Enhancement of white-noise-induced temporal regularity in a linear-optical system using coherent feedback

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The exploration of the constructive effects of noise is an interesting and important research topic. In this work, we investigate the influence of noise on a linear system using the model of a driven and dissipative linear-optical cavity. Results show that solely white noise can induce quasiregular dynamical oscillations, which would otherwise be in a steady state, owing to dissipation in the absence of noise. The regularity of noise-induced oscillations can be significantly enhanced by a linear coherent feedback loop. This is because the dissipation of the system is suppressed owing to the interference between the cavity field and feedback field. Our findings may provide a way to efficiently control white-noise-induced oscillations in a linear system and lead to further investigation of the interaction between noise and linear systems.

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I. INTRODUCTION

Intensive experimental and theoretical research has been conducted on the constructive effects of noise in various systems [1–3]. Numerous examples of performance improvement due to noise have been reported, such as stochastic resonance [4–9], coherence resonance (CR) [10–14], quasicycles [15–18], noise-enhanced stability [19–24], noise-induced phase synchronization [25,26], and noise-sustained propagation of signals [27,28]. In particular, CR is a paradigmatic example of noise-induced regularity, which describes the resonantlike phenomenon of periodic dynamics induced purely by noise without an external periodic signal. Generally, these noise-induced phenomena occur in nonlinear systems, e.g., an excitable system or a system with limit cycles, and they are mainly caused by the interplay between noise and nonlinearity. Furthermore, noise can play a beneficial role in a linear system with specially engineered noise [29–31]. However, rare work [18,32] has reported noise-induced regularity in linear systems with pure white noise.

Based on the above considerations, in this work, we investigate the influence of Gaussian white noise on a linear-optical system, that is, a driven and dissipative linear-optical cavity. We find that white noise can sustain coherent oscillations to a certain extent, which would otherwise decay to a steady state. In fact, the phenomenon of oscillations induced or sustained by white noise is a natural phenomenon in which noise acts as an activation energy source to drive quasiperiodic oscillations at or near intrinsic frequency even in linear systems, as investigated in a mechanical system [32] and a spiral-sink system [18]. However, the periodicity of white-noise-induced oscillations in a linear system has not been widely observed owing to a low signal-to-noise ratio (SNR). Therefore, it has not attracted much attention. We show that a coherent feedback loop can considerably enhance the regularity of noise-induced oscillations, leading to a strong coherent signal in an output field. The enhancement and deterioration of noise-induced coherent behaviors using feedback control has already been demonstrated [33–35], where delayed feedback based on measurement is used. We employ coherent feedback control to benefit from feedback control and prevent the loss of coherence by avoiding measurements [36,37]. In other words, a part of a noisy output field is coherently sent back to a system without measurements to instantaneously control the system dynamics. The feedback field carries the periodicity of the system and serves as a seeding signal that is input to the cavity, resulting in the amplification of noise-induced temporal regularity. The enhancement of periodicity by feedback control can also be explained from another aspect: the interference between the original cavity field and the feedback greatly suppresses the dissipation of the cavity field, which extends the coherence time of the system and facilitates noise-induced coherent oscillations.

It should be noted that the phenomenon of the noise-induced regularity in our model differs from CR in two aspects. First, in CR, the noise-induced coherent behaviors are best at an optimal noise level, while the SNR of the output signal in our model is a monotonic function of noise intensity, implying that no optimal or resonant noise intensity exists. Second, CR occurs in a nonlinear system such as an excitable system and a critical bifurcation system, which contains deterministic periodic solutions in certain parameter regimes. The function of noise in CR is to drive or excite the system into the regime with periodic dynamics. However, the noise-induced temporal regularity in our model occurs in a linear system without requiring nonlinearity. In the absence of noise, the dissipation would make coherent oscillations decay into a stable fixed point, which lacks a deterministic periodic solution in all parameter regimes. The function of noise is to sustain the intrinsic coherent oscillations with fluctuating oscillation amplitude. These flexible features suggest that our model is suitable for use in experiments and can easily be applied to other linear oscillator systems.

The remainder of this paper is organized as follows. In Sec. II, we give a brief review of the theoretical approach that we used to formulate our model, that is, the *SLH* formalism. In Sec. III, we introduce our model and derive the dynamical equation of our system using the *SLH* formalism. In Sec. IV, we show the phenomena of white-noise-induced coherent oscillations and the enhancement by the coherent feedback control based on numerical simulation and analytical expressions. Finally, we conclude our paper in Sec. V.

II. BRIEF REVIEW OF *SLH* FORMALISM

Before introducing our model, we would like to give a brief introduction to the theoretical approach that we employ to formulate our model, that is, the *SLH* formalism or the *SLH* framework [38–40]. It was developed by Gough and James [38,39] as a theoretical framework to deal with quantum networks consisting of an arbitrary number of localized quantum systems interconnected by propagating bosonic fields. For instance, this formalism can conveniently formulate complex cascaded quantum systems and quantum networks with feedforward and feedback connections. The *SLH* formalism is based on earlier theories of open quantum systems, i.e., the input-output relation of an open quantum system by Collett and Gardiner [41–43], the theory for cascaded quantum systems by Gardiner [44] and Carmichael [45], and the Husson-Parthasarathy quantum stochastic differential Eq. [46].

In the *SLH* formalism, each element G of a quantum network is described by the operator triple $G = (\hat{S}, \hat{L}, \hat{H})$, which characterizes all static and dynamic properties of the subsystem. $\hat{S}$ is a unitary matrix ($\hat{S}^\dagger\hat{S} = \hat{S}\hat{S}^\dagger = I$), called scattering operator, describing the static input-output relation of the fields. $\hat{L}$ is the coupling operator, describing how the field interacts with the internal freedom of the localized system. $\hat{H}$ is the internal Hamiltonian, a Hermitian operator, governing the dynamics of the localized system in the absence of the fields. Based on how the elements in a quantum network are connected, one can obtain the (S, L, H) triple for the whole system by using the standard algebraic rules, i.e., the connection rules for the series product and the concatenation product [38]. Then, one can derive the master equation or equations of motion for system operators from the $(\hat{S}, \hat{L}, \hat{H})$ triple [40].

We start with a basic model for an open quantum system: a quantum system linearly interacting with a propagating bosonic field, whose Hamiltonian is given by ($\hbar = 1$)

$$\hat{H} = \hat{H}_{\mathrm{sys}} + \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{int}}, \tag{1}$$

$$\hat{H}_{\mathrm{B}} = \int_0^\infty \omega\hat{b}^\dagger(\omega)\hat{b}(\omega)d\omega, \tag{2}$$

$$\hat{H}_{\mathrm{int}} = i\int_0^\infty d\omega\kappa(\omega)[\hat{b}(\omega) + \hat{b}^\dagger(\omega)][\hat{c} - \hat{c}^\dagger], \tag{3}$$

where $\hat{H}_{\mathrm{B}}$ is the Hamiltonian for the field and $\hat{b}(\omega)$ is the annihilation operator of the quantized field mode with frequency ω, satisfying the canonical commutation relationship $[b(\omega), b^\dagger(\omega')] = \delta(\omega - \omega')$. $\hat{H}_{\mathrm{sys}}$ is the free Hamiltonian of the system in the absence of the field. $\hat{c}$ is the system operator coupled to the field. If the localized system is a single-mode optical cavity, $\hat{c}$ is the annihilation operator for the cavity mode with decay rate γ; if the localized system is a two-level natural or artificial atom, $\hat{c}$ is the lowing operator for the atomic transition and γ is the relaxation rate of the atom.

Under the rotating-wave approximation and the Markov approximation, the Hamiltonian [Eq. (1)] can be rewritten as

$$\hat{H} = \hat{H}_{\mathrm{sys}} + i[\hat{b}^\dagger(t)\hat{L} - \hat{L}^\dagger\hat{b}(t)], \tag{4}$$

where $\hat{L} = \sqrt{\gamma}\hat{c}$ is the Lindblad operator, with γ resulting from the Makov approximation $\kappa(\omega) \to \sqrt{\frac{\gamma}{2\pi}}$, as well as the coupling operator in the $(\hat{S}, \hat{L}, \hat{H})$ triple. The time-dependent field operator is defined as $\hat{b}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{b}(\omega)e^{-i\omega t}d\omega$, which satisfies $[\hat{b}(t), \hat{b}^\dagger(t')] = \delta(t - t')$.

The evolution of such a quantum system linearly and weakly interacting with a propagating bosonic quantum field described above is governed by the quantum stochastic differential equation for the unitary propagator $\hat{U}$ of the coupled system [46],

$$d\hat{U}(t) = \big[(\hat{S} - I)d\Lambda + \hat{L}d\hat{B}^\dagger(t) - \hat{L}^\dagger\hat{S}d\hat{B}(t) - \big(\tfrac{1}{2}\hat{L}^\dagger\hat{L} + i\hat{H}_{\mathrm{sys}}\big)dt\big]\hat{U}(t), \tag{5}$$

where $\hat{B}(t)$ and $\hat{B}^\dagger(t)$ are, respectively, the time-integrated versions of the annihilation and creation operators of the field, that is, $\hat{B}(t) = \int_0^t \hat{b}(s)ds$, $\hat{B}^\dagger(t) = \int_0^t b^\dagger(s)ds$. $\hat{\Lambda}(t)$ represents a quantum stochastic process of counting the number of quanta in the field that have interacted with the system up to time t, that is, $\hat{\Lambda}(t) = \int_0^t \hat{b}^\dagger(s)\hat{b}(s)ds$. From Eq. (5), one can observe that the evolution of the coupled system is determined by three operators, that is, $\hat{S}$, $\hat{L}$, and $\hat{H}$. $\hat{S}$ and $\hat{L}$ characterize the interface properties of the system to the propagating field, and $\hat{H}$ determines the internal properties of the localized system. This might be the reason that in the *SLH* formalism, they are grouped together as a $(\hat{S}, \hat{L}, \hat{H})$ triple to fully describe the composite system. There are a few conditions for the validity of Eq. (5), including weak and linear coupling between the localized system and the field and the Markov approximation [40].

In a realistic quantum network, there are usually multiple ports involved in one system, interacting with multiple fields. In such situation, Eq. (5) is generalized to a multiport version,

$$d\hat{U}(t) = \left[\sum_{j,k}(\hat{S}_{jk} - \delta_{jk})d\hat{\Lambda}_{jk} + \sum_j \hat{L}_j d\hat{B}_j^\dagger(t) - \sum_{j,k}\hat{L}_j^\dagger\hat{S}_{jk}d\hat{B}_k(t) - \left(\frac{1}{2}\sum_j \hat{L}^\dagger L + i\hat{H}_{\mathrm{sys}}\right)dt\right]\hat{U}(t), \tag{6}$$

where

$$\hat{S} = \begin{pmatrix} \hat{S}_{11} & \dots & \hat{S}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{S}_{n1} & \dots & \hat{S}_{nn} \end{pmatrix}, \quad \hat{L} = \begin{pmatrix} \hat{L}_1 \\ \vdots \\ \hat{L}_n \end{pmatrix}. \tag{7}$$

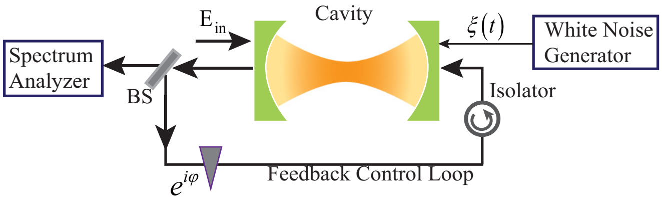

FIG. 1. Schematic of a two-sided driven optical cavity controlled by a coherent feedback loop, which contains a phase shifter and an isolator. A coherent optical field $E_{\rm in}$ and white noise are injected into the cavity through the left-sided and right-sided mirrors, respectively. The output field from the left input-output channel of the cavity is divided by a beam splitter (BS) into two parts: the reflected field as the feedback signal and the transmitted field as the final output signal of the system.

Here, $\hat{S}_{jk}$ describes the scattering from port j to k, and $\hat{L}_j$ is the system operator coupled to the field at port j.

Given the $(\hat{S},\hat{L},\hat{H})$ triple for each element in a quantum network and the feedforward or feedback relation between elements, one can obtain the $(\hat{S},\hat{L},\hat{H})$ triple for the whole system by using the cascading rule, $G_2 \triangleleft G_1 = [\hat{S}_2\hat{S}_1, \hat{L}_2 + \hat{S}_2\hat{L}_1, \hat{H}_1 + \hat{H}_2 + \frac{1}{2i}(\hat{L}_2^\dagger\hat{S}_2\hat{L}_1 - \hat{L}_1^\dagger\hat{S}_2^\dagger\hat{L}_2)]$, and the concatenation product $G_1 \boxplus G_2 = ([\begin{smallmatrix}\hat{S}_1 & 0\\ 0 & \hat{S}_2\end{smallmatrix}], [\begin{smallmatrix}\hat{L}_1\\ \hat{L}_2\end{smallmatrix}], \hat{H}_1 + \hat{H}_2)$. Then, one can derive the equation of motion for system operators or the master equation from the $(\hat{S},\hat{L},\hat{H})$ triple of the whole system [40]. This offers a standard procedure to formulate a quantum network consisting of any number of localized systems interacting with multiple bosonic fields, provided that the following three conditions hold: (i) the coupling between localized systems and fields are linear and Markovian; (ii) the propagation time between localized systems is negligible, i.e., the connection between localized systems is approximately instantaneous feedforward or instantaneous feedback; (iii) the input fields are vacuum. It is worth mentioning that although the validity of the *SLH* formalism requires linear coupling between localized systems and their connecting bosonic fields, it is applicable to both linear and nonlinear quantum networks, for instance, a nonlinear system with instantaneous coherent feedback control [47].

III. MODEL

As shown in Fig. 1, we consider a linear-optical cavity and its coherent feedback loop, in which a phase shifter is the control element and an isolator ensures the unidirectional propagation of the feedback field. This model is similar to that used in our previous work [48], where the focus was the coherent-feedback-induced transparency effect of light. The cavity contains two input-output channels, that is, a left-sided mirror with decay rate κ_1 and a right-sided mirror with decay rate κ_2. A coherent optical signal field with amplitude $E_{\rm in}$ is injected into the cavity through the left-sided mirror. Gaussian white noise is added to the system through the right-sided mirror to better control and examine noise-induced dynamics. A beam splitter with transmission coefficient η is used to divide the output signal of the cavity into two parts. The transmitted part is used as the final signal for detection and spectrum analysis, whereas the reflected part travels through the feedback loop and is sent back to the cavity through the right-sided mirror. A change in η is equivalent to the manipulation of the proportion of the cavity field for feedback or the amplitude of the feedback field.

We use the *SLH* quantum network formalism to describe our model for conveniently formulating the interaction between the system and feedback control loop. The $(\hat{S},\hat{L},\hat{H})$ triples used in our model are given by

$$G_{\rm in} = (1, E_{\rm in}, 0), \tag{8}$$

$$G_{a1} = (\hat{1}, \sqrt{\kappa_1}\hat{a}, \Delta_a\hat{a}^\dagger\hat{a}), \tag{9}$$

$$G_{a2} = (\hat{1}, \sqrt{\kappa_2}\hat{a}, 0), \tag{10}$$

$$G_{\rm BS} = \left[\begin{pmatrix}\sqrt{1-\eta^2} & -\eta\\ \eta & \sqrt{1-\eta^2}\end{pmatrix}, 0, 0\right], \tag{11}$$

$$G_\varphi = (e^{i\varphi}, 0, 0), \tag{12}$$

$$\mathbb{I}_1 = (1, 0, 0), \tag{13}$$

where $\hat{a}$ $(\hat{a}^\dagger)$ represents the annihilation (creation) operators of the cavity mode and $\Delta_a = \omega_c - \omega_{\rm in}$ is the detuning of the cavity resonance from the input field. Similar to the notations used in [48], $G_{\rm in}$ represents the coherent light field applied to the system, G_{a1} (G_{a2}) represents the left-sided (right-sided) mirror of the cavity, $G_{\rm BS}$ represents the beam splitter, and $\mathbb{I}_1$ is a padding component that describes a simple pass-through from an input to an output. Then, by using the cascading rule and the concatenation product introduced in the last section, we derive the $(\hat{S},\hat{L},\hat{H})$ triple for the entire system, G:

$$\begin{aligned}G &= (G_{a2} \boxplus \mathbb{I}_1)\triangleleft(G_\varphi \boxplus \mathbb{I}_1)\triangleleft G_{\rm BS}\triangleleft(G_{a1} \boxplus \mathbb{I}_1)\triangleleft(G_{\rm in} \boxplus \mathbb{I}_1)\\ &= (\hat{S},\hat{L},\hat{H}),\end{aligned} \tag{14}$$

where

$$\hat{S} = \begin{pmatrix}e^{i\varphi}\sqrt{1-\eta^2} & -\eta e^{i\varphi}\\ \eta & \sqrt{1-\eta^2}\end{pmatrix}, \tag{15}$$

$$\hat{L} = \begin{pmatrix}\sqrt{\kappa_2}\hat{a} + e^{i\varphi}\sqrt{1-\eta^2}(\sqrt{\kappa_1}\hat{a} + E_{\rm in})\\ \eta(\sqrt{\kappa_1}\hat{a} + E_{\rm in})\end{pmatrix}, \tag{16}$$

$$\begin{aligned}\hat{H} &= \Delta_a\hat{a}^\dagger\hat{a} + \frac{\sqrt{\kappa_1}E_{\rm in}}{2i}(\hat{a}^\dagger - \hat{a}) + \sqrt{\kappa_1\kappa_2}\sqrt{1-\eta^2}\sin(\varphi)\hat{a}^\dagger\hat{a}\\ &\quad + \frac{1}{2i}\sqrt{1-\eta^2}\sqrt{\kappa_2}E_{\rm in}(\hat{a}^\dagger e^{i\varphi} - \hat{a}e^{-i\varphi}).\end{aligned} \tag{17}$$

With the $(\hat{S},\hat{L},\hat{H})$ triple for the whole system obtained above, one is able to obtain the quantum Langevin equation for the cavity mode using the procedure in [40],

$$\begin{aligned}\frac{d\hat{a}}{dt} &= -(i\Delta_{\rm eff} + \kappa_{\rm eff})\hat{a} + \eta\sqrt{\kappa_2}e^{i\varphi}\hat{a}_{\rm in,2}\\ &\quad - (\sqrt{\kappa_1} + \sqrt{\kappa_2}e^{i\varphi}\sqrt{1-\eta^2})(E_{\rm in} + \hat{a}_{\rm in,1}),\end{aligned} \tag{18}$$

where $\hat{a}_{\rm in,1}$ and $\hat{a}_{\rm in,2}$ are, respectively, the vacuum inputs at the left-sided cavity mirror and the dark port of the beam splitter, $\kappa_{\rm eff} = (\kappa_1 + \kappa_2)/2 + \cos(\varphi)\sqrt{\kappa_1\kappa_2}\sqrt{1-\eta^2}$ is the effective decay rate of the cavity field under coherent feedback

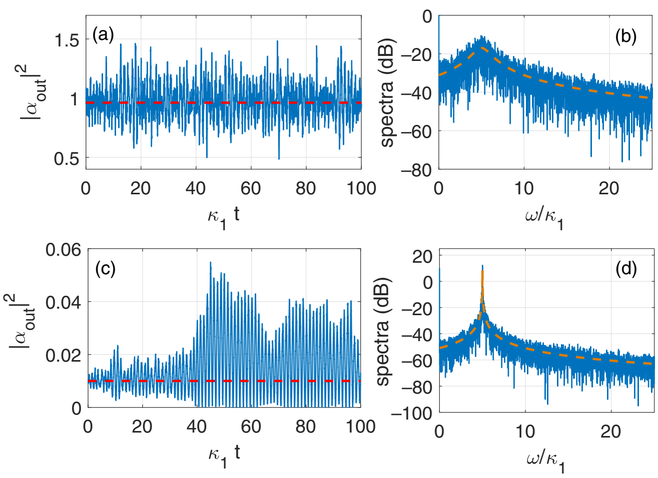

FIG. 2. Noise-induced quasiregular oscillations in the (a) absence and (c) presence of coherent feedback. (b) and (d) show the power spectra obtained from sufficiently long trajectories using the same parameters as those used in (a) and (c), respectively. The red dashed lines in (a) and (c) show the intensity of the output field for the noise-free case. The dashed orange curves in (b) and (d) show the power spectra calculated from the analytical expression [Eq. (20)]. The parameters are as follows: $\kappa_2 = \kappa_1$, $E_{\rm in} = \kappa_1$, $\Delta_a = -5\kappa_1$, $\varphi = \pi$, $D = 10^{-2}$, $\eta = 1$ for (a) and (b), and $\eta = 0.1$ for (c) and (d).

control, and $\Delta_{\rm eff} = \Delta_a + \sin(\varphi)\sqrt{\kappa_1\kappa_2}\sqrt{1-\eta^2}$ is the effective detuning. Under the assumption of a strong cavity field in our system, quantum fluctuations of the cavity field can be approximately neglected and the quantum operator for the cavity mode in the Langevin equation [Eq. (18)] can be approximately replaced by a classical complex amplitude of the cavity field, i.e., $\hat{a} \to \alpha$. In addition, in order to investigate the influence of white noise on the system dynamics, we assume that a white noise with tunable noise intensity is added to the system through the right-sided cavity mirror. Then, we arrive at the classical equation of motion for the mean amplitude of the cavity field,

$$\frac{d\alpha}{dt} = -(i\Delta_{\rm eff} + \kappa_{\rm eff})\alpha - (\sqrt{\kappa_1} + \sqrt{\kappa_2}e^{i\varphi}\sqrt{1-\eta^2})E_{\rm in} - \sqrt{\kappa_2}\xi(t). \quad (19)$$

Here we have phenomenologically added a white-noise term $\xi(t)$, which represents the external white noise injected on the right-sided cavity mirror. It satisfies $\langle\xi(t)\xi(t')\rangle = 2D\delta(t-t')$, where D is the dimensionless noise intensity. This equation [Eq. (19)] is the starting point for our analytical and numerical analyses of noise-induced behaviors in our system.

IV. RESULTS AND DISCUSSION

Figure 2 shows noise-induced quasiperiodic oscillations in the time and frequency domains. Figures 2(a) and 2(c) show typical trajectories of the intensity of the output field, $|\alpha_{\rm out}|^2 = |\eta(\sqrt{\kappa_1}\alpha + E_{\rm in})|^2$, in the absence and presence of coherent feedback, respectively. Figures 2(b) and 2(d) show the power spectra corresponding to Figs. 2(a) and 2(c) obtained using the trajectories with a long evolution time. To show noise-induced coherent dynamics from a steady state, for all results shown in Fig. 2, the initial mean amplitude of the cavity field is considered to be the steady-state value. As shown in Fig. 2(a), it is difficult to observe the regularity in the noisy trajectory from the time evolution without feedback. However, a low and wide peak in the spectrum is visible in Fig. 2(b). The central frequency of the peak in the spectrum is the same as $|\Delta_a|$, which is the deterministic oscillation frequency of the system before decaying to the steady state if the system is not initially at the steady state. This indicates that the coherent dynamics of the system determined by its intrinsic frequency are not fully eliminated via dissipation; instead, the noise along with the dissipation sustains coherent oscillations. When we switch the coherent feedback control on by varying η from 1 to 0.1, the oscillations of $|\alpha_{\rm out}|^2$ exhibit evident periodicity in the time domain, although the amplitude of the oscillations varies continuously, as shown in Fig. 2(c). The periodicity can also be observed in the power spectrum [Fig. 2(d)], where a considerably higher and narrower peak appears at $\omega/\kappa_1 = |\Delta_a|$. This is because the output noisy signal before entering the feedback loop carries the periodicity information. Therefore, the feedback signal serves as a seed, thereby amplifying the periodicity in the original signal.

To confirm the numerical simulation results shown in Fig. 2, we perform the Fourier transformation of Eq. (19) and obtain the analytical expression of the power spectrum for the output field,

$$\begin{aligned} S(\omega) &= \int \langle\alpha_{\rm out}(\omega)\alpha^*_{\rm out}(\omega')\rangle d\omega' \\ &= \frac{2D\eta^2\kappa_1\kappa_2}{(\omega-\omega_p)^2 + W^2} \\ &\quad + \left|1 - \frac{\kappa_1 + e^{i\varphi}\sqrt{\kappa_1\kappa_2(1-\eta^2)}}{i(\omega-\omega_p)+W}\right|^2 \eta^2 E_{\rm in}^2\delta(\omega), \end{aligned} \quad (20)$$

where $\omega_p = -\Delta_{\rm eff}$, $W = \kappa_{\rm eff}$, and $\alpha_{\rm out}(\omega)$ is the Fourier transformation of $\alpha_{\rm out}(t)$. There are two terms in the power spectrum. The first term is the noise spectrum and it is a Lorentzian function, where ω_p is the central frequency of the signal peak and W is the width of the peak. The second term is the spectrum caused by the input coherent field, which is a δ function at $\omega = 0$. As we are interested in the signal around a nontrivial frequency, i.e., the intrinsic frequency $|\Delta_a|$, only the first term provides an effective contribution; hence, the effective power spectrum obeys the Lorentzian profile. The analytical results based on Eq. (20) are shown by the dashed orange curves in Figs. 2(b) and 2(d), and they are in good agreement with the numerical results. We then study the influence of noise strength D on the properties of noise-induced oscillations. Figure 3(a) shows the spectra of noise-induced oscillations at several different noise levels, ranging from $D = 10^{-4}$ to $D = 1$. The height of the peak increases with the noise strength, whereas the width of the peak remains constant for all noise strengths. Following the method used in [3,11,12] to quantitatively measure the regularity and amplitude of noise-induced oscillations, we define the SNR as $\text{SNR} = \omega_p H/W$ and the height of the peak as $H = 2D\eta^2\kappa_1\kappa_2/W^2$.

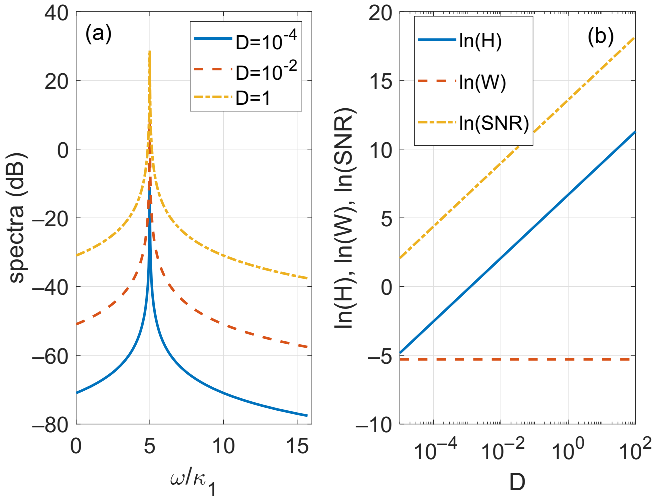

FIG. 3. (a) Spectra of the output field, $\alpha_{\rm out}$, at three different noise intensities $D = 10^{-4}, 10^{-2}, 1$. (b) Peak height (H), peak width (W), and SNR as a function of noise intensity D (in logarithmic units). The parameters are as follows: $\kappa_2 = \kappa_1$, $E_{\rm in} = \kappa_1$, $\Delta_a = -5\kappa_1$, $\varphi = \pi$, and $\eta = 0.1$.

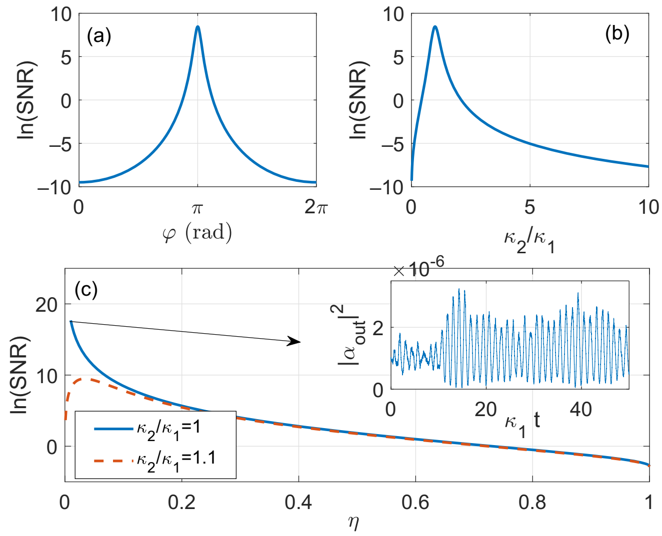

FIG. 4. SNR as function of the (a) phase shift, (b) ratio of the decay rates of two channels, and (c) transmission coefficient of the beam splitter. The parameters are as follows: $\Delta_a = -5\kappa_1$, $E_{\rm in} = \kappa_1$, $D = 10^{-2}$, $\varphi = \pi$ for (b) and (c), $\kappa_2 = \kappa_1$ for (a) and solid curve in (c), $\kappa_2 = 1.1\kappa_1$ for the dashed curve in (c), $\eta = 0.1$ for (a) and (b), and $\eta = 0.001$ for the inset shown in (c).

Then, we have the following expression for the SNR:

$$\mathrm{SNR} = \frac{2D\eta^2\kappa_1\kappa_2\omega_p}{\kappa_{\rm eff}^3}. \tag{21}$$

Figure 3(b) shows the plot of the height of the peak (H), width of the peak (W), and SNR versus D. W remains constant as noise increases. In contrast, the logarithms of H and the SNR increase linearly with noise, that is, H and the SNR increase exponentially with noise. This is a difference between the noise-induced temporal regularity in our model and that in CR, where an optimal noise intensity is required to achieve the best noise-induced constructive effect. This implies that our proposal does not require the precise manipulation of the noise intensity to satisfy the resonance condition; thus, it is more flexible to observe and realize.

In addition to the noise strength, other parameters can be used to control noise-induced temporal regularity. Based on Eq. (21), we plot the SNR of the noise-induced quasiregular signal versus phase shift φ, κ_2, and η in Fig. 4. As shown in Fig. 4(a), the SNR reaches its maximum at $\varphi = \pi$. This can be directly explained using Eq. (21). Width W and the denominator of the SNR are the minimum at $\varphi \to \pi$, whereas the numerator of the SNR does not depend on φ. Therefore, the maximum SNR is obtained at this point. At $\varphi = \pi$, the effective decay rate of the cavity field, $\kappa_{\rm eff}$, becomes the minimum because of the interference between the original cavity field and feedback field. Moreover, it tends to zero for $\eta \to 0$ and $\kappa_2 \to \kappa_1$. The almost zero dissipation causes the system to sustain coherent oscillations for a long time. Similarly, there is a single peak on the SNR $-\ \kappa_2$ curve. The SNR reaches its maximum at $\kappa_2/\kappa_1 = 1$, where the effective decay rate of the system is the lowest and the signal peak is the narrowest. A large value of κ_2 is beneficial for coupling the feedback with the cavity field, whereas a small value of κ_2 is required for maintaining a low dissipation of the system. Therefore, an optimal decay rate κ_2 is required to achieve the best quasiregular oscillations.

The dependence of the SNR on η, or the proportion of the field used for feedback, is presented in Fig. 4(c). For $\kappa_2 = \kappa_1$, the SNR increases as η decreases, even when η is extremely small [the solid curve in Fig. 4(c)]. For $\kappa_2 \neq \kappa_1$, the maximal SNR is achieved at an optimal value of η [the dashed curve in Fig. 4(c)]. In the first case, the numerator and denominator of the SNR tend to become zero under the condition $\varphi = \pi$. In addition, the denominator decreases faster than the numerator, leading to an extremely high SNR when η approaches zero. This is because a smaller value of η indicates a stronger feedback signal and better SNR. It should be noted that the amplitude of the output field of the system, $\alpha_{\rm out}$, is proportional to η. Thus, an extremely small value of η provides an extremely weak output signal (see inset in Fig. 4), which makes it difficult for detection devices to respond. Therefore, a moderate value of η should be selected to ensure a good SNR and detectable signal. In the case of $\kappa_2 \neq \kappa_1$, the denominator of the SNR never becomes zero under the condition $\varphi = \pi$. There is an optimal value of κ_2 for the best SNR because of the compromise between the strong feedback field (small η) and strong output field (large η).

Finally, let us compare the noise-induced oscillations in our work with two interesting noise-induced phenomena, that is, quasicycles and noise-induced stability. (i) A quasicycle is a phenomenon of oscillatory activity which only exists with the presence of noise and it can occur in both linear [18] and nonlinear [15] systems with spiral-sink dynamics, i.e., the Jacobian matrix has a complex conjugate pair of eigenvalues. Different from quasicycles, our system is linear and it has one stable fixed point without a spiral feature in its deterministic dynamics. The common features of our proposal and quasicycles are the reliance of quasiperiodic oscillations on the existence of noise and the absence of periodic orbits in the deterministic dynamics. (ii) The phenomenon of the noise-enhanced stability takes place in various systems ranging from open quantum systems [20] to medical systems [23] and to financial systems [21], in which the lifetime of a system variable at a metastable state can be increased via manipulation of the coupling strength with reservoir or noise intensity. This phenomenon results from the interplay between system nonlinearities and thermal noises, but it has similarity with our results in the sense that noise can help to maintain the system property, although in our work what the noise sustains are quasiperiodic oscillations which would decay to a stable fixed point in a finite time in the absence of noise.

V. CONCLUSION

We have investigated white-noise-induced temporal regularity in a linear optical system with a linear feedback control loop. Weak regularity is considerably enhanced by coherent feedback because the interference between the original cavity field and feedback field significantly reduces the effective decay of the system. The enhancement of noise-induced regularity is sensitive to the phase shift in the feedback loop. Furthermore, the SNR of the system output is a monotonic function of noise intensity, which is different from the phenomenon of CR. Our model does not rely on system nonlinearity and the results are valid for a large range of parameters. Therefore, it is feasible for experimental realization and could be extended to other linear systems, i.e., a mechanical oscillator system with coherent feedback control.

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