Viscous flow in a one-dimensional spin-polarized Fermi gas: The role of integrability on viscosity

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The transport properties of one-dimensional Fermi gases at low temperatures are often described by the Luttinger liquid (LL) model. However, to study dissipative effects, one needs to examine interactions beyond the LL model. In this work, we provide a simple model that allows for a direct microscopic calculation of the bulk viscosity, namely, the one-dimensional spin polarized p-wave Fermi gas. To leading order in the finite interaction strength, we find that the bulk viscosity is finite and consistent with the requirement of scale symmetry. We further show that the bulk viscosity satisfies the Bose-Fermi duality relating the weakly interacting limit of the spin polarized Fermi gas to the strongly interacting limit of the Lieb-Liniger model and vice versa. This work establishes the bulk viscosity to leading order in scale-breaking interactions in both the strongly and weakly interacting limits for both high and low temperatures.

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I. INTRODUCTION

Transport in integrable models has attracted much attention recently as they exhibit behavior that differs fundamentally from their nonintegrable counterparts [1–3]. In integrable models, a large number of conservation laws qualitatively changes the long-wavelength dynamics, rendering the standard hydrodynamic approach inapplicable. However, for certain integrable models, such as the Lieb-Liniger model, a generalized hydrodynamic description is possible and has been proven to be very successful in describing the dynamics of such integrable systems at the ideal level without viscous effects [4–10]. Including dissipation effects in generalized hydrodynamics still remains a theoretical challenge.

On the other hand, the long-wavelength dynamics of a generic many-body system are still governed by a few transport coefficients, such as the conductivity, thermal conductivity, and bulk viscosity [11]. Although these quantities were first defined in the hydrodynamic context, there has been a substantial amount of care in defining these transport quantities using Kubo formulas [12–15] that are valid for both integrable and nonintegrable systems. As an example, it is well known that these definitions show that the conductivity, thermal conductivity, and shear viscosity (for higher-dimensional models) are infinite when the models are integrable, and become finite with the inclusion of interactions that break the integrability [16–19].

Amongst those transport coefficients, the bulk viscosity [for both one-dimensional (1D) and higher-dimensional models] is special in that it is constrained to be zero [20] for scale and Galilean invariant systems, due to a hidden SO(2,1) symmetry [21–24]. This conclusion is independent of whether the system is integrable or not. On the other hand, not all integrable models have scale and Galilean invariance. For such models without Galilean invariance, one expects a zero-frequency bulk viscosity (Drude weight) as in the case of kinetic calculations. In addition, it has been shown from basic arguments that the zero-frequency bulk viscosity lacks a Drude peak for a large class of many-body systems, i.e., the bulk viscosity spectral function cannot diverge [14]. It thus remains an important question to explore the interplay between integrability and scale invariance on the behavior of bulk viscosity.

In this work, we focus on a particularly simple integrable model, the Cheon-Shigehara model of spinless fermions with p-wave interactions in one dimension [25]. We compute its bulk viscosity directly via Kubo formalism using the low-energy Luttinger liquid (LL) formalism, which should be applicable to both the weak and strong coupling regimes. The Cheon-Shigehara model is described by a single low-energy scattering parameter, the 1D p-wave scattering length ℓ. In both the noninteracting limit, $\ell = 0$, and the resonantly interacting limit, $\ell = \infty$, this model possesses scale and Galilean symmetry, and should thus have zero bulk viscosity. In addition, it is also known that this model is dual to the 1D Lieb-Liniger model via a strong-weak duality. For these reasons, this 1D system provides an ideal platform for studying the interplay between scale symmetry, integrability, and the bulk viscosity. As we shall show later, one can also establish explicitly the relation between bulk viscosities in these two models.

We find that indeed the zero-frequency bulk viscosity vanishes at the scale and Galilean invariant point $\ell = 0$ and $\ell = \infty$. In the vicinity of the noninteracting point $\ell = 0$, one finds a nonzero bulk viscosity that scales as ℓ^2 in the weak interacting limit and as $1/\ell^2$ close to the resonant limit. To the leading order that we are working at, we find no Drude weight for the bulk viscosity spectral function. We note, however, that a recent calculation that incorporates higher-order diagrams does indicate the existence of a finite Drude weight that vanishes in the scale and Galilean invariant point [26].

The Cheon-Shigehara model can be approximately realized in experiments by using confined gases in 1D optical traps [27–35]. The bulk viscosity can then be extracted by examining the entropy production from a periodically modulated interaction strength [15], or from the damping of large-amplitude monopole excitations after a quench in the 1D harmonic trapping potential [24]. In actual experiments, however, the effects of the p-wave effective range can also be significant. At the same time, it is important to note that unlike the p-wave Fermi gases in three dimensions, the effective range term in one dimension is irrelevant in the 1D effective field theory description while the three-body interactions are in fact marginal. In principle, a finite effective range will certainly break the scale symmetry and renders a finite bulk viscosity when $\ell = 0$ or ∞, but we expect the related bulk viscosity to have a weaker temperature dependence compared with those for finite ℓ [36]. For clarity of presentation, we shall neglect both the effect of the effective range and three-body interactions in the following calculation.

The remainder of our paper is organized as follows. In Sec. II we discuss the derivation of the bulk viscosity and its application to the Cheon-Shigehara model, and present the low-temperature LL theory for weakly interacting spin-polarized Fermi gas. We then proceed to calculate the bulk viscosity in Sec. III in the weakly interacting limit according to the low-temperature LL theory, and in the high-temperature limit according to the virial expansion. In Sec. IV we establish the Bose-Fermi duality for the bulk viscosity, which allows us to obtain a complete picture for the bulk viscosity in both the weakly and strongly interacting limits at low and high temperatures. We then compare our results to previous kinetic theory calculations based on three-body processes in Sec. V and give our conclusions in Sec. VI.

II. THE 1D SPIN-POLARIZED FERMI GAS AND BULK VISCOSITY

In this work, we shall focus our attention on a particularly simple model of a spinless Fermi gas interacting in one dimension. Let us thus consider the following Hamiltonian:

$$H = \int dx\, \psi^\dagger(x)\left(-\frac{1}{2m}\partial_x^2 - \mu\right)\psi(x) - \frac{g}{4}\int dx \psi^\dagger(x)\overleftrightarrow{\partial_x}\psi^\dagger(x)\psi(x)\overleftrightarrow{\partial_x}\psi(x), \quad (1)$$

where $\psi(x)$ is the fermionic annihilation operator, g is the odd-wave coupling constant, m is the atomic mass, μ is the chemical potential, and we have set $\hbar = 1$. The two-body interaction depends on the derivative $\overleftrightarrow{\partial_x} = (\overleftarrow{\partial_x} - \overrightarrow{\partial_x})/2$, which is the symmetrized and Galiliean-invariant derivative that acts both on the immediate left and right. The relation between the bare coupling constant g and the p-wave scattering length ℓ can be established by computing the two-body scattering T-matrix at low energy, and the details are discussed in Appendix A, where it is shown that

$$\frac{m}{4\ell} = \frac{1}{g} + \frac{m\Lambda}{2\pi}, \quad (2)$$

with Λ being a large momentum cutoff. Physically the cutoff is of the order of r_0^{-1}, with r_0 being the range of the potential. However, as the effective range is an irrelevant quantity to the physics, we will focus on the zero range limit. In the weak coupling limit, the coupling constant is proportional to the scattering volume: $g = 4\ell/m$.

To exhibit the relevance of scale invariance in the discussion of the bulk viscosity, we first derive an expression for bulk viscosity for our model Hamiltonian, Eq. (1), that makes the effects of scale transformation explicit. Our starting point is the Kubo formula in the stress-stress form (see Eq. (3.4) in Ref. [14]).

The bulk viscosity spectral function $\zeta(\omega)$ can be defined in terms of the response of a fluid to a time-dependent strain [14],

$$\zeta(\omega) = \frac{\chi_{\Pi,\Pi}(\omega)}{iL(\omega+i\delta)} + \frac{1}{iL(\omega+i\delta)}\left[\left\langle \frac{d\Pi_b}{db}\bigg|_{b=0}\right\rangle - \frac{d}{db}\langle\Pi\rangle_b\bigg|_{b=0}\right], \quad (3)$$

where L is the volume of the system, $\chi_{\Pi\Pi}(\omega)$ is the retarded correlation function,

$$\chi_{\Pi\Pi}(\omega) = i\int_0^\infty e^{i(\omega+i\delta)t}\langle[\Pi(t), \Pi(0)]\rangle, \quad (4)$$

and $\Pi(t)$ is the (trace of the) stress tensor,

$$\Pi(t) = 2H + \frac{1}{\ell}C_\ell(t), \quad (5)$$

where again H is the Hamiltonian in Eq. (1) and we also define the thermodynamic contact [35]:

$$C_\ell = -\frac{\partial H}{\partial \ell^{-1}} = -\frac{g^2}{4}\int dx\psi^\dagger(x)\overleftrightarrow{\partial_x}\psi^\dagger(x)\psi(x)\overleftrightarrow{\partial_x}\psi(x). \quad (6)$$

The quantities in the second line of Eq. (3) depend on how the stress-tensor operator changes under a scale transformation and how the expectation value of the stress tensor changes under a scale transformation. These terms can be evaluated by examining how the field operators change under a scale transformation,

$$\psi_b(x) = e^{-b}\psi(xe^{-b}). \quad (7)$$

From this transformation, the stress tensor changes according to

$$\Pi_b = 2e^{-2b}H(\ell e^{-b}) + \frac{e^{-b}}{\ell}C_\ell(\ell e^{-b}). \quad (8)$$

Note also that we have explicitly included the dependence of ℓ in the Hamiltonian and the contact to emphasize how these operators change under a scale transformation. Equation (8) takes into account the renormalization group flow via the two-body solution of the coupling constant, Eq. (2), but no further renormalization effects, i.e., there are no anomalous dimensions. It readily follows that

$$\left\langle \frac{d\Pi_b}{db}\bigg|_{b=0}\right\rangle = -2PL - \frac{1}{\ell}\langle C_\ell\rangle + \frac{1}{\ell^2}\left\langle\frac{\partial C_\ell}{\partial \ell^{-1}}\right\rangle, \quad (9)$$

where we have used the fact that the expectation value of the stress tensor is related to the pressure P via $\langle\Pi\rangle = PL$. The last term is related to how the contact operator changes with ℓ^{-1} and can be ignored, as it vanishes in the zero range limit as Λ^{-1}.

To evaluate the change in the expectation value we use the relation between the stress tensor and the pressure P. In general the pressure can be written as a function of the volume L, the entropy S, the particle number N, and the scattering length ℓ,

$$P = \frac{1}{L^2}\bar{p}\left(\frac{L}{\ell}, S, N\right), \tag{10}$$

for some dimensionless function $\bar{p}$. The scale transformation can be achieved by dilating the volume by a factor $L_b = Le^b$. After dilating the system volume, one finds

$$\frac{d}{db}\langle\Pi\rangle = -2PL - \frac{1}{\ell}\langle C_\ell\rangle - \frac{1}{\ell^2}\frac{\partial}{\partial\ell^{-1}}\langle C_\ell\rangle. \tag{11}$$

The combination of Eqs. (9) and (11) with Eq. (3) shows that the second line in Eq. (3) is related to the change in the contact with respect to the scattering volume. This quantity is naturally related to the stress-tensor correlation function and linear response theory. This is evident when one considers how the contact changes under an infinitesimal time-independent change in the scattering volume, $\ell \to \ell + \delta\ell$:

$$\frac{\partial\langle C_\ell\rangle}{\partial\ell} = -\chi_{\Pi\Pi}(0). \tag{12}$$

This leads us to the final result that the bulk viscosity spectral function is given by

$$\zeta(\omega) = \frac{\chi_{\Pi\Pi}(\omega) - \chi_{\Pi\Pi}(0)}{i(\omega + i\delta)}, \tag{13}$$

where we have muted the dependence on the volume of the system. Equation (13) will be the starting point for our study of the bulk viscosity using the Luttinger liquid formalism.

A. Low-energy Luttinger liquid Hamiltonian

In the low-temperature and weakly interacting limits, the dominant low-energy degrees of freedom are from momentum modes near the two Fermi points, $\pm k_F$, where k_F is the Fermi wave number which is related to the chemical potential $\mu = k_F^2/2m$. In this regime, the dynamics can be formulated in terms of the Luttinger liquid model [37–39]. Here we present some of the details for the derivation of the Luttinger liquid model associated with Eq. (1). We begin with the standard procedure of bosonizing the fields. To this end we expand the fermionic fields around the two Fermi points as

$$\psi(x) = \sum_{r=\pm1} e^{irk_F x}\psi_r(x), \tag{14}$$

where $\psi_r(x)$ is the chiral fermionic mode near the rk_F Fermi point ($r = \pm1$). In terms of these chiral fermionic modes, the Hamiltonian becomes a sum of various terms: $H = H_0 + V_0 + V_1 + V_2 + V_2'$:

$$H_0 = \sum_r \int dx\, \psi_r^\dagger(x)\left[-irv_F\partial_x - \frac{\partial_x^2}{2m}\right]\psi_r(x), \tag{15}$$

$$V_0 = \int dx g k_F^2 n_R(x) n_L(x), \tag{16}$$

$$V_1 = \sum_r \int dx\, irgk_F[\psi_r^\dagger(x)\overleftrightarrow{\partial_x}\psi_r(x)n_{\bar{r}}(x)], \tag{17}$$

$$V_2 = \int dx\, g\psi_R^\dagger(x)\overleftrightarrow{\partial_x}\psi_L^\dagger(x)\psi_L(x)\overleftrightarrow{\partial_x}\psi_R(x), \tag{18}$$

$$V_2' = \sum_r \int dx\, g\psi_r^\dagger(x)\overleftrightarrow{\partial_x}\psi_r^\dagger(x)\psi_r(x)\overleftrightarrow{\partial_x}\psi_r(x). \tag{19}$$

Here $n_r(x) = \psi_r^\dagger(x)\psi_r(x)$ is the density of r-moving fermions. We also note that all these terms should be normal ordered.

In this representation, the interaction V_0 is marginal in a renormalization group sense and provides the standard LL physics. The leading irrelevant interactions are given by the band curvature term in H_0 and the interaction V_1. The interaction terms V_2 and V_2' are even more irrelevant than the previous terms, and shall be neglected in the following calculation.

For our purpose, it is most convenient to write the fermion model in terms of the so-called renormalized chiral mode defined by $\varphi_R(x)$ and $\varphi_L(x)$. For details see Appendix B. In terms of these bosonic modes our effective low-energy Hamiltonian is given by

$$H \approx H_{\rm LL} + V_- + V_+, \tag{20}$$

$$H_{\rm LL} = \frac{v}{2}\int dx\{[\partial_x\varphi_L(x)]^2 + [\partial_x\varphi_R(x)]^2\}, \tag{21}$$

$$V_- = \frac{\sqrt{2\pi}}{6}\tilde{\eta}_-\int dx[(\partial_x\varphi_L)^3(x) - (\partial_x\varphi_R)^3(x)], \tag{22}$$

$$V_+ = \frac{\sqrt{2\pi}}{6}\tilde{\eta}_+\int dx[(\partial_x\varphi_L)^2(x)\partial_x\varphi_R(x) - L \leftrightarrow R], \tag{23}$$

where $H_{\rm LL}$ is the standard LL Hamiltonian, and $\varphi_r(x)$ is a dressed chiral bosonic mode defined in Appendix B.

In Eqs. (21)–(23), v is the velocity of the chiral bosonic mode, $\varphi_r(x)$ with $r = -\bar{r} = \pm1$ for right and left movers, respectively. The values for v and the LL parameter $\mathcal{K}$ are

$$v = v_F\sqrt{1 - \left(\frac{mgk_F}{2\pi}\right)^2}, \tag{24}$$

$$\mathcal{K} = \sqrt{\frac{1 - mgk_F/2\pi}{1 + mgk_F/2\pi}}. \tag{25}$$

The interactions V_- and V_+ are derived from the fermionic operators V_1 and the band curvature portion of H_0. They are the only leading terms beyond the LL model allowed by parity: $\varphi_r(x) \to \varphi_{\bar{r}}(-x)$. V_- couples the like chiral modes while V_+ couples the opposite ones [40]. We label such interactions as intraband and interband scattering, respectively. The explicit expressions for the coupling constants are

$$\eta_- = \frac{\sqrt{2\pi}\tilde{\eta}_-}{6} = \sqrt{\frac{2\pi}{\mathcal{K}}}\frac{1}{24m}\left[3 + \mathcal{K}^2 + \frac{3mgk_F}{2\pi}(\mathcal{K}^2 - 1)\right], \tag{26}$$

$$\eta_+ = \frac{\sqrt{2\pi}\tilde{\eta}_+}{6} = \sqrt{\frac{2\pi}{\mathcal{K}}}\frac{1}{8m}\left[(1 - \mathcal{K}^2) - \frac{mgk_F}{2\pi}(1 + 3\mathcal{K}^2)\right]. \tag{27}$$

Although Eqs. (26) and (27) may look complex, they take simple forms to leading order in ℓ: $\tilde{\eta}_- \approx 1/m$ and $\tilde{\eta}_+ \approx -3k_F\ell/(\pi m)$.

In this representation the density fluctuations are given by

$$n(x) = -\sqrt{\frac{\mathcal{K}}{2\pi}}\sum_{r=\pm 1} r\partial_x\varphi_r(x). \tag{28}$$

We can also define the current and stress-tensor operators according to conservation of number and momentum:

$$0 = \partial_t n(x,t) + \partial_x J(x,t), \tag{29}$$

$$0 = \partial_t J(x,t) + \partial_x \Pi(x,t). \tag{30}$$

These definitions provide a direct evaluation of the current $J(x)$ and the local stress tensor $\Pi(x)$ operators via the Heisenberg equations of motion:

$$J(x) = \sum_r -\sqrt{\frac{\mathcal{K}}{2\pi}}\left\{v\partial_x\varphi_r(x) - \frac{\sqrt{2\pi}r}{6}(3\tilde{\eta}_- + \tilde{\eta}_+)[\partial_x\varphi_r(x)]^2\right\}, \tag{31}$$

$$\Pi(x) = \sum_r -\sqrt{\frac{\mathcal{K}}{2\pi}}\Big[v^2 r\partial_x\varphi_r(x) - 2v\frac{\sqrt{2\pi}\tilde{\eta}_+}{6}\{\partial_x\varphi_r(x)\partial_x\varphi_{\bar{r}}(x) - [\partial_x\varphi_r(x)]^2\} - \frac{4\sqrt{2\pi}}{6}(3\tilde{\eta}_- + \tilde{\eta}_+)\mathcal{H}(x)\Big] \tag{32}$$

The stress tensor employed in Eq. (3) is related to the local stress tensor in Eq. (32) via integration over space: $\Pi = \int dx\Pi(x)$. This gives us our final expression for the stress tensor for the LL model of a zero-ranged spin-polarized Fermi gas:

$$\Pi = -\sqrt{\frac{\mathcal{K}}{2\pi}}\Big[\sum_r\int dx v^2 r\partial_x\varphi_r(x) - 2v\frac{\sqrt{2\pi}\tilde{\eta}_+}{6}\sum_r\int dx\{\partial_x\varphi_r(x)\partial_x\varphi_{\bar{r}}(x) - [\partial_x\varphi_r(x)]^2\} - \frac{4\sqrt{2\pi}}{6}(3\tilde{\eta}_- + \tilde{\eta}_+)H\Big] \tag{33}$$

III. EVALUATION OF BULK VISCOSITY FOR SPIN-POLARIZED FERMI GAS

The calculation of the bulk viscosity spectral function can then be done by substituting Eq. (33) into Eq. (13), and evaluating the resulting response functions perturbatively. Before even performing the calculation, one can immediately see a difference between the intra- and interband couplings.

The intraband coupling constant only enters Π in conjunction with the Hamiltonian H [41]. Hence, this term will not produce a finite bulk viscosity, as any commutator involving an operator and H vanishes in thermal equilibrium. This fact is consistent with a previous calculation, the density-density correlation function [40], where one can evaluate the density-density correlation function perturbatively with respect to V_- and show that it reproduces the noninteracting free fermion result with a renormalized Fermi velocity v and a renormalized mass $1/\tilde{\eta}_-$. Since the noninteracting gas has a vanishing bulk viscosity, the bulk viscosity for a LL with only band curvature corrections is zero. This result is intuitive since in the noninteracting limit $\tilde{\eta}_- = 1/m$, while $\tilde{\eta}_+ = -3k_F\ell/(\pi m)$ vanishes [41].

The true nontrivial interaction effects are contained in V_+, the interband scattering. This interaction introduces a new term into $\Pi(t)$. When evaluating Eq. (13), only this term contributes to the bulk viscosity, as the commutator between H and any operator vanishes in thermal equilibrium. The nonvanishing term is proportional to $\tilde{\eta}_+^2$ at leading order, with higher-order corrections that can be calculated perturbatively using finite temperature field theory. In doing so we replace the retarded correlation function in Eq. (4) with the imaginary time-ordered Green's function and evaluate it to leading order in the scattering volume. The result is (details in Appendix C)

$$\chi_{\Pi,\Pi}(q, i\omega_n) = \mathcal{K}\frac{v^2\tilde{\eta}_+^2}{9}\sum_r\frac{1}{\beta}\sum_{i\eta_n}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\times[D_r(p, i\eta_n)D_{\bar{r}}(-p, -i\eta_n + i\omega_n) + 3D_r(p, i\eta_n)D_r(-p, -i\eta_n + i\omega_n)], \tag{34}$$

where $\beta = 1/T$ the inverse temperature with Boltzmann's constant set to unity, and the bare bosonic propagator, $D_r(q, i\omega_n)$, is given by

$$D_r(q, i\omega_n) = \frac{-rq}{i\omega_n - rvq}. \tag{35}$$

The frequency summation in Eq. (34) can be done analytically, and only has a finite contribution due to the first term,

$$\chi_{\Pi,\Pi}(q, i\omega_n) = -\mathcal{K}\frac{v^2\tilde{\eta}_+^2}{9}\sum_r\times\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{p^2}{i\omega_n - 2rvp}[1 + 2n_B(rvp)], \tag{36}$$

where $n_b(x) = [\exp(\beta x) - 1]^{-1}$ is the Bose-Einstein distribution.

The bulk viscosity spectral function can then be evaluated by substituting the result of Eq. (36) back into Eq. (13) and performing the analytic continuation back to real frequencies,

$i\omega_n \to \omega + i\delta$. This procedure gives the final expression for the bulk viscosity:

$$\zeta(\omega) = \mathcal{K}\frac{\tilde{\eta}_+^2}{72v}\frac{\omega}{\tanh\left(\frac{\beta\omega}{4}\right)}. \tag{37}$$

In the weakly interacting limit this gives

$$\zeta(\omega) \approx \frac{m\ell^2 k_F}{8\pi^2}\frac{\omega}{\tanh\left(\frac{\omega}{4T}\right)}. \tag{38}$$

In Eq. (38) we have added an additional factor of m^2, as our definition of Π differs from the traditional version by a factor of m, see Appendix B.

Equations (37) and (38) are valid to leading order in perturbation theory with respect to V_+. In terms of the scattering volume ℓ, the zero-frequency limit of the bulk viscosity at finite temperatures is

$$\zeta(\omega \to 0) = 1/(2\pi)^2 m k_F \ell^2 T. \tag{39}$$

On the other hand, at strictly zero temperature, the bulk viscosity depends linearly on the frequency,

$$\lim_{T\to 0}\zeta(\omega) = (mk_F/8\pi^2)\ell^2\omega, \tag{40}$$

which is consistent with the vanishing of the density of states near the Fermi surface, $\rho(\omega) \propto \omega^{\frac{1}{2}(\mathcal{K}+1/\mathcal{K})-1} \approx \omega^{2(k_F\ell/\pi)^2}$ [39], i.e., there is a lack of flow.

Moving towards the strong coupling limit, V_- and V_+ are still the only allowed leading irrelevant interactions consistent with parity constraint. In this limit, we do not know the explicit density and interaction dependencies of the speed of sound v, the LL parameter $\mathcal{K}$, or the coupling constant $\tilde{\eta}_+$. However, both the LL parameter and the speed of sound ought to be well behaved around resonance, namely, they can be expanded in powers of $(n\ell)^{-1}$, where n is the density:

$$\mathcal{K} = \bar{\mathcal{K}}\left(\frac{n^2}{mT}\right), \quad v = \frac{n}{m}\,\bar{v}\left(\frac{n^2}{mT}\right). \tag{41}$$

Similarly, we expect that $\tilde{\eta}_+$ should also be to leading order proportional to $1/(n\ell)$ in order to be consistent with scale symmetry arguments:

$$\tilde{\eta}_+ = \frac{1}{n\ell}\bar{\eta}_+\left(\frac{n^2}{mT}\right). \tag{42}$$

From these scaling relations, the bulk viscosity in the strongly interacting regime is of the form

$$\zeta(\omega \to 0) = \frac{1}{n\ell^2}\bar{\zeta}\left(\frac{n^2}{mT}\right) \tag{43}$$

to leading order in the scale-breaking interactions. The behavior of these scaling functions can in principle be obtained from Bethe ansatz calculations [42].

At high temperatures, $T \gg k_F^2/2m$, one can also obtain explicit expressions for the bulk viscosity using the virial expansion in both weak and strong coupling limits. In both limits, Eq. (3) reduces to the evaluation of the contact-contact correlation function just as in the three-dimensional (3D) case [36,43–45]. The calculation is presented in Appendix C. In the weak coupling limit, one finds

$$\zeta(\omega \to 0) \approx \frac{2}{\pi^{5/2}}(k_F\ell)^2 T^{1/2}. \tag{44}$$

Combining Eq. (44) with the result from the low-temperature LL calculation for weak coupling, Eq. (37), we see that the zero-frequency bulk viscosity is proportional to ℓ^2 and vanishes in the limit $\ell \to 0$, consistent with the requirement of scale symmetry. In addition, $\zeta(\omega \to 0)$ exhibits a smooth temperature dependence since there is no finite temperature phase transition in one dimension. This is explicitly shown in Fig. 1.

Near resonance when $k_F\ell \gg 1$, the high-temperature bulk viscosity is given by

$$\zeta(\omega \to 0) \propto \frac{k_F^2}{\ell^2 T^{3/2}}\ln\left(\frac{4}{\ell^2\omega e^{-\gamma_E}}\right). \tag{45}$$

Equation (45) is consistent with the general scaling form presented in Eq. (43), up to a logarithmic factor. In this limit $\bar{\zeta}(x) \propto x^{3/2}$.

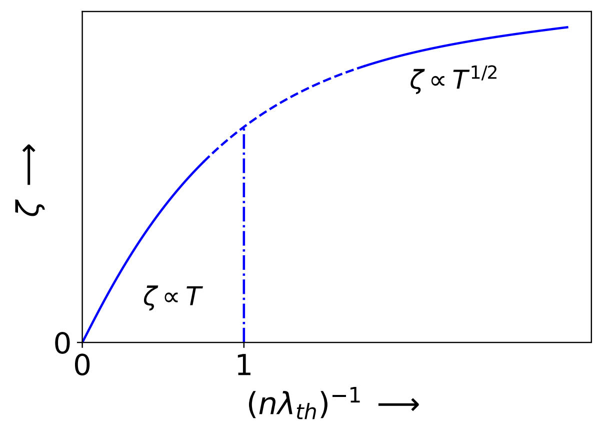

FIG. 1. Bulk viscosity for a weakly interacting spin-polarized 1D Fermi gas with odd-wave interactions as a function of density n. $\lambda_{\rm th} = \sqrt{2\pi/mT}$ is the thermal de Broglie wavelength with $\hbar$ and k_B set to unity. The bulk viscosity at low temperatures is evaluated via the LL model, Eq. (37), while the high-temperature result follows the virial expansion, Eq. (44). The dashed-dotted line where $n\lambda_{\rm th} = 1$ approximates the transition between the LL and virial expansion results. In both limits the bulk viscosity is proportional to ℓ^2 as required by conformal symmetry. Our results in the high- and low-temperature limits smoothly connect with one another, consistent with the lack of a finite-temperature superfluid transition in one dimension,

IV. VISCOSITY IN THE DUAL LIEB-LINIGER MODEL

It is well known that the Cheon-Shigehara model in Eq. (1) is dual to the Lieb-Liniger model of interacting bosons [25,46–49]:

$$H_{\rm LL} = \int dx\,\psi^\dagger(x)\left(-\frac{1}{2m}\partial_x^2 - \mu\right)\psi(x) - \frac{g}{4m}\int dx\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x), \tag{46}$$

where $g = -4/ma$ and a is the 1D scattering length. In particular, this duality relates the strongly interacting limit of

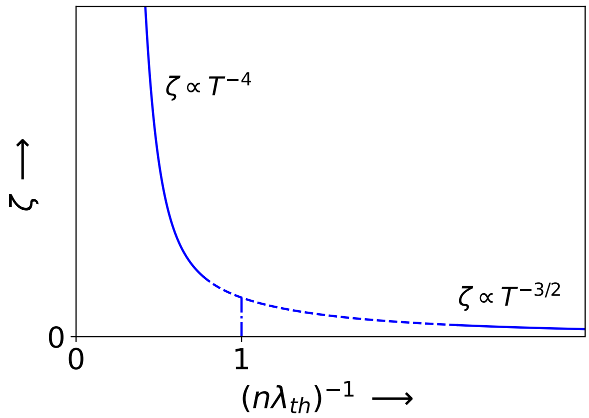

FIG. 2. Bulk viscosity of a spin-polarized Fermi gas near resonance. At high temperatures the bulk viscosity vanishes like $T^{-3/2}$, Eq. (45), while for low temperatures the bulk viscosity diverges as T^{-4}, Eq. (48). The dashed line acts as a guide for the eye, while the vertical dashed-dotted line separates the high- and low-temperature behaviors. From the duality relations, this plot also describes the bulk viscosity for a weakly interacting 1D Bose gas.

one model to the weakly interacting limit of the other. Since the Cheon-Shigehara model is dual to the Lieb-Liniger model of interacting bosons, it is natural to ask whether the Lieb-Liniger model also has a finite bulk viscosity.

To address this, we first consider the high-temperature limit. In the high-temperature limit Eq. (3) can be evaluated analytically to second order in the virial expansion for arbitrary interaction strengths [36,43–45]. Such a calculation for the bulk viscosity of the Lieb-Liniger model suggests the following duality relation (see Appendix D):

$$\zeta_F(\ell) = \zeta_B\left(\frac{1}{a}\right), \tag{47}$$

where $\zeta_F(\ell)$ and $\zeta_B(a)$ are the bulk viscosities for the fermionic Cheon-Shigehara model and the bosonic Lieb-Liniger model, respectively.

Although we calculated the duality explicitly in the high-temperature limit, we expect this duality to hold at arbitrary temperature. As a result, we can deduce that the bulk viscosity of the Lieb-Liniger model near resonance and at low temperatures is given by Eq. (38) with the replacement of ℓ with a^{-1} via Eq. (47).

Similarly, the bulk viscosity for the spin-polarized Fermi gas near resonance and at low temperatures can be determined by evaluating the bulk viscosity for a weakly interacting Bose gas in one dimension. This is done explicitly in Appendix D:

$$\zeta_B(\omega \to 0) \approx \frac{16}{\pi^2}\frac{n^7}{a^2T^4}\log\left(16\frac{T^2}{n^2\omega}\right). \tag{48}$$

From Eqs. (47) and (48) we can infer that the bulk viscosity for the spin-polarized Fermi gas near resonance and at low temperatures is consistent with Eq. (43) with $\bar{\zeta}(x) = x^4$, i.e., the bulk viscosity near resonance diverges as T^{-4} [50]. In Fig. 2 we provide a schematic for the bulk viscosity at low temperatures for the strongly interacting Fermi gas, or equivalently the weakly interacting Bose gas in 1D.

V. COMPARISON TO TRADITIONAL KINETIC THEORY

The results of Secs. III–V were derived using the microscopic Kubo formula for the bulk viscosity, Eq. (3). Although our results are consistent with scale symmetry, they do not conform to the expectations of an integrable model, i.e., a finite Drude peak. Previously in the literature there have been kinetic theory calculations which were consistent with integrability, but not scale symmetry [51,52]. In this section we discuss the discrepancy between the microscopic approach starting from Eq. (3) and kinetic theory.

In the kinetic theory approach, one examines the response of fermionic quasiparticles to a slowly varying velocity gradient. In order to observe dissipation and a finite bulk viscosity, it is necessary to consider interactions between three quasiparticles as the two-body scattering only leads to forward and backward scattering, which does not relax the momentum [53,54]. This argument leads to a bulk viscosity $\zeta \propto k_F^5 \ell^{-2} T^{-3}$ at low temperatures. The kinetic theory then predicts a divergent bulk viscosity in the weakly interacting limit at low temperatures which is inconsistent with the requirement of conformal symmetry [20]. A similar analysis [54] for high temperatures with zero-range p-wave interactions also gives a vanishing bulk viscosity for arbitrary ℓ, because the real part of the quasi-particle self-energy leads only to the renormalization of the chemical potential and the effective mass. In both scenarios, these calculations do not satisfy the symmetry requirements.

The failure of the kinetic theory in describing the bulk viscosity has been addressed in some recent works [55,56]. The conclusion is that the bulk viscosity at heart is related to the propagation of pairs. Hence, it is quite difficult for a single quasi-particle kinetic theory to produce qualitatively correct predictions. Our calculation provides concrete evidence where such a discrepancy is present. It remains an open question whether some modification of kinetic theory is possible to fully quantify the bulk viscosity [56].

VI. CONCLUSIONS

In this article we have shown that the bulk viscosity spectral function is finite for the Cheon-Shigehara model and its dual Lieb-Liniger model to leading order in the interaction. Our results are based on a microscopic Kubo formula for the bulk viscosity that does not depend on the assumptions of hydrodynamics.

The absence of a Drude peak at leading order is striking, as it suggests that integrability does not require the bulk viscosity to be infinite. However, our study does not preclude the existence of a Drude peak at higher orders in the two-body scale breaking interactions. In fact, a recent work suggests that this is indeed the case [26]. The Drude peak is present, but is a higher-order contribution to the Bulk viscosity spectral function. That being said, the Drude peak is still consistent with scale symmetry, and will necessarily vanish when the interactions are scale invariant. Such a conclusion is beyond the previous kinetic theory calculations [51,52], emphasizing the importance of evaluating the bulk viscosity using a microscopic approach.

Note added in proof. Recently, we learned of two works currently in preparation on this subject [26,50]. Reference

[50] provides qualitatively identical results to our own, while Ref. [26] provides evidence for the Drude peak at higher orders in perturbation theory.

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APPENDIX A: TWO-BODY SCATTERING OF THE SPIN-POLARIZED FERMI GAS

The Hamiltonian for this system is given by

$$H = \int dx\, \psi^\dagger(x)\left(-\frac{\partial_x^2}{2m} - \mu\right)\psi(x) - \int dx\, \frac{g}{4}\psi^\dagger(x)\overleftrightarrow{\partial_x}\psi^\dagger(x)\psi(x)\overleftrightarrow{\partial_x}\psi(x), \quad \text{(A1)}$$

where $\psi^{(\dagger)}(x)$ is the fermionic annihilation (creation) operator, m is the atomic mass, μ is the chemical potential, $\overleftrightarrow{\partial_x} = (\overleftarrow{\partial_x} - \overrightarrow{\partial_x})/2$ is the symmetrized derivative that acts on both the immediate left and right, and g is the bare p-wave coupling constant that depends on the ultraviolet cutoff for the theory Λ. In this work we have set $\hbar$ to unity.

In order to regularize the theory, we solve for the two-body T-matrix. Consider two fermions with momenta $Q/2 \pm k$ and total energy $E = Q^2/4m + k^2/m$ scattering into a state of two fermions with momenta $Q/2 \pm k'$. The corresponding matrix element for the two-body T-matrix is

$$\left\langle \frac{Q}{2} \pm k \middle| T \middle| \frac{Q}{2} \pm k' \right\rangle = kk'\tilde{T}(E)$$
$$\frac{1}{\tilde{T}(E)} = \frac{1}{g} - \frac{1}{2}\int_{-\Lambda}^{\Lambda} \frac{dl}{2\pi}\frac{l^2}{E - \frac{Q^2}{4m} - \frac{l^2}{m} + i\delta} = \frac{1}{g} + \frac{m\Lambda}{2\pi} - \frac{m}{4}\sqrt{-E + Q^2/4 - i\delta}. \quad \text{(A2)}$$

Given Eq. (A2), one can show that the scattered relative wave function is given by

$$\psi(x) = \sin(kx) + f_k\frac{x}{|x|}e^{ik|x|}$$
$$f_k = -ik\frac{m\tilde{T}(k^2/m)}{4}, \quad \text{(A3)}$$

where f_k is the scattering amplitude. According to the effective range expansion, the scattering amplitude can be written as

$$f_k = ik\left[-\frac{1}{\ell} - ik + O(k^2)\right]^{-1}, \quad \text{(A4)}$$

where ℓ is the one-dimensional p-wave scattering volume. Comparing Eq. (A2) to Eq. (A4), one can identify

$$\frac{m}{4\ell} = \frac{1}{g} + \frac{m\Lambda}{2\pi}. \quad \text{(A5)}$$

APPENDIX B: LINEARIZATION AND BOSONIZATION OF THE SPIN-POLARIZED FERMI GAS

In order to describe the low-temperature and low-energy properties of the system, we follow the standard procedure of linearizing the fermionic operators around the Fermi surface and bosonizing the result [39]. First we write the fermionic operators as

$$\psi(x) = \sum_{r=\pm 1} e^{irk_F x}\psi_r(x)$$
$$\psi_r(x) = \frac{1}{\sqrt{L}}\sum_k \psi(rk_F + k)e^{ikx}, \quad \text{(B1)}$$

where k_F is the Fermi wave number and $\mu = k_F^2/2m$. In terms of these chiral modes the Hamiltonian in Eq. (A1) is written as

$$H_0 = \sum_r \int dx\, \psi_r^\dagger(x)\left[-irv_F\partial_x - \frac{\partial_x^2}{2m}\right]\psi_r(x), \quad \text{(B2)}$$
$$V_0 = \int dx g k_F^2 n_R(x) n_L(x), \quad \text{(B3)}$$
$$V_1 = \sum_r \int dx\, irgk_F[\psi_r^\dagger(x)\overleftrightarrow{\partial_x}\psi_r(x)n_{\bar{r}}(x)], \quad \text{(B4)}$$
$$V_2 = \int dx\, g\psi_R^\dagger(x)\overleftrightarrow{\partial_x}\psi_L^\dagger(x)\psi_L(x)\overleftrightarrow{\partial_x}\psi_R(x), \quad \text{(B5)}$$
$$V_2' = \sum_r \int dx\, g\psi_r^\dagger(x)\overleftrightarrow{\partial_x}\psi_r^\dagger(x)\psi_r(x)\overleftrightarrow{\partial_x}\psi_r(x). \quad \text{(B6)}$$

In Eqs. (B2)–(B6), $n_r(x) = \psi_r^\dagger(x)\psi_r(x)$ is the density of r-moving fermions. We have also neglected writing the implicit normal ordering operator for clarity.

Secondly, we write the chiral fermionic operators $\psi_r(x)$ as

$$\psi_r(x) = \frac{1}{\sqrt{2\pi\alpha}}e^{\sqrt{2\pi}i\phi_r(x)} = \frac{1}{\sqrt{L}}e^{-\sqrt{2\pi}i\phi_r^+(x)}e^{-\sqrt{2\pi}i\phi_r^-(x)}, \quad \text{(B7)}$$

where $\phi_r(x)$ is the chiral bosonic operator, α is a short distance cutoff that we take to zero at the end of the calculation, and L is the length of the system. We have also neglected the Klein factors for simplicity. In the second equality we write the bosonic field in terms of their annihilation $\phi_r^-(x)$ and creation $\phi_r^+(x)$ parts. The commutation relation between the creation and annihilation operators is

$$[\phi_r^-(x), \phi_{r'}^+(y)] = -\frac{1}{2\pi}\ln\left[\frac{-2\pi ir}{L}(x - y + i\alpha)\right]\delta_{r,r'}. \quad \text{(B8)}$$

From Eqs. (B7) and (B8), one can then write Eqs. (B2)–(B6) in terms of the bosonic field $\phi_r(x)$ by the method of point-splitting:

$$H \approx H_{\rm LL} + H_{bc} + V_1$$
$$H_{\rm LL} = \int dx\left\{\frac{v_F}{2}[(\partial_x\phi_R)^2 + (\partial_x\phi_L)^2] - \frac{gk_F^2}{2\pi}(\partial_x\phi_R)(\partial_x\phi_L)\right\}, \quad \text{(B9)}$$

$$H_{bc} = \frac{\sqrt{2\pi}}{6m}\int dx[(\partial_x\phi_L)^3 - (\partial_x\phi_R)^3], \tag{B10}$$

$$V_1 = \frac{gk_F}{2\sqrt{2\pi}}\int dx[(\partial_x\phi_R)^2(\partial_x\phi_L) - L \leftrightarrow R]. \tag{B11}$$

As we will discuss below, Eq. (B9) gives the standard Luttinger liquid model that describes sound waves with linear dispersions. The contributions from Eqs. (B10) and (B11) give the leading irrelevant interactions.

In defining H we have neglected the contributions due to V_2 and V_2'. These terms are more irrelevant than H_{bc} and V_1. This point can be illustrated by considering the bosonized form of V_2:

$$\begin{aligned}V_2 = &-\frac{g}{24\pi^2}\int dx\big[\partial_x^3\phi_R\partial_x\phi_L + \phi_x\phi_R\partial_x^3\phi_L\big]\\ &+\frac{g}{4\pi}\int dx\big(\partial_x^2\phi_R\big)\big(\partial_x^2\phi_L\big)\\ &+\frac{g}{4\pi}\int dx(\partial_x\phi_R)^2(\partial_x\phi_L)^2\\ &-\frac{g}{6\pi}\int dx[(\partial_x\phi_R)^3\partial_x\phi_L + \partial_x\phi_R(\partial_x\phi_L)^3].\end{aligned} \tag{B12}$$

V_2 has two distinct effects. First, V_2 modifies the dispersion of the sound waves. Equivalently, one can say that the LL parameter and the renormalized sound velocity, defined below, become momentum dependent. As we will discuss later, this momentum dependence is not important to the discussion of the bulk viscosity and can be neglected. Secondly, V_2 generates interactions that are quartic in the bosonic fields. These interactions are more irrelevant than the cubic interactions and can also be ignored.

To bring Eqs. (B9)–(B11) to the standard LL form, define

$$\phi_L = \frac{\tilde\theta + \tilde\phi}{\sqrt{2\pi}} \quad \phi_R = \frac{\tilde\theta - \tilde\phi}{\sqrt{2\pi}}. \tag{B13}$$

In terms of these new fields, one obtains

$$\begin{aligned}H_{\rm LL} &= \int dx\,\frac{v}{2\pi}\left[\mathcal{K}(\partial_x\tilde\theta)^2 + \frac{1}{\mathcal{K}}(\partial_x\tilde\phi)^2\right]\\ H_{bc} &= \int dx\,\frac{1}{6\pi m}[3(\partial_x\tilde\theta)^2(\partial_x\tilde\phi) + (\partial_x\tilde\phi)^3]\\ V_1 &= \int dx\,\frac{gk_F}{(2\pi)^2}[-(\partial_x\tilde\theta)^2(\partial_x\tilde\phi) + (\partial_x\tilde\phi)^3],\end{aligned} \tag{B14}$$

where the renormalized v and LL parameter $\mathcal{K}$ are defined as

$$v = v_F\sqrt{1 - \left(\frac{mgk_F}{2\pi}\right)^2} \quad \mathcal{K} = \sqrt{\frac{1 - mgk_F/2\pi}{1 + mgk_F/2\pi}}. \tag{B15}$$

For our purposes it is more convenient to define renormalized chiral modes via [40]

$$\varphi_L(x) = \sqrt{\frac{1}{2\pi}}\left(\mathcal{K}\tilde\theta(x) + \frac{1}{\mathcal{K}}\tilde\phi(x)\right), \tag{B16}$$

$$\varphi_R(x) = \sqrt{\frac{1}{2\pi}}\left(\mathcal{K}\tilde\theta(x) - \frac{1}{\mathcal{K}}\tilde\phi(x)\right). \tag{B17}$$

In terms of the dressed chiral modes, Eqs. (B16) and (B17), the Hamiltonian becomes

$$H_{\rm LL} = \int dx\frac{v}{2}[(\partial_x\varphi_R)^2 + (\partial_x\varphi_L)^2], \tag{B18}$$

$$V_- = \eta_-\int dx[(\partial_x\varphi_L)^3 - (\partial_x\varphi_R)^3], \tag{B19}$$

$$V_+ = \eta_+\int dx[(\partial_x\varphi_L)^2\partial_x\varphi_R - (\partial_x\varphi_R)^2\partial_x\varphi_L], \tag{B20}$$

$$\eta_- = \frac{\sqrt{2\pi}\tilde\eta_-}{6} = \sqrt{\frac{2\pi}{\mathcal{K}}}\frac{1}{24m}\left[3 + \mathcal{K}^2 + \frac{3mgk_F}{2\pi}(\mathcal{K}^2 - 1)\right], \tag{B21}$$

$$\eta_+ = \frac{\sqrt{2\pi}\tilde\eta_+}{6} = \sqrt{\frac{2\pi}{\mathcal{K}}}\frac{1}{8m}\left[(1 - \mathcal{K}^2) - \frac{mgk_F}{2\pi}(1 + 3\mathcal{K}^2)\right]. \tag{B22}$$

In this representation the density fluctuations are given by

$$n(x) = -\sqrt{\frac{\mathcal{K}}{2\pi}}\sum_{r=\pm1} r\partial_x\varphi_r(x). \tag{B23}$$

Similarly, we define the current and stress tensor according to

$$\begin{aligned}0 &= \partial_t n(x,t) + \partial_x J(x,t)\\ 0 &= \partial_t J(x,t) + \partial_x\Pi(x,t).\end{aligned} \tag{B24}$$

From these definitions, and the Heisenberg equations of motion, the current $J(x)$ and stress tensor $\Pi(x)$ operators are found to be

$$\begin{aligned}J(x) = \sum_r -\sqrt{\frac{\mathcal{K}}{2\pi}}\bigg\{ & v\partial_x\varphi_r(x)\\ & - \frac{\sqrt{2\pi}r}{6}(3\tilde\eta_- + \tilde\eta_+)[\partial_x\varphi_r(x)]^2\bigg\},\end{aligned} \tag{B25}$$

$$\begin{aligned}\Pi(x) = \sum_r -\sqrt{\frac{\mathcal{K}}{2\pi}}\bigg[& v^2 r\partial_x\varphi_r(x)\\ & - 2v\frac{\sqrt{2\pi}\tilde\eta_+}{6}\{\partial_x\varphi_r(x)\partial_x\varphi_{\bar r}(x) - [\partial_x\varphi_r(x)]^2\}\\ & - \frac{4\sqrt{2\pi}}{6}(3\tilde\eta_- + \tilde\eta_+)\mathcal{H}(x)\bigg].\end{aligned} \tag{B26}$$

The current form of $\Pi(x)$ cannot be transparently connected to Eq. (5). That said, when one performs the trace in the noninteracting limit, the stress tensor satisfies

$$\lim_{\ell\to 0}\int dx\Pi(x) = \frac{2H}{m}. \tag{B27}$$

This is the relation expected by conformal symmetry, up to a factor of m.

APPENDIX C: CALCULATION OF THE BULK VISCOSITY USING MICROSCOPIC THEORY

In this section we present a derivation of the bulk viscosity in the high-temperature limit by using the full microscopic theory. We begin with Eq. (13). Although the bulk viscosity is defined in terms of the stress-tensor correlation function, we note that the commutator of the Hamiltonian with any operator vanishes in thermal equilibrium. Equivalently, the bulk viscosity can also be defined in terms of the the contact correlation function [36,43–45],

$$\zeta(\omega) = \frac{\text{Im}[\chi_{CC}(\omega) - \chi_{CC}(0)]}{\ell^2 \omega}, \tag{C1}$$

where

$$\chi_{CC}(\omega) = \int_{-\infty}^{\infty} dt e^{i(\omega+i\delta)t} i\theta(t)\langle[C_\ell(t), C_\ell(0)]\rangle \tag{C2}$$

and the contact operator C_ℓ is given by Eq. (6).

The evaluation of Eq. (C2) can be calculated using finite-temperature field theory. According to the virial expansion, the leading contribution to Eq. (C1) comes from the pair-pair propagator,

$$\zeta(\omega) \approx \int_{-\infty}^{\infty} \frac{dQ}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{n_B(x) - n_B(x+\omega)}{\ell^2 \omega} \times \text{Im}\left[\frac{T(Q, x - i\delta)}{4}\right] \text{Im}\left[\frac{T(Q, x + \omega - i\delta)}{4}\right], \tag{C3}$$

where $n_B(x)$ is the Bose-Einstein distribution at temperature T, and $T(Q, z)$ is the T-matrix, Eq. (A2).

In the high-temperature limit the chemical potential is large and negative, and an expansion in terms of the fugacity, $z = e^{\beta\mu}$, is permissible. This gives the following result for the bulk viscosity spectral function:

$$\zeta(\omega) \approx z^2 \frac{1 - e^{-\beta\omega}}{\ell^2 \omega} \int_{-\infty}^{\infty} \frac{dQ}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{\pi} e^{-\beta\frac{Q^2}{4} - \beta x} \times \text{Im}\left[\frac{T(x - i\delta)}{4}\right] \text{Im}\left[\frac{T(x + \omega - i\delta)}{4}\right] \tag{C4}$$

and

$$\left(\frac{T(x - i\delta)}{4}\right)^{-1} = \frac{1}{\ell} + \sqrt{-x + i\delta}. \tag{C5}$$

In the weakly interacting limit one finds

$$\zeta(\omega \to 0) \approx \frac{2}{\pi^{5/2}} (k_F \ell)^2 T^{1/2}. \tag{C6}$$

To obtain Eq. (C6) we have used the equation of state to relate the fugacity to the density n and the Fermi momentum k_F,

$$z = n\sqrt{2\pi\beta} \quad n = \frac{k_F}{\pi}. \tag{C7}$$

Conversely, in the resonantly interacting limit, $\ell^{-1} = 0$, one obtains

$$\lim_{\ell^{-1}\to 0} \lim_{\omega\to 0} \zeta(\omega) \approx 2\frac{k_F^2}{\pi^{5/2}} \frac{1}{\ell^2} \frac{1}{T^{3/2}} \ln\left(m\ell^2 T\right), \tag{C8}$$

$$\lim_{\omega\to 0} \lim_{\ell^{-1}\to 0} \zeta(\omega) \approx 2\frac{k_F^2}{\pi^{5/2}} \frac{1}{\ell^2} \frac{1}{T^{3/2}} \ln\left(\frac{4T}{\omega}\right). \tag{C9}$$

We briefly note that Eq. (C3) also describes the low-temperature bulk viscosity to leading order in perturbation theory. A direct evaluation of Eq. (C3) using the microscopic model at low temperatures gives results consistent with Eq. (13) in the zero-frequency limit. However, one needs to use the two-body T-matrix in the presence of the many-body background. We will not provide the details here, but provide a similar analysis for the Lieb-Liniger model below. This further confirms that the LL model provides an accurate evaluation of the bulk viscosity.

APPENDIX D: BULK VISCOSITY FOR THE LIEB-LINIGER MODEL

In this Appendix we consider the bulk viscosity spectral function for the Lieb-Liniger model of interacting bosons:

$$H = \int dx \frac{1}{2m} \partial_x\phi^\dagger(x)\partial_x\phi(x) + \int dx \frac{g}{4} \phi^\dagger(x)\phi^\dagger(x)\phi(x)\phi(x), \tag{D1}$$

where $\phi(x)$ is the bosonic operator, m is the atomic mass, and $g = -4/a$ relates the interaction strength to the scattering length a.

In the high-temperature limit, the two-body T-matrix has the form

$$\left(\frac{T(x - i\delta)}{4}\right)^{-1} = -a + \frac{1}{\sqrt{-x + i\delta}}, \tag{D2}$$

where $\xi_k = \frac{k^2}{2m} - \mu$, and $n_B(x) = (e^{\beta x} - 1)^{-1}$ is the Bose-Einstein distribution.

We can again define the stress tensor according to

$$\Pi = 2H + aC_a, \tag{D3}$$

where the contact operator is

$$C_a = \frac{\partial H}{\partial a} = \int dx \frac{g^2}{16} \phi^\dagger(x)\phi^\dagger(x)\phi(x)\phi(x). \tag{D4}$$

We evaluate the bulk viscosity starting from Eq. (13). Substituting Eq. (D3) into Eq. (13), one finds that the bulk viscosity is again related to the contact-contact correlator. The calculation proceeds identically to the case of the LL model of spinless fermions. At leading order in the interaction strength the bulk viscosity can be written as

$$\zeta(\omega) \approx \frac{a^2}{\pi\beta} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{n_B(x) - n_B(x+\omega)}{\omega} \times \text{Im}\left[\frac{T(x - i\delta)}{4}\right] \text{Im}\left[\frac{T(x + \omega - i\delta)}{4}\right], \tag{D5}$$

where $T(x - i\delta)$ is the two-body T-matrix defined in the vacuum. Equation (D5) is valid in the high-temperature limit for arbitrary interaction strengths. Near resonance $na \ll 1$ the bulk viscosity can be analytically evaluated:

$$\zeta(\omega \to 0) \approx \frac{2}{\pi^{5/2}} (k_F a)^2 T^{1/2}, \tag{D6}$$

where we have used $n = k_F/\pi$, and similarly for the weakly interacting limit $na \gg 1$:

$$\lim_{\omega\to 0} \lim_{a^{-1}\to 0} \zeta(\omega) \approx 2\frac{k_F^2}{\pi^{5/2}} \frac{1}{a^2} \frac{1}{T^{3/2}} \ln\left(\frac{4T}{\omega}\right). \tag{D7}$$

Upon inspection of Eqs. (D6) and (D7), one finds that they are equivalent to Eqs. (C6)–(C9) in the opposing limit. This allows one to establish the duality relations

$$\zeta_F(\ell) = \zeta_B\left(\frac{1}{a}\right). \tag{D8}$$

Hence, the bulk viscosity for a strongly (weakly) interacting Fermi gas is equal to the bulk viscosity of a weakly (strongly) interacting Bose gas.

From the duality relation, Eq. (D8), we can obtain the bulk viscosity for the spin-polarized Fermi gas near resonance by considering the weakly interacting limit of the Lieb-Liniger model at low temperatures, which can be evaluated perturbatively. The only required modification to Eq. (D5) is to use the T-matrix calculated within the many-body background. To leading order in perturbation theory one can write the many-body T-matrix as

$$\mathrm{Im}\left[\frac{T(Q, x - i\delta)}{4}\right] \approx \frac{1}{a^2}\int \frac{dk}{\pi}\left[1 + n_B\left(\xi_{\frac{Q}{2}+k}\right) + n_B\left(\xi_{\frac{Q}{2}-k}\right)\right] \times \pi\delta\left(x - \xi_{\frac{Q}{2}+k} - \xi_{\frac{Q}{2}-k} - i\delta\right). \tag{D9}$$

From this expression, one can show that the integrand is dominated near $\epsilon \approx 0$ and $Q \approx 0$. Expanding near this point and evaluating the resulting integrals gives

$$\zeta_B(\omega \to 0) \approx \frac{16}{\pi^2}\frac{n^7}{a^2T^4}\log\left(16\frac{T^2}{n^2\omega}\right). \tag{D10}$$

In order to obtain Eq. (D10) we have used the noninteracting equation of state:

$$n = \frac{1}{\sqrt{2\pi\beta}}\mathrm{Li}_{1/2}(e^{\beta\mu}), \tag{D11}$$

where $\mathrm{Li}_s(x)$ is the polylogarithm function. As one approaches the zero-temperature limit for fixed density, one can show $\sqrt{2\beta}n = (-\beta\mu)^{-1/2}$, i.e., $\mu \to 0^-$ as $T \to 0$. In Ref. [50], a similar result to Eq. (D10) was found. There the authors worked at strictly zero frequency, and thus the logarithm in Eq. (D10) is regulated by the scattering length a instead of the frequency.

From Eqs. (C9) and (D10) one can construct the bulk viscosity for a 1D bosonic gas for arbitrary temperatures and for weak interactions, shown in Fig. 2. From the duality relation Fig. 2 describes the bulk viscosity for 1D Bose gas weak interactions and the strongly interacting spin-polarized Fermi gas.

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