Letter

Activating strong nonlocality from local sets: An elimination paradigm

Subhendu B. Ghosh,[1] Tathagata Gupta,[1] Ardra A. V.,[2] Anandamay Das Bhowmik,[1] Sutapa Saha,[1] Tamal Guha,[3] and Amit Mukherjee[4]

[1]*Physics and Applied Mathematics Unit, Indian Statistical Institute, Kolkata, 203 B. T. Road, Kolkata 700108, India*
[2]*School of Physics, IISER Thiruvananthapuram, Vithura, Kerala 695551, India*
[3]*Department of Computer Science, The University of Hong Kong, Pokfulam Road, Hong Kong*
[4]*Centre for Astroparticle Physics and Space Science (CAPSS), Bose Institute, Block EN, Sector V, Salt Lake, Kolkata 700091, India*

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Apart from the Bell nonlocality, which deals with the correlations generated from the local input-output statistics, quantum theory exhibits another kind of nonlocality that involves the indistinguishability of the locally preparable set of multipartite states. While Bell-type nonlocality cannot be activated from a given local correlation via local operations and shared randomness, it is already reported that the latter kind of nonlocality can be activated from a "local," i.e., locally distinguishable set of states. However, recently it is shown that a stronger notion of such a nonlocality, which deals with elimination instead of discrimination, can be activated from locally preparable bipartite states of dimension 7×8. The present work observes that the same notion can be demonstrated even in lower dimensional multipartite systems. Importantly, the possibly strongest version of such an activation is further depicted here, where none of the transformed product states can be eliminated, even if all but one of the parties come together.

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Introduction. The celebrated notion of Bell nonlocality [1,2] excludes any local-realistic description to substitute multipartite quantum correlations. This, in turn, identifies quantum correlations to be advantageous over its classical counterpart in several practical applications [3–16]. However, quantum theory admits a more elegant nonclassicality in question of state discrimination. While given a single copy every pure classical preparation can be distinctively identified from the others, only the orthogonal quantum states are perfectly distinguishable with a single copy. The volume of such distinguishable states for the multipartite scenario gets further decreased under the limited measurement setting, like local operations and classical communication (LOCC) [17–23]. Unlike most of the nonclassical aspects of quantum correlations obtained from the entangled quantum states, Bennett *et al.* [24] first reported that the LOCC indistinguishability holds even for orthogonal product quantum states and coined the term *quantum nonlocality without entanglement* for such a phenomenon. Consequently, a series of important results have been carried out in this direction [25–35] which have significant importance to understand the complex topology of quantum state space structure [36–41].

Limits on state discrimination in quantum theory further give rise to several interesting questions in the context of state elimination [42–45], where instead of identification the main goal is to rule out one or more quantum states from an ensemble of consideration. Now, if the performed measurement preserves the orthogonality of the remaining states after elimination, then it is further helpful in the context of the state discrimination. Motivated by this fact, recently Halder *et al.* have introduced a stronger notion of quantum nonlocality for product states which forbids elimination of any of the product states of a set under orthogonality preserving local measurements (OPLMs or local OPMs) [40]. Consequently, these fueled a plethora of interesting studies in the recent past from the stronger perspective of state indistinguishability, i.e., irreducibility under OPLM [23,46–49].

Apart from revealing the elegant intricacies of state space structure, local indistinguishability and irreducibility of quantum states also indicate the prospect of locking of information such that unlocking requires entangled resources. This characteristic certainly has a crucial significance in various quantum cryptographic schemes, viz., secret sharing and data hiding [50–54]. However, in the practical settings, the complexity to retrieve a hidden information should depend on their mutual trustworthiness. Also, it might be important for one of those agents to manipulate the complexity should their mutual trustworthiness change after they have shared the secret with each other. For instance, consider three agents Alice, Bob, and Charlie who agree to share a LOCC distinguishable quantum secret at first. However, in time, Charlie may distrust others and want to update the complexity of the secret, upon which the revealing of the secret must demand all of them to be in the same laboratory. This motivates one to propose another version of quantum nonclassicality, which deals with the activation of quantum nonlocality from the locally distinguishable quantum states. The framework has recently been reported in [55] for initially distinguishable entangled states and in [56] for product states. Note that the task can be trivially accomplished if the agents flag the indistinguishable ensemble with a distinguishable one and according to the trust update Charlie can discard his distinguishable share. This redundancy

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is termed as activation of nonlocality from a locally redundant set [55].

In the present work, we have dealt with the activation of a stronger nonlocality from a set of locally distinguishable product states, which are also free from local redundancy. Precisely speaking, besides the nonlocal aspects of state discrimination, we have further considered a stronger version, which is related to the impossibility of state elimination instead of state discrimination. Notably, the authors in [56] have introduced a similar notion for bipartite product states, however with a higher dimensional quantum state. In contrast, we have shown that such a feature is generic even in the smaller dimensional quantum systems. Further, we have extended this activation phenomenon in the multipartite scenario and have come up with the possibly strongest nonlocality activation. In particular, performing OPLMs on a $\mathbb{C}^{6^{\otimes 3}}$ we transform the set to tripartite orthogonal product qutrits, which is locally irreducible even if all but one player come together. Lastly, motivated by the practical situation of trust-updated secret sharing we propose a set of states in $\mathbb{C}^{3^{\otimes 2}} \otimes \mathbb{C}^6$, which can be used to activate the possibly strongest form of nonlocality by performing an OPLM only at the third party's possession. In addition, our results also draw a significant difference between the two types of quantum nonlocality—while the Bell nonlocality cannot be activated from a shared local correlation [57,58], the stronger version of nonlocality related to state identification can be activated from locally distinguishable product states. Importantly, our last example activates the possibly strongest version of nonlocality without involving any communication between the parties.

Genuine activation of strong quantum nonlocality without entanglement. We will start our exploration of genuinely activable sets by showing that a locally distinguishable set of bipartite product states can be transformed to a set of locally irreducible orthogonal states via local orthogonality preserving measurements. Let us begin with a precise definition for local irreducibility, which is a stronger version of state indistinguishability.

Definition 1. A set of multipartite states is said to be locally irreducible, if, given an unknown state from the set, there is no LOCC implementable measurement to eliminate the possibility of an element of the set, keeping the others mutually orthogonal to each other.

Notably, the absence of nontrivial OPLMs for each of the individual parties sufficiently characterizes the set to be locally irreducible [40].

Now, consider the set $\mathcal{G}_1 \equiv \{|\psi_i\rangle_{AB}\}_{i=1}^5 (\subset \mathbb{C}^3 \otimes \mathbb{C}^6)$, where

$$|\psi_1\rangle_{AB} = |0\rangle_A|\mathbf{0}-\mathbf{1}+\mathbf{4}-\mathbf{5}\rangle_B, \tag{1a}$$

$$|\psi_2\rangle_{AB} = |2\rangle_A\,|\mathbf{1}-\mathbf{2}+\mathbf{5}-\mathbf{3}\rangle_B, \tag{1b}$$

$$|\psi_3\rangle_{AB} = |1-2\rangle_A\,|\mathbf{0}-\mathbf{4}\rangle_B, \tag{1c}$$

$$|\psi_4\rangle_{AB} = |0-1\rangle_A\,|\mathbf{2}-\mathbf{3}\rangle_B, \tag{1d}$$

$$|\psi_5\rangle_{AB} = |0+1+2\rangle_A\,|\mathbf{0}+\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{4}+\mathbf{5}\rangle_B. \tag{1e}$$

Proposition 1. The set $\mathcal{G}_1$ is locally distinguishable and free from local redundancy.

Proof. It is quite straightforward to prove that the set $\mathcal{G}_1$ is without local redundancy. Here, Bob's system can be considered to be the composition of qubit and qutrit subsystems. Precisely, $|\mathbf{0}\rangle_B := |00\rangle_{b_1b_2}$, $|\mathbf{1}\rangle_B := |01\rangle_{b_1b_2}$, $|\mathbf{2}\rangle_B := |02\rangle_{b_1b_2}$, $|\mathbf{3}\rangle_B := |10\rangle_{b_1b_2}$, $|\mathbf{4}_B\rangle := |11\rangle_{b_1b_2}$, $|\mathbf{5}\rangle_B := |12\rangle_{b_1b_2}$. Take two states, $|\psi_3\rangle_{AB}$ and $|\psi_4\rangle_{AB}$. When any of the subparts (qubit or qutrit) of Bob's system for both states is discarded the reduced states will be nonorthogonal.

Furthermore, $\mathcal{G}_1$ can also be shown to be locally distinguishable. The players pursue the following distinguishability protocol. First Bob performs a measurement $N_B \equiv \{N_1 := P[|\mathbf{0}-\mathbf{4}\rangle_B], N_2 := P[|\mathbf{2}-\mathbf{3}\rangle_B], N_3 := P[|\mathbf{0}+\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{4}+\mathbf{5}\rangle_B], N_4 := \mathbb{I} - (N_1+N_2+N_3)\}$. Here, $P[(|i\rangle, |j\rangle)_\#] := (|i\rangle\langle i| + |j\rangle\langle j|)_\#$, and # denotes the party. When N_1 clicks, the given state must be $|\psi_3\rangle$. Similarly, for the click N_2, the state is $|\psi_4\rangle$, and for N_3 it is $|\psi_5\rangle$. Whenever N_4 clicks the given state can be either $|\psi_1\rangle$ or $|\psi_2\rangle$. However, in that case, Alice can perform a measurement to distinguish between these two [17,59]. This concludes the local distinguishability protocol for the set $\mathcal{G}_1$. ■

In the following, we will demonstrate a protocol to activate the strong nonlocality without entanglement from the set $\mathcal{G}_1$.

Theorem 1. The locally distinguishable set $\mathcal{G}_1$ can be transformed deterministically to a locally irreducible set via OPLM.

Proof. Consider that Bob performs a local OPM on the subsystem B: $K_B \equiv \{K_1 := P[(|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{2}\rangle)_B], K_2 := P[(|\mathbf{3}\rangle, |\mathbf{4}\rangle, |\mathbf{5}\rangle)_B]\}$. If K_1 clicks they end up in one of

$$\left\{\begin{matrix} |0\rangle_A|\mathbf{0}-\mathbf{1}\rangle_B,\ |2\rangle_A|\mathbf{1}-\mathbf{2}\rangle_B, \\ |1-2\rangle_A|\mathbf{0}\rangle_B,\ |0-1\rangle_A|\mathbf{2}\rangle_B, \\ |0+1+2\rangle_A|\mathbf{0}+\mathbf{1}+\mathbf{2}\rangle_B. \end{matrix}\right\}$$

On the other hand, if Bob gets K_2, they are then left with one of the following five states:

$$\left\{\begin{matrix} |0\rangle_A|\mathbf{4}-\mathbf{5}\rangle_B,\ |2\rangle_A|\mathbf{5}-\mathbf{3}\rangle_B, \\ |1-2\rangle_A|\mathbf{4}\rangle_B,\ |0-1\rangle_A|\mathbf{3}\rangle_B, \\ |0+1+2\rangle_A|\mathbf{3}+\mathbf{4}+\mathbf{5}\rangle_B. \end{matrix}\right\}$$

It is clear that the five updated states when K_1 clicks form the celebrated unextendable product basis (UPB) [36,37] in $\mathbb{C}^3 \otimes \mathbb{C}^3$. The states in the case of the K_2 outcome also form the same UPB where $\{|\mathbf{3}\rangle_B, |\mathbf{4}\rangle_B, |\mathbf{5}\rangle_B\}$ span Bob's three-dimensional Hilbert space. It has been well established that this orthogonal set of product states is locally indistinguishable [24,37], and also locally irreducible (see Supplemental Material [60]). ■

This is certainly an example of genuine activation of bipartite quantum nonlocality without entanglement. However, orthogonal sets of bipartite product states that show activable nonlocality have already been reported [56]. But in some of the protocols, mentioned there, a different outcome of a single local OPM provides different dimensional sets of nonlocal product states. Moreover, the dimension requirement to activate such nonlocality in [56] is minimum 7×8 for bipartite systems, while our elegant example shows that such an activation is possible even with lower dimensional quantum systems.

The question of genuine activation of multipartite quantum nonlocality from orthogonal sets of product states also has not been explored yet. In the following, we delve into this question and answer in the affirmative with an explicit example. Consider the orthogonal set of tripartite product states $\mathcal{G}_2 \equiv \{|\phi_i\rangle_{ABC}\}_{i=1}^4 (\subset \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^4)$ where

$$|\phi_1\rangle_{ABC} = |0\rangle_A\,|0-1\rangle_B\,|\mathbf{1}+\mathbf{2}\rangle_C, \tag{4a}$$
$$|\phi_2\rangle_{ABC} = |0-1\rangle_A\,|1\rangle_B\,|\mathbf{0}+\mathbf{3}\rangle_C, \tag{4b}$$
$$|\phi_3\rangle_{ABC} = |1\rangle_A\,|0\rangle_B\,|\mathbf{0}-\mathbf{1}+\mathbf{2}-\mathbf{3}\rangle_C, \tag{4c}$$
$$|\phi_4\rangle_{ABC} = |0+1\rangle_A\,|0+1\rangle_B\,|\mathbf{0}+\mathbf{1}+\mathbf{2}+\mathbf{3}\rangle_C. \tag{4d}$$

Proposition 2. The set $\mathcal{G}_2$ is free from local redundancy and discriminable under LOCC.

Proof. It is straightforward to show that the set $\mathcal{G}_2$ is free form local redundancy. Here, Charlie's system can be considered as two composite qubits. Let us denote those subsystems by c_1 and c_2: $|\mathbf{0}\rangle_C := |00\rangle_{c_1c_2}$, $|\mathbf{1}\rangle_C := |01\rangle_{c_1c_2}$, $|\mathbf{2}\rangle_C := |10\rangle_{c_1c_2}$, $|\mathbf{3}\rangle := |11\rangle_{c_1c_2}$. Consider the states $|\phi_1\rangle$ and $|\phi_2\rangle$. Note that discarding the subsystem c_i we will have $\rho_{c_j}^k := \mathrm{Tr}_{c_i}|\phi_k\rangle\langle\phi_k|$, for $i, j, k \in \{1, 2\}$. It is quite evident that $\rho_{c_j}^1$ and $\rho_{c_j}^2$ are nonorthogonal for $j = 1, 2$.

We will now show that the set $\mathcal{G}_2$ is locally distinguishable. The distinguishability protocol is as follows. First, Charlie performs a measurement $K_C \equiv \{K_1 := P[|\mathbf{0}+\mathbf{3}\rangle_C], K_2 := P[|\mathbf{0}-\mathbf{3}\rangle_C], K_3 := P[|\mathbf{1}+\mathbf{2}\rangle_C], K_4 := P[|\mathbf{1}-\mathbf{2}\rangle_C]\}$. If K_1 clicks the given state must be one of $\{|\phi_2\rangle_{ABC}, |\phi_4\rangle_{ABC}\}$ which are perfectly locally distinguishable [17,59]. If K_2 clicks, the given state must be $|\phi_3\rangle_{ABC}$. When K_3 clicks, the given state is one of $\{|\phi_1\rangle_{ABC}, |\phi_4\rangle_{ABC}\}$ which can always be perfectly distinguished via LOCC. For the click K_4, the given state is certainly $|\phi_3\rangle_{ABC}$. ■

Now, we are in a position to show that the set $\mathcal{G}_2$ can be transformed, with certainty, to a set of orthogonal states which are impossible to distinguish locally.

Theorem 2. The set $\mathcal{G}_2$ can be converted to a set of tripartite locally irreducible product states, *a.k.a*, the SHIFTS UPB [36] using local OPM.

Proof. Let us consider that Charlie performs a local OPM on the subsystem C: $R_B \equiv \{R_1 := P[(|\mathbf{0}\rangle, |\mathbf{1}\rangle)_C], R_2 := P[(|\mathbf{2}\rangle, |\mathbf{3}\rangle)_C]\}$. If R_1 clicks they end up in one of

$$\{|0\rangle_A|0-1\rangle_B|\mathbf{1}\rangle_C,\ |0-1\rangle_A|1\rangle_B|\mathbf{0}\rangle_C,$$
$$|1\rangle_A|0\rangle_B|\mathbf{0}-\mathbf{1}\rangle_C,\ |0+1\rangle_A|0+1\rangle_B|\mathbf{0}+\mathbf{1}\rangle_C\}.$$

On the other hand, if R_2 clicks, they will be left with one of

$$\{|0\rangle_A\,|0-1\rangle_B\,|\mathbf{2}\rangle_C,\ |0-1\rangle_A|1\rangle_B|\mathbf{3}\rangle_C,$$
$$|1\rangle_A|0\rangle_B|\mathbf{2}-\mathbf{3}\rangle_C,\ |0+1\rangle_A|0+1\rangle_B|\mathbf{2}+\mathbf{3}\rangle_C\}.$$

It is evident that both the above sets are basically equivalent to the SHIFTS UPB in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$ [24,36,37]. Furthermore, states belonging to SHIFTS UPB are known to be perfectly indistinguishable via LOCC [37]. In the Supplemental Material [60] it is shown that these states are locally irreducible if all the parties perform local operations on their respective individual subsystems and communicate classically amongst themselves. This completes our proof. ■

Notably, any orthogonal multipartite set of product states that can be thought of as a bipartition of $\mathbf{C}^2 \otimes \mathbb{C}^d$ with $d \geqslant 2$, can always be shown as locally distinguishable [37]. As a consequence, the above-mentioned SHIFTS UPB is distinguishable and hence reducible under LOCC, whenever two of the parties come together. This motivates us to activate even a stronger (possibly the strongest) notion of nonlocality in the following.

Genuine activation of possibly strongest quantum nonlocality without entanglement. After demonstrating the activation of genuine nonlocality without entanglement in the multipartite scenario, the pertinent question is whether or not there exists any orthogonal set which can show genuine activation of the possibly strongest form of quantum nonlocality without entanglement as demonstrated in [40]. While the stronger version of quantum nonlocality without entanglement characterizes the set of locally irreducible states (see Definition 1), the possibly strongest version identifies those multipartite states, none of which can be eliminated even if all but one party perform a joint measurement. Note that strong quantum nonlocality without entanglement [40] cannot be obtained in mere three-qubit systems. The minimum dimension required to show such phenomena is at least three qutrit. In the following, we provide a set which answers this question in affirmation. Consider the orthogonal set $\mathcal{G}_3$ that contains the following of 27 tripartite product states[1] $|\xi_i^{\pm}\rangle$, $i \in \{1, \ldots, 4, 6, \ldots, 9, 11, \ldots, 14\}$ and $|\xi_j\rangle$, $j \in \{5, 10, 15\}$ in $\mathbb{C}^6 \otimes \mathbb{C}^6 \otimes \mathbb{C}^6$.

$$|\xi_1^{\pm}\rangle = |\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle, \tag{5a}$$
$$|\xi_2^{\pm}\rangle = |\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle, \tag{5b}$$
$$|\xi_3^{\pm}\rangle = |\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle, \tag{5c}$$
$$|\xi_4^{\pm}\rangle = |\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle, \tag{5d}$$
$$|\xi_5\rangle = |\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{0}-\mathbf{4}\rangle, \tag{5e}$$
$$|\xi_6^{\pm}\rangle = |\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle\,|\mathbf{0}-\mathbf{4}\rangle, \tag{5f}$$
$$|\xi_7^{\pm}\rangle = |\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle\,|\mathbf{0}-\mathbf{4}\rangle, \tag{5g}$$
$$|\xi_8^{\pm}\rangle = |\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle\,|\mathbf{1}-\mathbf{5}\rangle, \tag{5h}$$
$$|\xi_9^{\pm}\rangle = |\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle\,|\mathbf{2}-\mathbf{3}\rangle, \tag{5i}$$
$$|\xi_{10}\rangle = |\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{1}-\mathbf{5}\rangle, \tag{5j}$$
$$|\xi_{11}^{\pm}\rangle = |\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle\,|\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{1}-\mathbf{5}\rangle, \tag{5k}$$
$$|\xi_{12}^{\pm}\rangle = |\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle\,|\mathbf{0}-\mathbf{4}\rangle\,|\mathbf{2}-\mathbf{3}\rangle, \tag{5l}$$
$$|\xi_{13}^{\pm}\rangle = |\mathbf{0}\pm\mathbf{1}+\mathbf{4}\pm\mathbf{5}\rangle\,|\mathbf{1}-\mathbf{5}\rangle\,|\mathbf{2}-\mathbf{3}\rangle, \tag{5m}$$
$$|\xi_{14}^{\pm}\rangle = |\mathbf{0}\pm\mathbf{2}+\mathbf{4}\pm\mathbf{3}\rangle\,|\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{1}-\mathbf{5}\rangle, \tag{5n}$$
$$|\xi_{15}\rangle = |\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{2}-\mathbf{3}\rangle\,|\mathbf{2}-\mathbf{3}\rangle. \tag{5o}$$

Proposition 3. The set $\mathcal{G}_3$ is not locally redundant and is distinguishable under LOCC, even when all the parties are separated.

Proof. We first provide a brief outline of the proof that the above set of states are free from local redundancy. The detailed proof is given in the Supplemental Material [60]. Note that the quantum system possessed by each individual can only be composed of a qubit and qutrit subsystem.

[1]For the sake of better readability, here we drop the party notation (A, B, C) in the subscripts.

Therefore, for each player we can write $|\mathbf{0}\rangle := |00\rangle$, $|\mathbf{1}\rangle := |01\rangle$, $|\mathbf{2}\rangle := |02\rangle$, $|\mathbf{3}\rangle := |10\rangle$, $|\mathbf{4}\rangle := |11\rangle$, $|\mathbf{5}\rangle := |12\rangle$. First consider the players discard their subsystems in such a way that they ultimately get the dimension of the whole tripartite system below 27. This is possible when more than one player discards their qutrits (i.e., $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$, or, $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^6$, or, $\mathbb{C}^2 \otimes \mathbb{C}^3 \otimes \mathbb{C}^3$, etc.). In this case, it is clear that all the states in Eq. (5) will not retain their orthogonality. Other possible cases of discarding the subparts are as follows: any one player discards their qutrit, any one player discards their qubit, and more than one player discards their qubits. Though cumbersome, it is quite straightforward to show that in all these cases the reduced states' set will not be orthogonal anymore. Therefore, we can conclude that the set $\mathcal{G}_3$ does not have local redundancy.

Now, we move to the proof that the set $\mathcal{G}_3$ can be distinguished with the help of LOCC alone. Due to the symmetries present in the set $\mathcal{G}_3$, each player may need to perform any of the following three measurements at different steps of the protocol.

$$
\begin{aligned}
\mathcal{M}_1 \;&\equiv\; \{P_1 := P[|\mathbf{0}-\mathbf{4}\rangle], P_2 := P[|\mathbf{1}-\mathbf{5}\rangle], P_3 := P[|\mathbf{2}-\mathbf{3}\rangle],\\
&P_4 := \mathbb{I} - (P[|\mathbf{0}-\mathbf{4}\rangle] + P[|\mathbf{1}-\mathbf{5}\rangle] + P[|\mathbf{2}-\mathbf{3}\rangle])\},\\
\mathcal{M}_2 \;&\equiv\; \{Q_1 := P[|\mathbf{0}+\mathbf{1}+\mathbf{4}+\mathbf{5}\rangle], Q_2 := P[|\mathbf{0}-\mathbf{1}+\mathbf{4}-\mathbf{5}\rangle],\\
&Q_3 := \mathbb{I} - (P[|\mathbf{0}+\mathbf{1}+\mathbf{4}+\mathbf{5}\rangle] + [|\mathbf{0}-\mathbf{1}+\mathbf{4}-\mathbf{5}\rangle])\},\\
\mathcal{M}_3 \;&\equiv\; \{R_1 := P[|\mathbf{0}+\mathbf{2}+\mathbf{4}+\mathbf{3}\rangle], R_2 := P[|\mathbf{0}-\mathbf{2}+\mathbf{4}-\mathbf{3}\rangle],\\
&R_3 := \mathbb{I} - (P[|\mathbf{0}+\mathbf{2}+\mathbf{4}+\mathbf{3}\rangle] + P[|\mathbf{0}-\mathbf{2}+\mathbf{4}-\mathbf{3}\rangle])\}.
\end{aligned}
$$

The detailed protocol is pictorially described in the Supplemental Material [60]. ■

Theorem 3. The set $\mathcal{G}_3$ can be deterministically transformed via local OPMs to an orthogonal set of tripartite product states which are locally irreducible even if all but one player come together.

Proof. Suppose each player performs a specific orthogonality preserving local measurement: $\mathcal{K} \equiv \{K_1 := P[|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{2}\rangle], K_2 := P[|\mathbf{3}\rangle, |\mathbf{4}\rangle, |\mathbf{5}\rangle]\}$. Here, the notation we follow is as follows: K_j^i is the jth projector (K_j) that clicks when the ith player performs the measurement $\mathcal{K}$. Therefore, after the measurement a total of eight possibilities can occur as each player can get any one of two possible outcomes K_1^i or K_2^i. In each of these eight cases it is straightforward to see that the updated set of 27 states will be of the following generic form.

$$
\left\{
\begin{array}{c}
|p\rangle\,|q\rangle\,|\eta_\pm\rangle,\; |q\rangle\,|\eta_\pm\rangle\,|p\rangle,\; |\eta_\pm\rangle\,|p\rangle\,|q\rangle,\\
|p\rangle\,|r\rangle\,|\kappa_\pm\rangle,\; |r\rangle\,|\kappa_\pm\rangle\,|p\rangle,\; |\kappa_\pm\rangle\,|p\rangle\,|r\rangle,\\
|q\rangle\,|r\rangle\,|\eta_\pm\rangle,\; |r\rangle\,|\eta_\pm\rangle\,|q\rangle,\; |\eta_\pm\rangle\,|q\rangle\,|r\rangle,\\
|r\rangle\,|q\rangle\,|\kappa_\pm\rangle,\; |q\rangle\,|\kappa_\pm\rangle\,|r\rangle,\; |\kappa_\pm\rangle\,|r\rangle\,|q\rangle,\\
|p\rangle\,|p\rangle\,|p\rangle,\; |q\rangle\,|q\rangle\,|q\rangle,\; |r\rangle\,|r\rangle\,|r\rangle.
\end{array}
\right\} \tag{6}
$$

Here, $|\eta_\pm\rangle := (|p\rangle \pm |q\rangle)/\sqrt{2}$ and $|\kappa_\pm\rangle := (|p\rangle \pm |r\rangle)/\sqrt{2}$. In each of the eight outcomes, for all 27 states $p,\ q$, and r will have some specific values from $p \in \{\mathbf{0}, \mathbf{4}\},\ q \in \{\mathbf{1}, \mathbf{5}\}$, and $r \in \{\mathbf{2}, \mathbf{3}\}$. For example, if for all the players the outcomes are K_1 throughout, then the reduced set of states will be of the above form with $p = \{\mathbf{0}\},\ q = \{\mathbf{1}\}$, and $r = \{\mathbf{2}\}$.

Note that the above set of states is basically the orthogonal set that manifests strong quantum nonlocality without entanglement [40]. ■

At this end, one may be further curious to activate such a possibly strongest genuine quantum nonlocality without entanglement by performing a measurement on the possession of a single party. This has vivid importance in the framework of data hiding and secret sharing between all but one untrusted party. Precisely speaking, in such a scenario the particular trusted agent (personified as Charlie) has full authority to judge how trustworthy are the other parties and depending upon that he may compel others to meet him in person to decode a hidden secret. As an example consider the following set $\mathcal{G}_4$ of 27 orthogonal product states $|\zeta_i^\pm\rangle$, $i \in \{1, \ldots, 4, 6, \ldots, 9, 11, \ldots, 14\}$ and $|\zeta_j\rangle$, $j \in \{5, 10, 15\}$ in $\mathbb{C}^{3^{\otimes 2}} \otimes \mathbb{C}^6$,

$$|\zeta_1^\pm\rangle = |0\rangle\,|1\rangle\,|\mathbf{0} \pm \mathbf{1} + \mathbf{4} \pm \mathbf{5}\rangle, \tag{7a}$$
$$|\zeta_2^\pm\rangle = |0\rangle\,|2\rangle\,|\mathbf{0} \pm \mathbf{2} + \mathbf{4} \pm \mathbf{3}\rangle, \tag{7b}$$
$$|\zeta_3^\pm\rangle = |1\rangle\,|2\rangle\,|\mathbf{0} \pm \mathbf{1} + \mathbf{4} \pm \mathbf{5}\rangle, \tag{7c}$$
$$|\zeta_4^\pm\rangle = |2\rangle\,|1\rangle\,|\mathbf{0} \pm \mathbf{2} + \mathbf{4} \pm \mathbf{3}\rangle, \tag{7d}$$
$$|\zeta_5\rangle = |0\rangle\,|0\rangle\,|\mathbf{0} - \mathbf{4}\rangle, \tag{7e}$$
$$|\zeta_6^\pm\rangle = |1\rangle\,|0 \pm 1\rangle\,|\mathbf{0} - \mathbf{4}\rangle, \tag{7f}$$
$$|\zeta_7^\pm\rangle = |2\rangle\,|0 \pm 2\rangle\,|\mathbf{0} - \mathbf{4}\rangle, \tag{7g}$$
$$|\zeta_8^\pm\rangle = |2\rangle\,|0 \pm 1\rangle\,|\mathbf{1} - \mathbf{5}\rangle, \tag{7h}$$
$$|\zeta_9^\pm\rangle = |1\rangle\,|0 \pm 2\rangle\,|\mathbf{2} - \mathbf{3}\rangle, \tag{7i}$$
$$|\zeta_{10}\rangle = |1\rangle\,|1\rangle\,|\mathbf{1} - \mathbf{5}\rangle, \tag{7j}$$
$$|\zeta_{11}^\pm\rangle = |0 \pm 1\rangle\,|0\rangle\,|\mathbf{1} - \mathbf{5}\rangle, \tag{7k}$$
$$|\zeta_{12}^\pm\rangle = |0 \pm 2\rangle\,|0\rangle\,|\mathbf{2} - \mathbf{3}\rangle, \tag{7l}$$
$$|\zeta_{13}^\pm\rangle = |0 \pm 1\rangle\,|1\rangle\,|\mathbf{2} - \mathbf{3}\rangle, \tag{7m}$$
$$|\zeta_{14}^\pm\rangle = |0 \pm 2\rangle\,|2\rangle\,|\mathbf{1} - \mathbf{5}\rangle, \tag{7n}$$
$$|\zeta_{15}\rangle = |2\rangle\,|2\rangle\,|\mathbf{2} - \mathbf{3}\rangle. \tag{7o}$$

Proposition 4. The set $\mathcal{G}_4$ is distinguishable under LOCC and free from local redundancy.

Proof. The proof that the set $\mathcal{G}_4$ does not have redundancy is quite straightforward. One may consider that the subsystem of Charlie ($\mathbb{C}^6$) consists of a qubit and qutrit. Now, if we discard any of the qubit or qutrit, not all pairs that remain would be orthogonal. The proof is quite evident from the proof of Proposition 3.

We will now move to describe a local discrimination protocol of the set $\mathcal{G}_4$. We will provide here a brief outline of the protocol.

Charlie first performs a measurement

$$
\begin{aligned}
\mathcal{M}_1^C \;&\equiv\; \{P_1 := P[|\mathbf{0}-\mathbf{4}\rangle_C], P_2 := P[|\mathbf{1}-\mathbf{5}\rangle_C],\\
&P_3 := P[|\mathbf{2}-\mathbf{3}\rangle_C],\\
&P_4 := \mathbb{I} - (P[|\mathbf{0}-\mathbf{4}\rangle_C] + P[|\mathbf{1}-\mathbf{5}\rangle_C] + P[|\mathbf{2}-\mathbf{3}\rangle_C])\}.
\end{aligned}
$$

Now, depending upon different outcomes, Bob and Charlie will perform some suitable measurements at their local sites to distinguish the set. A step by step detailed analysis is provided in the Supplemental Material [60]. ■

Theorem 4. The set $\mathcal{G}_4$ can be deterministically transformed, via a single local OPM at Charlie's site, to an orthogonal set of tripartite product states which is locally irreducible in every bipartition.

Proof. Consider that Charlie performs a local OPM, $\mathcal{K}^C \equiv \{K_1^C := P[|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{2}\rangle_C], K_2^C := P[|\mathbf{3}\rangle, |\mathbf{4}\rangle, |\mathbf{5}\rangle_C]\}$. For different outcomes of $\mathcal{K}^C$, the post measurement state will turn out to be any of the following set:

$$\left\{\begin{array}{c} |0\rangle|1\rangle|\tilde{\nu}_{\pm}\rangle,\ |1\rangle\,|\nu_{\pm}\rangle\,|p\rangle,\ |\nu_{\pm}\rangle|o\rangle|q\rangle, \\ |0\rangle|2\rangle|\tilde{\tau}_{\pm}\rangle,\ |2\rangle|\tau_{\pm}\rangle|p\rangle,\ |\tau_{\pm}\rangle|o\rangle|r\rangle, \\ |1\rangle|2\rangle|\tilde{\nu}_{\pm}\rangle,\ |2\rangle|\nu_{\pm}\rangle|q\rangle,\ |\nu_{\pm}\rangle|1\rangle|r\rangle, \\ |2\rangle|1\rangle|\tilde{\tau}_{\pm}\rangle,\ |1\rangle|\tau_{\pm}\rangle|r\rangle,\ |\tau_{\pm}\rangle\,|2\rangle\,|q\rangle, \\ |0\rangle\,|0\rangle\,|p\rangle,\ |1\rangle\,|1\rangle\,|q\rangle,\ |2\rangle\,|2\rangle\,|r\rangle, \end{array}\right\} \tag{8}$$

where here, $|\nu_{\pm}\rangle := (|0\rangle \pm |1\rangle)/\sqrt{2}$, $|\tau_{\pm}\rangle := (|0\rangle \pm |2\rangle)/\sqrt{2}$, $|\tilde{\nu}_{\pm}\rangle := (|p\rangle \pm |q\rangle)/\sqrt{2}$ and $|\tilde{\tau}_{\pm}\rangle := (|p\rangle \pm |r\rangle)/\sqrt{2}$. p, q, and r can have any value $\{\mathbf{0},\mathbf{4}\}$, $\{\mathbf{1},\mathbf{5}\}$, and $\{\mathbf{2},\mathbf{3}\}$, respectively. Now, when K_1^C clicks, the post measurement state can be any of the set Eq. (8) with $(p,q,r) = (\mathbf{0},\mathbf{1},\mathbf{2})$. Otherwise, if K_2^C clicks, they are left with any of set Eq. (8) where $(p,q,r) = (\mathbf{4},\mathbf{5},\mathbf{3})$. It is evident that the set Eq. (8) shows strong quantum nonlocality without entanglement [40,46]. This completes our proof. ■

Conclusion. In summary, we have studied the genuine activation of nonlocality from several sets of local states. However, the phrases "local" and "nonlocal" have been used from the state discrimination perspective. More precisely, we have dealt with two different sets of locally distinguishable multipartite product states in $\mathbb{C}^3 \otimes \mathbb{C}^6$ and $\mathbb{C}^{2^{\otimes 2}} \otimes \mathbb{C}^4$, which can be transformed to the set of locally indistinguishable states in $\mathbb{C}^{3^{\otimes 2}}$ and $\mathbb{C}^{2^{\otimes 3}}$, respectively, by choosing an appropriate measurement in possession of one of the parties. Furthermore, we have considered a stronger notion of the state discrimination problem, namely, the orthogonality preserving reducibility and have shown to activate such a notion from a set of multipartite locally distinguishable product states of dimension $\mathbb{C}^{6^{\otimes 3}}$. It is observed that under LOCC, the set can be transformed deterministically to a set of states in $\mathbb{C}^{3^{\otimes 3}}$, which is even irreducible in all possible bipartitions. Further, we have moved to a stricter notion of such activation where the transformation is achieved by a single agent only. Such an example is demonstrated to transform a locally distinguishable set of states of $\mathbb{C}^{3^{\otimes 2}} \otimes \mathbb{C}^6$ to a strongly irreducible one in $\mathbb{C}^{3^{\otimes 3}}$. The elegance of state construction and the transformation protocol makes it trivial to extend in any arbitrary higher dimensional set of product states exhibiting nonlocality in terms of local discrimination and orthogonality preserving elimination. Besides its foundational interest to understand the topology of the state spaces of composite quantum systems, our work deserves significant importance from the practical perspective. It has mimicked an interesting framework of secured data hiding between several parties, where the distributor is flexible to update the distinguishability of the secured data hidden in the correlation of the given states. Recently, this approach of genuinely activating quantum nonlocality has also been extended [61] to show generation of some stronger resources, for example, local quantum state unmarkability [62].

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