Performance of quantum batteries with correlated and uncorrelated chargers

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(Received 8 June 2022; revised 14 August 2022; accepted 29 November 2022; published 12 December 2022)

Energy can be stored in quantum batteries by electromagnetic fields as chargers. In this paper, the performance of a quantum battery with single and double chargers is studied. It is shown that by using two independent charging fields, prepared in coherent states, the charging power of the quantum battery can be significantly improved, though the average number of embedded photons are kept the same in both scenarios. The results reveal that for the case of initially correlated states of the chargers, the amount of extractable energy, measured by ergotropy, is more than initially uncorrelated ones, with appropriate degrees of field intensity. Though the correlated chargers lead to greater reduction in the purity of the quantum battery, more energy and in turn more ergotropy are stored in this case. In addition, we study the battery-charger mutual information and von Neumann entropy and by using their relation we find that both quantum and classical correlations are generated between the quantum battery and chargers. We also study quantum consonance of the battery as the nonlocal coherence among its cells and find some qualitative relations between the generation of such correlations and the capability of energy storage in the quantum battery.

DOI: 10.1103/PhysRevA.106.062609

I. INTRODUCTION

Batteries as portable energy storage devices play a significant role in our modern life. Nearly all aspects of this new era from household appliances to medical instruments, including navigation, transportation, and so on, depend, in a way, on the performance of these systems. A typical battery contains one or more electrochemical cells in which an energy conversion occurs from a chemical type to an electrical one, by means of the so-called reduction-oxidation reactions [1–3].

On the other hand, quantum effects become important as the size scale of electronic devices is quickly decreasing. So there may be an advantage from such quantum effects upon which one may build devices that show a better performance than their classical analogs [4–14]. One such device can be a quantum storage system or quantum battery (QB).

A QB is a collection of one or more quantum systems (mostly two-level ones) with the ability of energy storage. The notion of a QB has been brought into the spotlight after the work by Alicki and Fannes [15].

Generally, the study of QB performance has been divided into two distinct scenarios: charging and (self-)discharging. The latter indicates a situation in which the QB loses its energy due to interaction with the surrounding environment [16–18]. Actually, this is also prevalent among traditional batteries [19,20]. In the former, usually an external charging field acts as a charger in order to store energy in the QB [21,22].

Some figures of merit are proposed by which one can evaluate the useful capabilities and performance of a special QB. The most studied quantities are, for example, stored energy [23,24], ergotropy [22,25,26], and charging power [27–29].

First, Alicki and Fannes claimed that global entangling operations end in an increment of energy extractable from the QB [15]. However, this idea appeared to be imperfect since Hovhannisyan *et al.* [30] proved that the optimal energy extraction can be achieved using indirect consecutive permutation operations by which no entanglement is dynamically generated, but it is worth noting that the performance of such operations is a time-consuming process. In fact, it is argued that collective charging operations (with entanglement creation) might offer a considerable speedup of the charging process compared to a parallel charging scenario in which each cell of the QB is individually charged [31].

Charging power can also be enhanced by disordered interactions between quantum cells of the QB [32]. Moreover, the effect of disorder and localization on QBs has been studied [33]. It has been shown that the batteries which are in a many-body localized phase gain more stability and a shorter optimal timescale of the charging process in comparison with those being ergodic and Anderson localized phases.

The role of quantum entanglement and coherence for two- and three-cell QBs was investigated in [34]. Kamin *et al.* showed that while entanglement seems to be effectless or even destructive, quantum coherence presents a qualitative relation with the efficiency of the QB. However, more effort is required to address the impact of quantum correlations in this field since a universal relation between the performance of the quantum battery and its correlation contents has not yet been found [35].

As stated before, energy storage in a QB can be done using an external charging field. Andolina *et al.* considered three

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well-known quantum optical states, i.e., Fock, coherent, and squeezed vacuum, as the initial state of the charger [22]. They confirmed that the coherent state is optimal in the sense of energy deposition and extraction because it results in a smaller amount of charger-battery entanglement.

In this paper we consider a QB including four two-level quantum cells (qubit) interacting with one or two photonic cavities as chargers, each prepared in a coherent initial state. The results reveal that, regarding the charging power of the QB, employing two independent chargers is significantly superior to the single-charger case, while the average photon number is kept the same for both cases. Furthermore, quantum correlated bipartite chargers, e.g., semi-Bell states, result in more ergotropy in comparison with uncorrelated chargers. Moreover, we study the purity of the QB and find that the QB with initial correlated chargers becomes more mixed than that with uncorrelated chargers during the charging process. This more mixedness in turn results in more energy storage in the QB. In addition, we numerically prove that the QB and its charging fields can share both quantum and classical correlations with the same values. Also, the results show that such correlations between the QB and chargers along with the quantum consonance of the QB have some role to play as they positively interplay with the performance of the QB.

This paper is organized as follows. Section II presents the physical model and its dynamics. Section III is dedicated to the definition of some figures of merit and concepts which are used in this paper. In Sec. IV the results of different charging scenarios and some discussions can be found. Section V summarizes the paper.

II. MODEL

In this paper we consider an array of identical two-level quantum systems (a qubit) as a QB, interacting with one or two independent single-mode photonic cavities as the charging device [36–38]

$$H = H_B + H_C + H_I, \tag{1}$$

where

$$H_B = \frac{\omega_0}{2}\sum_{i}^{N_B}\sigma_i^z, \tag{2a}$$

$$H_C = \sum_{j}^{N_C}\omega_j a_j^\dagger a_j, \tag{2b}$$

$$H_I = \sum_{i,j}^{N_B,N_C} g_{ij}(\sigma_i^+ a_j + \sigma_i^- a_j^\dagger) \tag{2c}$$

are battery, charger, and interaction Hamiltonians, respectively. In these equations, N_B and N_C are the numbers of quantum cells of the QB and the chargers, respectively, σ_i^k ($k = x, y, z$) denotes the well-known Pauli matrices, $\sigma_i^\pm$ are raising and lowering operators for the ith qubit, $a_j^\dagger$ and a_j are creation and annihilation operators of jth charger, respectively, ω_0 is the energy splitting between the ground and excited states of each qubit, ω_j is the frequency of photons in each cavity, and g_{ij} is the coupling strength between the ith qubit and jth charger. Here we consider the resonant condition $\omega_j = \omega_0$ and also assume that the coupling strengths are independent of battery and chargers, i.e., $g_{ij} = g = 2\omega_0$ which in turn means that we focus on strong-coupling regimes [39–41]. Suppose the QB and chargers undergo a unitary evolution $U = \exp(-iHt)$, where H is given by Eq. (1), and for simplicity we set $\hbar = 1$,

$$\rho_{B,C}(t) = \mathrm{Tr}_{C,B}[U\rho_B(0)\otimes\rho_C(0)U^\dagger], \tag{3}$$

where $\rho_B(0)$ and $\rho_C(0)$ are initial states of the battery and charger(s), respectively. Throughout this paper we focus on an initial four-cell empty battery in which all qubits are prepared in their ground states so $\rho_B(0) = (|g\rangle\langle g|)^{\otimes 4}$. Meanwhile, we set each charger to be in a coherent optical state $|\alpha\rangle$, where α is a complex number identifying the average number of embedded photons in the cavity as $\bar{n} = |\alpha|^2$ [42]. In fact, it is shown that the coherent state is the optimal state to charge a quantum battery since it causes less battery-charger entanglement [22].

III. FIGURES OF MERIT

In this section we provide some figures of merit by which one can evaluate the performance of a QB.

A. Thermodynamic performance

Assume a quantum battery with density matrix ρ and Hamiltonian H. Then its total amount of stored energy is simply given by

$$E(\rho, H) = \mathrm{Tr}(\rho H). \tag{4}$$

Mostly, for different reasons, one is not able to entirely extract this energy through unitary operations. So we need to consider ergotropy, i.e., the maximum amount of energy which can be unitarily extracted [43]. It is defined as the difference between internal energies of quantum state ρ and its corresponding passive state η:

$$\mathcal{E}(\rho, H) = E(\rho, H) - E(\eta, H). \tag{5}$$

If η is passive, it provides no energy by means of unitary operations. The passive state of a quantum system possesses nonincreasing population with respect to its Hamiltonian H and also $[H, \eta] = 0$. Consider the spectral decompositions of the battery state and Hamiltonian as $\rho = \sum_{i=1}^d p_i|p_i\rangle\langle p_i|$ and $H = \sum_{i=1}^d \epsilon_i|\epsilon_i\rangle\langle\epsilon_i|$ so that $p_1 \geqslant p_2 \geqslant \cdots \geqslant p_d$ and $\epsilon_1 \leqslant \epsilon_2 \leqslant \cdots \leqslant \epsilon_d$, where d denotes the dimension of the Hilbert space. Then the passive state is $\eta = \sum_{i=1}^d p_i|\epsilon_i\rangle\langle\epsilon_i|$ and ergotropy can be written as

$$\mathcal{E}(\rho, H) = \sum_{i,j}^{d} p_i\epsilon_j(|\langle p_i|\epsilon_j\rangle|^2 - \delta_{i,j}), \tag{6}$$

where $\delta_{i,j}$ is the Kronecker delta function. As ergotropy is the maximum amount of extractable energy using unitary operations, the most successful extracting operations are those which transform the quantum state ρ to the passive state η. Another important figure of merit that determines how fast a quantum battery can be charged is power

$$P(t) = \frac{E(\rho_B(t), H)}{t}, \tag{7}$$

where t denotes the charging time.

B. Correlations

As previously mentioned, most effort has been dedicated to the role of quantum entanglement in the performance of quantum batteries. It appears to be effective [25,44], destructive [22,25], or effectless [29,34]. So it makes sense to seek another quantifier of quantum correlations which may show more correspondence with the performance of quantum batteries. Moreover, entanglement is not the only quantifier of quantum correlations. In this paper we consider quantum consonance, which estimates the global coherence in quantum systems and for a general multipartite density matrix is defined by [45]

$$C(\rho) = \sum_{k_1k_2,\ldots,k_n,l_1l_2,\ldots,l_n} \left| \rho^c_{k_1k_2,\ldots,k_n,l_1l_2,\ldots,l_n} \prod_m (1-\delta_{k_m,l_m}) \right|, \tag{8}$$

where $\rho^c = U\rho U^\dagger$ is the transformed density matrix by using some unitary operations which eliminate the local coherence of ρ. So unlike entanglement, quantum consonance can be easily calculated even for multipartite quantum systems. For a two-qubit system ρ_{AB} the above relation reduces to

$$C(\rho_{AB}) = \sum_{k_1k_2,l_1l_2} \left| \rho^c_{k_1k_2,l_1l_2}(1-\delta_{k_1,l_1})(1-\delta_{k_2,l_2}) \right|, \tag{9}$$

where $\rho^c_{k_1k_2,l_1l_2} = (U_A \otimes U_B)\rho_{AB}(U_A \otimes U_B)^\dagger$, in which U_A and U_B are some transformations that remove the local coherence of the reduced states $\rho_{A,B} = \mathrm{Tr}_{B,A}(\rho_{AB})$. The subindices k_1, l_1 and k_2, l_2 label the basis of the two qubits as $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Using Eq. (9), it can be easily shown that quantum consonance includes only the elements related to $|00\rangle\langle 11|$, $|01\rangle\langle 10|$, $|10\rangle\langle 01|$, and $|11\rangle\langle 00|$ of the transformed state $\rho^c_{k_1k_2,l_1l_2}$. The generalization to the four-qubit case is also straightforward.

Moreover, one may want to know how much the battery and charger(s) become correlated during the charging process. It can be quantified by battery-charger mutual information. It evaluates the total amount of correlations (classical plus quantum) between two parts. Let us consider ρ_{BC} to be the joint density matrix of the battery and charger. Then mutual information is given by [46,47]

$$I(\rho_{BC}) = \chi(\rho_{BC}) + Q(\rho_{BC}) = S(\rho_B) + S(\rho_C) - S(\rho_{BC}), \tag{10}$$

in which $\chi(\rho_{BC})$ and $Q(\rho_{BC})$ denote the classical and quantum correlation between the QB and chargers, respectively, and $S(\rho) = \mathrm{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy.

IV. TWO SCENARIOS: CHARGING WITH SINGLE OR DOUBLE PHOTONIC CAVITIES

As we previously mentioned, we consider each charger to be in a coherent state $|\alpha\rangle$. In this section we investigate two distinct charging scenarios (Fig. 1): charging with (i) one single-mode cavity $|\alpha\rangle$ and (ii) two single-mode cavities $\{|\alpha_1\rangle, |\alpha_2\rangle\}$ with different configurations (correlated or uncorrelated).

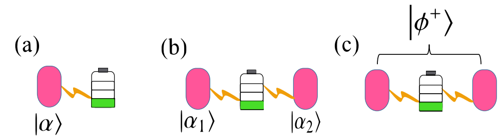

FIG. 1. Schematic of the three charging schemes: (a) single, (b) double uncorrelated, and (c) double correlated chargers.

A. Boosting the charging power

Now we compare two charging cases: In the first there is only one charger ($N_C = 1$) in state $|\alpha\rangle$, while the second case contains two uncorrelated chargers ($N_C = 2$) in state $|\alpha_1\rangle|\alpha_2\rangle$. For an unbiased comparison between these charging schemes we consider an equal average number of photons in both cases ($|\alpha|^2 = |\alpha_1|^2 + |\alpha_2|^2$). Figure 2 shows the charging power for these two charging protocols with $\alpha = 0.5$, 1.5, and 2.5. Clearly, in all cases, charging with two chargers appears to have a better performance in terms of power, regardless of field intensities. This is an interesting result: Though the average number of photons in both charging approaches is the same, it is possible to charge the battery with a higher rate if one use two cavities. Moreover, raising the average number of embedded photons in the cavities leads to the growth of maximum power. In fact, the gain in ability of faster charging in double cavities compared to the single cavity can be up to 76% for $\alpha = 2.5$. One may think of a monotonic enhancement of charging power for double chargers with respect to the single one when α is arbitrarily increased. However, this is not the case because we plot the maximum of charging power versus the field intensity in Appendix A and it shows a periodic dependence on α. In addition, the double-charger case needs a shorter timescale to reach the maximum power compared to the single-charger scheme for which the peaks take longer times to emerge.

We should mention that single and double uncorrelated chargers show no considerable advantage over each other

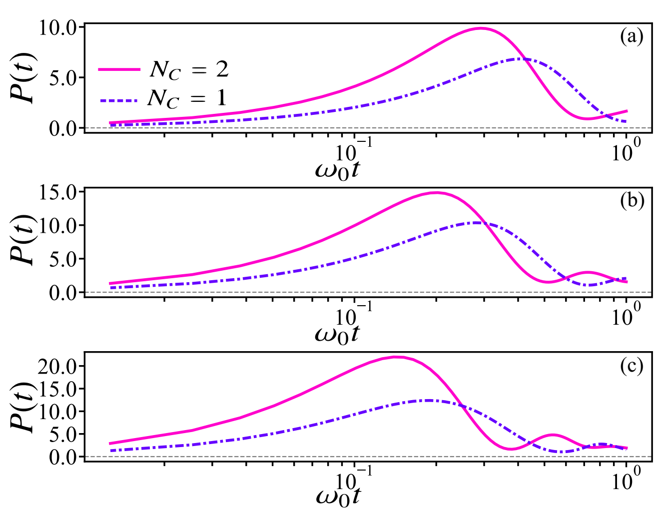

FIG. 2. Charging power of the QB with one ($N_C = 1$) and two ($N_C = 2$) independent chargers, respectively, in states $|\alpha\rangle$ and $|\alpha_1\rangle|\alpha_2\rangle$ with the condition $|\alpha|^2 = |\alpha_1|^2 + |\alpha_2|^2$ with various amounts of field intensity. (a) $\alpha = 0.5$, (b) $\alpha = 1.5$, and (c) $\alpha = 2.5$.

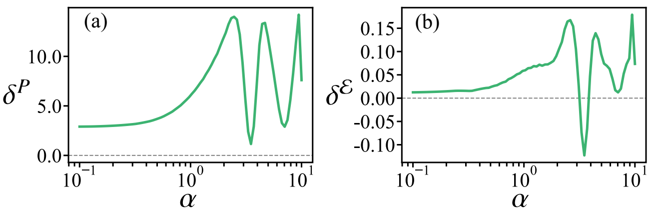

FIG. 3. Behavior of (a) δ^P and (b) $\delta^{\mathcal{E}}$ versus the degree of field intensity.

regarding the ergotropy storage, though the latter is slightly better. To show this we consider the difference between the maximum amount of ergotropy over time for these charging schemes, i.e., $\delta^{\mathcal{E}} = \mathcal{E}_{\max}^{N_C=2} - \mathcal{E}_{\max}^{N_C=1}$, where $\mathcal{E}_{\max}^{N_C=2}$ and $\mathcal{E}_{\max}^{N_C=1}$ are the maximal ergotropy of the QB with double uncorrelated and single chargers, respectively. We define the same quantity for charging power as δ^P. In Fig. 3 we plot these differences versus the degree of field intensity. While δ^P is considerably large for various amounts of α [Fig. 3(a)], as expected from Fig. 2, $\delta^{\mathcal{E}}$ is small and even negative for some field intensities [Fig. 3(b)] denoting $\mathcal{E}_{\max}^{N_C=1} > \mathcal{E}_{\max}^{N_C=2}$.

In what follows we want to see if different configurations of chargers with or without correlations play any significant role in the charging of the QB.

B. Correlated and uncorrelated chargers

One may ask if the existence of correlations between chargers can result in a considerable effect during the charging process. So here we aim to address this point. We consider two double-charger schemes. In the first one, the state of two chargers is a product state $|\psi\rangle = |\alpha_1\rangle|\alpha_2\rangle$, while in the second, the two chargers are prepared to be in an entangled semi-Bell state [48,49]

$$|\phi^+\rangle = \frac{1}{\sqrt{N_+}}(|\alpha_1\rangle|\alpha_1\rangle + |\alpha_2\rangle|\alpha_2\rangle), \tag{11}$$

where $N_+ = 2(1 + e^{-2(|\alpha_1|^2+|\alpha_2|^2)})$ is a normalization constant. The appellation of such states as semi-Bell, quasi-Bell, or Bell-like states comes from their similarity to the well-known Bell states $|\phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ in which $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ are the standard orthogonal basis of a two-qubit system. However, the semi-Bell states are made of continuous-variable states, namely, coherent states without the orthogonality condition, i.e., $\langle\alpha_1||\alpha_2\rangle \neq 0$ for $\alpha_1 \neq \alpha_2$. Various schemes of their experimental preparation have been proposed [50–52]. Throughout the rest of paper, we set $\alpha_1 = -\alpha_2 = \alpha$. In Appendix C we provide the results of other choices of semi-Bell states to prove the universality of our results.

Figure 4 shows the ergotropy of the quantum battery, i.e., $\mathcal{E}(t) = \mathcal{E}(\rho_B(t), H_B)/N_B\omega_0$, versus time for the two initial states $|\psi\rangle$ and $|\phi^+\rangle$ of the cavities. A significant capability of the chargers in the semi-Bell state in charging the quantum battery can be inferred when α is large enough compared to the uncorrelated state $|\psi\rangle$ [Fig. 4(c)]. In fact, when the cavities share no initial correlation, raising the intensities of the charging field reduces the overall stored ergotropy in the quantum battery, though there is no considerable difference between the two types of chargers for smaller values of α [Fig. 4(a)]. However, the dependence of the maximum ergotropy on the degree of field intensity α is periodic, as shown in Appendix A. In addition, it can be qualitatively seen that the maximum amount of ergotropy takes a shorter time to be deposited in the quantum battery [Figs. 4(b) and 4(c)] if one uses an initially correlated state for the photonic cavities. Therefore, this kind of charging scheme is also able to provide some advantages in terms of charging power.

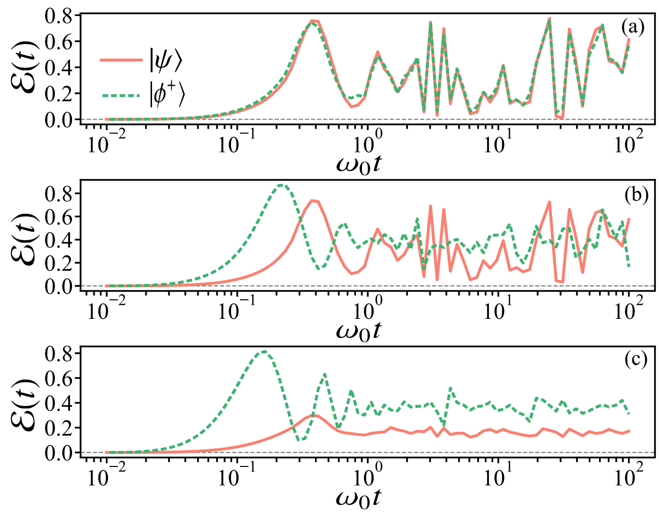

FIG. 4. Ergotropy of the QB with initially correlated ($|\phi^+\rangle$) and uncorrelated ($|\psi\rangle$) chargers for (a) $\alpha = 0.5$, (b) $\alpha = 1.5$, and (c) $\alpha = 2.5$.

The ratio of ergotropy and internal energy, i.e., $\Gamma(t) = \frac{\mathcal{E}(\rho_B(t),H)}{E(\rho_B(t),H)}$, is also an interesting quantity since it determines what fraction of the total stored energy is extractable. In Fig. 5 this ratio is depicted versus time for the two types of cavity initial states. A behavior similar to ergotropy can be seen from this figure. While the ratios obtained by the two charging settings are almost the same for $\alpha = 0.5$, increasing the intensity of charging fields results in the distinction of the

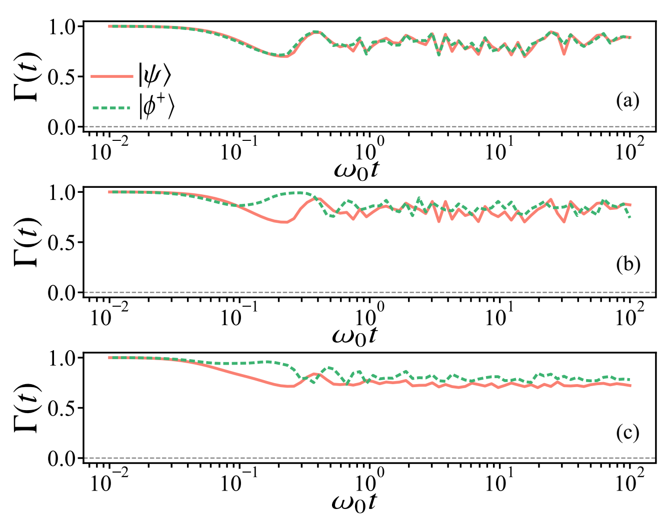

FIG. 5. Ratio of extractable energy of the QB with initially correlated and uncorrelated chargers for (a) $\alpha = 0.5$, (b) $\alpha = 1.5$, and (c) $\alpha = 2.5$.

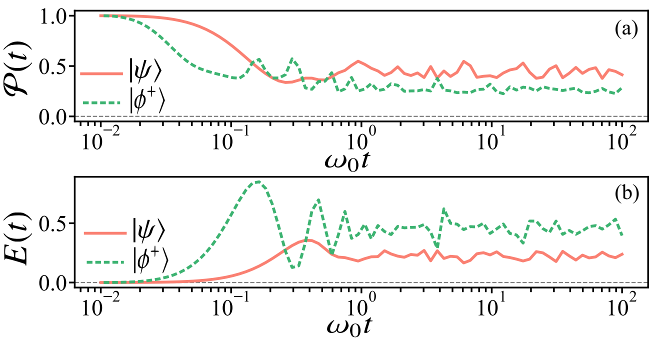

FIG. 6. Dynamics of (a) purity and (b) stored energy of the QB with the two states $|\psi\rangle$ and $|\phi^+\rangle$ as the chargers for $\alpha = 2.5$.

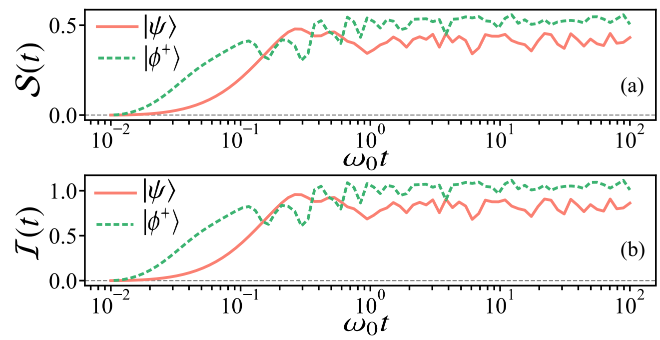

FIG. 7. Dynamics of battery-charger (a) entanglement and (b) mutual information for $|\psi\rangle$ and $|\phi^+\rangle$ with $\alpha = 2.5$.

extractable energy ratio deposited in the quantum battery by the correlated cavities.

Now let us take a careful look at the purity of the QB defined by

$$\mathcal{P}(t) = \mathrm{Tr}[\rho_B(t)^2]. \tag{12}$$

For the sake of brevity, we only consider the result with $\alpha = 2.5$ for which more distinction between the two considered charging schemes emerges. As shown by Fig. 6, the purity of the QB somehow ends up with behavior opposite to ergotropy [Fig. 4(c)]. Here the separable chargers make the QB possess larger purity during the time evolution with respect to the correlated ones, while this is not the case for ergotropy. On the other hand, it is known that if a quantum state is pure, it can be unitarily transformed into its ground state. So, by using Eq. (5) we have $\mathcal{E}(\rho_B, H_B) = E(\rho_B, H_B)$, i.e., the whole of energy can be extracted in the form of ergotropy (with the assumption of zero energy for the ground state), but for mixed states $\mathcal{E}(\rho_B, H_B) < E(\rho_B, H_B)$ [22]. Consequently, QBs with higher purity also offer larger extractable energy. One may think this is violated by our results, but this is not true. In fact, for a given energy, the more pure the QB is, the closer its ergotropy will be to its energy. In our cases, different charging schemes result in different amounts of stored energy, as seen from Fig. 6(b). Obviously, the stored energy in the QB with entangled chargers is more than that for the separable chargers and this leads in turn to the greater amount of ergotropy.

In addition, the higher purity of the QB with separable chargers is due to less battery-charger entanglement generated. Figure 7(a) shows the von Neumann entropy of the QB, i.e., $\mathcal{S}(t) = S(\rho_B(t))/N_B$, as the measure of total system (QB plus chargers) entanglement for $\alpha = 2.5$. Clearly, the QB becomes more entangled (and in turn mixed) with correlated chargers than separable ones. Overall, the battery-charger entanglement appears with two distinct roles: It results in more energy injection into the QB and consequently more ergotropy, but it allows smaller fractions of energy to be extractable. In our work, at least, the former role tends to be dominant.

We are also interested in the total amount of correlation between the QB and chargers, i.e., mutual information $\mathcal{I}(t) = \mathcal{I}(\rho_{BC}(t))/N_B$ [Fig. 7(b)]. From Fig. 7 it is easy to show that $\mathcal{I}(t) = 2\mathcal{S}(t)$. This is the case for general pure states and implies the existence of both classical and quantum correlations between the QB and chargers. In fact, noting Eq. (10), it is well known that for pure states quantum entanglement and quantum discord are the same [53], so $\mathcal{S}(t)$ can quantify the total amount of quantum correlations [$\mathcal{S}(\rho_{BC}) = Q(\rho_{BC})$]. Therefore, as the total state of the system (QB plus chargers) remains pure during the time evolution, $\mathcal{I}(t) = 2\mathcal{S}(t)$ means that the QB and chargers share both quantum and classical correlations, each one being the same and making up half of the total correlations (mutual information), meaning that $\mathcal{S}(\rho_{BC}) = \chi(\rho_{BC})$ [54]. Moreover, all previous discussion for the von Neumann entropy holds true for the mutual information as well. Comparing Figs. 7(a) and 7(b) with Fig. 4(c), one can confirm that the generation of correlation between the QB and chargers positively contributes to the deposition of ergotropy in the QB. To a certain extent, whenever the QB and chargers are more correlated, larger amounts of ergotropy is stored in the QB. In Fig. 8 we plot the ergotropy of the QB along with the battery-charger mutual information to make their relevance more apparent.

The dynamics of generated quantum consonance (QC) in the quantum battery is shown in Fig. 9. As stated before, the battery is initially prepared in its pure ground state $\rho_B(0) = |g\rangle^{\otimes 4}\langle g|$ without any local and global coherence or correlations between qubits. Then QC is generated in the QB due to the simultaneous interaction of the qubits with the chargers. Generally, both types of cavity initial states are able to create QC among qubits of the QB. However, if one prepares the correlated state $|\phi^+\rangle$ as the chargers, a larger amount of QC appears in the QB, similar to the stored ergotropy.

V. CONCLUSION

In this paper we considered a four-cell QB which can interact with one or two photonic cavities as chargers. We

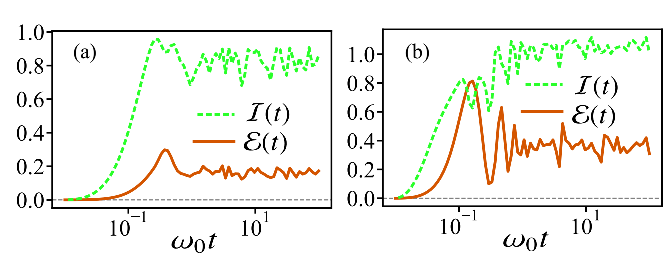

FIG. 8. Ergotropy of the QB and battery-charger mutual information for (a) $|\psi\rangle$ and (b) $|\phi^+\rangle$ with $\alpha = 2.5$.

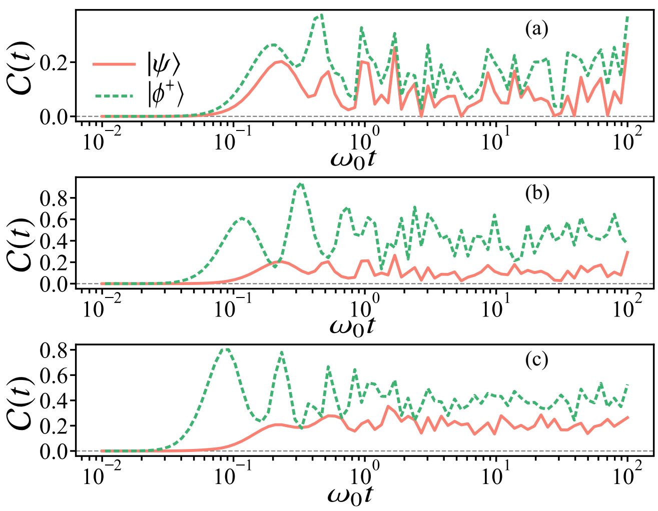

FIG. 9. Quantum consonance of the QB with initially correlated and uncorrelated chargers for (a) $\alpha = 0.5$, (b) $\alpha = 1.5$, and (c) $\alpha = 2.5$.

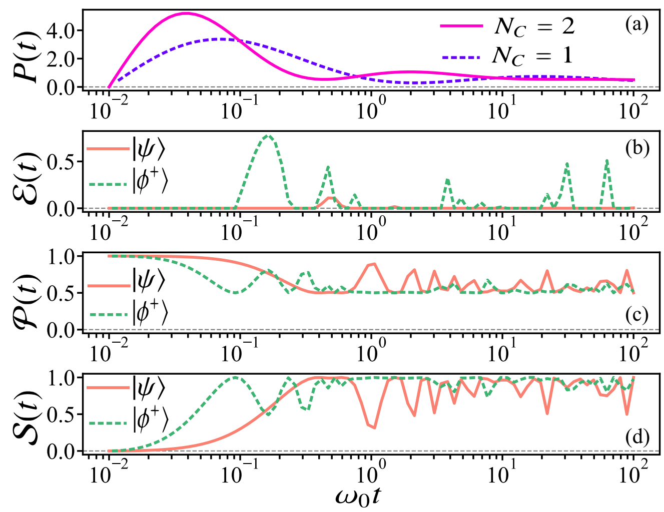

FIG. 11. Dynamics of (a) charging power, (b) ergotropy, (c) purity, and (d) von Neumann entropy for a single-qubit QB with chargers in states $|\psi\rangle$ and $|\phi^+\rangle$ for $\alpha = 2.5$.

demonstrated that the simultaneous utilization of two independent chargers in coherent states considerably increases the charging power of the QB. This result was achieved while the single- and double-charger scenarios contain the same average number of photons.

Moreover, it was shown that when the chargers are initially correlated, namely, are in a semi-Bell state, and by using a sufficient amount of field intensity, more ergotropy and the ratio of useful extractable energy are stored in the QB in comparison to the case in which the chargers are in a product state without any correlations. We made this more evident by preparing the results for various types of semi-Bell states. These results provide motivation for considering the role of different correlations between the chargers in the energy storage.

On the other hand, the QB with correlated chargers is less pure than that with uncorrelated chargers, during the time evolution. However, the former results in more energy and ergotropy storage in the QB.

In addition, as the state of the system (QB plus chargers) undergoes a unitary evolution, it remains pure and we showed that the total correlation between the QB and chargers comprises both classical and quantum types with the same values. Finally, we saw that the battery-charger correlations and nonlocal coherence of the battery positively interplay with the performance of the QB.

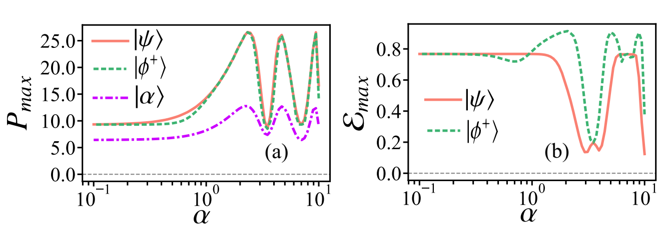

FIG. 10. (a) Maximum amount of charging power versus α for single, double separable, and double correlated chargers and (b) maximum stored ergotropy of the QB versus α for double separable and correlated chargers.

APPENDIX A: MAXIMUM AMOUNT OF ERGOTROPY AND POWER VERSUS FIELD INTENSITY

It has already been seen that (Fig. 2) the charging power of the QB benefits from double chargers (uncorrelated) for all values of α. Now, in this Appendix we explore the dependence of maximum charging power ($P_{\max} = \max_t[P(t)]$) and ergotropy ($\mathcal{E}_{\max} = \max_t[\mathcal{E}(t)]$) on the field's intensity α. Figure 10(a) shows $P_{\max}$ versus α for three different initial states of chargers. The double-charger cases appear with similar performances, regardless of being correlated or not. This is why the result for charging power with a correlated charger is not presented in the paper. Furthermore, a periodic dependence of maximum power on α is observed for all initial instances of chargers. While double chargers result in a considerable advantage over the single charger, regarding the maximum power, the amount of such an advantage oscillates with the degree of intensity. A similar discussion holds true for maximum ergotropy of the QB over time evolution, as shown in Fig. 10(b). Almost for all degrees of field intensity, the initially correlated chargers are able to enhance the maximum amount of ergotropy deposited in the QB compared to the uncorrelated chargers, but the amount of this enhancement is α dependent.

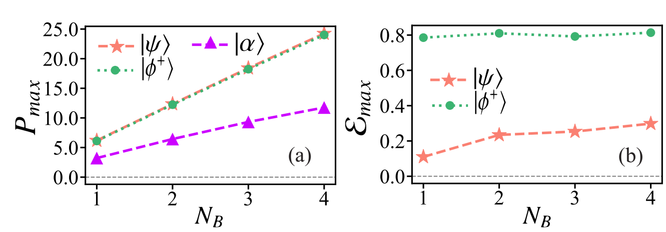

FIG. 12. Maximum of (a) charging power and (b) ergotropy as a function of N_B with $\alpha = 2.5$.

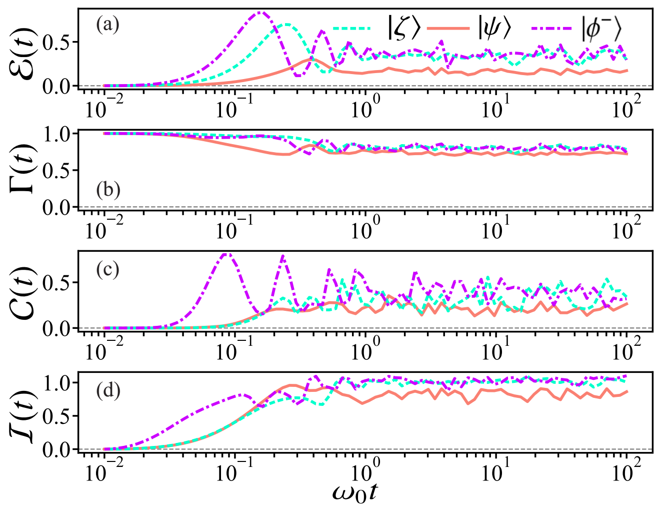

FIG. 13. Dynamics of (a) ergotropy, (b) ratio of extractable energy, (c) quantum consonance of the QB, and (d) battery-charger mutual information with chargers in states $|\psi\rangle$, $|\zeta\rangle$, and $|\phi^-\rangle$ for $\alpha = 2.5$.

APPENDIX B: SCALING OF MAXIMUM POWER AND ERGOTROPY WITH THE SIZE OF THE QB

It can be interesting to see the scaling trend of maximum power and ergotropy with the number of embedded cells (the qubit). However, first, let us check our results for a single-cell QB. For the sake of brevity, we only consider the results for $\alpha = 2.5$. As can be seen from Fig. 11, almost all previous results for the four-cell QB are also valid here, one exception being the inability of the separable chargers in ergotropy storage. Next we plot the maximum power and ergotropy as a function of QB size (N_B) in Fig. 12. As shown in Fig. 12(a), enlarging the QB provides a faster charging process since the maximum power grows when N_B is increased for all initial states of the charger(s). The increasing rate of maximum charging power is the same for double correlated and uncorrelated chargers. Moreover, the double-charger scheme offers a sharper growth of maximum power with N_B compared to the single-charger case. On the other hand, different scaling behavior is observed for the maximum amount of ergotropy [Fig. 12(b)]. Comparing the results for correlated and uncorrelated chargers, while the former demonstrate a far better performance for all N_B, for this case the maximum ergotropy is almost fixed for varying system sizes. For the uncorrelated chargers, however, a jump is shown from $N_B = 1$ to $N_B = 2$ and then a slight increase for other numbers of embedded qubits. We should note that in order to determine the scaling trend with more certainty, one needs to go beyond such limited numbers of qubits; however, we can at least rely on our results for the system sizes considered here.

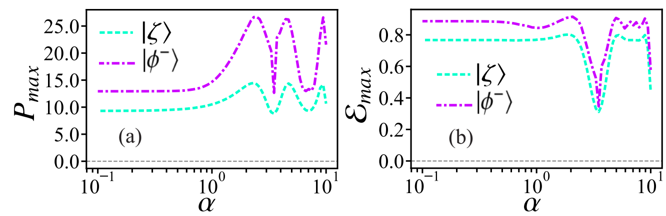

FIG. 14. Maximum of (a) charging power and (b) ergotropy of the QB as a function of α for correlated chargers $|\zeta\rangle$ and $|\phi^-\rangle$.

APPENDIX C: OTHER TYPES OF SEMI-BELL STATES

In this Appendix we provide complementary materials to prove the universality of our results. We consider other types of semi-Bell states as the initial state of the photonic cavities, namely, $|\phi^-\rangle = \frac{1}{\sqrt{N_-}}(|\alpha\rangle|\alpha\rangle - |-\alpha\rangle|-\alpha\rangle)$ and $|\zeta\rangle = \frac{1}{\sqrt{\kappa}}(|\alpha\rangle|0\rangle + |0\rangle|\alpha\rangle)$, with $N_- = 2(1 - e^{-4|\alpha|^2})$ and $\kappa = 2(1 + e^{-|\alpha|^2})$ [55]. As shown by Fig. 13, both $|\phi^-\rangle$ and $|\zeta\rangle$ store higher amounts of ergotropy and QC in the QB compared to $|\psi\rangle$, with $|\phi^-\rangle$ being the best choice. In the other words, when the chargers are initially correlated, one can see better performance of the QB regarding the stored ergotropy and generated quantum correlation. This is even the case if one considers the useful extractable energy Γ [Fig. 13(b)]. Here $|\zeta\rangle$ initially deposits a larger value of Γ, though in the steady limit $|\phi^-\rangle$ and $|\zeta\rangle$ have no considerable advantage over each other and both result in greater Γ compared to uncorrelated initial chargers. For these two states also we plot the maximum power and ergotropy as a function of α in Fig. 14. A periodic dependence on α similar to that for the previous semi-Bell state ($|\phi^+\rangle$) is apparent. Overall, $|\phi^-\rangle$ emerges with the best performance among the semi-Bell states regarding both the maximum power and ergotropy.

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