

## Asymmetry and nonlinearity of current-bias characteristics in superfluid–normal-state junctions of weakly interacting Bose gases

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We uncover current-bias characteristics of superfluid-normal state junctions with weakly-interacting Bose gases. It is shown that in the presence of a chemical potential bias the characteristics can strongly be asymmetric for origin. The salient feature that is absent in the fermionic counterpart arises from a tunneling process associated with a condensate and a bosonic Andreev reflection process. It turns out that such processes are intrinsically nonlinear and therefore do not obey Ohm's law even at a low bias. In addition, the remaining processes are found to obey Ohm's law and become dominant for transport driven by a temperature bias.

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Recent years have witnessed spectacular development of atomtronics [1,2], where quantum transport phenomena with ultracold atomic gases have been studied under various geometries such as quantum point contact [3], planar junction [4,5], and junction on spin space [6]. Owing to controllability of ultracold atomic gases, one can utilize quantum states whose realizations are elusive with typical condensed matter systems including metals, semiconductors, and superconductors. A weakly-interacting Bose-Einstein condensate (BEC) is the stellar example in that our understanding on it has significantly deepened since the first realization with ultracold atomic gases [7,8]. When it comes to quantum transport of the BECs, Josephson and dissipative currents passing through barriers [9–13] and tubes [14,15] have already been measured.

In quantum transport, it is known that junction systems in which different quantum states are attached play a crucial role in condensed matter physics. A celebrated example is the  $p$ - $n$  junction, which has a direct relevance to electronics and has been applied for semiconductor devices such as diodes and transistors [16]. Another famous example, which is related to the present paper, is the superconductor-normal metal (or semiconductor) junction [17]. In this system, one can observe the current-bias characteristics that originate from suppression of the quasiparticle transport process [18] and presence of the effective pair transport process due to the Andreev reflection [19]. Especially, the latter process is associated with the conversion between particle and hole, which is imprinted as particle-hole mixing in superconductors described by Bogoliubov-de-Gennes Hamiltonian. Since weakly-interacting BECs at low temperature are described by the Bogoliubov Hamiltonian that has the  $2 \times 2$  matrix structure analogous to one in superconductors, the existence of bosonic Andreev reflections in such BECs is expected [20–22]. However, transport characteristics of a superfluid-normal state (SN) junction composed of weakly-interacting bosons and the role of the bosonic Andreev reflection process in terms of mesoscopic transport have yet to be understood.

In this paper, we microscopically investigate transport characteristics of SN junctions made of weakly-interacting Bose gases [Fig. 1(a)]. By analyzing typical situations such as planar junction and point contact via the tunneling Hamiltonian formalism, we demonstrate that unlike the superconductor-normal metal junctions, the current-chemical potential bias characteristics show an asymmetry for origin [Fig. 1(b)], which is significant in the vicinity of the critical temperature of a BEC. This notable feature of the bosonic junction is ascribed by presences of a tunnel coupling between condensate and normal boson and bosonic Andreev reflection process, both of which remarkably do not obey Ohm's law. Due to chemical potential bias ( $\Delta\mu$ ) and tunnel-coupling dependencies, the former is found to be more dominant than the latter over a low-bias regime in which  $\Delta\mu$  is close to the chemical potential of noninteracting bosons. We also show that the remaining processes that appear in the corresponding fermionic system are conventional in that an Ohmic response is obtained in the presence of a chemical potential bias and become dominant in transport induced by a temperature bias.

We consider a two-terminal configuration where the left (right) reservoir is filled by normal (superfluid) bosons and the coupling between reservoirs can be captured with the following tunneling term:

$$H_T = \sum_{\mathbf{p}, \mathbf{k}} t_{\mathbf{p}, \mathbf{k}} b_{\mathbf{p}, R}^\dagger b_{\mathbf{k}, L} + \text{H.c.} \quad (1)$$

Here  $t_{\mathbf{p}, \mathbf{k}}$  is the tunneling amplitude, and  $b_{\mathbf{k}, L}$  ( $b_{\mathbf{p}, R}^\dagger$ ) the annihilation (creation) operator of a boson with momentum  $\mathbf{k}$  ( $\mathbf{p}$ ) in the left (right) reservoir. In order to obtain comparable results with superconductor-normal metal junctions in which the Bogoliubov-de-Gennes approach is applicable, we focus on a situation that an interaction between bosons in each reservoir is short range and weakly repulsive. In such a situation, the superfluid reservoir at low temperature and the normal reservoir can be described by the Bogoliubov theory and the Hartree-Fock (HF) theory [8], respectively.

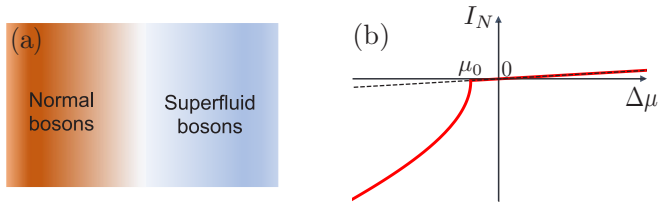


FIG. 1. (a) Superfluid-normal state junction of a weakly-interacting Bose gas. We focus on a situation that the transport properties can be described by the tunneling Hamiltonian formalism. (b) Typical particle current behavior as a function of a chemical potential bias  $\Delta\mu$  in the corresponding junctions (solid-red curve). The characteristic deviates from symmetric one (dashed line) due to presences of tunneling process associated with a condensate and bosonic Andreev reflection process, both of which emerge for  $\Delta\mu < \mu_0$  with chemical potential of noninteracting bosons  $\mu_0$ .

With these understandings, we now evaluate the particle current in SN junctions of bosons. By taking the convention that a particle flow from left to right reservoir is positive, the particle current at time  $\tau$  is expressed as

$$I_N = 2 \sum_{\mathbf{p}, \mathbf{k}} \text{Re}[t_{\mathbf{p}, \mathbf{k}} G_{\mathbf{k}, \mathbf{p}, LR}^<(\tau, \tau)], \quad (2)$$

where  $G_{\mathbf{k}, \mathbf{p}, LR}^<(\tau, \tau') = -i \langle b_{\mathbf{p}, R}^\dagger(\tau') b_{\mathbf{k}, L}(\tau) \rangle$  is lesser Green's function [23]. We are especially interested in mesoscopic transport induced by chemical potential or temperature bias between reservoirs. Therefore, we turn to adapt the Keldysh formalism [24,25], which provides an efficient program for calculation of mesoscopic currents. In addition, since in the SN system an AC component is absent in the presence of a static bias unlike superfluid-superfluid junctions [26,27], it is also convenient to work in the frequency space, where the current is expressed as

$$I_N = \int_{-\infty}^{\infty} d\omega I_N(\omega), \quad (3)$$

with the integrand  $I_N(\omega)$  with respect to frequency  $\omega$ . By solving the Dyson equation for lesser Green's function with the help of the Langreth rule [24], we find that the current including all processes up to infinite order in tunneling amplitude is obtained as [28]

$$I_N = I_c + I_1 + I_2 + I_3 + I_A. \quad (4)$$

The expression above shows that the particle current consists of five different processes. First,  $I_c$  is unique to the present system being absent in the superconducting junction case and describes a process in which a particle in the normal phase is converted into a condensate in the superfluid phase. We point out that this process resembles a conversion process between condensate and Bogoliubov mode appearing in the superfluid-superfluid junction [4,26,27,29–31]. In such a superfluid junction, the presence of the conversion process between condensate and normal boson (Bogoliubov mode) is inevitable for tunneling where the momentum conversation is broken. In addition,  $I_1$ ,  $I_2$ , and  $I_3$  are processes, which are also present in the fermionic case, describe normal particle transfer between reservoirs, transfer of a particle with creation or annihilation of pairs, and a process in which a particle in

the normal phase is converted into a hole in the superfluid phase, respectively [32,33]. Finally,  $I_A$  is the bosonic Andreev reflection process where a particle in the normal phase is reflected as a hole.

In order to see a qualitative difference between bosonic and fermionic systems in a simple manner, we first look at the planar junction case in which the current analysis up to second order in tunneling amplitude is reasonable. In this case, the particle current passing through the junction is expressed as

$$I_N \approx I_c + I_1. \quad (5)$$

Here,  $I_2$ ,  $I_3$ ,  $I_A$  are extinguished, since these are composed of the higher-order processes in tunneling amplitude [28]. Whereas  $I_1$  is the only contribution in the tunneling junction of the fermionic system, the contribution of  $I_c$  also appears in the present bosonic system. The presence of the additional contribution and the difference of quasiparticle excitations between bosons and fermions give rise to a nontrivial current response. To demonstrate this, we focus on the particle current in a low chemical-potential bias ( $\Delta\mu$ ) regime where the low-energy excitations play dominant roles [26,27]. In this case, we find that  $I_1$  in the bosonic junction obeys Ohm's law and therefore its differential conductance  $G_1$  is independent of  $\Delta\mu$ . This implies that  $I_1$  component in the current-bias curve is symmetric for origin in a similar manner to the corresponding fermionic junction. We also point out that  $G_1$  shows a peculiar temperature dependence depending on the temperature regime of the normal bosons in which the chemical potential of noninteracting bosons  $\mu_0$  is a key parameter within the HF theory;  $G_1$  is proportional to  $T^{7/2}$  near the critical temperature of BECs  $T_c$  where  $\mu_0 \rightarrow 0^-$ , whilst at high temperature where  $\mu_0/T \rightarrow -\infty$  it is suppressed by an exponential factor whose argument is proportional to  $\mu_0/T$  [28]. Especially, the fractional temperature dependence near  $T_c$  reflects on the density of states in normal bosons  $\rho_N(\omega) \propto \sqrt{\mu_0 + \omega}$ . Such a behavior in  $I_1$  is different from the fermionic junction in which the quasiparticle current as a function of  $\Delta\mu$  is suppressed by the factor  $e^{-\Delta/T}$  below the superconducting gap  $\Delta$  and shows the abrupt enhancement near  $\Delta$  [17]. The difference between bosonic and fermionic junctions arises from facts that the quasiparticle excitation in the fermions has the gap while that in the bosons is gapless, and frequency dependence of density of states in normal fermions is ignorable owing to the Fermi level while that in normal bosons is non-negligible.

On the other hand,  $I_c$  being inherent to the bosonic junction shows an asymmetric behavior in  $\Delta\mu$ , since it vanishes for  $\Delta\mu > \mu_0$ . This property arises from the fact that  $I_c$  is proportional to  $\rho_N(-\Delta\mu)$ , which becomes zero unless  $\mu_0 - \Delta\mu > 0$ . Physically, this means that for  $\mu_0 - \Delta\mu < 0$  the conversion between the condensation element and normal boson does not occur, as a particle in the normal phase cannot find the corresponding condensation element. Notice that for the low- $\Delta\mu$  regime  $I_c$  is absent at high temperature where  $\mu_0$  takes a large negative value. In addition, the appearance of the factor  $\rho_N(-\Delta\mu)$  implies that  $I_c$  is intrinsically nonlinear and has a square root dependence in  $\Delta\mu$ . The presence of the nonlinear curve due to  $I_c$  is peculiar to the bosonic SN junction and is absent in the superfluid-superfluid junction of bosons [4,26,27,29–31].

Now that the particle current in the tunneling regime has been demonstrated, we turn to look at the higher-order tunneling effects. To see this, we consider the single-mode point-contact system in which tunneling occurs between the single points in each reservoir [6,27,28,33–35]. For the point-contact system, higher-order tunneling processes are generated by controlling the gate potential applied to constriction [3] or increasing an atom-impurity potential via the Feshbach resonance [6], and therefore, all the contributions in the right-hand side of Eq. (4) are facilitated. The detailed calculation based on the Keldysh formalism reveals that the qualitative change of  $I_c$  compared to the planar junction case does not occur in that  $I_c$  is proportional to  $\rho_N(-\Delta\mu)$ , whereas the higher-order tunneling processes give an impact on its proportional coefficient [28]. Thus, it follows that as in the planar junction case  $I_c$  in the point contact case is relevant near  $T_c$ . In addition, we find  $I_1 \approx G_1 \Delta\mu$  for low  $\Delta\mu$ , where  $G_1$  is independent of  $\Delta\mu$  similar to the planar junction. Due to the difference in  $t_{\mathbf{p},\mathbf{k}}$  between the planar junction and the point contact, however,  $G_1$  in the point contact is found to be proportional to  $T^{3/2}$  near  $T_c$ , which is different from one in the planar junction [28]. With increasing the coupling between reservoirs,  $I_2$  and  $I_3$  whose leading order contributions are respectively proportional to fourth order and sixth order in  $t_{\mathbf{p},\mathbf{k}}$  are generated. By performing an explicit calculation for low  $\Delta\mu$ , we find that  $I_2$  and  $I_3$  also obey Ohm's law where the corresponding conductances  $G_2$  and  $G_3$  are proportional to  $T^{3/2}$  near  $T_c$  and are exponentially suppressed at high temperature as in  $G_1$  [28].

In contrast,  $I_A$  whose leading contribution is proportional to the fourth order in  $t_{\mathbf{p},\mathbf{k}}$  turns out to possess a non-Ohmic character. To see this, we point out that the integrand in  $I_A$  contains the product of densities of states in particle and hole branches of normal bosons, that is,

$$\rho_N(\omega - \Delta\mu)\rho_N(-\omega - \Delta\mu). \quad (6)$$

Thus, it follows that in order for  $I_A$  to be nonzero, there must be frequencies that simultaneously satisfy  $\mu_0 + \omega - \Delta\mu > 0$  and  $\mu_0 - \omega - \Delta\mu > 0$ . This indicates that  $I_A$  can be nonzero for  $\Delta\mu < \mu_0$  and contribute to low-energy transport near  $T_c$  similar to  $I_c$ . In addition, the fact that a particle is reflected as a hole in the Andreev reflection process implies that the integrand in  $I_A$  also contains the factor

$$n_L(\omega - \Delta\mu) - n_L(\omega + \Delta\mu), \quad (7)$$

where  $n_L$  is the Bose distribution function in (normal) reservoir  $L$ . Equation (7) is linear in  $\Delta\mu$  for the low-bias regime. However, Ohm's law is prohibited in  $I_A$  due to the dependence in densities of states [Eq. (6)], which introduces an additional  $\Delta\mu$  dependence in the integrand of  $I_A$ . An explicit numerical analysis shows that  $I_A \propto (\Delta\mu - \mu_0)^2 \theta(\mu_0 - \Delta\mu)$  for small  $\Delta\mu$  and  $\mu_0$  [28]. This is in sharp contrast to the Andreev reflection contribution in the superconductor where the Andreev current obeys Ohm's law for the low-bias regime [32,33]. We also point out that up to this order of the expression,  $I_A$  has no temperature dependence as in  $I_c$ . Since the leading order contribution in  $I_c$  is the second order in  $t_{\mathbf{p},\mathbf{k}}$  and  $I_c \propto \sqrt{\mu_0 - \Delta\mu}$ , however,  $I_c$  is more dominant than  $I_A$  over the low-bias regime.

Besides transport induced by a chemical potential bias, we can discuss one induced solely by a temperature bias between

reservoirs  $\Delta T$ . Regarding the particle current, it turns out that both  $I_c$  and  $I_A$  become zero. The former contribution is absent since the density of states in the normal bosons vanishes at zero frequency. On the other hand, the latter one is absent due to the fact that the Andreev reflection process contains the factor Eq. (7) in the integrand, which trivially vanishes at  $\Delta\mu = 0$ . In other words, the remaining processes  $I_1$ ,  $I_2$ , and  $I_3$  are generated by  $\Delta T$ . It is then straightforward to show that in the presence of small  $\Delta T$  these contributions are proportional to  $\Delta T$ . Furthermore, we discuss the heat current  $I_Q$  induced by  $\Delta T$ . By using lesser Green's function, it is expressed as

$$I_Q = 2 \sum_{\mathbf{p},\mathbf{k}} \lim_{\tau' \rightarrow \tau} \text{Re}[it_{\mathbf{p},\mathbf{k}} \partial_{\tau} G_{\mathbf{k},\mathbf{p},LR}^<(\tau, \tau')]. \quad (8)$$

As in the case of the particle current, the heat current can efficiently be calculated in the frequency space. Then,  $i\partial_{\tau}$  in Eq. (8) is replaced by  $\omega$  in the frequency representation and the integrand of Eq. (8) is given by one of Eq. (2) multiplied by  $\omega$ , i.e.,  $I_Q = \int_{-\infty}^{\infty} d\omega \omega I_N(\omega)$ . The direct evaluation of  $I_Q$  shows that the contributions coming from the conversion process between condensate and normal boson and the Andreev reflection process vanish, and the remaining processes corresponding to  $I_1$ ,  $I_2$ , and  $I_3$  in the particle current become dominant. In addition, we can show that for the low- $\Delta T$  regime,  $I_Q$  is proportional to  $\Delta T$ , implying the presence of Fourier's law.

We now discuss possible realizations of the bosonic SN junctions with ultracold atomic gases. Unlike condensed-matter setups where superconductor and normal metal consist of different materials, junction systems in cold-atom experiments are made of an atomic gas originally trapped in one [3]. By considering that the critical temperature of the BEC depends on an atomic density, in order to verify the predictions discussed in this paper, one has to make a two-terminal system where the chemical potential difference between reservoirs is small and density difference between reservoirs is sufficiently large such that the thermodynamic properties of the left and right reservoirs are described by the HF and the Bogoliubov theories, respectively. To achieve this, one can use a repulsive external potential, which is only added in the left reservoir and compensates the chemical potential difference arising from the large density difference between reservoirs [36]. Another way to make the similar system is to use the synthetic-reservoir technique available in ultracold atomic gases with internal degrees of freedom. There, the hyperfine or nuclear spin index of each atom is regarded as the reservoir index, and tunneling and chemical-potential shift between reservoirs can be introduced as coupling and energy-level shift between spins, respectively. Such a technique has already been implemented to confirm the Josephson effect [37] and point contact transport [6].

For realizations of the SN junctions, it is instructive to point out that due to the critical fluctuation the HF theory is broken down in the very vicinity of  $T_c$ , where  $\mu_0$  approaches to zero. By using the Ginzburg criterion, such a fluctuation is important around the critical chemical potential  $\mu_c = (4\pi a)^2 M T_c^2$  with the mass of an atom  $M$  and the  $s$ -wave scattering length  $a$  [8]. In order to see the existence of the regime where the theoretical analysis addressed in this paper is justified in a concrete

way, we consider a system of  $^{87}\text{Rb}$  atoms and assume that the atomic densities in the normal and superfluid reservoirs are respectively of the order of  $10^{11}\text{ cm}^{-3}$  and  $10^{13}\text{ cm}^{-3}$ . In this case,  $T_c \sim 10^{-9}\text{ K}$  and  $\mu_c \sim 10^{-11}\text{ K}$  in the normal reservoir, while  $T_c \sim 10^{-8}\text{ K}$  in the superfluid reservoir. Thus, if the temperature of the system is  $T \sim 10^{-9}\text{ K}$ , we can reach the regime in which the superfluid reservoir is described by the Bogoliubov theory and normal reservoir is done by the HF theory while avoiding the critical fluctuation.

To summarize, we addressed the current-bias characteristics of the bosonic SN systems. Owing to the presence of the tunneling process inherent to the bosonic systems and the excitation properties, the transport characteristics turn out to be drastically different from the fermionic counterparts. Especially, we revealed that in the vicinity of the critical temperature of a BEC, the current-chemical potential bias characteristics can be nonlinear due to the conversion process between condensate and normal boson and the bosonic Andreev process. At the same time, such processes yielding the nonlinear character are found to be irrelevant for transport induced by a temperature bias.

The results shown in this paper may encourage a variety of related works. An analysis in terms of the scattering formalism with the Bogoliubov equation is an interesting direction for future research. Such an analysis may play a

key role in connecting the tunneling process such as the conversion process discussed in this paper with the so-called quantum condensation and evaporation [38–42]. In addition, the formalism explored in this work can be applied to various BEC systems discussed in ultracold atomic gases. Weakly-interacting Bose gases with spin could be one of the systems. There, the current-bias characteristics may further be nontrivial, since such BECs possess a variety of Bogoliubov excitations depending on the phases realized [43]. Similarly, junction systems of the binary mixtures with or without spin-orbit coupling are promising in terms of comparable superconducting systems [44]. Furthermore, the SN junction in two-component Fermi gases may be interesting in terms of Bardeen-Cooper-Schrieffer (BCS)-BEC crossover [45]. There, questions would be how evolution of the transport characteristics as a function of the  $s$ -wave scattering length and crossover between fermionic and bosonic Andreev reflections are. Another promising direction may be to look at optical lattice systems in which an effect of commensurability is significant near the Mott-insulator transition [46].

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