Quantum thermodynamics in nonequilibrium reservoirs: Landauer-like bound and its implications

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We study quantum thermodynamics in nonequilibrium reservoirs (NERs) that are prepared from initially thermal ones via unitary driving processes. Based on the formulation of entropy production in NERs, we establish a Landauer-like bound and show that the bound can be violated under a definite condition depending on the states of NERs. It is found that the breakdown of this Landauer-like bound implies occurrences of several anomalous phenomena in NERs, namely, the efficiency enhancement of quantum thermal machines beyond the Carnot limit, the extraction of work from a single reservoir, and the spontaneous heat flow from a cold reservoir to a hot one. The results are illustrated through a physical model taking the preparation processes of NERs into account. Our work sheds light on the reason and condition of some unusual phenomena occurring in NERs and is helpful for one to prepare effective NERs that can be used as quantum resources to realize certain thermodynamic tasks.

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I. INTRODUCTION

Recent years have seen a growing development of quantum thermodynamics (QT) [1–6] which incorporates the latest achievements of quantum theory, such as quantum information and the dynamics of open quantum systems, into classical thermodynamics. The main aims of QT include extensions of conventional thermodynamic laws to quantum domains and designs of quantum thermal machines (QTMs) utilizing quantum resources and/or quantum effects. The pursuit of quantum advantage of QTMs over their classical counterparts is an ongoing issue in QT. In a pioneering work [7], Scully *et al.* showed that, by replacing atoms (bath) in regular thermal states with nonequilibrium states containing certain coherence, the cavity field (system) can arrive at a higher temperature allowing the efficiency of a photonic Carnot engine to surpass the limit imposed by the thermal bath. Since then, the nonequilibrium reservoir (NER) [8–15], as a type of quantum resource, is extensively used to power thermodynamic tasks. Reservoirs with quantum coherence or correlation are shown to be able to enhance the performance of QTMs [16–25], improve the extraction of work [26–28], and increase the thermalization temperature of quantum systems [29–31]. For a configuration in which two NERs are bridged by coupled systems, the heat in the steady-state regime can be transferred spontaneously from the cold reservoir to the hot one for specific phase differences of initial coherent states of reservoirs [32]. The Otto engine in contact with squeezed reservoirs can be led to surpass the Otto and Carnot limits [33–37]. It also turns out that depending on whether energy exchanges in the preparation process of NERs are involved, QTMs in the NERs can perform different functions, and a complete consideration is always consistent with thermodynamic laws, whereas a partial scenario may not [38]. Hence, a complete picture involving initial preparations of NERs is necessary for reconciling some thermodynamic inconsistencies that arise in the presence of NERs.

Being different from the relatively simple thermal reservoir, the NER is generally characterized by many factors, such as coherence and squeezing parameters in their states. To realize certain tasks resorting to the power of NERs, in particular the ones that cannot be achieved by thermal reservoirs, one has to choose suitable NERs by adjusting various parameters. The construction of a general condition under which some unusual tasks can be realized by the NERs is thus useful not only in understanding of characters of NERs but also in their applications. To shed some light on this issue, in this work, we establish a Landauer-like inequality for NERs generated from thermal reservoirs through driving processes. We show that the violation of this inequality under a definite condition signifies the appearance of several unusual results in NERs including the excess of Carnot efficiency for QTMs, the extraction of work from a single reservoir, and the spontaneous heat flow from a cold reservoir to a hot one.

A quantum master equation is a popular approach for describing the dynamics of an open quantum system; however, it may bring about thermodynamic inconsistencies [39–44] in the treatment of QT if the assumptions needed for its derivation are not respected. The collision model [45], as an alternative tool, enabling one to keep track of the information of reservoirs, is widely applied in simulating the dynamics of open quantum systems [46–54] as well as dealing with QT [55–71]. In the framework of the collision model, the reservoir is modeled as a series of identical ancillas, and the system of interest interacts/collides with them successively. At each step of the collisions, a fresh ancilla is used to interact with the system and then discarded. Due to the simplicity of the mechanism, collision models are suitable for dealing with thermodynamics in the presence of nonthermal reservoirs

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[29–34,38,67–71]. In Ref. [68], the authors developed weakly coherent collision models, by which the first and second laws of thermodynamics were formulated with a nontrivial contribution of quantum coherence. In Ref. [69], thanks to collision models, the authors demonstrated advantages of utilizing coherence in nonthermal baths as a resource in enhancing the performance of thermal machines. The collision model can also be utilized to describe the micromaser, which is an experimental setup with a lossy cavity being pumped by a beam of atoms [46]. In this work, we shall adopt a collision model to describe the dynamics of a system interacting with NERs and construct thermodynamic quantities and laws.

The paper is organized as follows. Section II is devoted to the construction of a Landauer-like bound for NERs and the condition for its breakdown. After introducing the preparation process of a NER in Sec. II A, we describe the dynamics of the system in NERs and define associated thermodynamic quantities in Sec. II B by virtue of a collision model. The entropy production is derived in Sec. II C, which takes the driving process for a NER into account. In analogy to the conventional Landauer bound for thermal baths, a Landauer-like bound for the NERs is established, and its violating condition is presented in Sec. II D. In Sec. III, we address the implications of violation of the Landauer-like bound for several unusual phenomena. It turns out that under the condition of the breakdown of this Landauer-like bound, the efficiency of QTMs can be enhanced to surpass the Carnot limit (Sec. III A), the work can be extracted from a single reservoir (Sec. III B), and the heat can be transferred from a cold reservoir to a hot one spontaneously (Sec. III C). The obtained results are illustrated in Sec. IV by considering a concrete physical model in which both the system and the NERs are modeled as qubits. In Sec. IV A, it is found that in addition to the usual machines with efficiency being less than the Carnot limit, the system can work as a superengine or as a superrefrigerator with efficiency surpassing the Carnot limit when the Landauer-like bound is violated. In Sec. IV B, we illustrate the extraction of work from a single NER and the spontaneous heat flow from a cold reservoir to a hot one under exactly the same condition of the violation of our Landauer-like bound. The conclusion is made in Sec. V.

II. THERMODYNAMIC QUANTITIES AND LANDAUER-LIKE BOUND IN NERs

A. Preparing NERs via unitary driving process

In our model, the system S interacts with M reservoirs labeled by R_i with $i = 1, 2, \ldots, M$ and each R_i is modeled as an ensemble of identical ancillas in the sense of a collision model (see Fig. 1 for $M = 1$). Albeit our purpose is to explore the thermodynamics of NERs, we do not regard them as free resources but as produced from thermal reservoirs through driving processes. Therefore, our model consists of two stages, i.e., the preparation stage for NERs (cf. the left part of Fig. 1) and the collision stage between the system and NERs (cf. the right part of Fig. 1).

As the starting point of R_i, it is initially prepared in a thermal state being of the form

$$\rho_{R_i}^{th} = e^{-\beta_i \hat{H}_{R_i}^0}/Z_i^0, \tag{1}$$

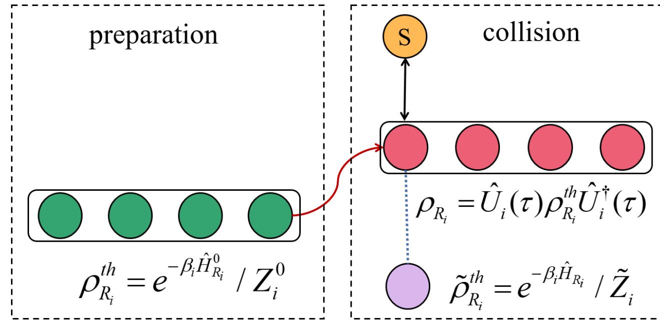

FIG. 1. Sketch of our model (with $M = 1$) which consists of two stages, i.e., the preparation stage for the NER and the collision stage between the system and the NER. Each ancilla in R_i with Hamiltonian $\hat{H}_{R_i}^0$ is initially prepared in thermal equilibrium state $\rho_{R_i}^{\text{th}}$ at inverse temperature β_i (labeled by green circles), which is then driven out of equilibrium by a protocol g_t to the nonequilibrium state $\rho_{R_i} = \hat{U}_i(\tau)\rho_{R_i}^{\text{th}}\hat{U}_i^\dagger(\tau)$ (labeled by red circles). We associate the nonequilibrium state ρ_{R_i} with a reference thermal state $\tilde{\rho}_{R_i}^{\text{th}}$ (labeled by purple circles) which possesses the same inverse temperature β_i as $\rho_{R_i}^{\text{th}}$ and the final Hamiltonian $\hat{H}_{R_i}$ after the driving.

where $Z_i^0 = \text{Tr}[e^{-\beta_i \hat{H}_{R_i}^0}]$ is the partition function with the inverse temperature $\beta_i = 1/T_i$. We set $k_B = \hbar = 1$ throughout the paper. To acquire a nonequilibrium state, a driving protocol characterized by g_t is exerted on R_i, which transforms the Hamiltonian $\hat{H}_{R_i}(t) = \hat{H}_{R_i}(g_t)$ of R_i from $\hat{H}_{R_i}^0 \equiv \hat{H}_{R_i}(0)$ at $t = 0$ to $\hat{H}_{R_i} \equiv \hat{H}_{R_i}(\tau)$ at $t = \tau$. The thermal state $\rho_{R_i}^{\text{th}}$ of R_i in Eq. (1) after the driving process becomes

$$\rho_{R_i} = \hat{U}_i(\tau)\rho_{R_i}^{\text{th}}\hat{U}_i^\dagger(\tau), \tag{2}$$

in which the unitary time-evolution operator $\hat{U}_i(\tau) = \mathcal{T}\exp(-\frac{i}{\hbar}\int_0^\tau \hat{H}_{R_i}(t)dt)$ with $\mathcal{T}$ the time-ordering operator.

Generally, the stronger the driving process, the farther the state ρ_{R_i} in Eq. (2) deviates from the thermal equilibrium state $\rho_{R_i}^{\text{th}}$ in Eq. (1). We adopt quantum relative entropy, defined for states ϱ_1 and ϱ_2 as $D(\varrho_1 \parallel \varrho_2) = \text{Tr}[\varrho_1 \ln \varrho_1] - \text{Tr}[\varrho_1 \ln \varrho_2]$, to quantify the distance between two states. Therefore, the driving process can be characterized by the relative entropy $D(\rho_{R_i} \parallel \rho_{R_i}^{\text{th}})$. Apart from it, an equivalent depiction for the driving process is the so-called irreversible entropy $D(\rho_{R_i} \parallel \tilde{\rho}_{R_i}^{\text{th}})$ [72–75] with $\tilde{\rho}_{R_i}^{\text{th}} = e^{-\beta_i \hat{H}_{R_i}}/\tilde{Z}_i$ (with $\tilde{Z}_i = \text{Tr}[e^{-\beta_i \hat{H}_{R_i}}]$ the partition function) being a reference thermal state associated with ρ_{R_i}. The state $\tilde{\rho}_{R_i}^{\text{th}}$ could be achieved through a quasistatic isothermal evolution from the initial thermal state $\rho_{R_i}^{\text{th}}$ in Eq. (1), which features the same inverse temperature β_i as $\rho_{R_i}^{\text{th}}$ on the one hand, and the Hamiltonian $\hat{H}_{R_i}$ of R_i after the actual driving process on the other hand.

B. Dynamics of the system and thermodynamic quantities

After the nonequilibrium preparation for reservoirs, the system-reservoirs collisions are turned on. At each shot of the collision, the system S collides simultaneously with M fresh ancillas for a short duration τ' with each ancilla being taken from a different reservoir. The ancillas after collisions are discarded and the procedure is iterated. A generic Hamiltonian governing system-reservoirs collisions reads

$$\hat{H}_{SR} = \sum_{i=1}^{M} \hat{H}_{SR_i} = \sum_{i=1}^{M} \sum_{\alpha} \lambda_{i,\alpha} \hat{A}^{\alpha} \otimes \hat{B}_i^{\alpha}, \tag{3}$$

where $\hat{A}^{\alpha}$ and $\hat{B}_i^{\alpha}$ are Hermitian operators acting on S and the ancilla in R_i, respectively, while $\lambda_{i,\alpha}$ denotes the interaction strength. Although in the collision stage we assume that the free Hamiltonians of both the system and the reservoirs are time independent, the system-reservoirs collisions are actually time dependent since they exist only in time intervals $[(n-1)\tau', n\tau']$ (with $n \geqslant 1$ denoting the number of collisions) and vanish otherwise. The total Hamiltonian for the collision process can thus be formulated as

$$\hat{H}_{\text{tot}}(t) = \hat{H}_S + \hat{H}_R + h(t)\hat{H}_{SR}, \tag{4}$$

where $\hat{H}_S$ and $\hat{H}_R = \sum_{i=1}^{M} \hat{H}_{R_i}$ are free Hamiltonians of the system and total reservoirs, respectively, and the Heaviside function $h(t)$ is equal to 1 for $t \in [(n-1)\tau', n\tau']$ and zero otherwise. After the nth step of the collision, the system's state $\rho_S^{n-1} \equiv \rho_S[(n-1)\tau']$ is transformed to $\rho_S^n \equiv \rho_S(n\tau')$ as

$$\rho_S^n = \text{Tr}_R \rho_{SR}^n = \text{Tr}_R\{\hat{U}_{SR}(\tau')\rho_{SR}^{n-1}\hat{U}_{SR}^{\dagger}(\tau')\}, \tag{5}$$

where $\rho_{SR}^{n-1} = \rho_S^{n-1} \otimes \rho_R$ with $\rho_R = \prod_{i=1}^{M} \rho_{R_i}$ and $\hat{U}_{SR}(\tau') = \exp[-i(\hat{H}_S + \hat{H}_R + \hat{H}_{SR})\tau']$. The state ρ_{R_i} of the ancilla in R_i is correspondingly transformed to $\rho_{R_i}^n = \text{Tr}_{S\bar{R}_i}\rho_{SR}^n$ with $\bar{R}_i$ denoting $M-1$ ancillas other than R_i. Note that the state of the ancilla after collision is n dependent.

In the collision model, the energetic cost in terms of work is generally required to switch on and off the system-reservoirs collisions unless the collisions fulfill strict energy conservation given by $[\hat{H}_{SR}, \hat{H}_S + \hat{H}_R] = 0$ [76]. The work in the nth round of collision injected by an external agent can be formulated as

$$\begin{aligned}
\Delta W^n &= \int_{(n-1)\tau'}^{n\tau'} dt \frac{d}{dt}\text{Tr}_{SR}[\hat{H}_{\text{tot}}(t)\rho_{SR}(t)] \\
&= \int_{(n-1)\tau'}^{n\tau'} \text{Tr}_{SR}\left[\frac{d\hat{H}_{\text{tot}}(t)}{dt}\rho_{SR}(t)\right]dt \\
&= \text{Tr}_{SR}(\hat{H}_{SR}\{\rho_{SR}[(n-1)\tau'] - \rho_{SR}(n\tau')\}),
\end{aligned} \tag{6}$$

where we have used $\text{Tr}_{SR}[\hat{H}_{\text{tot}}(t)\frac{d\rho_{SR}(t)}{dt}] = 0$. Note that in Eq. (6) $\rho_{SR}[(n-1)\tau'] \equiv \rho_{SR}^{n-1} = \rho_S^{n-1} \otimes \rho_R$ and $\rho_{SR}(n\tau') \equiv \rho_{SR}^n$. The heat flowing out of reservoir R_i in the nth collision is defined as

$$\Delta Q_i^n = -\text{Tr}\big[\hat{H}_{R_i}\big(\rho_{R_i}^n - \rho_{R_i}\big)\big], \tag{7}$$

which is just the energy change of R_i during the collision. Here, we have attributed all the energy changes of reservoir R_i whether in thermal state or in nonequilibrium state to heat. This could be regarded as an assumption since a generic system in a nonequilibrium state interacting with another one can also exchange energy in the form of work [68]. As $\text{Tr}_{SR}[(\hat{H}_S + \hat{H}_R + \hat{H}_{SR})(\rho_{SR}^n - \rho_{SR}^{n-1})] = 0$, we can obtain by the quantities in Eqs. (6) and (7) that the change in internal energy of the system is

$$\Delta U_S^n = \text{Tr}\big[\hat{H}_S\big(\rho_S^n - \rho_S^{n-1}\big)\big] = \sum_i^M \Delta Q_i^n + \Delta W^n, \tag{8}$$

which complies with the first law of thermodynamics.

C. Entropy production

The second law is embodied by the non-negativity of the entropy production, which can be expressed for the nth collision as [77]

$$\Sigma = I_{\rho_{SR}^n}(S:R) + D\big[\rho_R^n \big\| \rho_R\big] \geqslant 0. \tag{9}$$

The first term of Eq. (9) is quantum mutual information quantifying correlations between the system and all the reservoirs established in the nth collision, which is defined as $I_{\rho_{SR}^n}(S:R) = S(\rho_S^n) + S(\rho_R^n) - S(\rho_{SR}^n)$ with $S(\varrho) = -\text{Tr}[\varrho \ln \varrho]$ being the von Neumann entropy of ϱ and $\rho_R^n = \text{Tr}_S\rho_{SR}^n$. The second term of Eq. (9) is quantum relative entropy for the states of reservoirs before and after the collision. Therefore, the irreversibility of a collision process originates from cutting off correlations of the system and reservoirs, as well as discarding the information of reservoirs. Since the system and reservoirs are initially uncorrelated and the von Neumann entropy remains invariant in unitary dynamics, we obtain that $I_{\rho_{SR}^n}(S:R) = -\Delta\widetilde{S}(\rho_S^n) + \Delta S(\rho_R^n)$, with $\Delta\widetilde{S}(\rho_S^n) = S(\rho_S^{n-1}) - S(\rho_S^n)$ the entropy decrease of the system and $\Delta S(\rho_R^n) = S(\rho_R^n) - S(\rho_R)$ the entropy increase of reservoirs. The entropy production can thus be updated to

$$\Sigma = -\Delta\widetilde{S}\big(\rho_S^n\big) + \Delta S\big(\rho_R^n\big) + D\big[\rho_R^n \big\| \rho_R\big] \geqslant 0. \tag{10}$$

For the later construction of a Landauer-like bound, we integrate the heat current ΔQ_i^n into the expression of entropy production by associating the state ρ_R with reference thermal state $\tilde{\rho}_R^{\text{th}} = \prod_{i=1}^{M} \tilde{\rho}_{R_i}^{\text{th}}$ with $\tilde{\rho}_{R_i}^{\text{th}} = e^{-\beta_i \hat{H}_{R_i}}/\tilde{Z}_i$ being introduced previously in Sec. II A. With the state $\tilde{\rho}_R^{\text{th}}$, the entropy increase $\Delta S(\rho_R^n)$ of reservoirs can be expanded as

$$\begin{aligned}
\Delta S(\rho_R^n) &= -\text{Tr}\big[\rho_R^n \ln \rho_R^n\big] + \text{Tr}\big[\rho_R \ln \rho_R\big] \\
&= -\text{Tr}\big[\rho_R^n \ln \rho_R^n\big] + \text{Tr}\big[\rho_R^n \ln \tilde{\rho}_R^{\text{th}}\big] - \text{Tr}\big[\rho_R^n \ln \tilde{\rho}_R^{\text{th}}\big] \\
&\quad + \text{Tr}\big[\rho_R \ln \tilde{\rho}_R^{\text{th}}\big] - \text{Tr}\big[\rho_R \ln \tilde{\rho}_R^{\text{th}}\big] + \text{Tr}\big[\rho_R \ln \rho_R\big] \\
&= -D\big[\rho_R^n \big\| \tilde{\rho}_R^{\text{th}}\big] + \sum_i^M \beta_i \Delta\widetilde{Q}_i^n + D\big[\rho_R \big\| \tilde{\rho}_R^{\text{th}}\big],
\end{aligned} \tag{11}$$

where $\Delta\widetilde{Q}_i^n = -\Delta Q_i^n$ is the heat flowing from S to R_i. By inserting the expression $\Delta S(\rho_R^n)$ in Eq. (11) into Eq. (10), the entropy production can be reformulated as

$$\begin{aligned}
\Sigma = \sum_i^M \beta_i \Delta\widetilde{Q}_i^n - \Delta\widetilde{S}(\rho_S^n) + D\big[\rho_R^n \| \rho_R\big] \\
+ D\big[\rho_R \big\| \tilde{\rho}_R^{\text{th}}\big] - D\big[\rho_R^n \big\| \tilde{\rho}_R^{\text{th}}\big] \geqslant 0.
\end{aligned} \tag{12}$$

In Eq. (12), the term $D[\rho_R \| \tilde{\rho}_R^{\text{th}}]$ characterizes the driving process for the preparation of NERs whose influences on the entropy production in the collision process can be examined more precisely when we identify two contributions to a nonequilibrium state: one is due to population mismatch

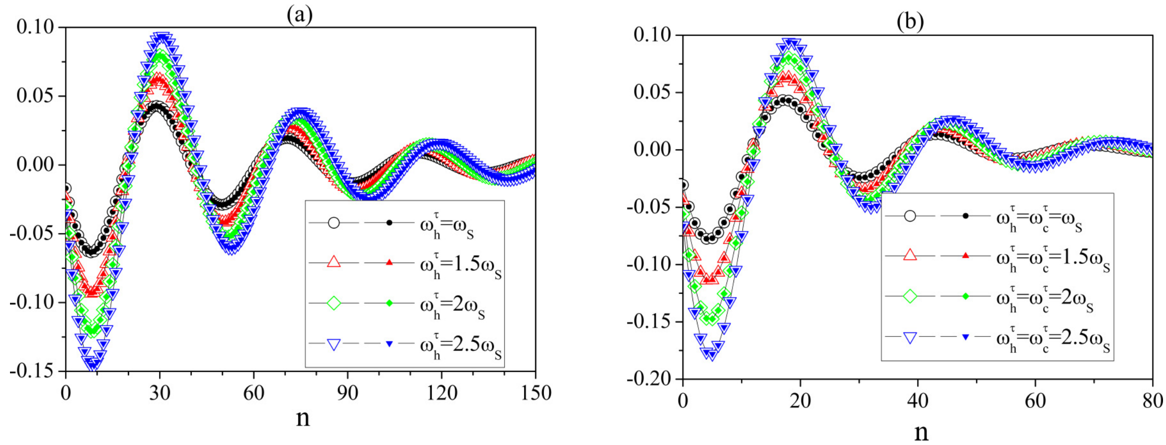

FIG. 2. Plots of the LHS (solid symbols) and RHS (hollow symbols) of our Landauer-like principle in the equality form, Eq. (16), against the collision number n for the system in contact with a reservoir R_h ($M = 1$) (a) and two reservoirs R_h and R_c ($M = 2$) (b). We set $\omega_h^0 = 2\omega_S$, $\beta_h = \omega_S$, and $g_h^x = g_h^y = 10\omega_S$ for the case of $M = 1$, while $\omega_h^0 = \omega_c^0 = 2\omega_S$, $\beta_h = 0.5\omega_S$, $\beta_c = \omega_S$, and $g_{h(c)}^x = g_{h(c)}^y = 10\omega_S$ for the case of $M = 2$. The system is initially prepared in its ground state and the other parameters are set as $\tau = \omega_S$ and $\tau' = 0.01\omega_S$.

with respect to the thermal state and the other one is due to the coherence generated in the preparation process. Therefore, $D[\rho_R\|\tilde{\rho}_R^{\text{th}}]$ can be decomposed into two components being of the forms [72–75]

$$D\big[\rho_R\big\|\tilde{\rho}_R^{\text{th}}\big] = D\big[\Delta[\rho_R]\big\|\tilde{\rho}_R^{\text{th}}\big] + C(\rho_R), \tag{13}$$

where $D[\Delta[\rho_R]\|\tilde{\rho}_R^{\text{th}}]$ denotes the imbalance of population between ρ_R and $\tilde{\rho}_R^{\text{th}}$ with $\Delta[\varrho]$ meaning a dephasing map for ϱ by which only diagonal matrix elements are left, while $C(\rho_R)$ is the relative entropy of coherence of ρ_R generated in the driving process defined as

$$C(\rho_R) = D[\rho_R\|\Delta[\rho_R]] = S(\Delta[\rho_R]) - S(\rho_R). \tag{14}$$

D. Landauer-like bound

Having obtained explicit forms of entropy production, we proceed to explore the possible constraint on the dissipated heat of the system in NERs by information-theoretic entropy in the sense of the Landauer principle. To provide a benchmark for NERs, we temporarily return to the scenario without the driving process, namely, the reservoirs are prepared in thermal states with $\rho_R = \rho_R^{\text{th}} = \tilde{\rho}_R^{\text{th}}$, under which the usual Landauer principle, in forms of both equality and inequality, can be recovered from Eqs. (9) and (12) as [78]

$$\sum_i^M \beta_i \Delta\widetilde{\mathcal{Q}}_i^n = \Delta\widetilde{\mathcal{S}}(\rho_S^n) + \mathcal{I}_{\rho_{SR}^n}(S:R) + \mathcal{D}\big[\rho_R^n\big\|\rho_R\big] \geqslant \Delta\widetilde{\mathcal{S}}(\rho_S^n), \tag{15}$$

where we use the calligraphy fonts to represent the thermodynamic quantities in thermal reservoirs. Turning back to NERs, a Landauer-like principle in the equality form can also be constructed by means of Eqs. (9) and (12) as

$$\sum_i^M \beta_i \Delta\widetilde{Q}_i^n = \Delta\widetilde{S}(\rho_S^n) + I_{\rho_{SR}^n}(S:R) + D\big[\rho_R^n\big\|\tilde{\rho}_R^{\text{th}}\big] - D\big[\rho_R\big\|\tilde{\rho}_R^{\text{th}}\big]. \tag{16}$$

Here, we should point out that although a NER cannot be endowed with a definite temperature (β_i is the temperature of thermal reservoir R_i before driving), we still call equality (16) the Landauer-like principle in the presence of NERs, as an analogy of the conventional one in the presence of thermal reservoirs. We will numerically verify the validity of equality (16) by comparing its left-hand side (LHS) with the right-hand side (RHS) for a concrete model (see Fig. 2). Even though the Landauer-like principle of equality from (16) in NERs is established, the corresponding inequality form

$$\sum_i^M \beta_i \Delta\widetilde{Q}_i^n \geqslant \Delta\widetilde{S}(\rho_S^n) \tag{17}$$

cannot always hold and will be violated under the condition

$$D\big[\rho_R\big\|\tilde{\rho}_R^{\text{th}}\big] > I_{\rho_{SR}^n}(S:R) + D\big[\rho_R^n\big\|\tilde{\rho}_R^{\text{th}}\big]. \tag{18}$$

Recall that ρ_R is the nonequilibrium state of reservoirs driven from initial thermal state ρ_R^{th}, while $\tilde{\rho}_R^{\text{th}}$ is the associated reference thermal state. From the condition (18), we find that the bound (17) is more likely to be violated when the distance of ρ_R from the state $\tilde{\rho}_R^{\text{th}}$ in terms of $D[\rho_R\|\tilde{\rho}_R^{\text{th}}]$ is larger. In other words, whether the condition (18) can be reached depends on the state ρ_R of NERs which in turn is determined by the costs of the driving process in our scheme. Therefore, the Landauer-like bound (17) can still work for NERs that do not arrive at the condition (18), while being violated for ones satisfying it. The implications of this latter case are studied in the following. Here, we should clarify that although the condition (18) can indicate qualitatively the costs of preparation of NERs that may induce the violation of Landauer-like bound (17) via the term $D[\rho_R\|\tilde{\rho}_R^{\text{th}}]$, the concrete costs in terms of thermodynamic quantities, such as work, for creating the NERs is yet to be addressed.

III. IMPLICATIONS OF VIOLATION OF LANDAUER-LIKE BOUND IN NERs

In the following, we address the implications of breakdown of the Landauer-like bound (17) in several anomalous phenomena occurring in the presence of NERs. We show that the violation of Eq. (17) signifies the possibility of increasing the efficiency of QTMs beyond the Carnot limit, extracting work from a single reservoir as well as transferring heat from a cold reservoir to a hot one spontaneously. That is, these unusual results appear when the state of NERs satisfies the condition (18) under which the Landauer-like bound (17) breaks down. As an illustration, we consider that the system S (i.e., the working substance) is coupled to two NERs, R_h and R_c, and both of them are driven from thermal states with inverse temperatures β_h and β_c ($\beta_h < \beta_c$), respectively.

A. Enhancing efficiency of QTMs beyond Carnot limits

We first show that the efficiency of QTMs operated in NERs can be enhanced to exceed the Carnot bound for corresponding thermal reservoirs. It is known that in the presence of thermal reservoirs, i.e., R_h and R_c are in thermal states when interacting with S, the efficiency $\eta_{\rm th}$ of an engine and the coefficient of performance (COP) $\zeta_{\rm th}$ of a refrigerator are bounded by Carnot efficiency $\eta_c = 1 - \beta_h/\beta_c$ and Carnot COP $\zeta_c = \beta_h/(\beta_c - \beta_h)$, respectively, in the sense of $\eta_{\rm th} = \eta_c - \eta_e$ and $\zeta_{\rm th} = \zeta_c - \zeta_e$ with η_e and ζ_e being strictly non-negative. The explicit forms of η_e and ζ_e can be expressed as

$$\eta_e = \frac{\mathcal{I}_{\rho^*_{SR}}(S:R) + \mathcal{D}\big[\rho^*_R\big\|\rho_R\big]}{\beta_c \Delta Q^*_h}, \tag{19a}$$

$$\zeta_e = \frac{\mathcal{I}_{\rho^*_{SR}}(S:R) + \mathcal{D}\big[\rho^*_R\big\|\rho_R\big]}{(\beta_c - \beta_h)\Delta \mathcal{W}^*}, \tag{19b}$$

in which the notation X^* represents quantity X in the steady-state regime and the numerator $\mathcal{I}_{\rho^*_{SR}}(S:R) + \mathcal{D}[\rho^*_R\|\rho_R]$ is just the entropy production in thermal reservoirs [cf. Eq. (15)] implying that $\eta_{\rm th} < \eta_c$ and $\zeta_{\rm th} < \zeta_c$ are guaranteed by the second law of thermodynamics. As to be shown, however, in the presence of NERs, the efficiency (COP) of an engine (a refrigerator) can be enhanced to exceed the Carnot efficiency η_c (COP ζ_c) as long as the state of NERs fulfills the condition (18) under which the Landauer-like bound (17) is violated.

The efficiency η of an engine and the COP ζ of a refrigerator are defined as

$$\eta = \frac{|\Delta W^*|}{\Delta Q^*_h}, \tag{20a}$$

$$\zeta = \frac{\Delta Q^*_c}{\Delta W^*}. \tag{20b}$$

By combining the relation $\Delta Q^*_h + \Delta Q^*_c + \Delta W^* = 0$ and our Landauer-like principle (16), the efficiency (20a) and COP (20b) can be further decomposed into two terms (see the Appendix for the derivation)

$$\eta = \eta_c - \eta_{ne}, \tag{21a}$$

$$\zeta = \zeta_c - \zeta_{ne}, \tag{21b}$$

with η_{ne} and ζ_{ne} being of the forms

$$\eta_{ne} = \frac{I_{\rho^*_{SR}}(S:R) + D\big[\rho^*_R\big\|\tilde{\rho}^{\rm th}_R\big] - D\big[\rho_R\big\|\tilde{\rho}^{\rm th}_R\big]}{\beta_c \Delta Q^*_h}, \tag{22a}$$

$$\zeta_{ne} = \frac{I_{\rho^*_{SR}}(S:R) + D\big[\rho^*_R\big\|\tilde{\rho}^{\rm th}_R\big] - D\big[\rho_R\big\|\tilde{\rho}^{\rm th}_R\big]}{(\beta_c - \beta_h)\Delta W^*}. \tag{22b}$$

Obviously, only when $\eta_{ne} \geqslant 0$ ($\zeta_{ne} \geqslant 0$) can the efficiency of an engine (the COP of a refrigerator) be bounded by the Carnot efficiency η_c (Carnot COP ζ_c). However, we find that $\eta_{ne} < 0$ ($\zeta_{ne} < 0$), namely, $\eta > \eta_c$ ($\zeta > \zeta_c$), if $I_{\rho^*_{SR}}(S:R) + D[\rho^*_R\|\tilde{\rho}^{\rm th}_R] - D[\rho_R\|\tilde{\rho}^{\rm th}_R] < 0$, which is just the condition (18) for the breakdown of Landauer-like bound (17). Therefore, the Carnot efficiency (Carnot COP) can be surpassed in the presence of NERs when the Landauer-like bound (17) is violated under the condition (18).

B. Extracting work from a single NER

To show the extraction of work from a single NER, we set $\Delta Q^*_c = 0$, i.e., the system is only in contact with R_h. In this case, the first law $\Delta Q^*_h + \Delta Q^*_c + \Delta W^* = 0$ in the steady-state regime is reduced to $\Delta W^* = -\Delta Q^*_h$, in which ΔW^* is the work performed on the system by an external agent so that $\Delta W^* < 0$ means the extraction of work from R_h. If the reservoir R_h is in the thermal state, the usual Landauer principle (15) can be simplified as

$$\beta_h \Delta \mathcal{W}^* = -\beta_h \Delta Q^*_h = \mathcal{I}_{\rho^*_{SR}}(S:R) + \mathcal{D}[\rho^*_R\|\rho_R] \geqslant 0, \tag{23}$$

which shows that no work can be extracted from a single thermal bath. Nevertheless, when R_h is a NER, our Landauer-like principle (16) is reduced to

$$\begin{aligned}\beta_h \Delta W^* = -\,\beta_h \Delta Q^*_h = I_{\rho^*_{SR}}(S:R)\\ + D\big[\rho^*_R\big\|\tilde{\rho}^{\rm th}_R\big] - D\big[\rho_R\big\|\tilde{\rho}^{\rm th}_R\big],\end{aligned} \tag{24}$$

which implies that work can be extracted from R_h with $\Delta W^* < 0$ under precisely the condition (18), i.e., $D[\rho_R\|\tilde{\rho}^{\rm th}_R] > I_{\rho^*_{SR}}(S:R) + D[\rho^*_R\|\tilde{\rho}^{\rm th}_R]$.

C. Transferring heat from a cold reservoir to a hot one spontaneously

Here, we show that breakdown of the Landauer-like bound (17) leads to spontaneous heat flow from a cold reservoir to a hot one. To exclude the involvement of work, we assume $\Delta W^* = 0$ so that $\Delta Q^*_h = -\Delta Q^*_c$. If both reservoirs R_h and R_c are in thermal states with $\beta_c > \beta_h$, one obtains from the usual Landauer principle (15) the expression

$$(\beta_c - \beta_h)\Delta Q^*_c = -\mathcal{I}_{\rho^*_{SR}}(S:R) - \mathcal{D}[\rho^*_R\|\rho_R] \leqslant 0, \tag{25}$$

which indicates that the heat is transferred in the normal manner with $\Delta Q^*_c \leqslant 0$. However, when the thermal reservoirs are driven to NERs, our Landauer-like principle (16) brings about

$$(\beta_c - \beta_h)\Delta Q^*_c = -I_{\rho^*_{SR}}(S:R) - D\big[\rho^*_R\big\|\tilde{\rho}^{\rm th}_R\big] + D\big[\rho_R\big\|\tilde{\rho}^{\rm th}_R\big], \tag{26}$$

which implies that the spontaneous heat flow from R_c to R_h with $\Delta Q^*_c > 0$ is possible under the condition (18), i.e., $D[\rho_R\|\tilde{\rho}^{\rm th}_R] > I_{\rho^*_{SR}}(S:R) + D[\rho^*_R\|\tilde{\rho}^{\rm th}_R]$.

So far, we have shown the implications of breakdown of the Landauer-like bound (17) for three unusual phenomena that arise in the presence of NERs. The underlying reason for these phenomena can be attributed to the fact that the NERs are taken as free resources. When energetic costs for the preparations of NERs are reasonably taken into account, these unconventional results could be recovered [7]. On the other hand, if the preparation costs are acceptable, the NERs can be regarded as useful resources to power thermodynamic tasks that are prohibitive by means of thermal reservoirs. Our findings provide guidelines for the preparation of effective NERs that can achieve certain tasks in practical applications.

IV. ILLUSTRATION VIA A CONCRETE MODEL

In this section, we illustrate the obtained results through a concrete physical model in which both the system and the reservoirs are modeled as qubits (see Fig. 1). To implement the driving process, we set a time-dependent Hamiltonian to a generic ancilla in the reservoir R_i being of the form [79,80]

$$\hat{H}_{R_i}(t) = \frac{\omega_i(t)}{2}\left[\cos\left(\frac{\pi t}{2\tau}\right)\sigma_i^x + \sin\left(\frac{\pi t}{2\tau}\right)\sigma_i^z\right], \qquad (27)$$

where the frequency of each ancilla varies linearly as $\omega_i(t) = \omega_i^0(1 - t/\tau) + (\omega_i^\tau t)/\tau$ and $\{\sigma_i^x, \sigma_i^y, \sigma_i^z\}$ are Pauli operators. At initial time $t = 0$, the ancillas in R_i are prepared in identical thermal states $\rho_{R_i}^{\text{th}} = e^{-\beta_i \hat{H}_{R_i}^0}/Z_i^0$ with $\hat{H}_{R_i}^0 \equiv \hat{H}_{R_i}(0) = (\omega_i^0\sigma_i^x)/2$ at inverse temperature β_i. By exerting a driving protocol for a duration τ, the initial thermal state $\rho_{R_i}^{\text{th}}$ of R_i is transformed to $\rho_{R_i} = \hat{U}_i(\tau)\rho_{R_i}^{\text{th}}\hat{U}_i^\dagger(\tau)$ with the time-evolution operator $\hat{U}_i(\tau) = \mathcal{T}\exp(-\frac{i}{\hbar}\int_0^\tau \hat{H}_{R_i}(t)dt)$ generated by the Hamiltonian (27). The reference thermal state associated with ρ_{R_i} reads $\tilde{\rho}_{R_i}^{\text{th}} = e^{-\beta_i \hat{H}_{R_i}}/\tilde{Z}_i$ with $\hat{H}_{R_i} \equiv \hat{H}_{R_i}(\tau) = (\omega_i^\tau\sigma_i^z)/2$. By varying the parameter ω_i^τ in the Hamiltonian $\hat{H}_{R_i}(t)$ for a fixed τ, we can obtain distinct NERs, which allows us to show that only when the state of NERs fulfills condition (18) can those unconventional phenomena appear.

The system S is governed by $\hat{H}_S = (\omega_S\sigma_S^z)/2$ with the frequency ω_S and the interaction Hamiltonian of system reservoirs in the collision process is chosen as

$$\hat{H}_{SR} = \sum_{i=1}^{M}\hat{H}_{SR_i} = \sum_{i=1}^{M}\left(g_i^x\sigma_S^x\otimes\sigma_i^x + g_i^y\sigma_S^y\otimes\sigma_i^y\right), \qquad (28)$$

where g_i^x and g_i^y stand for coupling strengths between S and R_i. Only when both $g_i^x = g_i^y$ and $\omega_S = \omega_i^\tau$ for $i = 1, 2, \ldots, M$ are satisfied can the strict energy conservation of the system-reservoirs interaction be ensured, otherwise the work is required to sustain system-reservoirs collisions.

With this physical model, we first numerically verify the validity of equality (16), i.e., the Landauer-like principle in NERs. We plot the LHS and RHS of (16) in the dynamical evolution for the system interacting with a reservoir R_h ($M = 1$) and two reservoirs R_h and R_c ($M = 2$) in Figs. 2(a) and 2(b), respectively. One can see that the LHS exactly coincides with the RHS for all the cases confirming the validity of equality (16).

A. Performances of QTMs

In this section, we illustrate the performances of QTMs in NERs by considering a configuration in which the system S is coupled to two reservoirs R_h and R_c and only one of them is driven out of equilibrium. Here, for simplicity we assume that if the initial thermal reservoir does not undergo a driving

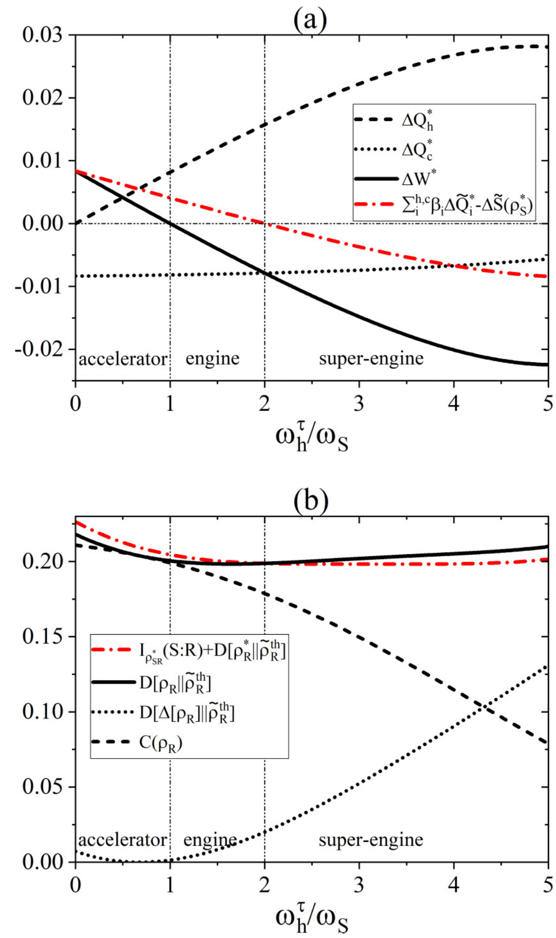

FIG. 3. Three operational regimes of QTMs in the steady state of the system for the hot reservoir R_h being driven out of equilibrium. (a) Thermodynamic quantities ΔQ_h^*, ΔQ_c^*, ΔW^*, and Landauer-like bound (17) in terms of the difference $\sum_i^{h,c}\beta_i\Delta\widetilde{Q}_i^* - \Delta\widetilde{S}(\rho_S^*)$ as a function of ω_h^τ/ω_S. (b) The terms $D[\rho_R\|\tilde{\rho}_R^{\text{th}}]$ and $I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\text{th}}]$ of the LHS and RHS of the condition (18) as well as two components of $D[\rho_R\|\tilde{\rho}_R^{\text{th}}]$, namely, nonequilibrium population $D[\Delta[\rho_R]\|\tilde{\rho}_R^{\text{th}}]$ and relative entropy of coherence $C(\rho_R)$, against ω_h^τ/ω_S. The other parameters are set as $\omega_h^0 = 3\omega_S$, $\omega_c = \omega_S$, $\tau = \omega_S$, $\tau' = 0.01\omega_S$, $g_{h(c)}^x = g_{h(c)}^y = 10\omega_S$, $\beta_h = 0.5\omega_S$, and $\beta_c = \omega_S$.

process, its Hamiltonian is given as $\hat{H}_{R_{h(c)}} = \omega_{h(c)}\sigma_{h(c)}^z/2$, otherwise it is governed by the time-dependent Hamiltonian in Eq. (27).

First, we consider that the hot reservoir R_h, being prepared initially in thermal state $\rho_{R_h}^{\text{th}} = e^{-\beta_h\omega_h^0\sigma_h^x/2}/Z_h^0$ with inverse temperature β_h, undergoes a driving process, while the cold one R_c remains in its thermal state $\rho_{R_c}^{\text{th}} = e^{-\beta_c\omega_c\sigma_c^z/2}/Z_c$ with $\beta_c = 2\beta_h$ before colliding with the system. We show in Fig. 3(a) the steady-state heat currents ΔQ_h^*, ΔQ_c^*, and the steady-state work ΔW^* with respect to ω_h^τ/ω_S. It is found that the machine can operate as an accelerator (with $\Delta W^* > 0$, $\Delta Q_h^* > 0$, and $\Delta Q_c^* < 0$) or as an engine (with $\Delta W^* < 0$, $\Delta Q_h^* > 0$, and $\Delta Q_c^* < 0$) depending on intervals of ω_h^τ/ω_S. Remarkably, when $\omega_h^\tau > 2\omega_S$, there appears an engine with efficiency being larger than the Carnot value $\eta_c = 1 - \beta_h/\beta_c = 0.5$, which is called here superengine to distinguish it from the usual engine. Therefore, we can identify three operating regimes of the machine, i.e., accelerator, usual engine, and superengine, as labeled in Fig. 3. In Fig. 3(a), we also examine the Landauer-like bound (17) by plotting the difference $\sum_i^{h,c} \beta_i \Delta\widetilde{Q}_i^* - \Delta\widetilde{S}(\rho_S^*)$, which is positive, i.e., the bound still holds, in the regimes of accelerator and engine, whereas it becomes negative, i.e., the bound breaks down, in the domain of superengine (i.e., $\omega_h^\tau > 2\omega_S$). Therefore, the appearance of superengine is related to the breakdown of the Landauer-like bound (17). The violating condition (18) is manifested in Fig. 3(b), in which one can see that the value of $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ becomes larger than that of $I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$ once $\omega_h^\tau > 2\omega_S$ is consistent with the interval of occurrence of super-engine shown in Fig. 3(a). In Fig. 3(b), we also display two contributions of $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ [see Eq. (13)], i.e., the relative entropy of coherence $C(\rho_R)$ and the nonequilibrium population $D[\Delta[\rho_R]\|\tilde{\rho}_R^{\rm th}]$ of ρ_R. As can be visualized, the coherence of ρ_R in terms of $C(\rho_R)$ generated in the driving process makes a major contribution to $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ for the relatively small values of ω_h^τ/ω_S, while the nonequilibrium population dominates when the values of ω_h^τ/ω_S become large.

Next, we assume that the cold reservoir R_c is driven out of equilibrium from initial thermal state $\rho_{R_c}^{\rm th} = e^{-\beta_c\omega_c^0\sigma_c^x/2}/Z_c^0$ with inverse temperature β_c, while the hot one R_h remains in thermal state $\rho_{R_h}^{\rm th} = e^{-\beta_h\omega_h\sigma_h^z/2}/Z_h$ with $\beta_c = 2\beta_h$ before colliding with the system. In this case, as shown in Fig. 4(a), the machine can work as a refrigerator ($\Delta W^* > 0$, $\Delta Q_c^* > 0$, and $\Delta Q_h^* < 0$) but with the COP being either smaller than the Carnot limit $\zeta_c = \beta_h/(\beta_c - \beta_h) = 1$ in the interval $0 < \omega_c^\tau/\omega_S < 0.5$ or larger than ζ_c in the interval $0.5 < \omega_c^\tau/\omega_S < 1$. We call the refrigerator in the latter case superrefrigerator to distinguish it from the usual refrigerator. In the region of $\omega_c^\tau/\omega_S > 1$, the machine also exhibits anomalous functions, namely, it can extract work from the system with $\Delta W^* < 0$ and at the same time transfer heat from R_c to R_h with $\Delta Q_c^* > 0$ and $\Delta Q_h^* < 0$, which is thus called hybrid refrigerator. Both the superrefrigerator and the hybrid refrigerator appear in the intervals in which the Landauer-like bound (17) is violated, as shown in Fig. 4(a), implying the connection between these two events. Figure 4(b) displays two sides of the condition (18) confirming that $D[\rho_R\|\tilde{\rho}_R^{\rm th}] > I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$ in the region of $\omega_c^\tau/\omega_S > 0.5$ being consistent with that of occurrences of the superrefrigerator and the hybrid refrigerator. Two contributions of $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$, i.e., the relative entropy of coherence $C(\rho_R)$ and the nonequilibrium population $D[\Delta[\rho_R]\|\tilde{\rho}_R^{\rm th}]$ of ρ_R, are also illustrated in Fig. 4(b).

By means of a simple model consisting of a qubit in contact with two reservoirs, we have illustrated that, by changing the states of the NER, the system as a thermal machine can exhibit not only normal functions but also those that cannot be realized in thermal reservoirs once the Landauer-like bound in Eq. (17) breaks down. Since our results are derived without putting any restriction on the dimension of the system and the number of reservoirs, they can apply to other configurations such as a multilevel system in contact with multiple reservoirs. It can be expected that more interesting operating regimes with greatly enhanced performances would occur for those complex scenarios.

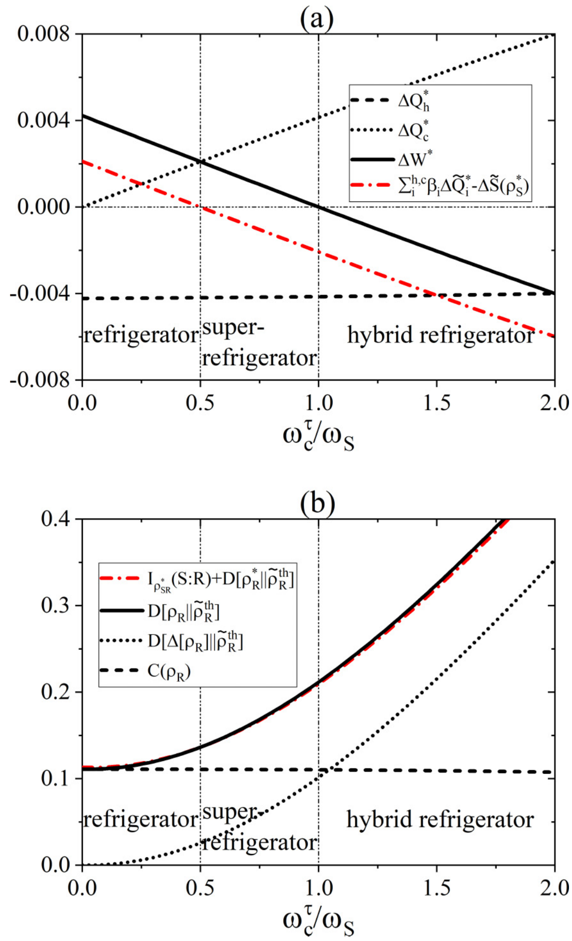

FIG. 4. The same as Fig. 3 but for the case of driving the cold reservoir R_c out of equilibrium so that all the plotted quantities are against ω_c^τ/ω_S. The parameters are set here as $\omega_c^0 = \omega_S$, $\omega_h = \omega_S$, $\tau = \omega_S$, $\tau' = 0.01\omega_S$, $g_{h(c)}^x = g_{h(c)}^y = 10\omega_S$, $\beta_h = 0.5\omega_S$, and $\beta_c = \omega_S$.

B. Work extraction from a single NER and spontaneous heat transfer from a cold reservoir to a hot one

In this section, we first show the extracting work from a single NER by assuming that the system S is coupled only to R_h. From Fig. 5(a), we can see that the work can be extracted from the single NER R_h characterized by $\Delta W^* < 0$ in certain intervals of ω_h^τ/ω_S. In Fig. 5(b), we illustrate the LHS and the RHS of the condition (18), i.e., $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ and $I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$, against ω_h^τ/ω_S. A comparison between Figs. 5(a) and 5(b) clearly indicates that the work extraction from a single reservoir is possible when the condition (18) is reached, namely, $D[\rho_R\|\tilde{\rho}_R^{\rm th}] > I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$.

Next, we turn to the unconventional phenomenon of spontaneous heat flow from the cold reservoir R_c to the hot one R_h. We assume that both R_c and R_h are transformed to NERs from initially thermal ones. In order to ensure that no work is involved in the process of heat transfer, we set strict energy conservation for system-reservoirs interactions in terms of $\omega_S = \omega_h^\tau = \omega_c^\tau$ and meanwhile $g_{h(c)}^x = g_{h(c)}^y$. We display in Fig. 6(a) steady-state heat currents ΔQ_c^* and ΔQ_h^* and in Fig. 6(b) the values of $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ and $I_{\rho_{SR}^*}(S:R) + D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$

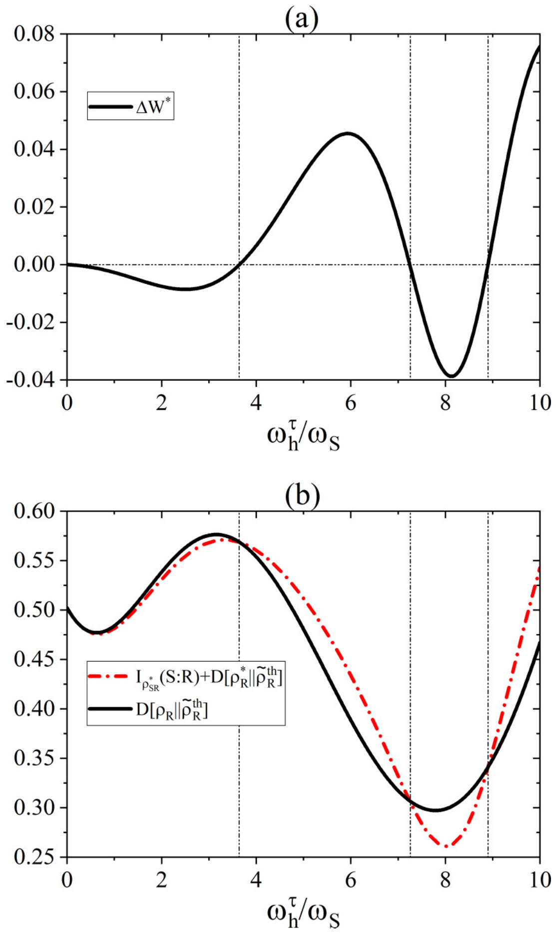

FIG. 5. The work ΔW^* (a) and the LHS and the RHS of the condition (18), i.e., $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ and $I_{\rho_{SR}^*}(S:R)+D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$ (b), as a function of ω_h^τ/ω_S for the system being coupled to a single NER R_h. The remaining parameters are $\omega_h^0=3\omega_S$, $\tau=\omega_S$, $\tau'=0.01\omega_S$, $g_h^x=10\omega_S$, $g_h^y=5\omega_S$, and $\beta_h=\omega_S$.

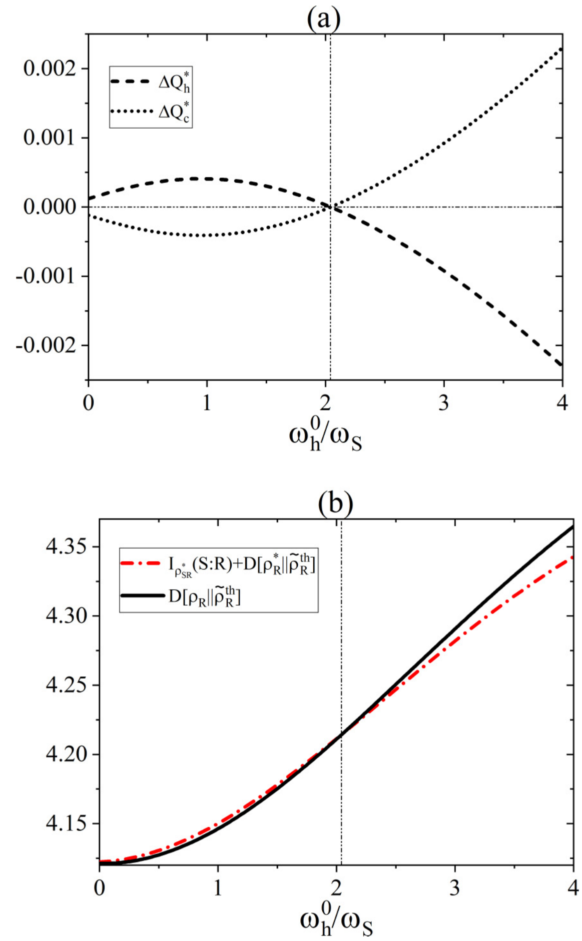

FIG. 6. The steady-state currents ΔQ_c^* and ΔQ_h^* (a) and the LHS and the RHS of the condition (18), i.e., $D[\rho_R\|\tilde{\rho}_R^{\rm th}]$ and $I_{\rho_{SR}^*}(S:R)+D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$ (b), as a function of ω_h^0/ω_S for the system being coupled to two NERs R_h and R_c. The remaining parameters are $\omega_h^\tau=\omega_c^\tau=\omega_S$, $\omega_c^0=2\omega_S$, $\tau=\omega_S$, $\tau'=0.01\omega_S$, $g_{h(c)}^x=g_{h(c)}^y=10\omega_S$, $\beta_h=0.5\omega_S$, and $\beta_c=10\omega_S$.

with respect to ω_h^0/ω_S, respectively. It turns out that spontaneous heat flow from R_c to R_h with $\Delta Q_c^*>0$ occurs under the condition (18), i.e., $D[\rho_R\|\tilde{\rho}_R^{\rm th}]>I_{\rho_{SR}^*}(S:R)+D[\rho_R^*\|\tilde{\rho}_R^{\rm th}]$.

V. CONCLUSION

In conclusion, we have studied quantum thermodynamics in the presence of NERs by considering a two-stage scheme, in which the NERs are firstly prepared from initial thermal reservoirs through a unitary driving process and then proceed to interact with the system of interest. On the analogy of conventional Landauer principle in thermal baths [see Eq. (15)] [78], we establish a similar equality (16) for NERs connecting the dissipated heat with information theory entropy. From the equality (16), it is found that a Landauer-like bound, Eq. (17), cannot always hold in NERs. The condition (18) for its violation is derived, which proves to be closely related to the states of NERs generated in the prior preparation process. We then show that the breakdown of Landauer-like bound (17) implies several anomalous phenomena occurring in NERs, namely, the excess of Carnot efficiency for QTMs, the extraction of work from a single reservoir, and the spontaneous heat flow from a cold reservoir to a hot one. By considering a concrete physical model, we have illustrated and confirmed the obtained results. Our work sheds some light on the reason and condition for several unusual phenomena occurring in NERs and is helpful for one to prepare and exploit effective NERs to realize thermodynamic tasks.

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APPENDIX: DERIVATIONS OF EQS. (21a)–(22b)

In the steady-state regime, the first law and the Landauer-like principle (16) are reduced, respectively, to

$$\Delta Q_h^*+\Delta Q_c^*+\Delta W^*=0 \tag{A1}$$

and

$$\beta_h \Delta Q_h^* + \beta_c \Delta Q_c^* = -I_{\rho_{SR}^*}(S:R) - D\big[\rho_R^* \big\| \tilde{\rho}_R^{\rm th}\big] + D\big[\rho_R \big\| \tilde{\rho}_R^{\rm th}\big]. \tag{A2}$$

The efficiency of an engine given by (20a) can be written as

$$\begin{aligned}\eta &= \frac{|\Delta W^*|}{\Delta Q_h^*} = \frac{\Delta Q_h^* + \Delta Q_c^*}{\Delta Q_h^*} = 1 + \frac{\beta_c \Delta Q_c^*}{\beta_c \Delta Q_h^*} \\ &= \eta_c + \frac{\beta_h \Delta Q_h^* + \beta_c \Delta Q_c^*}{\beta_c \Delta Q_h^*}\end{aligned} \tag{A3}$$

with $\eta_c = 1 - \frac{\beta_h}{\beta_c}$. By inserting Eq. (A2) into Eq. (A3), we obtain

$$\eta = \eta_c - \frac{I_{\rho_{SR}^*}(S:R) + D\big[\rho_R^* \big\| \tilde{\rho}_R^{\rm th}\big] - D\big[\rho_R \big\| \tilde{\rho}_R^{\rm th}\big]}{\beta_c \Delta Q_h^*}. \tag{A4}$$

In the same way, we can express the COP of a refrigerator given in (20b) as

$$\begin{aligned}\zeta &= \frac{\Delta Q_c^*}{\Delta W^*} = \frac{\beta_h(-\Delta Q_c^* - \Delta Q_h^*) + \beta_c \Delta Q_c^* + \beta_h \Delta Q_h^*}{(\beta_c - \beta_h)\Delta W^*} \\ &= \zeta_c + \frac{\beta_c \Delta Q_c^* + \beta_h \Delta Q_h^*}{(\beta_c - \beta_h)\Delta W^*} \\ &= \zeta_c - \frac{I_{\rho_{SR}^*}(S:R) + D\big[\rho_R^* \big\| \tilde{\rho}_R^{\rm th}\big] - D\big[\rho_R \big\| \tilde{\rho}_R^{\rm th}\big]}{(\beta_c - \beta_h)\Delta W^*},\end{aligned} \tag{A5}$$

with $\zeta_c = \frac{\beta_h}{\beta_c - \beta_h}$.

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