

Two-level quantum walkers on directed graphs. II. Application to quantum random access memory

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(Received 4 May 2022; accepted 20 January 2023; published 13 February 2023)

This is the second paper in a series of two. Using a multiparticle continuous-time quantum walk with two internal states, which was formulated in the first paper [R. Asaka *et al.*, preceding paper, *Phys. Rev. A* **107**, 022415 (2023)], we physically implement a quantum random access memory (QRAM). Data with address information are dual-rail encoded into quantum walkers. The walkers pass through perfect binary trees to access the designated memory cells and copy the data stored in the cells. A roundabout gate allocated at each node serves as a router to move the walker from the parent node to one of two child nodes, depending on the internal state of the walker. In this process, the address information is sequentially encoded into the internal states so that the walkers are adequately delivered to the target cells. The present QRAM, which processes 2^n m -qubit data, is implemented in a quantum circuit of depth $O(n \log(n + m))$ and requires $O(n + m)$ qubit resources. This is more efficient than the conventional bucket-brigade QRAM that requires $O(n^2 + nm)$ steps and $O(2^n + m)$ qubit resources for processing. Moreover, since the walkers are not entangled with any device on the binary trees, the cost of maintaining coherence can be reduced. Notably, by simply passing quantum walkers through binary trees, data can be automatically extracted in a quantum superposition state. In other words, any time-dependent control is not required.

DOI: [10.1103/PhysRevA.107.022416](https://doi.org/10.1103/PhysRevA.107.022416)

I. INTRODUCTION

This is the second paper in a series of two in which we consider a multiparticle continuous-time quantum walk with two internal states. In the present paper we propose a physical implementation of a quantum random access memory, using some devices developed in the preceding paper [1], in which an architecture of universal quantum computation using the quantum walk was provided.

A number of quantum algorithms exploiting quantum-mechanical effects have been proposed to achieve significant speedups over their classical analogs [2]. Algorithms for quantum phase estimation [3,4], quantum amplitude amplification [5–8], and quantum Hamiltonian simulation [9–18] are the most notable and are used as subroutines in, for example, Shor’s algorithm for factorizing large integers [3] and Grover’s algorithm for searching unsorted databases [7]. However, one should be careful about claiming that quantum algorithms are superior to classical counterparts in some cases. As an example, let us take the search problem of finding a particular item in an unstructured set consisting of N items. Grover’s algorithm incorporates the process of accessing and querying the database as an oracle (a black box that answers yes or no) and completes the search with only $O(\sqrt{N})$ oracle

queries, achieving a quadratic speedup over a classical exhaustive search. In practice, however, the oracle subroutines, i.e., converting data into a quantum superposition state and accessing and reading them, may be a cumbersome overhead that offsets the quantum speedup [19]. Namely, reducing the cost to the oracle is crucial for applications of quantum computation to search problems, Hamiltonian simulations, and machine learning for big data sets [20–38].

A quantum random access memory (QRAM) was introduced as a quantum counterpart of a random access memory, promising to efficiently access data and convert them into superposition states [39,40]. Conceptually, a QRAM is a quantum device comprising the following three principal schemes: (i) a routing scheme to access the specified memory cells whose addresses are given by an n -qubit superposition state

$$\sum_a |a\rangle_A = \sum_{\{a_j\}} |a_{n-1} \cdots a_0\rangle_A \in (\mathbb{C}^2)^{\otimes n},$$

$$a \in \mathbb{Z}_{\geq 0}, a_j \in \{0, 1\} \quad (0 \leq j \leq n-1), \quad (1.1)$$

(ii) a querying scheme to read the classical information $x^{(a)} \in \mathbb{Z}_{\geq 0}$ stored in the a th cell,¹ and (iii) an output scheme to retrieve the data in an m -qubit superposition state $\sum_a |x^{(a)}\rangle_D$. Here the subscripts A and D stand for the quantum versions of an address register and a data register, respectively. Explicitly, a QRAM is defined as a function

$$\text{QRAM} : \sum_a |a\rangle_A |0\rangle_D \mapsto \sum_a |a\rangle_A |x^{(a)}\rangle_D. \quad (1.2)$$

¹A QRAM can also process the quantum information where $|x^{(a)}\rangle$ consists of a superposition of states (see Sec. V). For the moment, however, we restrict ourselves to the classical case for convenience

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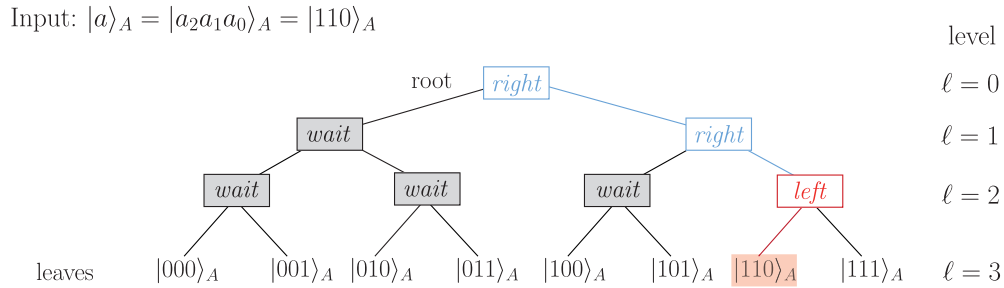


FIG. 1. The GLM bucket-brigade scheme defined on a binary tree with depth $n = 3$. A qutrit is installed at each node to route to the specified memory cells. For instance, to route to the cell at address $|110\rangle_A$, the three qutrits must be activated from *wait* to *left* or *right*.

Giovannetti, Lloyd, and Maccone proposed a remarkable QRAM architecture using the so-called bucket-brigade routing scheme [39,40]. The Giovannetti-Lloyd-Maccone (GLM) architecture is defined on a perfect binary tree with depth n on which $N = 2^n$ data are stored in the memory cells placed on the leaves of the binary tree (see Fig. 1). Each node in the tree is equipped with a qutrit with three energies labeled *wait*, *left*, and *right*, and all the qutrits are initially in the wait state. The qutrit acts as a router: The value $a_{n-1-\ell} \in \{0, 1\}$ ($0 \leq \ell \leq n-1$) in the address register (1.1) is delivered to one of the 2^ℓ nodes at the ℓ th level of the binary tree and if $a_{n-1-\ell} = 0$ ($a_{n-1-\ell} = 1$) activates the qutrit from *wait* to *left* (*right*) to route the subsequent $a_{n-2-\ell}$ to one of the two child nodes. After $O(n^2) = O(\log^2 N)$ steps, a unique route is assigned from the root to the specified memory cell, as schematically depicted in Fig. 1. A quantum bus then arrives at the cell through the assigned route, the data stored in the cell is coherently loaded onto the bus, and the bus loaded with the data returns to the root via the route it came from. Finally, reverting the activated qutrits to *wait*, sequentially from the last level, yields output on the right-hand side of (1.1). For each memory call, the overall computational cost and qubit resources required to process $N = 2^n$ m -qubit data are $O(n^2 + nm)$ and $O(2^n + m)$, respectively.

It is worth noting that the number of qutrits to be activated is only $O(n)$, which drastically reduces the cost of maintaining the quantum coherence compared to the fan-out scheme (most commonly used in a classical random access memory) that activates $O(2^n)$ qutrits. In fact, a high resilience of the bucket-brigade QRAM to generic noise was recently proved in [41]. The GLM QRAM has been improved and is realized efficiently by quantum circuits as in [42–44]. Some experimental implementations have also been proposed in [40,45–48].

More recently, the authors of the present paper have provided a novel QRAM algorithm that works on a perfect binary tree but does not require entanglement with any quantum device on the nodes [49]. In this sense, this algorithm promises to reduce the cost of maintaining quantum coherence compared to the bucket-brigade scheme, but its implementation has remained open until now. The purpose of this paper is to physically implement this QRAM algorithm using a multipartite continuous-time quantum walk with two internal states.

Our QRAM architecture is roughly sketched as follows. First, quantum information is dual-rail encoded into quantum walkers moving on parallel paths; single-qubit data is represented by the presence of a walker on one of the two parallel

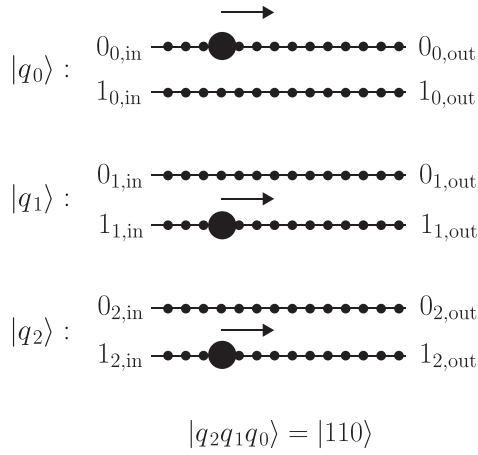
paths. Namely, the arbitrary m -qubit data associated with n -qubit address information is represented by a set of $n + m$ quantum walkers traveling on half of $2(n + m)$ paths. Second, each walker possesses two internal states (e.g., the spin-up and -down states of an electron). Depending on the internal state, the roundabout gate allocated at each node of the binary trees passes the walker to one of the two child nodes. The address information is sequentially encoded into the internal states so that the set of walkers is properly delivered to the designated memory cells. Finally, the data in the cell is copied by simply changing the positions of the walkers in the data register. The set of the walkers carrying the data is retrieved by the reverse operation of the routing scheme.

In the above implementation, the roundabout gate can be actually realized by the scattering of the walker from a directed graph [1]. The encoder, which converts the positional information of the path traveled by the specified walkers into the internal states of the walkers, is implemented by a combination of roundabout gates and single-qubit gates acting on the internal state of the walkers. The main advantages of our architecture are as follows. (i) The processing is fully parallelized without using any ancilla qubit and can access and retrieve the m -qubit data associated with n -qubit address information in $O(n \log(n + m))$ steps. The qubit resources necessary for the processing are $O(n + m)$. (ii) The walkers are not entangled with any device on the binary trees, thus reducing the cost of maintaining the quantum coherence. (iii) It does not require any time-dependent control: The QRAM process is automatically achieved by just passing the walkers through binary trees. (iv) Using the model developed in the preceding paper [1], it is possible to design a unified universal quantum computer that is compatible with the QRAM developed in this paper.

The rest of this paper is outlined as follows. Section II describes the general setup and gives an overview of our QRAM architecture. Some devices developed in the preceding paper [1], which are required in the present paper, are also summarized. A physical implementation of the QRAM is provided in Sec. III. In Sec. IV an alternative QRAM scheme that transforms a trivial state into a superposition of information stored in the specified memory cells,

$$\widetilde{\text{QRAM}} : |0\rangle_A |0\rangle_D \mapsto \sum_a |a\rangle_A |x^{(a)}\rangle_D \quad (1.3)$$

[cf. (1.2)], is proposed. Section V is devoted to a summary and discussion, where we briefly explain how to extract quantum


 FIG. 2. Dual-rail encoding of the state $|q_2q_1q_0\rangle = |110\rangle$

information (i.e., information in quantum superposition) in the designated cells instead of classical information.

II. PRELIMINARIES

This section gives an overview of the QRAM architecture, which is a physical realization of the algorithm developed in [49]. The architecture uses some quantum gates implemented by multiparticle continuous-time quantum walks [1].

A. Setup and layout of the QRAM

Our QRAM architecture employs a dual-rail encoding in which data and address information are represented as the positions of the paths on which the quantum walkers moving; a single-qubit state $|q_j\rangle \in \mathbb{C}^2$ is expressed by the presence of a quantum walker in one of two parallel paths

$$|q_j\rangle = \delta_{q_j,0}|2j\rangle_p + \delta_{q_j,1}|2j+1\rangle_p \quad (0 \leq j \leq n+m-1), \quad (2.1)$$

where $|2j\rangle_p \in \mathbb{C}^2$ ($|2j+1\rangle_p \in \mathbb{C}^2$) indicates that a walker is moving on the $2j$ th [($2j+1$)th] path. Correspondingly, an $(n+m)$ -qubit state is given by

$$\begin{aligned} |q_{n+m-1} \cdots q_0\rangle &= |q_{n+m-1}\rangle \otimes \cdots \otimes |q_0\rangle \\ &= \bigotimes_{j=0}^{n+m-1} (\delta_{q_j,0}|2j\rangle_p + \delta_{q_j,1}|2j+1\rangle_p) \in (\mathbb{C}^2)^{\otimes(n+m)}. \end{aligned} \quad (2.2)$$

Figure 2 shows an example of a dual-rail encoded state. For our purposes, we assign the first n qubits and the remaining m

qubits to the address and data registers, respectively:

$$\begin{aligned} |a\rangle_A &= |a_{n-1} \cdots a_0\rangle_A = |a_{n-1}\rangle_{A_{n-1}} \otimes \cdots \otimes |a_0\rangle_{A_0} = |q_{n-1} \cdots q_0\rangle \in (\mathbb{C}^2)^{\otimes n}, \\ |x^{(a)}\rangle_D &= |x_{m-1}^{(a)} \cdots x_0^{(a)}\rangle_D = |x_{m-1}^{(a)}\rangle_{D_{m-1}} \otimes \cdots \otimes |x_0^{(a)}\rangle_{D_0} = |q_{n+m-1} \cdots q_n\rangle \in (\mathbb{C}^2)^{\otimes m}. \end{aligned} \quad (2.3)$$

The $n+m$ quantum walkers (in superposition) access the specified memory cell(s) through half of the $2(n+m)$ parallel paths and retrieve the data stored in the cell(s). To this end, we prepare $2(n+m)$ parallel sheets on each of which two perfect binary trees of depth n are arranged so that the 2^n memory cells are sandwiched between the two sets of 2^n leaves, as schematically shown in Fig. 3. (See also Fig. 4 for a detailed description of a perfect binary tree.) A set of $n+m$ walkers (possibly in superposition) at input (output) terminals of the first (second) binary trees corresponds to the input (output) state. Let (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n$) be the w th node from the left at the ℓ th level of the perfect binary tree and let $|w, \ell\rangle_B \in \mathbb{C}^{2^{n+1}-1}$ denote that a set of $n+m$ walkers (called a bus) is moving toward the node (w, ℓ) from its parent node [the parent node for the root node $(0,0)$ denotes the input and output terminal (see Fig. 4)]. Namely, the bus that passes between these two nodes, carrying m -qubit data $|x^{(a)}\rangle_D$ associated with an n -qubit address $|a\rangle_A$, is represented as $|a\rangle_A |w, \ell\rangle_B |x^{(a)}\rangle_D$.

All the $n+m$ quantum walkers possess two internal states (e.g., the spin-up and -down states of an electron). Let $|c\rangle_{C_j} \in \mathbb{C}^2$ ($c \in \{0, 1\}$ and $0 \leq j \leq n+m-1$) be the internal state of the j th walker; we call the walker with $|0\rangle_{C_j}$ and $|1\rangle_{C_j}$ a red walker and a blue walker, respectively. In principle, we assume that the internal states are initialized to be red $|0\rangle_{C_j}$ ($0 \leq j \leq n+m-1$) before processes. All the walkers at the input and output terminals are colored red [$|0\rangle_C := \bigotimes_{j=0}^{n+m-1} |0\rangle_{C_j} \in (\mathbb{C}^2)^{\otimes(n+m)}$], and just before or just after passing through the nodes, all the walkers are colored red or blue [$|2^{n+m}-1\rangle_C := \bigotimes_{j=0}^{n+m-1} |1\rangle_{C_j} \in (\mathbb{C}^2)^{\otimes(n+m)}$] according to the address information (1.1). The address information is temporarily encoded in the internal states by a unitary gate $\mathcal{E}_{(w,\ell)}$ that intersects perpendicular to the paths between two levels $\ell-1$ and ℓ ($0 \leq \ell \leq n-1$) ($\ell = -1$ denotes the input terminals), as shown in Fig. 3. (See the next section for more details about $\mathcal{E}_{(w,\ell)}$.) A roundabout gate is set up at each node to move a red walker (blue walker) to the left child node (right child node) in the routing scheme and do exactly the opposite in the output scheme. The data stored in the memory cells are loaded to the walkers in the data register, which is simply realized by changing their positions, as described in the subsequent section. The walkers loaded with the data are retrieved in the output scheme, which is accomplished by reversing the routing scheme. That is a layout of our QRAM given by a function

$$\begin{aligned} \text{QRAM} : (\mathbb{C}^2)^{\otimes[2(n+m)+\log_2(2^{n+1}-1)]} &\longrightarrow (\mathbb{C}^2)^{\otimes[2(n+m)+\log_2(2^{n+1}-1)]} \\ \in &\in \\ \sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D &\longmapsto \sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |x^{(a)}\rangle_D, \end{aligned} \quad (2.4)$$

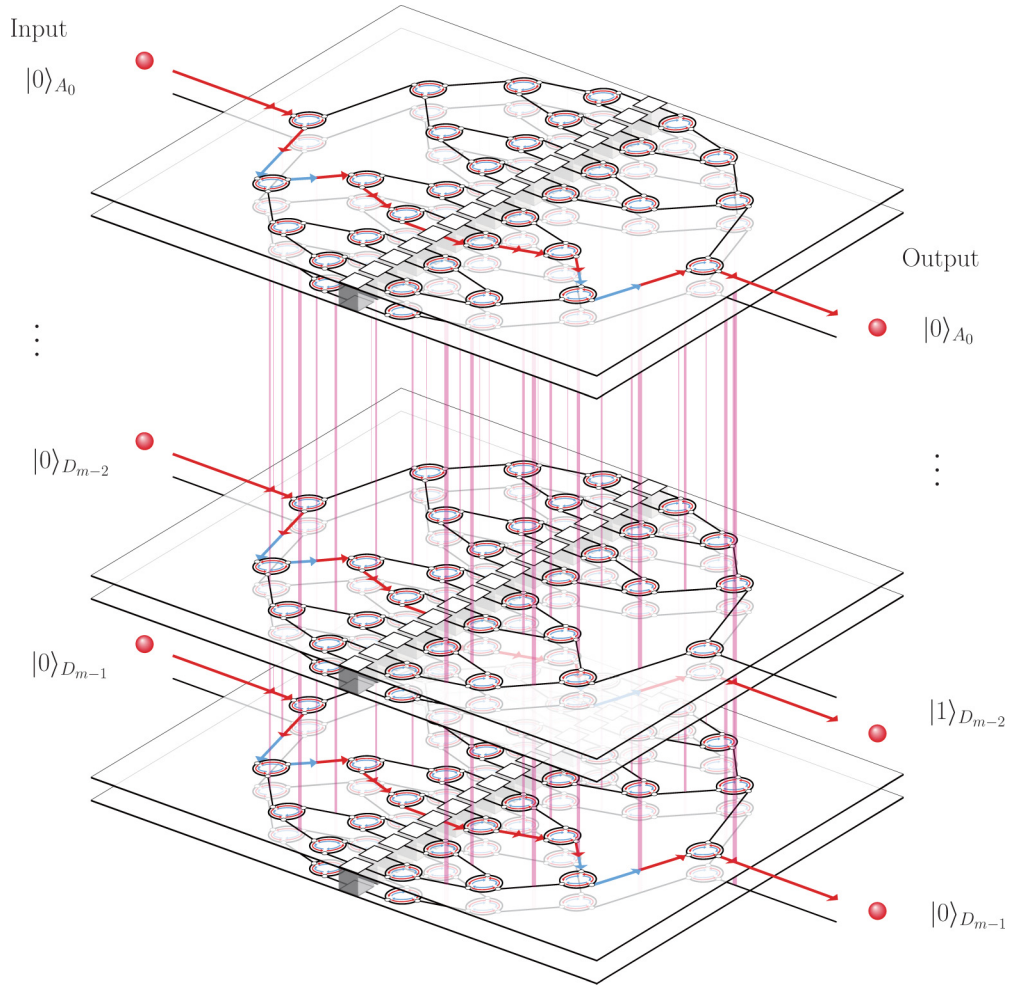


FIG. 3. Overview of the current QRAM architecture. Due to the dual-rail encoding [see (2.1), (2.2), and Fig. 2], the architecture is designed on $2(n + m)$ sheets. Each sheet has two perfect binary trees of depth n (cf. Fig. 4), with their 2×2^n leaves sandwiching the 2^n memory cells. The $n + m$ red quantum walkers at the left back (right front) represent the input (output) state. In the routing scheme, the roundabout gate is set up at each node of the trees so that it passes a red walker (blue walker) to the left (right) child node. The address information $|a_{n-1-\ell}\rangle_{A_{n-1-\ell}}$ is encoded in the internal state of all the $n + m$ walkers as $\otimes_{j=0}^{n+m-1} |a_{n-1-\ell}\rangle_{C_j}$ while they move to the node (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n - 1$) from its parent node [the parent node for the root node $(0,0)$ denotes the input terminal]. This process can be accomplished by the device $\mathcal{E}_{(w,\ell)}$ intersecting perpendicular to the paths between two levels $\ell - 1$ and ℓ ($0 \leq \ell \leq n - 1$) ($\ell = -1$ denotes the input terminals). The data $x^{(a)}$ stored in the memory cell at address a is loaded to the walkers arriving at the cell. By reversing the routing scheme, the walkers loaded with the data (in superposition) are retrieved as the output.

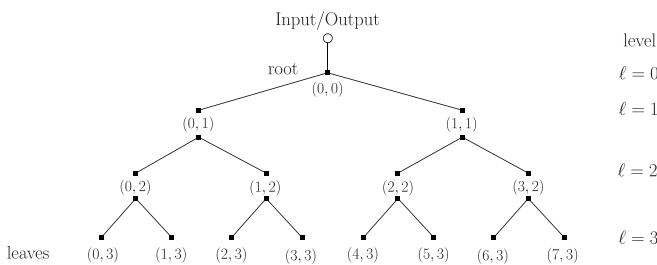


FIG. 4. Perfect binary tree with depth $n = 3$. At level ℓ ($0 \leq \ell \leq n$) of the binary tree, the w th node counting from the left is labeled as (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n$). Each node (w, ℓ) has two child nodes $(2w, \ell + 1)$ (left child) and $(2w + 1, \ell + 1)$ (right child) for $0 \leq \ell \leq n - 1$. The input or output terminal is connected to the root node $(0,0)$ by a path.

where $\mathcal{A} \subset \{0, \dots, 2^n - 1\}$ denotes the set of addresses of the specified memory cells.

B. Quantum gates

Next we briefly introduce several elementary quantum gates developed in the preceding paper [1] that are necessary for the design of the current QRAM architecture.

1. Single-qubit gates

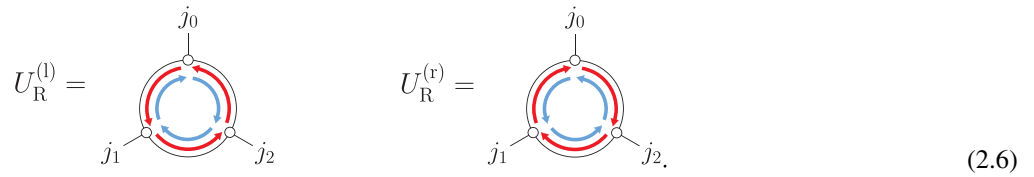
Arbitrary single-qubit gates are universally realized by a combination of roundabout gates and rotation gates acting on the internal states of the walker. The roundabout gate serves as a router that moves a walker either clockwise or counter-clockwise from one path to the next according to the internal

state of the walker:

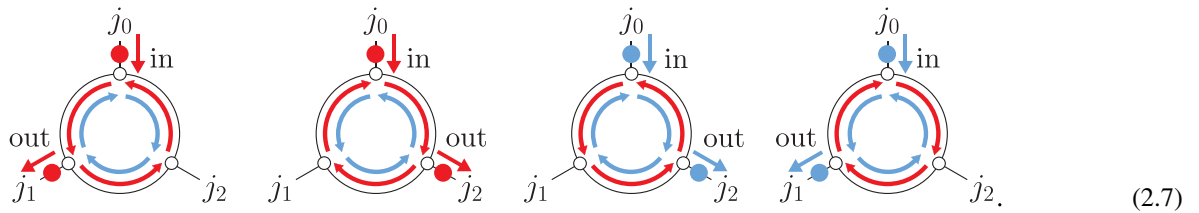
$$U_R^{(l)} = |0\rangle\langle 0|_{C_j} U_R + |1\rangle\langle 1|_{C_j} U_R^\dagger, \quad U_R^{(r)} = U_R^{(l)\dagger},$$

$$U_R = \sum_{k,l=0}^2 \delta_{l,k+1} |j_l\rangle\langle j_k|_p \quad (k, l \in \mathbb{Z}/3\mathbb{Z} = \{0, 1, 2\}). \quad (2.5)$$

Here $U_R^{(l)}$ and $U_R^{(r)}$ are unitary operators that move a red walker (a walker with the internal state $|0\rangle_{C_j}$) and a blue walker (a walker with $|1\rangle_{C_j}$) clockwise and counterclockwise, respectively, to the next path. Graphically, they are represented as



For example, the motion of a red or blue walker that enters the $U_R^{(l)}$ or $U_R^{(r)}$ gate from path j_0 is graphically given by

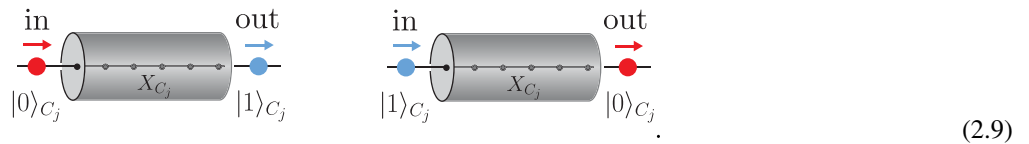


Physically, the roundabout gate can be implemented by a single-particle scattering from a directed graph as shown in Sec. III in [1].

Let us pictorially denote a quantum gate U_{C_j} acting on the internal state $|c\rangle_{C_j}$ of the j th quantum walker as

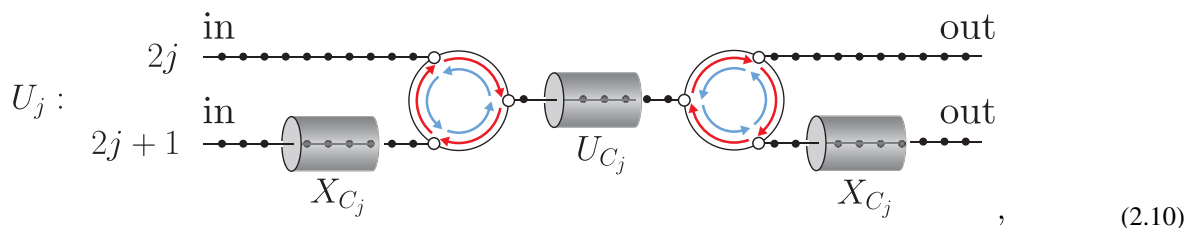


An arbitrary single-qubit gate U_{C_j} is universally realized by $U_{C_j} = e^{i\theta_0} R_z(\theta_1) R_y(\theta_2) R_z(\theta_3)$ [$\theta_k \in \mathbb{R}$ ($k = 0, 1, 2, 3$)] [2,50], where $R_y(\theta) := e^{-i\theta Y/2}$ [$R_z(\theta) := e^{-i\theta Z/2}$] is the operator that rotates the Bloch vector around the y axis (z axis) by a given angle θ . For example, the Pauli X gate is represented as $X_{C_j} = R_y(\pi)_{C_j}$ whose action on the states $|0\rangle_{C_j}$ and $|1\rangle_{C_j}$ are graphically represented as



For the spin-1/2 fermionic quantum walks, the operator $R_y(\theta)$ [$R_z(\theta)$] is physically realized by applying a magnetic field H in the direction of the y axis (z axis) with a specific strength depending on the angle θ . (See Fig. 5 in the preceding paper [1].)

Combining the roundabout gate and the gate U_{C_j} , one can construct the single-qubit gate U_j acting on the state $|q_j\rangle$, i.e., $U_j(|0\rangle_{C_j}|q_j\rangle) = |0\rangle_{C_j}(U_j|q_j\rangle)$ as given in [1],



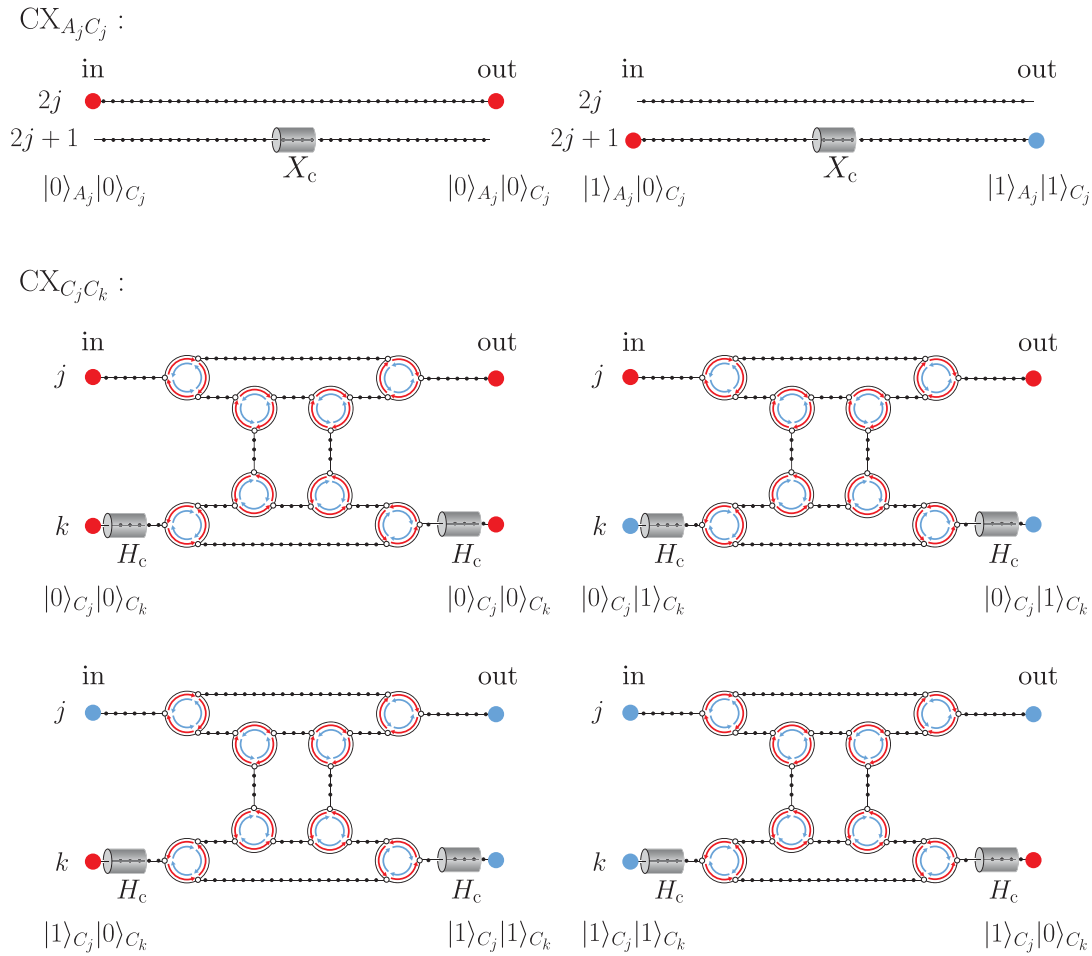
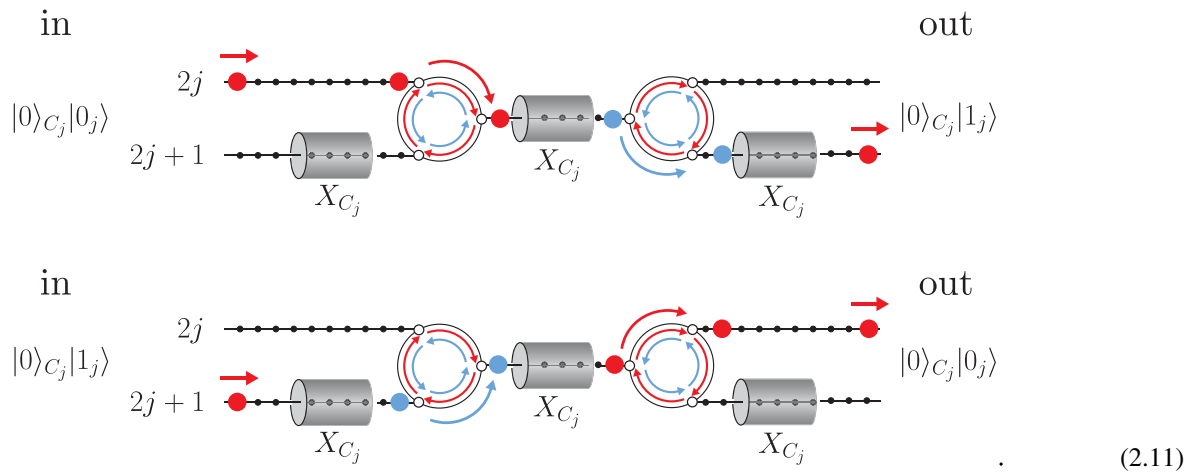


FIG. 5. Schematic description of the output states through the gates $CX_{A_j C_j}$ and $CX_{C_j C_k}$ and the corresponding input states.

where a red walker is considered as an input walker, i.e., $|0\rangle_{C_j}|q_j\rangle$. For instance, a walker passing through the Pauli X gate X_j is depicted as



2. Two-qubit gates

Any arbitrary quantum gate can be implemented by a proper combination of single-qubit gates described above and the controlled-NOT (CNOT) gate [2,50]. The CNOT gate CX_{jk}

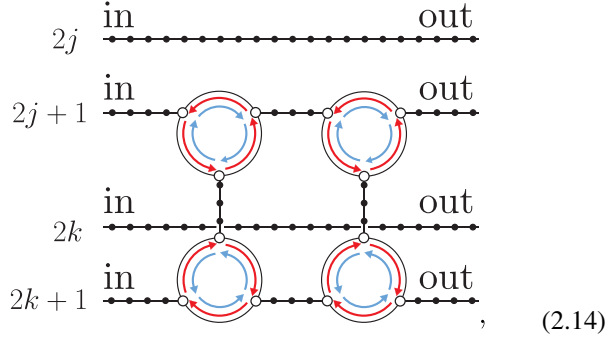
acting nontrivially on $|q_j\rangle \otimes |q_k\rangle$ is decomposed to

$$CX_{jk} = H_k C P_{jk} H_k, \tag{2.12}$$

where $H_k [=iR_y(\pi/2)_k R_z(\pi)_k]$ is the Hadamard gate acting on $|q_k\rangle$, which is achieved by setting $U_{C_k} = H_{C_k}$ in (2.10). Here CP_{jk} is a controlled-phase gate

$$CP_{jk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{jk}, \quad (2.13)$$

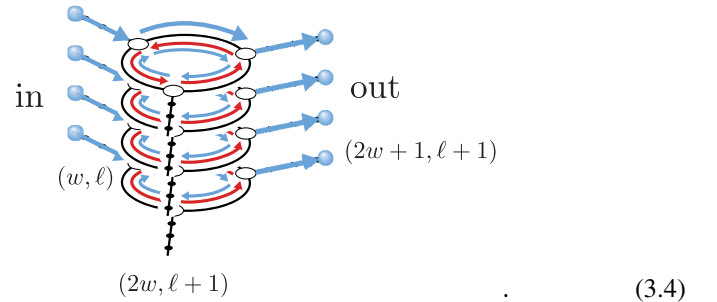
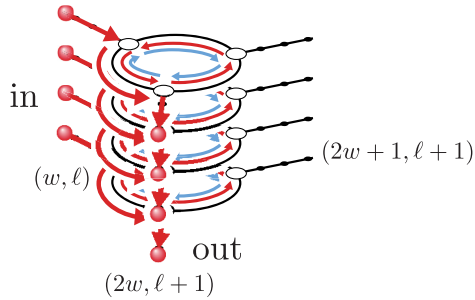
which is physically realized by the scattering of two walkers with the same internal state on an infinite path [1] (see Sec. IV in [1] for another controlled-phase gate),



where the input state is assumed to be $(|0\rangle_{C_k} |q_k\rangle) \otimes (|0\rangle_{C_j} |q_j\rangle)$.

III. PHYSICAL IMPLEMENTATION OF qRAM

Now we describe an implementation of the QRAM that realizes an algorithm formulated in [49]. Let us explain the



Formally, this process is given by the operator $\mathcal{R}_{(w,\ell)}$:

$$\mathcal{R}_{(w,\ell)} : |w, \ell\rangle_B \otimes \bigotimes_{j=0}^{n+m-1} |c\rangle_{C_j} \mapsto |2w+c, \ell+1\rangle_B \otimes \bigotimes_{j=0}^{n+m-1} |c\rangle_{C_j} \quad (c \in \{0, 1\}). \quad (3.5)$$

The internal states of all the walkers moving to the node (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n-1$) must be $|0\rangle_{C_j}$ ($|1\rangle_{C_j}$) ($0 \leq j \leq n+m-1$) for $a_{n-1-\ell} = 0$ ($a_{n-1-\ell} = 1$) so that the walkers passing through the routers at (w, ℓ) move to the left (right) node. Namely, the positional information of the path traveled by the $(n-1-\ell)$ th walker should be encoded to the internal states of all the walkers. This encoding process is formally written by the operator $\mathcal{E}_{(w,\ell)}$ ($0 \leq \ell \leq n-1$):

$$\mathcal{E}_{(w,\ell)} : \bigotimes_{j=0}^{n-1} |a_j\rangle_{A_j} \otimes \bigotimes_{j=0}^{n+m-1} |w \bmod 2\rangle_{C_j} \mapsto \bigotimes_{j=0}^{n-1} |a_j\rangle_{A_j} \otimes \bigotimes_{j=0}^{n+m-1} |a_{n-1-\ell}\rangle_{C_j}. \quad (3.6)$$

As shown immediately below, the operator $\mathcal{E}_{(w,\ell)}$ is achieved by a CNOT gate $CX_{A_j C_j}$ ($0 \leq j \leq n-1$),

$$CX_{A_j C_j} : |a_j\rangle_{A_j} \otimes |c_j\rangle_{C_j} \mapsto |a_j\rangle_{A_j} \otimes (\delta_{a_j,0} |c_j\rangle_{C_j} + \delta_{a_j,1} X_{C_j} |c_j\rangle_{C_j}), \quad (3.7)$$

details in the order of (i) the routing scheme \mathcal{F} , (ii) the querying scheme \mathcal{Q} , and (iii) the output scheme \mathcal{F}^\dagger . Our QRAM architecture is implemented by these schemes:

$$\text{QRAM} = \mathcal{F}^\dagger \mathcal{Q} \mathcal{F}. \quad (3.1)$$

(i) *Routing scheme \mathcal{F} .* The routing scheme is a scheme to deliver the $n+m$ quantum walkers (in superposition) to the desired memory cell(s):

$$\mathcal{F} : \sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D \mapsto \sum_{a \in \mathcal{A}} |a\rangle_A |a, n\rangle_B |0\rangle_C |0\rangle_D. \quad (3.2)$$

The input state

$$\sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D = \sum_{a \in \mathcal{A}} |a_{n-1} \cdots a_0\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D \quad (3.3)$$

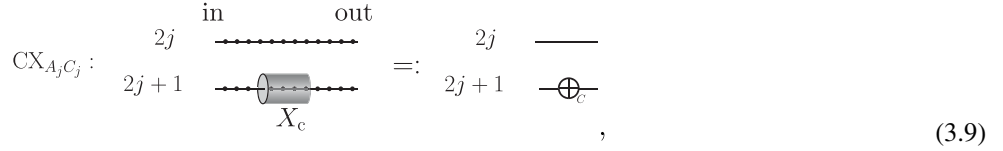
is dual-rail encoded into the positions of the $n+m$ red quantum walkers at input terminals as in Figs. 2 and 3. The $n+m$ walkers start moving simultaneously toward leaves.

The roundabout gate $U_R^{(l)}$ is installed at each node (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n-1$) so that it routes the red walkers (blue walkers) to the left (right) child node $(2w, \ell+1)$ [$(2w+1, \ell+1)$]:

and a multiple actions of a CNOT gate $CX_{C_j C_k}$ ($k \neq j$ and $0 \leq j \leq n + m - 1$), defined as

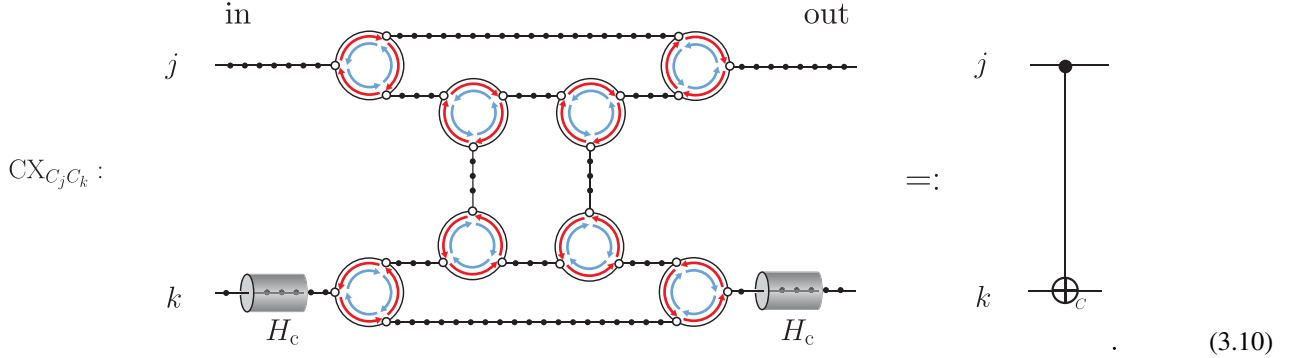
$$CX_{C_j C_k} : |c_j\rangle_{C_j} \otimes |c_k\rangle_{C_k} \mapsto |c_j\rangle_{C_j} \otimes (\delta_{c_j,0}|c_k\rangle_{C_k} + \delta_{c_j,1}X_{C_k}|c_k\rangle_{C_k}). \tag{3.8}$$

Their graphical representations are given, respectively, by



$$CX_{A_j C_j} : \begin{array}{c} \text{in} \quad \text{out} \\ 2j \text{ --- } \\ 2j+1 \text{ --- } \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} 2j \text{ ---} \\ 2j+1 \text{ ---} \end{array} \oplus_C, \tag{3.9}$$

and



$$CX_{C_j C_k} : \begin{array}{c} \text{in} \quad \text{out} \\ j \text{ --- } \\ k \text{ --- } \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} j \text{ ---} \\ k \text{ ---} \end{array} \oplus_C. \tag{3.10}$$

(Also see Fig. 5.) Now we describe an implementation of the operator $\mathcal{E}_{(w,\ell)}$. Let us assume that

$$n + m = 2^p, \quad p \in \mathbb{Z}_{\geq 0} \tag{3.11}$$

for simplicity. (An extension to generic n and m is straightforward.) Under this assumption, the operator $\mathcal{E}_{(w,\ell)}$ can be decomposed to

$$\begin{aligned} \mathcal{E}_{(w,\ell)} &= \mathcal{E}_\ell \mathcal{X}_{(w,\ell)}, \quad \mathcal{X}_{(w,\ell)} := \prod_{j=0}^{n+m-1} (\delta_{w \in 2\mathbb{Z}} + \delta_{w \in 2\mathbb{Z}+1} X_{C_j}), \\ \mathcal{E}_\ell &:= \mathcal{E}_\ell^{(p|p-1)} \dots \mathcal{E}_\ell^{(2|1)} \mathcal{E}_\ell^{(1|0)} CX_{A_{n-1-\ell} C_{n-1-\ell}}. \end{aligned} \tag{3.12}$$

The operator $\mathcal{X}_{(w,\ell)}$ resets the color of the blue walkers to red. The operator $CX_{A_{n-1-\ell} C_{n-1-\ell}}$ (3.7) encodes the information $a_{n-1-\ell}$ into the internal state of the $(n - 1 - \ell)$ th walker as $|a_{n-1-\ell}\rangle_{C_{n-1-\ell}}$, and the operator $\mathcal{E}_\ell^{(k+1|k)}$ copies the color $|a_{n-1-\ell}\rangle_{C_{n-1-\ell}}$ to another particle (see Fig. 6 to gain intuition), which is constructed by

$$\begin{aligned} \mathcal{E}_\ell^{(1|0)} &= [r, r + 2^{p-1}], \\ \mathcal{E}_\ell^{(k+1|k)} &= \prod_{s=0}^{2^{k-1}-1} [r - s2^{p-k}, r - s2^{p-k} - 2^{p-k-1}] [r + 2^{p-1} - s2^{p-k}, r + 2^{p-1} - s2^{p-k} - 2^{p-k-1}] \quad (k \geq 1). \end{aligned} \tag{3.13}$$

Here we set $r := n - 1 - \ell$, $q_{j+n+m} := q_j$, and $C_{j+n+m} := C_j$ and used the abbreviation

$$[j, k] := CX_{C_j C_k} \tag{3.14}$$

to simplify the notation. Note here that the colors of the walkers are mixed only during this encoding process; otherwise all they are set to either red or blue. Thus, the quantum walkers appropriately move to the paths connecting two nodes (w, ℓ) and $(2w + a_{n-1-\ell}, \ell + 1)$ ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n - 1$) by

$$\begin{aligned} \mathcal{F}^{(\ell+1|\ell)} &:= \sum_{w=0}^{2^\ell-1} \mathcal{R}_{(w,\ell)} \mathcal{E}_{(w,\ell)}, \\ \mathcal{F}^{(\ell+1|\ell)} &: |a\rangle_A |w, \ell\rangle_B \otimes \bigotimes_{j=0}^{n+m-1} |a_{n-\ell}\rangle_{C_j} \otimes |0\rangle_D \\ &\mapsto |a\rangle_A |2w + a_{n-1-\ell}, \ell + 1\rangle_B \otimes \bigotimes_{j=0}^{n+m-1} |a_{n-1-\ell}\rangle_{C_j} \otimes |0\rangle_D, \end{aligned} \tag{3.15}$$

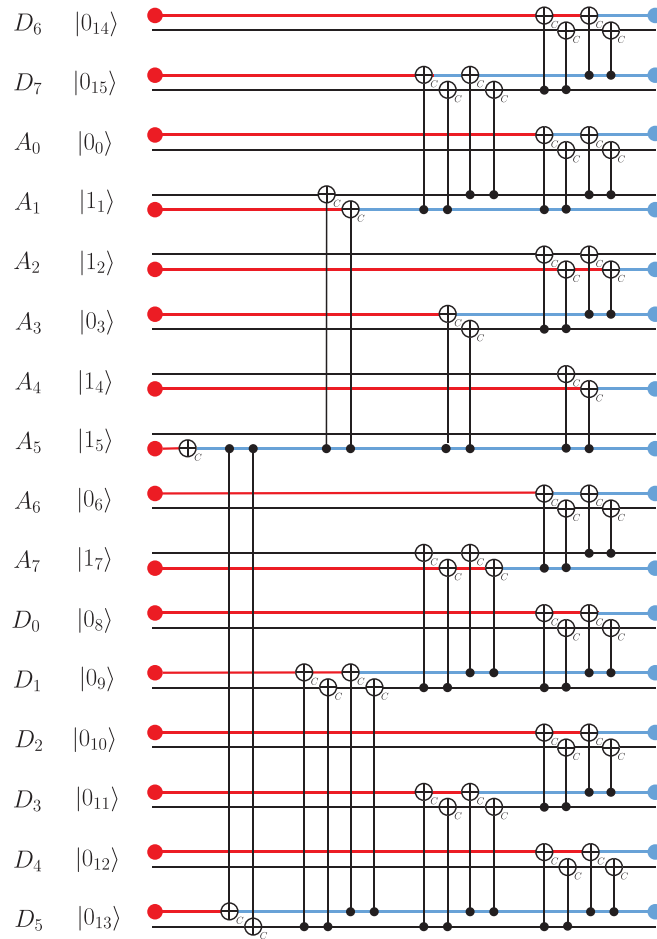


FIG. 6. Pictorial representation of the action of $\mathcal{E}_{(w,\ell)}$ defined by (3.12) [see also (3.6)] for $w \in 2\mathbb{Z}$, $\ell = 2$, $n = 8$, $m = 8$, $p = \log_2(n + m) = 4$, $|10110110\rangle_A$, and $|00000000\rangle_D$. After $O(p)$ steps, the positional information $|a_{n-1-\ell}\rangle_{A_{n-1-\ell}} = |1\rangle_{A_5}$ is encoded to the colors of all the walkers: $\bigotimes_{j=0}^{15} |0\rangle_{C_j} \mapsto \bigotimes_{j=0}^{15} |1\rangle_{C_j}$. The number of devices necessary for the processing is $O(n + m)$. Note that the colors of the walkers are mixed only during this encoding process; otherwise all they are set to either red or blue.

where $a_n := 0$. Recursively applying $\mathcal{F}^{\ell+1|\ell}$ to the walkers that started moving toward (w, ℓ) from its parent node and finally resetting the colors of the walkers to red, namely, performing the operator

$$\mathcal{F} = \sum_{w=0}^{2^n-1} \mathcal{X}_{(w,n)} \mathcal{F}^{(n|n-1)} \dots \mathcal{F}^{(2|1)} \mathcal{F}^{(1|0)}, \quad (3.16)$$

we properly deliver the walkers (in superposition) to the designated memory cells, as given by (3.2). The depth of the circuit required for the routing scheme is $O(np) = O(n \log(n + m))$.

(ii) *Querying scheme* \mathcal{Q} . The querying scheme \mathcal{Q} is a scheme that loads the data $x^{(a)}$ stored in the memory cell at the address a ,

$$\mathcal{Q} : \sum_{a \in \mathcal{A}} |a\rangle_A |a, n\rangle_B |0\rangle_C |0\rangle_D \mapsto \sum_{a \in \mathcal{A}} |a\rangle_A |a, n\rangle_B |0\rangle_C |x^{(a)}\rangle_D, \quad (3.17)$$

which is formally realized by

$$\mathcal{Q} = \sum_{a \in \mathcal{A}} |a, n\rangle \langle a, n|_B \otimes \bigotimes_{i=0}^{m-1} (X_{D_i})^{x_i^{(a)}}. \quad (3.18)$$

In our architecture, this scheme is implemented by simply exchanging the appropriate paths in the data register or alternatively by placing the Pauli X gates [see (2.10)], as pictorially shown in Fig. 7.

(iii) *Output scheme* \mathcal{F}^\dagger . The output scheme is a procedure to retrieve the data in superposition. In the present approach, this scheme is achieved by just applying the reverse operation of the routing scheme, i.e.,

$$\mathcal{F}^\dagger : \sum_{a \in \mathcal{A}} |a\rangle_A |a, n\rangle_B |0\rangle_C |x^{(a)}\rangle_D \mapsto \sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |x^{(a)}\rangle_D. \quad (3.19)$$

As shown in Fig. 3, the scheme can be implemented simply by arranging all devices used in the routing scheme so that their positions are perfect mirror images across the memory cells (without changing the direction of the arrows on the roundabout gates).

In summary, the present architecture processes 2^n m -qubit data in $O(n \log(n + m))$ steps, which requires $O(n + m)$ qubit resources and $O((n + m)2^n)$ quantum devices. Table I compares the number of computational steps and quantum resources required for the present QRAM architecture and the

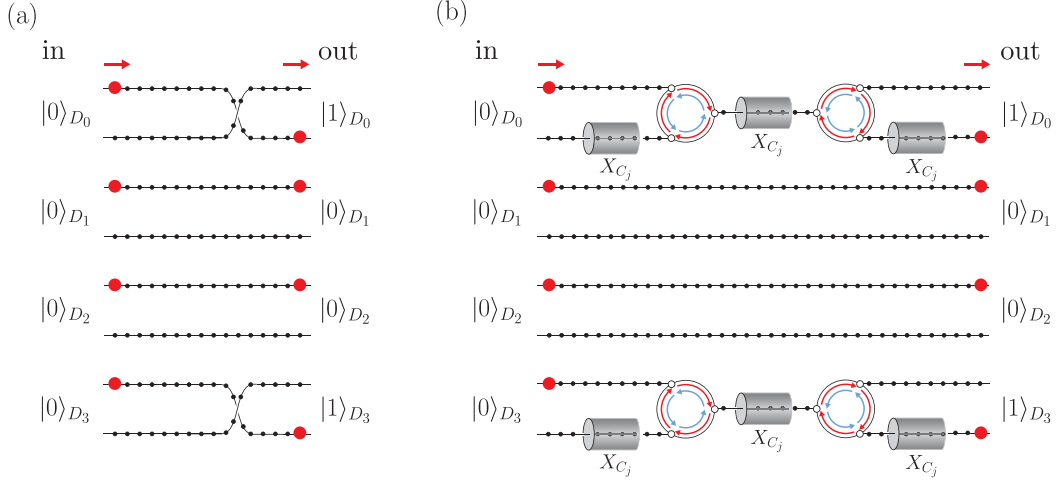


FIG. 7. Pictorial representation of the querying scheme (3.17) for $|0000\rangle_D \mapsto |1001\rangle_D$, which can be achieved by (a) exchanging paths or (b) placing the Pauli X gates.

original bucket-brigade architecture. Compared to the original bucket-brigade QRAM, the advantages of our architecture are that it requires fewer computation steps and qubit resources and does not require time-dependent control. On the other hand, the trade-off for these advantages is that it requires more space and quantum gates, as shown in Figs. 3 and 6.

IV. MODIFICATION OF THE ROUTING SCHEME

By the definition (1.2) of QRAM, the address information in superposition, namely, $\sum_a |a\rangle_A$, is prepared beforehand, with no mention of how it is actually constructed. Here we propose an alternative QRAM architecture that transforms a trivial state into a superposition of information stored in the desired memory cells

$$\begin{aligned} \widetilde{\text{QRAM}} : |0\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D \\ \mapsto \frac{1}{\sqrt{|\mathcal{A}|}} \sum_{a \in \mathcal{A}} |a\rangle_A |0, 0\rangle_B |0\rangle_C |x^{(a)}\rangle_D \end{aligned} \quad (4.1)$$

[cf. Eq. (2.4)], which is accomplished by modifying the routing and querying scheme slightly. Note that, in (4.1), the normalization factor $1/\sqrt{|\mathcal{A}|}$ is written down explicitly to improve the perspective of the discussion here.

First we construct the routing scheme $\tilde{\mathcal{F}}$,

$$\tilde{\mathcal{F}} : |0\rangle_A |0, 0\rangle_B |0\rangle_C |0\rangle_D \mapsto \frac{1}{\sqrt{|\mathcal{A}|}} \sum_{a \in \mathcal{A}} |0\rangle_A |a, n\rangle_B |0\rangle_C |0\rangle_D \quad (4.2)$$

[cf. (3.2)]. Note that, in this routing scheme $\tilde{\mathcal{F}}$, the address information is *not* encoded in the address state, which

actually remains $|0\rangle_A$ during the routing. Instead, to deliver the $n + m$ walkers to the memory cells at \mathcal{A} , the Hadamard-like gates are appropriately placed in the first binary tree on the top sheet, where the zeroth walker travels. Let $l_{(w,\ell)}$ ($r_{(w,\ell)}$) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n - 1$) be the number of designated memory cells whose ancestor is the left child node ($2w, \ell + 1$) [right child node ($2w + 1, \ell + 1$)] of (w, ℓ) (see Fig. 8 for a simple example). Then we define the Hadamard-like gate $\mathcal{H}_{(w,\ell)}$ acting on the internal state of the zeroth walker that moves to the node (w, ℓ) from its parent node,

$$\mathcal{H}_{(w,\ell)} := \frac{1}{\sqrt{l_{(w,\ell)} + r_{(w,\ell)}}} \begin{pmatrix} \sqrt{l_{(w,\ell)}} & \sqrt{r_{(w,\ell)}} \\ \sqrt{r_{(w,\ell)}} & -\sqrt{l_{(w,\ell)}} \end{pmatrix}_{C_0}, \quad (4.3)$$

which is given by $e^{i\theta} R_y(\theta) R_z(\pi)$ for $\theta = 2 \tan^{-1}(\sqrt{r_{(w,\ell)}/l_{(w,\ell)}})$ and is reduced to the standard Hadamard gate if $l_{(w,\ell)} = r_{(w,\ell)} = 1$ ($\theta = \pi/2$). Using this gate with $\mathcal{R}_{(w,\ell)}$ and $\mathcal{X}_{(w,\ell)}$ defined in (3.5) and (3.6), we can actually realize $\tilde{\mathcal{F}}$,

$$\begin{aligned} \tilde{\mathcal{F}} &= \sum_{w=0}^{2^n-1} \mathcal{X}_{(w,n)} \tilde{\mathcal{F}}^{(n|n-1)} \dots \tilde{\mathcal{F}}^{(2|1)} \tilde{\mathcal{F}}^{(1|0)}, \\ \tilde{\mathcal{F}}^{(\ell+1|\ell)} &:= \sum_{w=0}^{2^\ell-1} \mathcal{R}_{(w,\ell)} \tilde{\mathcal{E}}_{n-1} \mathcal{H}_{(w,\ell)} \mathcal{X}_{(w,\ell)}, \end{aligned} \quad (4.4)$$

where $\tilde{\mathcal{E}}_\ell$ is defined by slightly modifying \mathcal{E}_ℓ [Eq. (3.12)] as

$$\tilde{\mathcal{E}}_\ell := \mathcal{E}_\ell^{(p|p-1)} \dots \mathcal{E}_\ell^{(2|1)} \mathcal{E}_\ell^{(1|0)}. \quad (4.5)$$

TABLE I. Comparison of the number of computational steps and quantum resources required for the present method and the original bucket-brigade method (from [39,40]).

Method	No. of computational steps	No. of qubits	No. of quantum gates
quantum walk	$O(n \log(n + m))$	$O(n + m)$	$O((n + m)2^n)$
bucket-brigade	$O(n^2 + nm)$	$O(2^n + m)$	$O(2^n)$

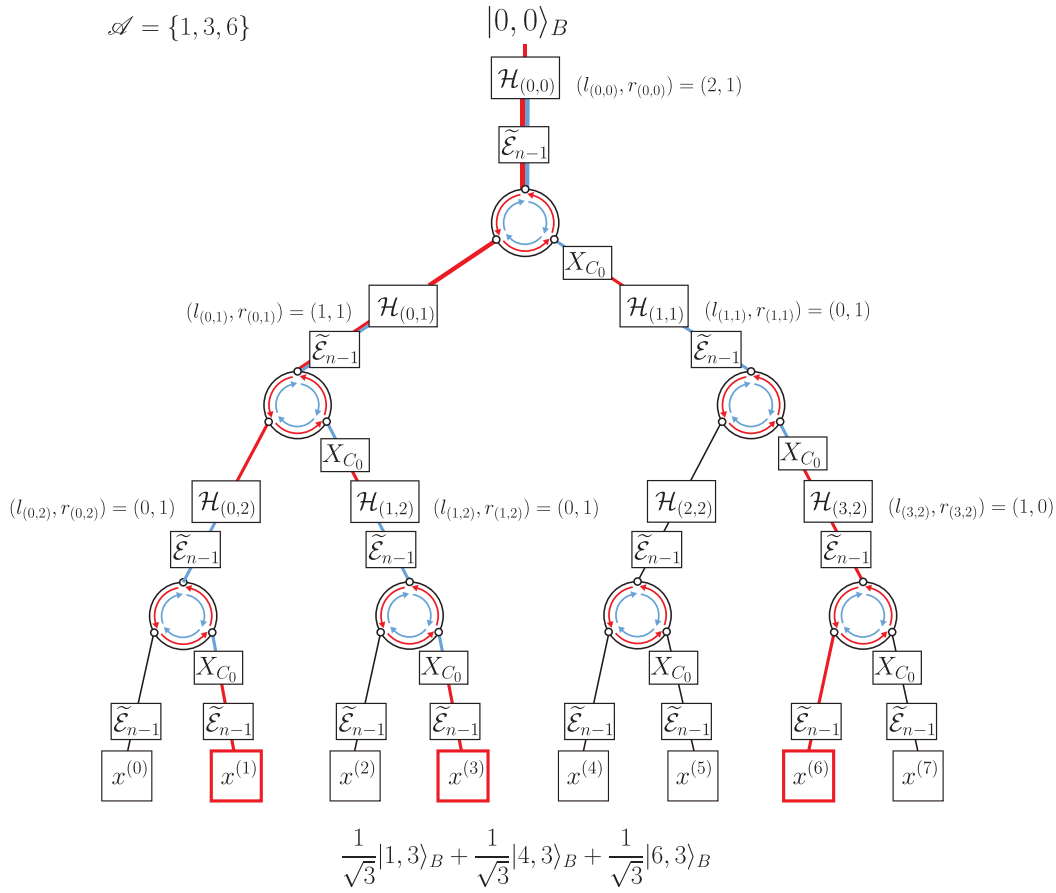


FIG. 8. Graphical representation of the modified routing scheme $\tilde{\mathcal{F}}$ defined in (4.2) on the top sheet for $\mathcal{A} = \{1, 3, 6\} = \{001, 011, 110\}$. On the top sheet, the Hadamard-like gate $\mathcal{H}_{(w,\ell)}$ [Eq. (4.3)] is equipped on each pass to the node (w, ℓ) ($0 \leq w \leq 2^\ell - 1$ and $0 \leq \ell \leq n - 1 = 2$). The internal state generated by passing through $\mathcal{H}_{(w,\ell)}$ is entangled with that of each walker by the gate $\tilde{\mathcal{E}}_{n-1}$.

Namely, $\tilde{\mathcal{E}}_{n-1}$ entangles the internal states of the zeroth walker with those of the other walkers:

$$\tilde{\mathcal{E}}_{n-1} : \sum_c |c\rangle_{C_0} \bigotimes_{j=1}^{n+m-1} |0\rangle_{C_j} \mapsto \sum_c \bigotimes_{j=0}^{n+m-1} |c\rangle_{C_j} \quad (c \in \{0, 1\}). \quad (4.6)$$

In Fig. 8 we pictorially show an example of the modified routing scheme on the top sheet.

To properly retrieve the walkers loaded with the data using output scheme \mathcal{F}^\dagger , the address information $|a\rangle_A$ must be encoded in the positions of the quantum walkers. Note that once the data has been loaded, it is no longer possible to retrieve the walkers using $\tilde{\mathcal{F}}^\dagger$. The querying scheme $\tilde{\mathcal{Q}}$ corresponding to (3.17) is modified to encode the data as well as the address of the cell where the data is stored:

$$\begin{aligned} \tilde{\mathcal{Q}} &: \frac{1}{\sqrt{|\mathcal{A}|}} \sum_{a \in \mathcal{A}} |0\rangle_A |a, n\rangle_B |0\rangle_C |0\rangle_D \\ &\mapsto \frac{1}{\sqrt{|\mathcal{A}|}} \sum_{a \in \mathcal{A}} |a\rangle_A |a, n\rangle_B |0\rangle_C |x^{(a)}\rangle_D. \end{aligned} \quad (4.7)$$

Explicitly it reads

$$\tilde{\mathcal{Q}} = \sum_{a \in \mathcal{A}} |a, n\rangle \langle a, n|_B \otimes \bigotimes_{i=0}^{n-1} (X_{A_i})^{a_i} \otimes \bigotimes_{i=0}^{m-1} (X_{D_i})^{x_i^{(a)}}, \quad (4.8)$$

which is accomplished by exchanging the specified paths in the address and data register or alternatively by placing the Pauli X gates, as explained in the preceding section.

Finally applying the output scheme \mathcal{F}^\dagger defined in (3.19), we obtain the desired QRAM architecture (4.1) in the form

$$\widetilde{\text{QRAM}} = \mathcal{F}^\dagger \tilde{\mathcal{Q}} \tilde{\mathcal{F}}. \quad (4.9)$$

V. SUMMARY AND DISCUSSION

A QRAM (2.4) or (4.1) has been physically realized by combinations of several elementary quantum devices, including the roundabout gate (2.6) developed in the preceding paper [1]. The 2^n m -qubit information can be retrieved in superposition by simply passing the $n + m$ quantum walkers through the perfect binary trees, as schematically shown in Fig. 3. The advantages of the present QRAM architecture compared to the original bucket-brigade QRAM are summarized as follows. (i) The procedure is completely parallelized without using any ancilla qubit. The 2^n m -qubit information can be retrieved after $O(n \log(n + m))$ steps. The qubit resources and the quantum gates required for the processing are $O(n + m)$ and $O(2^n(n + m))$. (ii) The walkers do not entangle with any device on the binary trees, which promises to reduce the cost of maintaining quantum coherence. (iii) Our QRAM architecture is free from any time-dependent control. In other words,

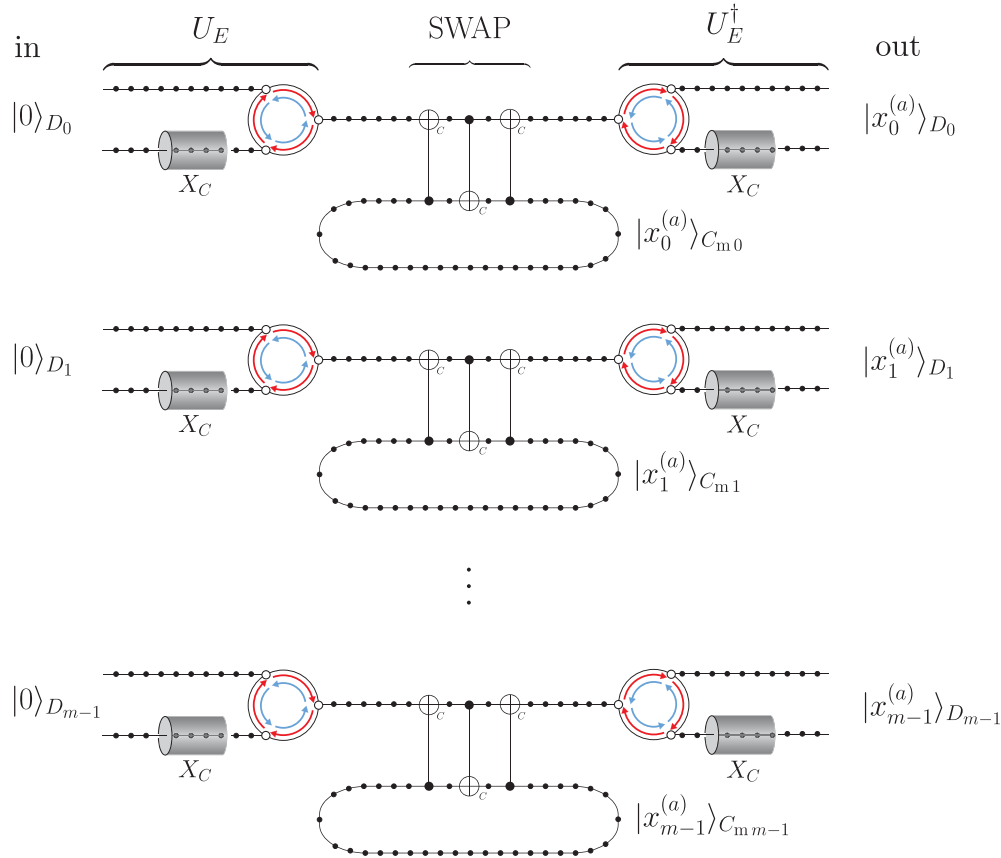


FIG. 9. Physical realization of the process (5.1). Quantum information is represented as the internal states of quantum walkers, which can be stored in the memory cell using a quantum memory developed in the preceding paper [1]. The information stored in the memory cell is transferred by the encoder (decoder) U_E (U_E^\dagger) and the SWAP gate consisting of the CNOT gates defined in (3.10).

information in quantum superposition can be composed by just passing the walkers through the binary trees. On the other hand, the trade-off for these advantages is that it requires more space and quantum gates. Since the actual implementation uses wave packets as quantum walkers, there will be errors due to their finite-size effects [51] as the quantum walkers pass through the devices. However, we can control this type of error with arbitrary precision by changing the system size (see Sec. V in the preceding paper [1] for details).

Finally, let us discuss how to generalize the present architecture to be able to process quantum information (i.e., information stored in the cell consists of a superposition of states). In the querying scheme described in Sec. III, m -bit classical information stored in a memory cell can be directly copied to an m -qubit state of the data register [see (3.17) and (3.18)], which can be achieved by exchanging the appropriate paths in the data register or by placing the Pauli X gates. For the quantum case, however, quantum information cannot be replicated due to the no-cloning theorem [52,53]. Instead, by a SWAP gate, the quantum information can be transferred to the state in the data register. In our architecture, m -qubit quantum information stored in the cell at address a can be represented as the internal states of m quantum walkers, each moving in a circle, as shown in Fig. 9. (This may be realized by a quantum memory developed in Sec. VI in the preceding paper [1].)

Explicitly, it is expressed as $\sum_x |x^{(a)}\rangle_{C_m}$, where the index C_m stands for the colors of the quantum walkers in the memory cell. As shown in Fig. 9, combining an encoder U_E [see Eq. (4.2) in the preceding paper [1] for a precise definition], which encodes the position of the quantum walker into the internal state of the walker, and the SWAP gate consisting of the three CNOT gates $CX_{C_j C_k}$ in (3.10), we can correctly transfer the quantum information in the memory cell to the data register:

$$\sum_x |x^{(a)}\rangle_{C_m} \mapsto \sum_x |x^{(a)}\rangle_D. \quad (5.1)$$

The combination of the model of universal quantum computation achieved in the preceding paper [1] with the current QRAM architecture is expected to enable efficient processing of quantum information such as the quantum phase estimation [3], Grover's algorithm for searching unsorted databases [7], and the quantum version of the fast Fourier transform [54].

ACKNOWLEDGMENTS

The present work was partially supported by Grant-in-Aid for Scientific Research (C) No. 20K03793 from the Japan Society for the Promotion of Science.

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