each mode of the bath. It is important to note that in this model $[H_S, H_{SR}] = 0$, implying that the system energy is always conserved, and the populations on each level of the qubit remain unchanged with time.

In order to obtain the exact dynamics of a single-qubit system, we begin with $\rho_{SR}(0) = \rho_S(0) \otimes \rho_R(0)$ and consider that the reservoir is initially in the squeezed thermal state, i.e., $\rho_R(0) = \prod_k \hat{S}_k(\zeta_k)\rho_{th}\hat{S}_k^\dagger(\zeta_k)$, where $\rho_{th} = \frac{1}{Z}e^{-(1/T)H_R}$, with T the temperature of the thermal state ρ_{th} and Z the normalization constant [73,81]. In addition, $\hat{S}_k(\zeta_k) = \exp[\frac{1}{2}(\zeta_k^*\hat{a}_k^2 - \zeta_k\hat{a}_k^{\dagger 2})]$ represents the squeezing operator for the mode $\hat{a}_k$, with $\zeta_k = r_k e^{i\theta_k}$, where r_k and θ_k are the squeezing strength and phase, respectively. As the composite system (qubit plus reservoir) is closed, it therefore obeys the unitary evolution, i.e., $\rho_{SR}(t) = U_I(t)\rho_{SR}(0)U_I^\dagger(t)$, where $U_I = \exp\{\sigma_z[\eta_k(t)\hat{a}_k^\dagger - \eta_k^*(t)\hat{a}_k]\}$ characterizes a unitary operator in the interaction picture with $\eta_k(t) = \lambda_k(1 - e^{i\omega_k t})/\omega_k$. Thus, by tracing over the reservoir variables, one can obtain the reduced density matrix for the dynamics of a single-qubit system in the basis $\{|e\rangle, |g\rangle\}$ as [78]

$$\rho_S(t) = \begin{pmatrix} \rho_S^{ee}(0) & \rho_S^{eg}(0)e^{-\Gamma(t)} \\ \rho_S^{ge}(0)e^{-\Gamma(t)} & \rho_S^{gg}(0) \end{pmatrix}. \tag{2}$$

Here $\Gamma(t)$ characterizes the decay factor which can be defined as (for detailed calculations see the Appendix)

$$\Gamma(t) = 2\int_0^\infty \frac{d\omega}{\pi} I(\omega)\coth\frac{\omega}{2T}\left(\frac{1-\cos\omega t}{\omega^2}\right) \times [\cosh 2r - \sinh 2r\cos(\omega t - \delta\theta)]. \tag{3}$$

In our study, we adopt the Ohmic spectral density, which takes the form $I(\omega) = \gamma\omega e^{-\omega/\omega_c}$, widely exploited in the spin-boson models [73,77–81]. The parameter γ is a dimensionless dissipative constant, indicating the coupling strength, $\delta\theta$ describes the phase difference between the squeezing phase relative to the phase of coupling parameter, and ω_c is the cutoff frequency.

Now we evaluate Eq. (3) for the two different cases of temperature. For example, in the zero-temperature limit ($T \to 0$) case, we have $\coth\frac{\omega}{2T} \to 1$. In this scenario, the reservoir is initially in the squeezed vacuum state. Thus, by solving the integral given in Eq. (3), we can obtain the analytical expression for the dephasing factor

$$\Gamma(t) = \frac{\gamma}{\pi}\{\alpha_1(t)\cosh 2r - \sinh 2r[\alpha_2(t)\cos\delta\theta + \alpha_3(t)\sin\delta\theta]\}, \tag{4}$$

with the time-dependent coefficients $\alpha_1(t) = \ln(1+\tau^2)$, $\alpha_2(t) = \ln(\frac{\sqrt{1+4\tau^2}}{1+\tau^2})$, and $\alpha_3(t) = 2\arctan\tau - \arctan 2\tau$, where $\tau = \omega_c t$. On the other hand, for the case of the high-temperature limit, we have $\coth\frac{\omega}{2T} \approx \frac{2T}{\omega}$, which we substitute into Eq. (3). Hence, by evaluating the above integral, it turns out that the dephasing factor still has the same form as given by Eq. (4); however, the time-dependent coefficients now become

$$\alpha_1(t) = \frac{2T}{\omega_c}[2\tau\arctan\tau - \ln(1+\tau^2)],$$
$$\alpha_2(t) = \frac{2T}{\omega_c}\left[2\tau(\arctan 2\tau - \arctan\tau) - \ln\left(\frac{\sqrt{1+4\tau^2}}{1+\tau^2}\right)\right],$$
$$\alpha_3(t) = \frac{2T}{\omega_c}\left[(\arctan 2\tau - 2\arctan\tau) + \tau\ln\left(\frac{1+4\tau^2}{1+\tau^2}\right)\right]. \tag{5}$$

We are interested in the dephasing dynamics of a two-qubit system, where each qubit is locally and independently coupled to the relevant squeezed thermal bath via the energy-preserving interaction. Therefore, based on the approach given in Ref. [82], we can easily evaluate the reduced density matrix for a bipartite system $\rho_{LB}(t)$ in the standard basis $\{|1\rangle = |ee\rangle, |2\rangle = |eg\rangle, |3\rangle = |ge\rangle, |4\rangle = |gg\rangle\}$ as

$$\rho_{LB}(t) = \begin{pmatrix} \rho^{11}(0) & \rho^{12}(0)e^{-\Gamma(t)} & \rho^{13}(0)e^{-\Gamma(t)} & \rho^{14}(0)e^{-2\Gamma(t)} \\ \rho^{21}(0)e^{-\Gamma(t)} & \rho^{22}(0) & \rho^{23}(0)e^{-2\Gamma(t)} & \rho^{24}(0)e^{-\Gamma(t)} \\ \rho^{31}(0)e^{-\Gamma(t)} & \rho^{32}(0)e^{-2\Gamma(t)} & \rho^{33}(0) & \rho^{34}(0)e^{-\Gamma(t)} \\ \rho^{41}(0)e^{-2\Gamma(t)} & \rho^{42}(0)e^{-\Gamma(t)} & \rho^{43}(0)e^{-\Gamma(t)} & \rho^{44}(0) \end{pmatrix}. \tag{6}$$

This equation characterizes the dynamics of a pair of two-level systems, where LB stands for local baths. Note that the diagonal elements remain constant for the pure dephasing case, while all the off-diagonal elements decrease with time as $\rho_{LB}(t)$ evolves. We consider both qubits initially prepared in the Bell-diagonal states, having the form

$$\rho(0) = \frac{1}{4}\left(I_A \otimes I_B + \sum_{i=1}^{3} c_i \sigma_A^i \otimes \sigma_B^i\right), \tag{7}$$

where I_A (I_B) indicates the identity operator of the subsystem A (B) and $\sigma^i_{A\,(B)}$ are the well-known Pauli operators acting on the subspace A (B). Further, $\{c_i = c_i(0)\}$ denotes the initial correlation parameters or functions with $0 \leqslant |c_i| \leqslant 1$. Remarkably, it includes the Werner ($|c_1| = |c_2| = |c_3| = c$) and Bell ($|c_1| = |c_2| = |c_3| = 1$) states.

B. Two qubits in a common bath

We now assume that two identical qubits are coupled to a common squeezed thermal reservoir via energy-preserving interactions. The total Hamiltonian is given by

$$H = \sum_{j=1}^{2}\left(\frac{\omega_0}{2}\hat{\sigma}_z^j + \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k + \hat{\sigma}_z^j \sum_k (\lambda_k \hat{a}_k + \lambda_k^* \hat{a}_k^\dagger)\right), \tag{8}$$

where indices j and k label the qubits and modes of the bath, respectively. In this case, we again consider that both qubits have the same coupling strengths λ_k to the environment. Exploiting the approach given in Ref. [77], we can calculate the reduced density matrix for the two qubits in the same standard basis as

$$\rho_{CB}(t) = \begin{pmatrix} \rho^{11}(0) & \rho^{12}(0)e^{-\Gamma(t)} & \rho^{13}(0)e^{-\Gamma(t)} & \rho^{14}(0)e^{-4\Gamma(t)} \\ \rho^{21}(0)e^{-\Gamma(t)} & \rho^{22}(0) & \rho^{23}(0) & \rho^{24}(0)e^{-\Gamma(t)} \\ \rho^{31}(0)e^{-\Gamma(t)} & \rho^{32}(0) & \rho^{33}(0) & \rho^{34}(0)e^{-\Gamma(t)} \\ \rho^{41}(0)e^{-4\Gamma(t)} & \rho^{42}(0)e^{-\Gamma(t)} & \rho^{43}(0)e^{-\Gamma(t)} & \rho^{44}(0) \end{pmatrix}, \tag{9}$$

where CB stands for common bath and $\Gamma(t)$ is defined in Eq. (4). Equation (9) describes the dynamics of two two-level systems subjected to a common squeezed thermal bath. We again assume that both qubits are initially in the Bell-diagonal states given by Eq. (7). It is worth mentioning that in the common bath scenario, diagonal elements and two off-diagonal elements $\rho^{32}(0)$ and $\rho^{23}(0)$ remain unchanged, while others decrease with time as $\rho_{CB}(t)$ evolves.

III. TRACE-DISTANCE QUANTUM AND CLASSICAL CORRELATIONS IN SQUEEZED THERMAL RESERVOIRS

In this section we first briefly introduce the concept of trace-norm geometric quantifiers to measure both quantum and classical correlations in a two-qubit system and recall certain known results from the literature. For this reason, let us assume a bipartite system is described by the density operator ρ and the closest classical-quantum state is characterized by ρ_c. The 1-norm geometric measures for quantum $\mathcal{Q}_G(\rho)$ and classical $\mathcal{C}_G(\rho)$ correlations between qubits A and B may be defined via trace distances as [26,27]

$$\mathcal{Q}_G(\rho) = \min_{\Pi_0} \|\rho - \rho_c\|_1 \tag{10}$$

and

$$\mathcal{C}_G(\rho) = \max_{\Pi_0} \|\rho_c - \pi\|_1, \tag{11}$$

respectively. Here $\|M\|_1 = \mathrm{Tr}[\sqrt{M^\dagger M}]$ is the 1-norm, Π_0 defines a set of classical-quantum states, having the generic form $\rho_c = \sum_j \mu^j \chi_A^j \otimes \rho_B^j$, with $0 \leqslant \mu^j \leqslant 1$ and $\sum_j \mu^j = 1$, and π is the product of the local marginals of ρ. Further, $\{\chi_A^j\}$ is a set of orthogonal projectors for the subsystem A and ρ_B^j refers to the reduced density operator of subsystem B. Interestingly, for the two-qubit X-type states [for example, the state given by Eq. (7)], the GQD-1 and its classical counterpart exhibit closed analytical expressions $\mathcal{Q}_G(\rho) = \mathrm{int}\{|c_1|, |c_2|, |c_3|\}$ and $\mathcal{C}_G(\rho) = \max\{|c_1|, |c_2|, |c_3|\}$, respectively, where int stands for the intermediate result among the absolute values of the correlation parameters [83–86].

The dephasing process preserves the general form of the Bell-diagonal states. Therefore, in this situation, the general patterns of the geometric quantum and classical correlations at any time can be directly obtained as

$$\mathcal{Q}_G[\rho(t)] = \mathrm{int}\{|c_1(t)|, |c_2(t)|, |c_3(t)|\} \tag{12}$$

and

$$\mathcal{C}_G[\rho(t)] = \max\{|c_1(t)|, |c_2(t)|, |c_3(t)|\}, \tag{13}$$

respectively. For the local baths case, the time-dependent correlation parameters (or functions) $c_1(t) = c_1 e^{-2\Gamma(t)}$, $c_2(t) = c_2 e^{-2\Gamma(t)}$, and $c_3(t) = c_3$ are straightforwardly computed from Eqs. (6) and (7). However, these evolved correlation parameters for the common bath take the forms $c_1(t) = \frac{1}{2}[c_1(1 + e^{-4\Gamma(t)}) + c_2(1 - e^{-4\Gamma(t)})]$, $c_2(t) = \frac{1}{2}[c_1(1 - e^{-4\Gamma(t)}) + c_2(1 + e^{-4\Gamma(t)})]$, and $c_3(t) = c_3$, which can be obtained easily from Eqs. (7) and (9).

It is quite prominent that the dynamical behaviors of the geometric correlations based on trace distance and the corresponding intriguing phenomena are strongly dependent on the initial value $c_i = c_i(0)$ and the nature of the environment being considered through the decay factor $\Gamma(t)$. For example, in the local baths scenario, the evolved correlation functions $|c_1(t)|$ and $|c_2(t)|$ present the same decay rate, signaling that they do not cross each other as a function of time, whereas in the dephasing process $|c_3(t)| = |c_3|$ remains constant throughout the entire dynamics. When $|c_1| > |c_2| > |c_3| \neq 0$ (or $|c_2| > |c_1| > |c_3| \neq 0$), only two crossings $|c_1(t)| = |c_3|$ and $|c_2(t)| = |c_3|$ are allowed among the correlation functions. Remarkably, these crossings cause at most two nonanalyticities (abrupt changes) in the intermediate value of $\mathcal{Q}_G[\rho(t)]$ which correspond to the nontrivial phenomenon of double sudden transitions. Nevertheless, these crossings among the correlation functions induce at most a single nonanalyticity in $\mathcal{C}_G[\rho(t)]$, associated with the emergence of the pointer-state basis. This reveals that for the Bell-diagonal states, the double sudden transitions is indeed a quantum effect, which is unattainable for the classical correlation.

Notably, the aforementioned phenomena hold for the Markovian noise models [36,37]. However, it was shown that for the case of non-Markovian random telegraph noise with both stationary and nonstationary stochastic properties, one can observe multiple crossings among the correlation functions [47–50]. Consequently, these crossings give rise to the occurrence of several successive sudden transitions in both quantum and classical correlations. In particular, due to the non-Markovian dynamics, classical correlation exhibits multiple abrupt changes before reaching a stable constant value which is associated with the appearance of a metastable pointer-state basis. These studies imply that dynamical behaviors of the correlations in the quantum system and their relevant unanticipated phenomena rely on the initial state of the system $[c_i = c_i(0)]$ and a detailed configuration of the noisy environment being considered. Therefore, it would be interesting to ask the following questions. First, what are the effects of the reservoir squeezing on the phenomena of freezing, double sudden transitions in GQD-1, and the appearance of the pointer-state basis in the local baths case? Second, how do the squeezing parameters influence the dynamical behaviors of the geometric correlations and their related intriguing phenomena when both qubits are exposed to a common bath? We will address these questions below.

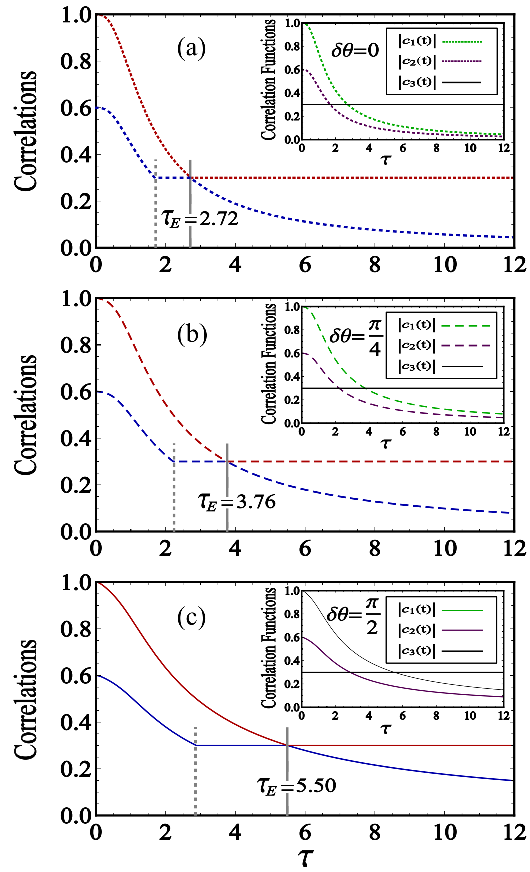

FIG. 1. Time evolution of quantum $\mathcal{Q}_G[\rho_{LB}(t)]$ (blue curves) and classical $\mathcal{C}_G[\rho_{LB}(t)]$ (red curves) correlations of two qubits locally subjected to thermal baths at zero temperature with different values of the phase difference: (a) $\delta\theta = 0$, (b) $\delta\theta = \frac{\pi}{4}$, and (c) $\delta\theta = \frac{\pi}{2}$. In all cases we consider initial correlation parameters $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, squeezing strength $r = 0.5$, and coupling constant $\gamma = 0.5$. The insets in each panel show the dynamics of the correlation parameters for different values of $\delta\theta$. In each panel the gray vertical solid line manifests the time instant τ_E for the emergence of the pointer-state basis.

A. Effects of the squeezing phase

The main focus of this section is to unveil the effects of the squeezing phase of a bath on 1-norm geometric correlations dynamics and intriguing phenomena associated with them. To this aim, in Fig. 1 we plot the time evolution of 1-norm geometric quantum correlations and their classical counterparts in a two-qubit system where each qubit is locally subjected to a squeezed thermal bath at zero temperature for different values of $\delta\theta$. Herein we consider the initial state parameters $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, squeezing strength $r = 0.5$, and coupling constant $\gamma = 0.5$. The inset in each panel of Fig. 1 clearly shows that the components $|c_1(t)|$ and $|c_2(t)|$ exhibit the same decay pattern, revealing that they will never cross each other as a function of timescale τ. However, due to the dephasing process, the function $|c_3(t)|$ stays constant throughout the entire dynamics. Therefore, the only allowed crossing are $|c_1(t)| = |c_3(t)|$ and $|c_2(t)| = |c_3(t)|$.

For the given correlation parameters, we initially have $\mathcal{Q}_G[\rho_{LB}(t)] = c_2(t) = c_2 e^{-2\Gamma(t)}$, implying that GQD-1 exhibits decay in the initial time, as illustrated by the blue dotted curve in Fig. 1(a). When the phase difference $\delta\theta$ between the squeezing phase relative to the coupling strength phase is zero, the first crossing $|c_2(t)| = |c_3(t)|$ occurs at the critical point $\tau = \tau_1^* = 1.68$, as shown by the purple dotted curve in the inset of Fig. 1(a). At this stage, GQD-1 becomes $\mathcal{Q}_G[\rho_{LB}(t)] = c_3(t) = c_3$ and experiences the first sudden transition during the temporal evolution, as indicated by the position of the gray vertical dashed line in Fig. 1(a). The GQD-1 becomes frozen (unaffected by decoherence) for a finite time interval, as displayed by the window between the gray vertical dashed and solid lines in Fig. 1(a). Finally, a second crossing $|c_1(t)| = |c_3(t)|$ appears at the point $\tau = \tau_2^* = 2.72$, as revealed by the green dotted curve in the inset of Fig. 1(a). From this point on, $\mathcal{Q}_G[\rho_{LB}(t)] = c_1(t) = c_1 e^{-2\Gamma(t)}$, signifying that the GQD-1 asymptotically decays to zero $\mathcal{Q}_G[\rho_{LB}(\infty)] = 0$, as shown by the blue dotted curve just after the gray vertical solid line in Fig. 1(a). On the other hand, the crossings $|c_2(t)| = |c_3(t)|$ and $|c_1(t)| = |c_3(t)|$ cause only one abrupt change in $\mathcal{C}_G[\rho_{LB}(t)]$, as shown by the red dotted curves in Fig. 1(a). Indeed, this reveals that the double sudden transitions phenomenon is genuinely a quantum effect that cannot be observed for classical correlations.

Recently, it was reported that dephasing in a single qubit could be suppressed by tuning the squeezing phase of the bath [73]. Therefore, when we induce a phase difference, for instance, $\delta\theta = \frac{\pi}{4}$, the decay in the correlation functions decreases, which causes a delay in the appearance of crossings $|c_2(t)| = |c_3(t)|$ and $|c_1(t)| = |c_3(t)|$, as displayed by the purple ($\tau_1^* = 2.18$) and green ($\tau_2^* = 3.76$) dashed curves, respectively, in the inset of Fig. 1(b). Consequently, the occurrence of the corresponding double sudden transitions in $\mathcal{Q}_G[\rho_{LB}(t)]$ and single abrupt change in $\mathcal{C}_G[\rho_{LB}(t)]$ are also retarded, as shown by the position of the gray vertical dashed and solid lines in Fig. 1(b). Strikingly, in Fig. 1(b), one can also observe an enhancement in the time interval over which the GQD-1 exhibits a constant magnitude. Furthermore, we analyzed these results for the other values of $\delta\theta$ and found that a maximum delay in the appearance of crossings $|c_2(t)| = |c_3(t)|$ and $|c_1(t)| = |c_3(t)|$ happens only when $\delta\theta = \frac{\pi}{2}$, as shown by the purple ($\tau_1^* = 2.88$) and green ($\tau_2^* = 5.50$) solid curves, respectively, in the inset of Fig. 1(c). As a result, one can observe a maximum increase in the critical time for the double sudden transitions in GQD-1; see the positions of the gray vertical (dashed and solid) lines in Fig. 1(c). Further, this also introduces maximum enhancement in the time interval for the frozen GQD-1, as shown by the blue solid curve in Fig. 1(c). This reveals that the squeezing phase of the bath renders a different avenue to suppress the dephasing rate in a two-qubit system and control the critical time for the occurrence of the double sudden transitions phenomenon in GQD-1. In addition, this also provides insights to prolong the time interval for frozen GQD-1.

Now we investigate how the reservoir squeezing phase influences the dynamical behaviors of geometric correlations and their corresponding nontrivial phenomena when both qubits are coupled to a common bath at zero temperature. In Fig. 2 we assume the same initial state and bath parameters as in Fig. 1. It is evident from each inset in Fig. 2 that, unlike the local baths setup, the correlation functions $|c_1(t)|$ and $|c_2(t)|$ do not have the same decay pattern in the common bath scenario. For example, after crossing $|c_2(t)| = |c_3(t)|$, one can observe that $|c_2(t)|$ starts a revival from a dark period (zero) to a stable constant value, i.e., $c_2(\infty)| = \frac{|c_1+c_2|}{2}$, as displayed by the purple dotted curve in the inset in Fig. 2(a). However, $c_1(t)$ asymptotically tends to the same constant level $\frac{|c_1+c_2|}{2}$ without any revival after the second crossing $|c_1(t)| = |c_3(t)|$, as illustrated by the green dotted curve in the inset in Fig. 2(a). Here we can still observe that these two crossings allow at most two sudden transitions in GQD-1 and one abrupt change in classical correlations, as shown by the blue and red dotted curves in Fig. 2(a), respectively. Due to this decay pattern of the correlation functions, GQD-1 tends to a nonzero steady state after the second sudden transition in the common bath, as displayed by the blue dotted curve, sharply after the gray vertical solid line in Fig. 2(a). Moreover, note that in the common bath case, for the given initial state, Eq. (9) manifests that there exists a decoherence-free subspace with two basis $|eg\rangle$ and $|ge\rangle$. This reveals that the common bath setup provides a promising avenue to generate steady-state GQD-1 and decoherence-free subspace.

In the common bath scenario, we also observe that by adequately tuning the phase difference $\delta\theta$, one can efficiently

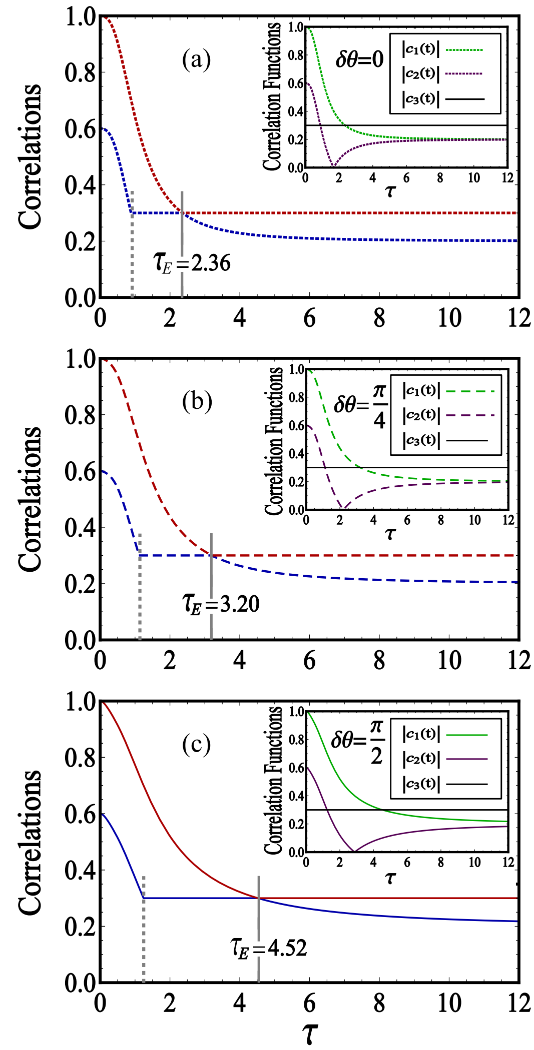

FIG. 2. Time evolution of quantum $\mathcal{Q}_G[\rho_{CB}(t)]$ (blue curves) and classical $\mathcal{C}_G[\rho_{CB}(t)]$ (red curves) correlations of two qubits subjected to a common thermal bath at zero temperature with different values of the phase difference: (a) $\delta\theta = 0$, (b) $\delta\theta = \frac{\pi}{4}$, and (c) $\delta\theta = \frac{\pi}{2}$. In all cases we consider initial correlation parameters $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, squeezing strength $r = 0.5$, and coupling constant $\gamma = 0.5$. The insets in each panel show the dynamics of the correlation parameters for different values of $\delta\theta$. In each panel the gray vertical solid line manifests the time instant τ_E for the emergence of the pointer-state basis.

harness the occurrence of abrupt changes in the 1-norm geometric correlations and prolong the time interval for frozen GQD-1. For instance, comparing Figs. 2(a) and 2(b), we can clearly see a delay in the appearance of the double sudden transitions phenomenon in GQD-1 when introducing a phase difference (e.g., $\delta\theta = \frac{\pi}{4}$), as indicated by the positions of gray vertical dashed and solid lines. In addition, we can introduce maximum retardation in the occurrence of double sudden transitions in GQD-1 and obtain optimal enhancement in the time interval for frozen GQD-1 when $\delta\theta = \frac{\pi}{2}$, as shown by the blue solid curve in Fig. 2(c). Moreover, Fig. 2 reveals that the time to achieve steady-state GQD-1 can be effectively controlled by properly adjusting the value of $\delta\theta$. This implies that our study renders a promising approach to control the generation of steady-state GQD-1 and decoherence-free subspace via the squeezing phase of the reservoir, which is crucial for improving the performance and reliability of quantum communication and computation in the presence of decoherence.

Effects of the squeezing phase on the emergence of the pointer-state basis

The measurement problem is at the heart of fundamental questions of quantum mechanics and the quantum-classical transition [87]. One way to deal with the classical limit is through the decoherence process [75], where a quantum apparatus $\mathcal{A}$ measures a system $\mathcal{S}$. The apparatus undergoes the environmental decoherence that collapses $\mathcal{A}$ into a possible set of classical states known as the pointer-state basis. Consequently, a classical observer can access information about the system through the pointer-state basis associated with the apparatus. Recently, it has been reported that the emergence of the pointer-state basis at a finite time is associated with a specific instant of time τ_E at which the geometric (or entropic) classical correlation between $\mathcal{A}$ and $\mathcal{S}$ becomes abruptly constant [36,45]. This highlights the significance of classical correlations in the investigation of the measurement process, although the composite $\mathcal{AS}$ state still has quantum features, as can be inferred from GQD-1 [36]. Moreover, the emergence of the pointer-state basis strongly relies on the detailed configuration of the decoherent environment [45–50]. Therefore, it is crucial to ask how the reservoir squeezing parameters influence the appearance of the pointer-state basis in both local baths and a common bath scenario.

To answer the aforementioned question, we consider that qubit A works as a quantum measurement apparatus $\mathcal{A}$ and qubit B indicates the system $\mathcal{S}$. First, we assume that the joint system $\mathcal{AS}$ is locally subjected to the dephasing noise induced by the squeezed thermal baths at zero temperature described by Eq. (6). The geometric classical correlation in $\mathcal{AS}$ can be exploited to define τ_E, which precisely corresponds to the critical time at which $\mathcal{C}_G[\rho_{LB}(t)]$ exhibits a sudden transition from a decaying regime to a constant nonvanishing value. In Fig. 1 we plot the dynamics of the classical correlations as a function of τ, with initial state $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, squeezing strength $r = 0.5$, and coupling constant $\gamma = 0.5$. In particular, when $\delta\theta = 0$, we observe that the sudden transition in the classical correlation occurs at a critical time $\tau = \tau_2^* = 2.72$, as displayed by the red dotted curve in Fig. 1(a). This signals that the pointer-state basis emerges at $\tau_E = \tau_2^* = 2.72$, as shown by the gray vertical solid line in Fig. 1(a). However, when we incorporate the phase difference, for example, $\delta\theta = \frac{\pi}{4}$, we notice a delay in the emergence of the pointer-state basis, i.e., $\tau_E = 3.76$ (characterized by the abrupt change in $\mathcal{C}_G[\rho_{LB}(t)]$ towards the constant nonzero level), as shown by the position of the gray vertical solid line in Fig. 1(b). Furthermore, we investigated the emergence of the pointer-state basis for the other values of $\delta\theta$; however, we obtained that the maximum retardation in the emergence of pointer-state basis occurs only when $\delta\theta = \frac{\pi}{2}$, that is, $\tau_E = 5.50$, as illustrated by the position of the gray vertical solid line in Fig. 1(c).

Now, in Fig. 2, we investigate the emergence of the pointer-state basis when the composite system $\mathcal{AS}$ interacts with a common squeezed thermal reservoir at zero temperature with the same set of parameters as given in Fig. 1. Specifically for $\delta\theta = 0$, the pointer-state basis emerges at $\tau_E = \tau_2^* = 2.36$, characterized by the sudden change in the classical correlations, as displayed by the position of the gray vertical solid line in Fig. 2(a). However, when we induce the phase difference, for instance, $\delta\theta = \frac{\pi}{4}$ and $\delta\theta = \frac{\pi}{2}$, we observe a delay in the emergence of the pointer-state basis, i.e., $\tau_E = \tau_2^* = 3.20$ and $\tau_E = \tau_2^* = 4.52$, as shown by the positions of the gray vertical solid lines in Figs. 2(b) and 2(c), respectively. Thus, by comparing the Figs. 1 and 2, we observe that the pointer-state basis emerges earlier in the common bath case than in the local baths case. In addition, we find that in both setups of the reservoir, the squeezing phase of the bath provides an efficient protocol to control the quantum-to-classical transition phenomenon (characterized by a pointer-state basis) without applying any operation on the main system. This finding can have potential implications for quantum technologies, where the control of the quantum-to-classical transition is critical for the performance and accuracy of the devices.

B. Effects of squeezing strength

In this section we investigate the impact of the squeezing strength r on the time evolution of the 1-norm geometric correlations and corresponding nontrivial phenomena in two qubits locally subjected to squeezed thermal baths or a common squeezed thermal bath at zero temperature. For this purpose, first, we consider the case of the local baths in Fig. 3 and illustrate the dynamics of quantum and classical correlations as a function of the timescale τ, setting the same initial state parameters and coupling constant γ as considered in Fig. 1, but different values of squeezing strength r with a fixed phase difference $\delta\theta = \frac{\pi}{2}$. Specifically, when the squeezing strength is $r = 0.1$, the first crossing $|c_2(t)| = |c_3(t)|$ is observed at a certain point $\tau = \tau_1^* = 3.10$, while the second crossing $|c_1(t)| = |c_3(t)|$ appears at $\tau = \tau_2^* = 7.24$, as displayed by the purple and green dotted curves in the inset of Fig. 3(a), respectively. These crossings among the correlation parameters induce the corresponding double and single sudden transitions in the decay rates of $\mathcal{Q}_G[\rho_{LB}(t)]$ and $\mathcal{C}_G[\rho_{LB}(t)]$, as shown by the blue and red dotted curves, respectively, in Fig. 3(a).

However, when we gradually increase the squeezing strength, for example, $r = 0.6$, the correlation functions decay fast, which shortens the critical time for the occurrence of first and second crossings, i.e., $\tau_1^* = 2.62$ and $\tau_2^* = 4.69$, as shown by the purple and green dashed curves, respectively, in the inset given in Fig. 3(b). As a consequence, the associated phenomenon of double sudden transitions in GQD-1 appears earlier in time, as evidenced by the positions of the gray vertical (dashed and solid) lines in Fig. 3(b). In addition, the size of the time interval for which GQD-1 exhibits a constant magnitude is also reduced, as displayed by the blue dashed curve in Fig. 3(b). Similarly, if we further increase the squeezing strength, i.e., $r = 1$, the decay in correlation functions $|c_1(t)|$ and $|c_2(t)|$ becomes more profound; see the green and purple solid curves, respectively, in the inset in Fig. 3(c). This implies that one can observe a further decrease in the critical time for the appearance of double sudden transitions in GQD-1, as displayed by the blue solid curve in Fig. 3(c). It

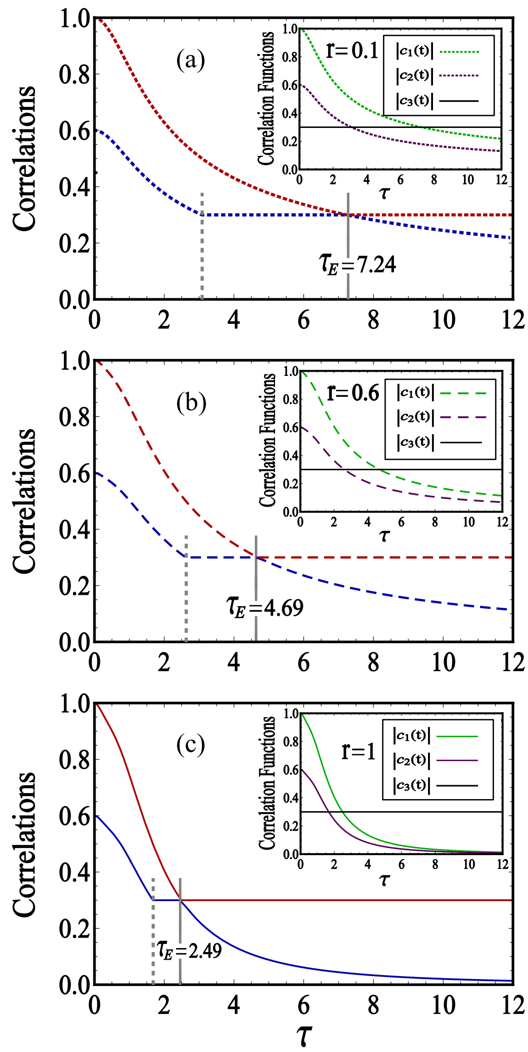

FIG. 3. Time evolution of quantum $\mathcal{Q}_G[\rho_{LB}(t)]$ (blue curves) and classical $\mathcal{C}_G[\rho_{LB}(t)]$ (red curves) correlations of two qubits locally subjected to thermal baths at zero temperature with different values of the squeezing strength: (a) $r = 0.1$, (b) $r = 0.6$, and (c) $r = 1$. In all cases we consider initial correlation parameters $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, $\delta\theta = \frac{\pi}{2}$, and coupling constant $\gamma = 0.5$. The insets in each panel show the dynamics of the correlation parameters for different values of r. In each panel the gray vertical solid line manifests the time instant τ_E for the emergence of the pointer-state basis.

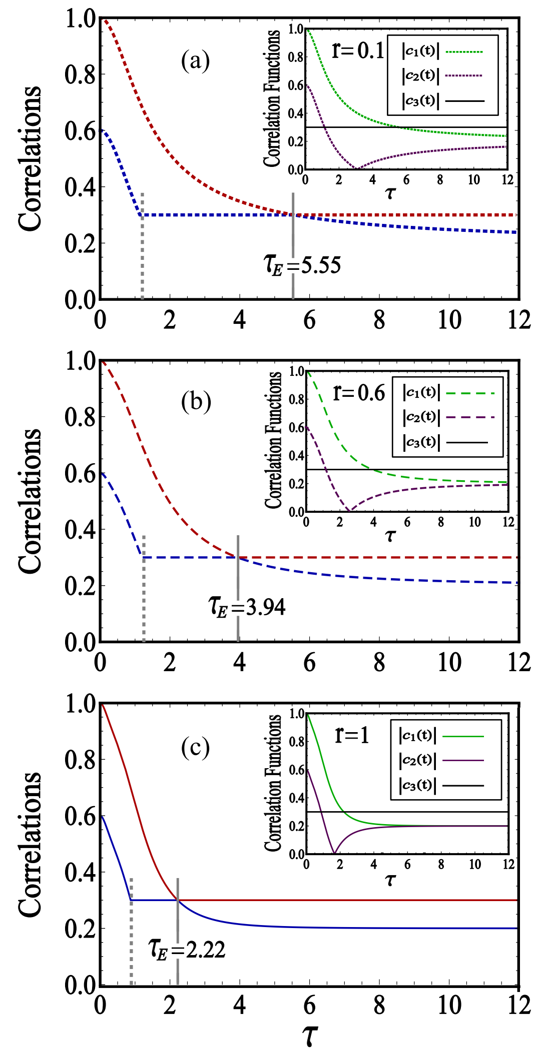

FIG. 4. Time evolution of quantum $\mathcal{Q}_G[\rho_{CB}(t)]$ (blue curves) and classical $\mathcal{C}_G[\rho_{CB}(t)]$ (red curves) correlations of two qubits subjected to a common thermal thermal bath at zero temperature with different values of the squeezing strength: (a) $r = 0.1$, (b) $r = 0.6$, and (c) $r = 1$. In all cases we consider initial correlation parameters $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.3$, $\delta\theta = \frac{\pi}{2}$, and coupling constant $\gamma = 0.5$. The insets in each panel show the dynamics of the correlation parameters for different values of r. In each panel the gray vertical solid line manifests the time instant τ_E for the emergence of the pointer-state basis.

also reduces the time interval for frozen GQD-1, as shown by the blue solid curve in Fig. 3(c). These findings reflect that the squeezing strength has a negative role in delaying the double sudden transitions in GQD-1 and enhancing the time interval over which the geometric quantum correlation stays constant.

In Fig. 4 we now assume the same set of parameters as given in Fig. 3 and reveal the influence of the squeezing strength r on the dynamics of geometric correlations in two qubits coupled to a common thermal squeezed bath with zero temperature. For example, when $r = 0.1$, the first crossing $|c_2(t)| = |c_3(t)|$ occurs at $\tau = \tau_1^* = 1.14$ and the second crossing $|c_2(t)| = |c_3(t)|$ at $\tau = \tau_2^* = 5.55$, as displayed by the purple and green dotted curves, respectively, in the inset in Fig. 4(a). Interestingly, in the common bath scenario, we still observe that these crossings induce double sudden transitions in $\mathcal{Q}_G[\rho_{CB}(t)]$ and a single abrupt change in $\mathcal{C}_G[\rho_{CB}(t)]$, as shown by the blue and red dotted curves, respectively, in Fig. 4(a). Furthermore, in the common bath setup, we found that GQD-1 tends to a stable nonvanishing value after the second sudden transition, as indicated by the blue dotted curve in Fig. 4(a). Like the local baths case, here we also observe that increasing the squeezing strength, e.g., $r = 0.6$ and $r = 1$, can reduce the critical time for the crossings among the correlation functions, as shown in the insets of Figs. 4(b) and 4(c), respectively. Indeed, this results in a reduction in time for the appearance of the double sudden transitions phenomenon in GQD-1 and also reduces the length of the time interval for frozen GQD-1, as illustrated by the positions of the gray vertical (dashed and solid) lines in Figs. 4(b) and 4(c). In addition, we found that the time required for achieving the steady-state GQD-1 decreases by increasing the squeezing strength of the bath, as shown by the blue [(a) $r = 0.1$, dotted; (b) $r = 0.6$, dashed; and (c) $r = 0.1$, solid] curves in Fig. 4. This reveals that the squeezing strength renders a promising approach for controlling the generation of steady-state GQD-1.

Effects of squeezing strength on the emergence of the pointer-state basis

We now examine the influence of the squeezing strength r on the emergence of the pointer-state basis that characterizes the quantum-to-classical transition. The classical correlations can be applied as a powerful tool to define well the critical time at which the pointer-state basis emerges. For this aim, in Figs. 3 and 4 we closely analyze the dynamical behaviors of the classical correlation in two qubits locally interacting with squeezed thermal baths or a single common squeezed thermal bath at zero temperature. In the case of local baths, when $r = 0.1$, we obtain an abrupt change from the initial decay regime to a constant stationary value in $\mathcal{C}_G[\rho_{LB}(t)]$ at the critical point $\tau_2^* = 7.24$, as displayed by the red dotted curve in Fig. 3(a). This infers that the pointer-state basis emerges at $\tau_E = \tau_2^* = 7.24$, as shown by the gray vertical solid line in Fig. 3(a). However, when we slowly increase the value of the squeezing strength, for example, $r = 0.6$ and 1, the pointer-state basis emerges earlier in time, i.e., $\tau_E = 4.69$ and 2.49, as illustrated by the positions of the gray vertical solid lines in Figs. 3(b) and 3(c), respectively. This reflects that increasing the squeezing strength leads to a decrease of the critical time for the emergence of the pointer-state basis. In other words, the phenomenon of the quantum-to-classical transition happens earlier if we enhance the squeezing strength.

In Fig. 4 we plot the dynamics of the classical correlation to investigate the effect of the squeezing strength on the emergence of the pointer-state basis in a common bath scenario. When $r = 0.1$, the classical correlation displays a sudden transition at the critical time $\tau = 5.55$, signaling that the pointer-state basis emerges at $\tau_E = \tau_2^* = 5.55$, as shown by the position of the gray vertical solid line in Fig. 4(a). Moreover, if we gradually increase the squeezing strength, for instance, $r = 0.6$ and 1, the time for the abrupt change in $\mathcal{C}_G[\rho_{CB}(t)]$ decreases, as shown by the red dashed and solid curves in Figs. 4(b) and 4(c), respectively. This reveals that the appearance time for a pointer-state basis also decreases, i.e., $\tau_E = \tau_2^* = 3.94$ and 2.22, as represented by the position of the gray vertical solid line in Figs. 4(b) and 4(c), respectively. By comparing Figs. 3 and 4, we can conclude that the squeezing strength has the same effects in both the local baths and common bath cases; however, the pointer-state basis emerges earlier in the common bath setup. This reflects that one can observe the phenomenon of quantum-to-classical transition earlier in the common bath case. Our findings provide insights into the investigation of the measurement problem in quantum mechanics.

Nevertheless, we further analyzed the above results for the squeezed thermal baths at the high-temperature limit. It is found that the squeezing parameters exhibit the same effects as obtained for a squeezed vacuum reservoir ($T = 0$). Therefore, we omitted the case of a high-temperature limit.

IV. CONCLUSION

We have investigated theoretically the dynamics of 1-norm geometric quantum correlations and their classical counterparts in a two-qubit system. Both qubits are initially prepared in Bell-diagonal states and locally coupled to spatially separated squeezed thermal reservoirs or a common squeezed thermal reservoir via energy-preserving interactions. We have unveiled the effects of reservoir squeezing on the abrupt changes in the evolution of geometric correlations. Strikingly, we have found that by properly tuning the squeezing phase of the reservoir, we can efficiently suppress the dephasing rate and retard the appearance of double sudden transitions in GQD-1 in both environmental setups. Further, we have shown that the squeezing phase of the bath provides an alternative promising tool for enhancing the finite time interval over which GQD-1 remains frozen, despite the deleterious effects of decoherence. On the other hand, in this framework, we have shown that the squeezing strength of the reservoir plays a negative role throughout the dynamics. Moreover, we have observed the generation of steady-state geometric correlations and decoherence-free subspace in the common bath scenario. Interestingly, their emergence can be controlled through the squeezing parameters of the bath. Nevertheless, we have found that adequately adjusting the squeezing parameters can significantly delay the emergence of a pointer-state basis (quantum-to-classical transition). Remarkably, we have shown that when $\delta\theta = \frac{\pi}{2}$, maximum retardation in the appearance of pointer-state basis can be observed. It is worth mentioning that the above phenomena appear earlier in the common bath scenario. Our study provides insights into controlling the geometric correlations and their corresponding nontrivial phenomena, which have vital applications in quantum information and measurement problems.

Finally, we would like to comment on the realization of our work in present-day experiments. The dephasing spin-boson model with an Ohmic-like spectrum can be realized in an ultracold hybrid system consisting of an impurity atom immersed in the Bose-Einstein condensates (BECs) environment [88–90]. In this configuration an impurity atom trapped in a double-well potential forms a qubit system, while the BECs are often referred to as a bosonic reservoir. The squeezing (Kerr-like nonlinearity) in the BECs (reservoir) can be induced due to the nonlinear atom-atom interactions inside the BECs atoms [91]. One can efficiently control squeezing parameters by adjusting the strength, duration, and phase of these nonlinear interactions [92–95]. Therefore, we hope that our results can be experimentally realized in the mentioned setup.

ACKNOWLEDGMENTS

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APPENDIX

The dynamical behavior of the off-diagonal elements in Eq. (2) is displayed as

$$e^{-2\Gamma(t)} = \mathrm{Tr}_R\left[\rho_R(0)\exp\left(2\sum_k[\eta_k(t)\hat{a}_k^\dagger - \eta_k^*(t)\hat{a}_k]\right)\right]. \tag{A1}$$

It is worth mentioning that the expression for $e^{-2\Gamma(t)}$ is just the characteristic function for the Wigner representation of $\rho_R(0)$ (which is a squeezed thermal state of all the modes of reservoir). Hence the decay factor can be written as [77–80]

$$\Gamma(t) = \sum_k \frac{1}{2}|2\eta_k(t)\cosh r_k + 2\eta_k^*(t)e^{i\theta_k}\sinh r_k|^2 \times \coth\frac{\omega_k}{2T}. \tag{A2}$$

Now substituting $\eta_k(t) = \lambda_k(1 - e^{i\omega_k t})/\omega_k$ into the expression (A2), the decay factor becomes

$$\Gamma(t) = \sum_k \frac{4|\lambda_k|^2}{\omega_k^2}(1 - \cos\omega_k t)\coth\frac{\omega_k}{2T} \times \{\cosh 2r_k - \sinh 2r_k\cos(\omega_k t - \Delta\theta_k)\}, \tag{A3}$$

where $\Delta\theta_k = \theta_k - 2\phi_k$ characterizes the phase difference between the squeezing phase θ_k corresponding to the coupling strength phase $\lambda_k = |\lambda_k|e^{i\phi_k}$.

For the sake of simplicity, we assume that all the bath modes have the same squeezing strength, i.e., $r_k = r$, and consider $\Delta\theta_k = \delta\theta$. We introduce a coupling spectral density $I(\omega) = 2\pi\sum_k|\lambda_k|^2\delta(\omega - \omega_k)$ [78,96] and take the continuum limit of the bath modes; thus we can easily obtain Eq. (3) from Eq. (A3).

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