





Concentrated subradiant modes in a one-dimensional atomic array coupled with chiral waveguides

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Non-Hermitian systems have recently attracted broad interest and exhibited intriguing physical phenomena, including the non-Hermitian skin effect, which have been widely studied in various fermionic and bosonic systems. Here we propose a non-Hermitian atom-waveguide system composed of a tilted one-dimensional atomic array coupled with two identical waveguides with opposite chiralities. Such system creates an effective lattice model including nonreciprocal long-range hoppings through the chiral-waveguide photon-mediated interactions. We find the excitations of the collective atomic states concentrate in the middle interface associated with subradiant modes, while, on the contrary, superradiant modes exhibit extended features. Such a unique feature in our proposed system is linked to the non-Hermitian skin effect. Simulation results present a subradiant funneling effect, with robustness against small atomic position disorders. Our work underpins the fundamental comprehension towards the non-Hermitian skin effect in open quantum systems and also provides prospective paths to study non-Hermitian systems in the area of quantum optics.

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I. INTRODUCTION

The non-Hermitian Hamiltonian has recently attracted great interest due to its describing the interactions between a physical system and the environment that are ubiquitous in nature. It has been extensively studied in versatile fields, including the measurement of dissipation in open quantum systems [1,2], the dynamics of the nonlinear instabilities in soft matter and quantum fluids [3,4], theoretical and experimental implementations in nonreciprocal coupling strengths [5–12], and many others [13–24]. The non-Hermitian skin effect (NHSE), in which the eigenstates are found to be concentrated near the interface [20,25–30], is one of the most remarkable phenomena in non-Hermitian systems in the past decade, and leads to many appealing physics in quantum systems [11,19–24,28–37]. For example, the non-Hermitian photonics mesh lattice with anisotropic couplings has been studied towards building up the light-harvesting platforms and optical sensors with enhanced sensitivity [30]. Moreover, NHSE is a key component in active arguments on the collapse of the conventional bulk-boundary correspondence, indicating the new perspective of exotic properties in non-Hermitian systems [31–35].

Exploring light-matter interactions and hence manipulating quantum states are critically fundamental in quantum optics, which have been profoundly studied in various systems [38]. Optical waveguides provide achievable platforms to control

efficient light-atom interactions, including providing single atom-photon couplings towards the photon transport process [39–45], and long-range atom-atom interactions in atomic arrays through mediating waveguide-guided propagating photons [46–50]. In particular, an ensemble of atoms coupled with one-dimensional (1D) waveguide systems has been studied to show fruitful properties such as sub- and superradiant states [51–54], the topologically enhanced photons absorption [55], nonlocal optical nonlinearities [56], and the electromagnetically induced transparency [57,58], which exhibit significant applications potentially in quantum state storage and quantum information processing. As a natural non-Hermitian system, the atom-waveguide system presents an interesting platform to explore the NHSE, which, however, has not been studied to the best of our knowledge. Fortunately, by virtue of recent developments in chiral quantum optics fields [5,59–65], it is possible to implement promising nonreciprocal control of light-atom interactions by employing the chiral waveguides [5,62,66–68], which provides the possibility to explore the NHSE in atom-waveguide systems.

In this work, we study a tilted one-dimensional atomic array coupled with two chiral waveguides, which exhibits subradiant states concentrated at the middle interface. By placing the atomic array in-between waveguides with a small angle, position-dependent dipole-dipole interactions have been built through mediating photons that propagate at opposite directions in two waveguides. Such system gives a non-Hermitian lattice model including long-range couplings. Opposite asymmetric hoppings between two atomic dipoles are supported in the lattice model, while the middle atom holds same decays into both waveguides and hence provides an artificial interface

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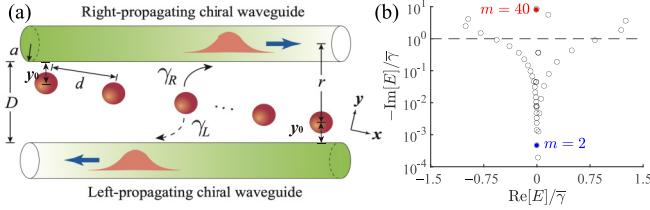


FIG. 1. (a) Schematic of a one-dimensional atomic array coupled with two chiral waveguides, with atoms uniformly arranged with a tilted small angle in-between the waveguides. Atoms emit photons into the right- (left-)propagating chiral waveguide, and the corresponding spontaneous decay rate is γ_R (γ_L), which depends on the distance $r - a$ (or $D - r + a$) from the atom to the top (or bottom) waveguide surface. (b) The complex eigenenergies E of the non-Hermitian lattice model including 41 atoms. The mode number m is arranged in descending order by the imaginary part of E . Blue (red) dot refers to the eigenvalue at $m = 2$ ($m = 40$).

at the center of the atomic array. Our system exhibits concentrated states with subradiant decays, but extended states with superradiant decays. Similar to the funneling effect of light in a photonic system that refers to any light field traveling toward an interface [30], we find that excitations at various positions in the atomic array evolve towards the center interface but hold the subradiant feature. During these processes, the superradiant states of the system dissipate fast. Such concentrated subradiant states show robust property against atomic position disorders. Our study therefore points towards the NHSE in the atom-waveguide system with possible applications in quantum-state harvesting and robust photon storage in the sense of quantum optics.

II. MODEL

We study a one-dimensional atomic array coupled with two chiral waveguides, as schematically shown in Fig. 1(a). In particular, N two-level atoms are aligned along the x axis with equal spacing d , each of which is labeled by j [for $j = -(N-1)/2, \dots, 0, \dots, +(N-1)/2$, for odd N] at positions x_j . Two chiral waveguides with spacing D , placed at a small angle to the x axis in xy plane, are identical except that photons are allowed in propagating in opposite directions, i.e., the top one is a right-propagating waveguide while the bottom one is left propagating [see Fig. 1(a)]. Here we only consider the spontaneous decay rate of guided modes, but ignore the effect caused by the environment since the loss to the environment leads to a background dissipation, and do not change the main feature of the findings in our proposed system (see Appendix A for details). The spontaneous decay rate γ_{Rj} (or γ_{Lj}) is dependent on the distance $r - a$ (or $D - r + a$) from the j th atom to the top (or bottom) waveguide surface, where r denotes the distance from each atom to the top waveguide cylinder axis and a is the radius of each waveguide. We take $r > a$ throughout this paper.

We next discuss the effective model that supports dipole-dipole interactions from the mediated waveguide photon. To this purpose, we take $N = 41$ atoms into consideration, and the term $\bar{\gamma}$ used as a normalization factor throughout this paper refers to the average spontaneous decay rate of this spe-

cific $N = 41$ system, i.e., $\bar{\gamma} = \sum_{j=-20}^{+20} (\gamma_{Rj} + \gamma_{Lj}) / (2N)$. We set $a = 250$ nm, $D = 1000$ nm, and $d = 9073.8$ nm ($d \gg \lambda$) for eliminating the effect caused by the environment in the interatomic interactions). $\lambda = 852$ nm is the wavelength of the waveguide photons. Atoms are uniformly arranged with a tilted angle $\theta \sim 0.002$ rad in-between the waveguides (i.e., the zeroth atom is placed in the middle of two waveguides). γ_{Rj} (γ_{Lj}) are calculated accordingly [69]. We notice that the atom experiences larger decay if it is close to one waveguide and gives relatively very small decay rates into both waveguides if it is near the middle between two waveguides [69–71]. Moreover, there emerges a generalized interface in the center of the atomic array, which is critically important to the later demonstrations of the non-Hermitian skin effect and the funnel-like behavior.

By taking the Born-Markov approximation and neglecting the retardation caused by the finite propagation velocity of photons, one can write the chiral master equation for the evolution of the system density operator [72],

$$\begin{aligned} \dot{\hat{\rho}}(t) = & -i[\hat{H}_{\text{sys}}, \hat{\rho}(t)] + \sum_{\lambda=R,L} \sum_j \frac{\gamma_{\lambda,j}}{2} \{[\hat{\sigma}_j, \hat{\rho}(t)\hat{\sigma}_j^\dagger] \\ & - [\hat{\sigma}_j^\dagger, \hat{\sigma}_j \hat{\rho}(t)]\} \\ & + \sum_{\substack{\lambda=R,L \\ k_\lambda x_j > k_\lambda x_l}} \sum_{j,l} \sqrt{\gamma_{\lambda,j} \gamma_{\lambda,l}} \{e^{-ik_\lambda(x_j - x_l)} [\hat{\sigma}_j, \hat{\rho}(t)\hat{\sigma}_l^\dagger] \\ & - e^{ik_\lambda(x_j - x_l)} [\hat{\sigma}_j^\dagger, \hat{\sigma}_l \hat{\rho}(t)]\}, \end{aligned} \quad (1)$$

where $\hat{\rho}(t)$ is the time-dependent system density operator. Here, $\hbar = 1$ for simplicity. The Hamiltonian of the two-level atoms system Hamiltonian reads $\hat{H}_{\text{sys}} = \sum_j \Delta_j \hat{\sigma}_j^\dagger \hat{\sigma}_j$, with j being the index of the j th atom and the operator $\hat{\sigma}_j^\dagger = |e_j\rangle\langle g_j|$ being the operator representing the transition from the ground state $|g_j\rangle$ to the excited state $|e_j\rangle$. $k_R = -k_L = k$, where k is the wave vector of the photon. \hat{H}_{eff} denotes the effective Hamiltonian.

To derive the effective Hamiltonian of our proposed system in Fig. 1(a), we rewrite the chiral master equation in explicit Lindblad form as [66,72]

$$\begin{aligned} \dot{\hat{\rho}}(t) = & -i[\hat{H}_{\text{sys}}, \hat{\rho}(t)] - i[\hat{H}_{\text{eff}} \hat{\rho}(t) - \hat{\rho}(t) \hat{H}_{\text{eff}}^\dagger] + \hat{c}_L \hat{\rho}(t) \hat{c}_L^\dagger \\ & + \hat{c}_R \hat{\rho}(t) \hat{c}_R^\dagger, \end{aligned} \quad (2)$$

where $\hat{c}_L = \sum_j \sqrt{\gamma_{Lj}} e^{ikx_j} \hat{\sigma}_j$ and $\hat{c}_R = \sum_j \sqrt{\gamma_{Rj}} e^{-ikx_j} \hat{\sigma}_j$. By comparing Eqs. (1) and (2), we obtain the long-range effective Hamiltonian of our model as

$$\begin{aligned} \hat{H}_{\text{eff}} = & -\frac{i}{2} \sum_j (\gamma_{Lj} + \gamma_{Rj}) \hat{\sigma}_j^\dagger \hat{\sigma}_j - i \sum_{j>l} \sqrt{\gamma_{Ll} \gamma_{Lj}} \hat{\sigma}_l^\dagger \hat{\sigma}_j e^{ik(x_j - x_l)} \\ & - i \sum_{j>l} \sqrt{\gamma_{Rl} \gamma_{Rj}} \hat{\sigma}_j^\dagger \hat{\sigma}_l e^{ik(x_j - x_l)}. \end{aligned} \quad (3)$$

The effective Hamiltonian in Eq. (3) is non-Hermitian, which is consistent with the Hamiltonians in Refs. [20,27–36,72,73], but includes nonuniform long-range dipole-dipole interactions where on-site decays in the first term and hopping coefficients in the second and third terms are dependent on the positions of the two atoms. In the following, we use the

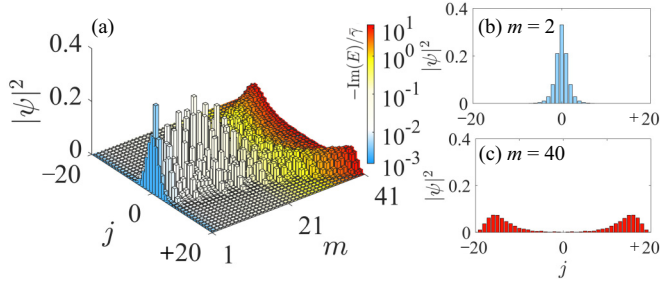


FIG. 2. (a) Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$. Colors for each eigenstate indicate the collective decay rates. (b),(c) The intensity distributions for the eigenstate with $m = 2$ and the eigenstate with $m = 40$, respectively.

Hamiltonian (3) to explore the NHSE phenomena in the model of Fig. 1(a).

III. RESULTS

The effective Hamiltonian in Eq. (3) includes $N = 41$ atoms at different positions between waveguides, and hence has no translational symmetry. We therefore diagonalize Eq. (3) with the open boundary directly in the spatial space. The resulting band structure is plotted in Fig. 1(b), which has 41 eigenenergies E at complex values. We use $m = 1, 2, \dots, 41$ to label the index of eigenenergies by increasingly sorting $[-\text{Im}(E)]$, the collective decay rate of the mode, from a slow decay rate to a fast decay rate. One can see that most of the modes exhibit a collective decay rate that is smaller than the average spontaneous decay rate of the system, i.e., $[-\text{Im}(E)] < \bar{\gamma}$, referring to subradiant modes, while the others have $[-\text{Im}(E)] > \bar{\gamma}$, giving the superradiant modes of the system. Notably, there are two modes having $[-\text{Im}(E)] \sim 10^{-3}\bar{\gamma}$ ($m = 1, 2$). The real parts of the eigenenergies are around zero for most subradiant modes, but diverge towards $\sim \pm\bar{\gamma}$ for larger m (larger collective decay rate).

We further plot the intensity distribution versus atom site j for all the eigenstates with $m = 1, 2, \dots, 41$, in Fig. 2(a). The remarkable feature is that the modes with very small collective decay rate (subradiant modes) exhibit concentrated intensity in the middle interface around the zeroth atom. When the collective decay rate of the modes increases, the concentration of intensity at the middle interface becomes weaker. Further increase of the collective decay rate to the superradiant regime results in the fact that the concentration disappears and the intensities of the modes extend gradually towards two ends of the atomic array. We show two examples of very opposite cases in Fig. 2(b) for the eigenstate at $m = 2$ and Fig. 2(c) for the eigenstate at $m = 40$. One can see that for the mode at $m = 2$ in Fig. 2(b), the intensity distribution is concentrated mostly at the middle of the atomic array, with exponentially decaying into both sides. The corresponding collective decay rate is $\sim 1.4 \times 10^{-3}\bar{\gamma}$, which is a subradiant mode. However, for the mode at $m = 40$ in Fig. 2(c), the collective decay rate gives a superradiant decay with $\sim 8\bar{\gamma}$, where the eigenstate exhibits the extended distributions with highest intensities at the ($j = \pm 17$)-th atoms.

Next we conduct numerical simulations to study the evolutions of the system under different conditions. We consider

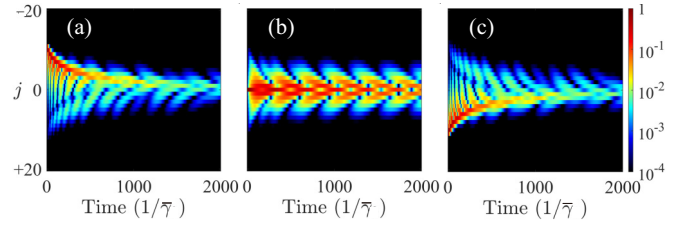


FIG. 3. Simulation results of the excited wave-packet evolutions with time, with three different initial excitation positions at (a) $j_s = -11$, (b) $j_s = 0$, and (c) $j_s = +11$, respectively.

the wave function of the excited wave packet as

$$|\phi(t)\rangle = \sum_j v_j(t) \hat{\sigma}_j^\dagger |0\rangle, \quad (4)$$

where $v_j(t)$ is the amplitude of the wave-packet state at the j th atom. By inserting Eq. (4) into the Schrödinger equation $H|\phi(t)\rangle = i \frac{d}{dt} |\phi(t)\rangle$, where H is defined in Eq. (3), we obtain the coupled time evolution equations at the position of the j th atom when exciting the j_s th atom ($-20 < j < +20$):

$$\begin{aligned} \dot{v}_j(t) = & -\frac{\gamma_{Lj} + \gamma_{Rj}}{2} v_j(t) - \sum_{l=-20}^{j-1} \sqrt{\gamma_{Rl} \gamma_{Rj}} e^{ik \cdot (j-l)d} v_l(t) \\ & - \sum_{l=j+1}^{+20} \sqrt{\gamma_{Lj} \gamma_{Ll}} e^{ik \cdot (l-j)d} v_l(t) + s(t) \delta_{j_s, j}. \end{aligned} \quad (5)$$

The j_s th atom is excited by a temporal Gaussian-shape excitation source $s(t)$,

$$s(t) = e^{-\frac{(t-t_s)^2}{2\tau^2}} e^{-i\omega_s t}, \quad (6)$$

where t_s and τ give the temporal center and the width of the Gaussian excitation, respectively, and ω_s represents the excitation frequency.

In simulations, we are aiming to excite the subradiant modes, which exhibits the concentration at the middle interface with a relatively long decay time [such as the mode at $m = 2$ in Fig. 2(b)]. We therefore set $\omega_s = -0.0032\bar{\gamma}$, $t_s = \bar{\gamma}^{-1}$, and $\tau = 2\bar{\gamma}^{-1}$. Note that the excitation frequency ω_s is chosen to be resonant with the eigenfrequency of the mode at $m = 2$. However, as shown in Fig. 1(b), the real parts of the eigenstates are around 0, so the excitation source here may excite many modes. Fortunately, the subradiant modes not only share the same features of concentration at the middle interface, but also exhibit long decay time, which makes it possible to observe the corresponding phenomena in simulations. We also take three different excited positions at $j_s = -11, 0$, and $+11$, respectively, where the simulation results showing the excited wave-packet evolutions with time are plotted in Fig. 3. One can see that for all three cases, the normalized intensity of the excitation gradually focuses at the middle interface during evolutions, while the energy of the excitation decays exponentially. Such phenomena are consistent with the eigenstate distributions in Fig. 2(a). After all superradiant modes quickly decay, the subradiant modes exhibit a slower decay with the funnel-like behavior that the

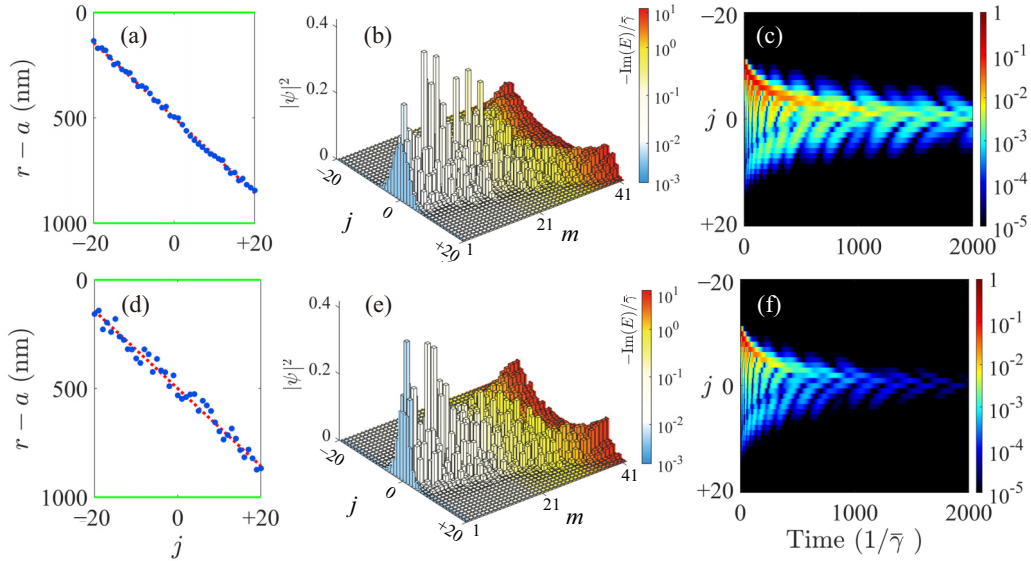


FIG. 4. Atomic positions with disorders at (a) $\delta = 2\theta d$ and (d) $\delta = 6\theta d$. The blue dots are the position arrangement of 41 atoms, the red dashed line denotes the original atomic positions without disorders, and the solid green lines indicate the two chiral-waveguide surfaces. (b),(e) The corresponding intensity distributions of all eigenstates, respectively. (c),(f) The corresponding simulation results of the excited wave-packet evolutions with time, with the initial excitation position at $j_s = -11$.

wave packet propagates to the middle interface. Moreover, compared with excitations near two sides of the atomic array, the initial excitation at the middle [see Fig. 3(b)] shows the confined wave packet with stronger intensities because the initial excitation largely overlaps with the eigenstates of subradiant modes and hence more such states are excited.

Different from Ref. [30], our model supports not only subradiant modes that concentrate the excited quantum states into the middle interface, but also includes superradiant modes that hold extended distributions but decay much faster. Therefore, in the funneling process in our proposed model, the total energy of the excitation decreases due to the energy loss not only from the background dissipation, but also mainly from these superradiant modes. Nevertheless, the concentrations of the wave packet at the middle interface associated with subradiant modes reveal the manifestation of the non-Hermitian skin effect in this atom-waveguide system.

To further explore the robustness of such funnel-like behavior in this non-Hermitian atom-waveguide system, we consider disorders of atomic positions, where each atom is deviated vertically from its original position by a factor of δR . Here, R is a random number chosen in a regime of $(-0.5, 0.5)$ and δ is a constant reflecting the disorder of the system. In simulations, we take $\delta = 2\theta d \approx 36$ nm and $\delta = 6\theta d \approx 108$ nm, which results in the disordered atomic positions shown in Figs. 4(a) and 4(d). The corresponding intensity distributions of all the eigenstates are presented in Figs. 4(b) and 4(e), respectively. One can see that in both cases, the subradiant modes are still concentrated near the interface, while the superradiant modes expand towards both sides of the atomic array. We also perform simulations at conditions similar as those in Fig. 3(a) with the excitation source applied on the -11 th atom. As seen in both Figs. 4(c) and 4(f), although larger disorders lead to faster decay of the collective excitation, excitations of the system are localized

into the middle interface of the atomic array. Simulations with disorders presented here therefore demonstrate robust NHSE associated with the subradiant modes.

The concentrated subradiant states in our proposed model are of a different nature from the localization effects under the topological protection [29,74–83], where only a few topological modes are selected to show the concentration. The subradiant states here are found to be associated with the NHSE, while the extended states are the results from the competition between fast single-atom losses and the NHSE (see Appendix C for details). Moreover, the interesting phenomena shown in our work mainly result from the nonreciprocal couplings but not the loss profile, while the losses are indeed crucial for the extended states. We further explore our proposed model over a broad range of parameters in Appendices C and E, and conclude that the existence that is found of concentrated states with subradiant features persists for the scale of the system, $N \rightarrow \infty$, which provides further evidence of connecting to NHSE [84]. Lastly, the occurrence of the interesting phenomena that are found mainly depends on the varying rate of $\gamma_{R(L)j}$ on the atomic positions, which is a result of the geometry of the atomic array. In other words, if $\gamma_{R(L)j}$ varies largely, the distributions of eigenstates show the phenomena of concentrated subradiant states and extended superradiant states. However, when the varying rate of $\gamma_{R(L)j}$ decreases, a transition occurs and the system exhibits all bulk states (refer to Appendix E).

IV. DISCUSSION AND CONCLUSION

Our proposal is potentially feasible for experimental realizations on optical platforms including nanofibers or nanophotonic waveguides coupled with atoms [53,54,85–89] and superconducting transmission lines with artificial atoms

[90–94]. Previous experiments have demonstrated that the nanofiber can effectively trap ~ 2000 atoms localized about 200 nm around the nanofiber surface [95], where the ratio between the guided mode decay rate (γ) and the vacuum decay rate of a single atom (γ_0) can reach $\gamma/\gamma_0 \sim 0.9 \pm 0.1$ [53]. Moreover, it has been shown that one can further reduce the impact from the losses emitted into the environment [90,96–99]. For example, recent developments in experiments have greatly strengthened the guided mode coupling coefficients $\gamma/\gamma_0 \sim 50$ for transmon qubits coupled to a 1D coplanar microwave transmission line [90]. On the other hand, chiral waveguides have been engineered in photonic nanostructures, where unidirectional transport of photons have been realized for chiral quantum optics [5,60–65,100]. All of these start-of-art technologies make the proposal of atoms localized in-between two chiral waveguides feasible in future experiments.

In summary, we have investigated a tilted one-dimensional atomic array coupled with two chiral waveguides, which supports a non-Hermitian lattice model with asymmetric long-range hoppings. Features of concentrated states with subradiant decays but extended states with superradiant decays are exhibited. By exciting the system at different atoms, we numerically show the atomic funnel-like behavior where the energy of the excitation is guided to the middle interface, with the robustness against small disorders. Our results reveal distinctive physics in a chiral atom-waveguide system, pointing to the NHSE associated with the subradiant modes, which paves a promising path to study intriguing non-Hermitian properties via quantum optics platforms. This theoretical proposal also shows potential applications towards the manipulation of quantum states, which is of great significance in the fields of quantum storage and quantum information processing.

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APPENDIX A: THE EFFECT OF THE ENVIRONMENT

The total spontaneous decay rate for a single atom placed near a waveguide can be written as

$$\gamma^{(\text{total})} = \gamma^{(\text{g})} + \gamma^{(\text{r})}, \quad (\text{A1})$$

where $\gamma^{(\text{g})}$ and $\gamma^{(\text{r})}$ refer to guided modes and radiation modes, respectively. The numerical results of $\gamma^{(\text{g})}$ can be obtained by the approach in Appendix B.

From many well-known works in Refs. [69–71], we know that as the distance increases, the decay rate of the guided modes exponentially decreases to zero, while the decay rate of the radiation modes exponentially decreases to the free-space decay γ_0 . In our main text, we take all parameters in Ref. [69], numerically calculate $\gamma^{(\text{g})}$ for a single waveguide, and exhibit our result in Fig. 7, which is consistent with Fig. 2(a) in Ref. [69]. Since we take the same parameters and refer to their calculation methods, there is a comparability of radiation modes in our work. We notice that the decay rate of the radiation modes stably reaches to free-space decay γ_0 at about $r - a = 125$ nm. Therefore, we can take the decay to the environment as γ_0 throughout the main text as long as we make sure the nearest distance between an atom and the surface of the waveguide is bigger than 125 nm. After making this approximation, the total spontaneous decay for a single atom is

$$\gamma^{(\text{total})} = \gamma^{(\text{g})} + \gamma_0. \quad (\text{A2})$$

Furthermore, we make our second reasonable approximation: the emission into the environment only provides independent single-atom decay, but does not contribute to dipole-dipole interactions. The same approximation is used in Ref. [101] when dealing with the effect caused by the environment. This approximation is valid when the interatomic distance (labeled as d in the main text) is much larger than the wavelength (calculated as $2\pi/k$ in the main text), i.e., $d \gg \lambda = 2\pi/k = 852$ nm. The reason is as follows: The waveguide mediates long-range interactions by photons, and

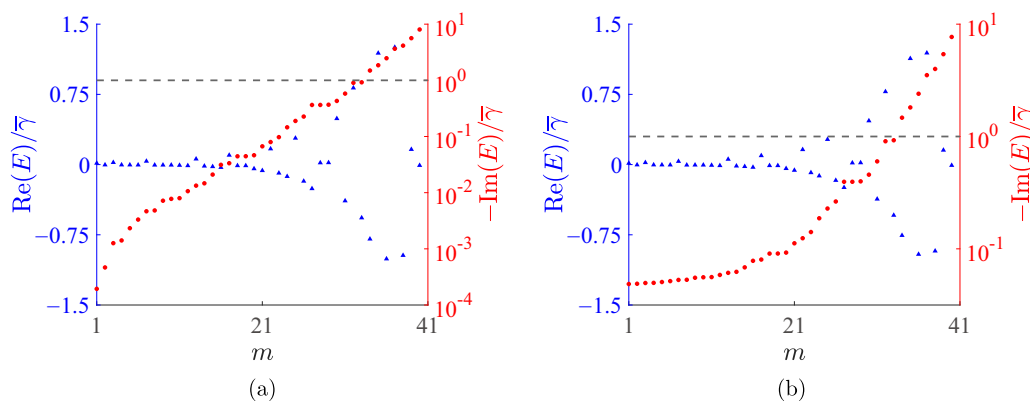


FIG. 5. (a) The complex eigenenergies E of the non-Hermitian lattice model including 41 atoms in the main text. (b) The complex eigenenergies E of the non-Hermitian lattice model including 41 atoms which takes into account the environment. The order of the mode number m is arranged in descending order by the imaginary part of E .

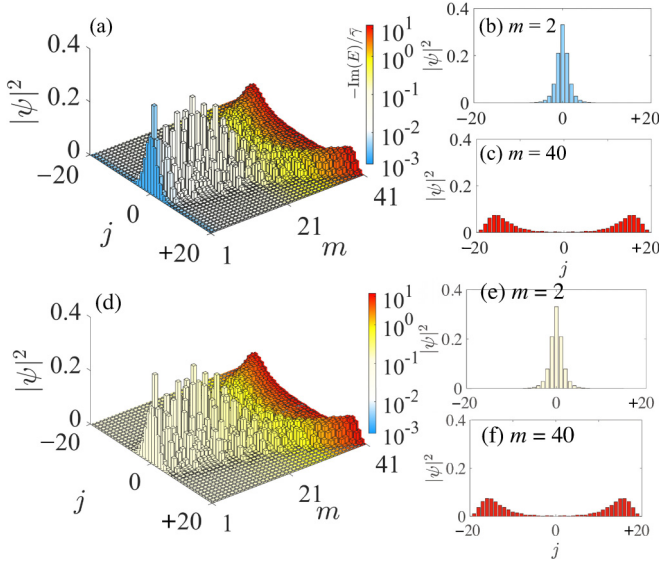


FIG. 6. The localization without taking into account the environment (upper panel) vs the localization that takes into account the environment (lower panel). (a), (d) Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$. Colors for each eigenstate indicate the collective decay rates. (b), (e) The intensity distributions for the eigenstate with $m = 2$. (c), (f) The intensity distributions for the eigenstate with $m = 40$.

thus the atoms interact with each other via the guided mode no matter what the atoms' spacing is. In contrast, the field decays depending on distance as a series of power laws in free space. Therefore, if the spacing of two atoms is too far, the cross-atom decay can be seen as zero.

From what has been discussed above, we obtain a new effective Hamiltonian including the effect by the environment which describes this non-Hermitian system written as

$$\begin{aligned} \hat{H}_{\text{eff}} = & -\frac{i}{2} \sum_j (\gamma_{Lj}^{(\text{total})} + \gamma_{Rj}^{(\text{total})}) \hat{\sigma}_j^\dagger \hat{\sigma}_j \\ & -i \sum_{j>l} \sqrt{\gamma_{Ll}^{(g)} \gamma_{Lj}^{(g)}} \hat{\sigma}_l^\dagger \hat{\sigma}_j e^{ik(x_j - x_l)} \\ & -i \sum_{j>l} \sqrt{\gamma_{Rl}^{(g)} \gamma_{Rj}^{(g)}} \hat{\sigma}_j^\dagger \hat{\sigma}_l e^{ik(x_j - x_l)}. \end{aligned} \quad (\text{A3})$$

We set the same parameters as the main text. Fig. 5 shows the energy spectra for this new Hamiltonian in Eq. (A3).

According to Fig. 5, one can see that the real part of the energies does not change, which indicates that adding the impact from the environment will not influence the exciting frequency in the later numerical simulation. In contrast, the imaginary part of the energies changes as one can expect. The losses of subradiant states inevitably increase. Fortunately, the number of sub- and superradiant states stays the same. However, the minimal loss is over $10^{-2} \bar{\gamma}^{(\text{total})}$ ($\bar{\gamma}^{(\text{total})} = \bar{\gamma}^{(g)} + \gamma_0$ due to linear algebra), where $\bar{\gamma}^{(\text{total})}$ and $\bar{\gamma}^{(g)}$ refer to the average total and guided spontaneous decay rate of this specific $N = 41$ system, respectively.

As illustrated in Fig. 6, the intensity distributions still exhibit similar patterns after the environment is included. We

can come to the similar conclusion as we have claimed in the main text: our system exhibits concentrated states with subradiant decays, but extended states with superradiant decays.

In summary, by making two reasonable approximations, we find the environment critically important to exert an impact on the energy spectra, especially the imaginary part, i.e., the losses. Nevertheless, the effect of environment does not change the main feature of our proposed system, i.e., the system has subradiant modes concentrated in the middle interface but also includes superradiant modes that hold extended distributions but decay much faster.

APPENDIX B: THE CALCULATION OF THE SPONTANEOUS DECAY RATE OF GUIDED MODES FOR A SINGLE WAVEGUIDE IN THE ATOM-WAVEGUIDE SYSTEM

In the main text, we use the spontaneous decay rate γ of guided modes in the atom-waveguide system. Here we provide details. We take the refractive index of the waveguide n_1 and the surrounding environment has the refractive index n_2 . The position-dependent rate of an atom decaying into the waveguide can be obtained by solving the time-dependent Schrödinger equation and then performing a standard Wigner-Weisskopf treatment, which has been attentively discussed in Ref. [71], and the resulting decay rate is

$$\gamma(r) = \frac{\Delta_j}{2\epsilon_0 \hbar} |\hat{\mathbf{d}}_j \cdot \hat{\mathbf{e}}(r)|^2, \quad (\text{B1})$$

where Δ_j denotes the atomic transition frequency; $\hat{\mathbf{d}}_j$ refers to the dipole moment for an atom, and $\hat{\mathbf{e}}(r)$ is the profile function for photon modes that propagate at the z axis along the waveguide direction, which reads [69]

$$\begin{aligned} e_r &= iC[(1-s)K_0(qr) + (1+s)K_2(qr)], \\ e_\varphi &= -C[(1-s)K_0(qr) - (1+s)K_2(qr)], \\ e_z &= C \frac{2q}{k} K_1(qr), \end{aligned} \quad (\text{B2})$$

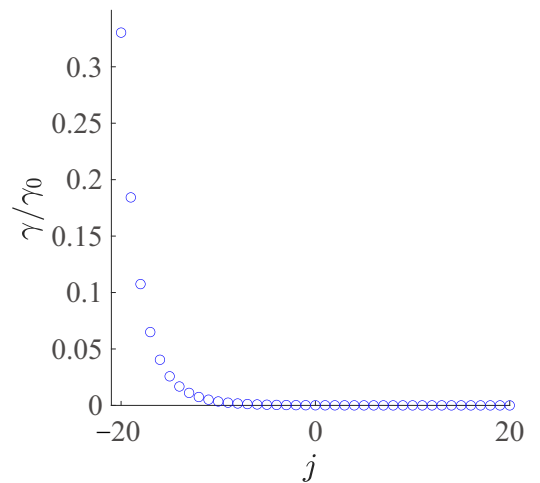


FIG. 7. The numerical results of the spontaneous decay rate determined by the distance $r - a$ (or $D - r + a$) from the atom to the top (or bottom) waveguide surface at the position of each atom $j = -20, -19, \dots, +19, +20$.

where K_p ($p = 0, 1, 2$) represents the modified Bessel functions of the second kind; C satisfies the normalization condition that $\int_0^{2\pi} d\varphi \int_0^\infty n_2^2 |\hat{\mathbf{e}}|^2 r dr = 1$. The parameters $q = \sqrt{k^2 - n_2^2 k_0^2}$ and $h = \sqrt{n_1^2 k_0^2 - k^2}$, and the propagation constant k is the solution of the waveguide eigenvalue equation that reads [102,103]

$$\begin{aligned} \frac{J_0(ha)}{haJ_1(ha)} = & -\frac{n_1^2 + n_2^2}{2n_1^2} \frac{K_1'(qa)}{qaK_1(qa)} + \frac{1}{h^2 a^2} \\ & - \left[\left(\frac{n_1^2 - n_2^2}{2n_1^2} \frac{K_1'(qa)}{qaK_1(qa)} \right)^2 \right. \\ & \left. + \frac{k^2}{n_1^2 k_0^2} \left(\frac{1}{q^2 a^2} + \frac{1}{h^2 a^2} \right)^2 \right]^{1/2}. \end{aligned} \quad (\text{B3})$$

Here, in Eqs. (B2) and (B3), the parameter s takes the form

$$s = \frac{1/h^2 a^2 + 1/q^2 a^2}{J_1'(ha)/haJ_1(ha) + K_1'(qa)/qaK_1(qa)}, \quad (\text{B4})$$

where J_1 is the Bessel functions of the first kind.

$$H = \begin{pmatrix} 0 & \sqrt{\gamma_{L-20}\gamma_{L-19}} & 0 & 0 & 0 & 0 \\ \sqrt{\gamma_{R-20}\gamma_{R-19}} & 0 & \sqrt{\gamma_{L-19}\gamma_{L-18}} & 0 & 0 & 0 \\ 0 & \sqrt{\gamma_{R-19}\gamma_{R-18}} & 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 & \sqrt{\gamma_{L+18}\gamma_{L+19}} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma_{R+18}\gamma_{R+19}} & 0 & \sqrt{\gamma_{L+19}\gamma_{L+20}} \\ 0 & 0 & 0 & 0 & \sqrt{\gamma_{R+19}\gamma_{R+20}} & 0 \end{pmatrix}. \quad (\text{C1})$$

By diagonalizing the Hamiltonian (C1), we can plot the distribution of all eigenstates, as shown in Fig. 8. One can find that here all modes are localized at the middle interface, which is similar to the results in Ref. [104].

We further add single-atom loss terms into the toy in Eq. (C1), and the new Hamiltonian is written as

$$H = \begin{pmatrix} -\frac{i}{2}(\gamma_{L-20} + \gamma_{R-20}) & \sqrt{\gamma_{L-20}\gamma_{L-19}} & 0 & 0 & 0 & 0 \\ \sqrt{\gamma_{R-20}\gamma_{R-19}} & -\frac{i}{2}(\gamma_{L-19} + \gamma_{R-19}) & \sqrt{\gamma_{L-19}\gamma_{L-18}} & 0 & 0 & 0 \\ 0 & \sqrt{\gamma_{R-19}\gamma_{R-18}} & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \sqrt{\gamma_{L+18}\gamma_{L+19}} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma_{R+18}\gamma_{R+19}} & -\frac{i}{2}(\gamma_{L+19} + \gamma_{R+19}) & \sqrt{\gamma_{L+19}\gamma_{L+20}} \\ 0 & 0 & 0 & 0 & \sqrt{\gamma_{R+19}\gamma_{R+20}} & -\frac{i}{2}(\gamma_{L+20} + \gamma_{R+20}) \end{pmatrix}. \quad (\text{C2})$$

The resulting intensity distribution of the eigenstates is shown in Fig. 9. One can notice that only when we add the single-atom losses, the extended states emerge while the subradiant states remain concentrated in the vicinity of the middle interface, as those in Fig. 8. By further adding back long-range terms towards the realistic model in Eq. (3) in the main text, one can obtain the result shown in Fig. 2 in the main text. From this perspective, we then conclude that the subradiant states are indeed associated with the non-Hermitian skin effect, and the extended states are results from the competition

Once we have all the information above, we can plot the decay rate by taking Eqs. (B1)–(B4) with parameters of the cesium atom and a silica cylinder nanofiber from Refs. [69,71], as illustrated in Fig. 7.

APPENDIX C: EXPLORE THE PHYSICAL CONNECTION BETWEEN CONCENTRATED SUBRADIANT STATES AND THE NON-HERMITIAN SKIN EFFECT

Let us further discuss the physical connection between the phenomena of the concentrated subradiant states and the extended superradiant states in our system and the non-Hermitian skin effect. In our proposed model, the extended modes are mainly the results of the losses profile, without which our system is supposed to exhibit the exact non-Hermitian skin effect. To be specific, the exact non-Hermitian skin effect which is supposed to emerge in our system will be destroyed by the quantity and the position of the losses; therefore, superradiant states behave to be extended on both sides of the array, while the states in the middle of the array, the subradiant modes, still behave like the skin effect.

To illustrate our argument, let us refer to the one-dimensional ring resonator system (see Fig. 1(e) in Ref. [104]) and consider our model by ignoring the loss profile and long-range couplings, but only taking the position-dependent nearest-neighbor couplings with real values, with the corresponding Hamiltonian as

between fast single-atom losses and the non-Hermitian skin effect.

Last but not the least, it is worth noting that the non-Hermitian effect does not mean all eigenstates have to localize near the interface. In fact, it only requires that partial eigenstates are localized near the interface. More specifically, it requires that the proportion of skin modes tends to be nonzero when the scale of the system $L \rightarrow \infty$ (see Fig. 2(b3) in Ref. [84]). We explore the impact from the system size, i.e., the atom number N , in Appendix E, and we find that with

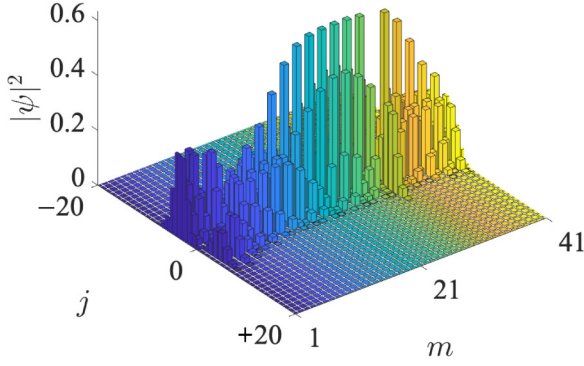


FIG. 8. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ from Eq. (C1).

the size of the system being enlarged, the main feature of the concentrated subradiant states and the extended superradiant states found in the main text persists, as illustrated in Fig. 20.

APPENDIX D: EXCLUSION OF THE EFFECT CAUSED BY THE LOSS PROFILE

Here we conduct more numerical simulations on an artificial reciprocal model, pointing out that the interesting phenomena shown in the main text mainly result from the nonreciprocal couplings but not the loss profile, while the losses are crucial for the extended states.

The Hamiltonian of our model is written as

$$\hat{H}_{\text{eff}} = -\frac{i}{2} \sum_j (\gamma_{Lj} + \gamma_{Rj}) \hat{\sigma}_j^\dagger \hat{\sigma}_j - i \sum_{j>l} \sqrt{\gamma_{Ll} \gamma_{Lj}} \hat{\sigma}_l^\dagger \hat{\sigma}_j e^{ik(x_j - x_l)} - i \sum_{j>l} \sqrt{\gamma_{Rl} \gamma_{Rj}} \hat{\sigma}_j^\dagger \hat{\sigma}_l e^{ik(x_j - x_l)}. \quad (\text{D1})$$

To make a comparison, we devise a toy model which has the same local loss profile as our proposed model and the

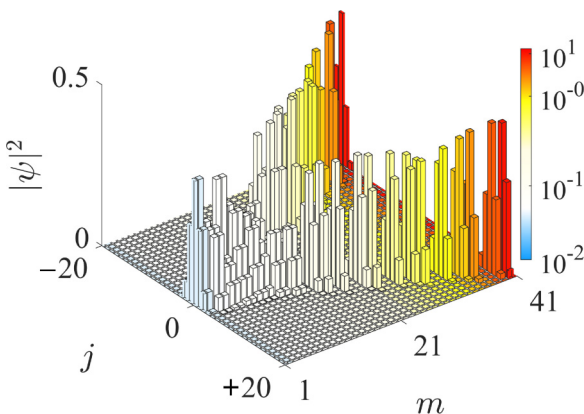


FIG. 9. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ from Eq. (C2).

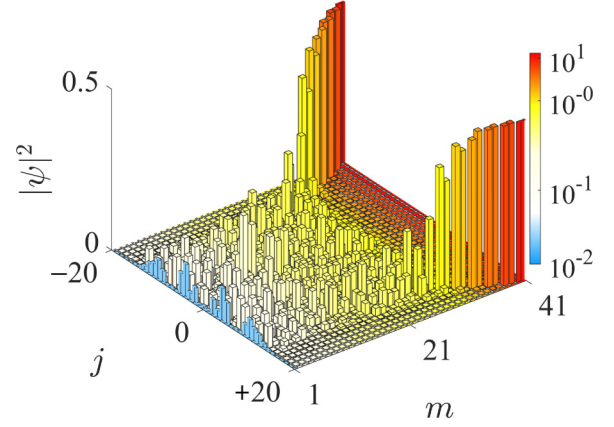


FIG. 10. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ of the toy model from Eq. (D2). Colors for each eigenstate indicate the collective decay rates.

corresponding Hamiltonian is written as

$$\hat{H}_{\text{eff}} = -\frac{i}{2} \sum_j (\gamma_{Lj} + \gamma_{Rj}) \hat{\sigma}_j^\dagger \hat{\sigma}_j - i \sum_{j \neq l} t_{jl} \hat{\sigma}_l^\dagger \hat{\sigma}_j e^{ik|x_j - x_l|}, \quad (\text{D2})$$

where $t_{jl} = (\sqrt{\gamma_{Ll} \gamma_{Lj}} + \sqrt{\gamma_{Rl} \gamma_{Rj}})/2$ is the reciprocal long-range coupling between the j th and l th atoms.

According to Eq. (D2), we plot the localization of the reciprocal lattice in Fig. 10. One can see that the reciprocal toy model exhibits mostly bulk modes, while it is noticeable that the eigenstates with the biggest losses have very strong intensities localized on both sides of the array. By comparing Fig. 2(a) in the main text and Fig. 10, one can exclude the possibility that the subradiant states solely reflect the loss profile, but have a strong connection with the nonreciprocal couplings, while the high losses are crucial for the extended states.

APPENDIX E: STUDY OF THE NON-HERMITIAN MODEL OVER A BROAD RANGE OF PARAMETERS

Here we conduct more numerical calculations over a range of parameters, which are summarized in the following.

As labeled in Fig. 1(a), we consider four variables in our proposed model, i.e., $y_0, H = D - 2y_0, d, N$, where d is the atoms' spacing. H represents the width of the space that the atomic array takes. y_0 refers to the nearest distance from an atom to the vertical surface of the waveguide, namely, the distance from the $[j = -\frac{(N-1)}{2}]$ -th ($[j = +\frac{(N-1)}{2}]$ -th) atom to the top (bottom) waveguide. N is the atom number.

At the very first, let us introduce three important factors, which quantify the proposed system and thus make it more convenient to study over a range of parameters.

(i) The average lifetime of all subradiant states, which reflects the average level of decay rate of all subradiant states, is labeled as $\langle \tau \rangle$.

(ii) The FWHM of the most concentrated state, which confers the quantity of the concentration of the most localized state, is labeled as FWHM_{min} . The bigger the FWHM_{min} is, the lower concentration the state is. We obtain this quantity by

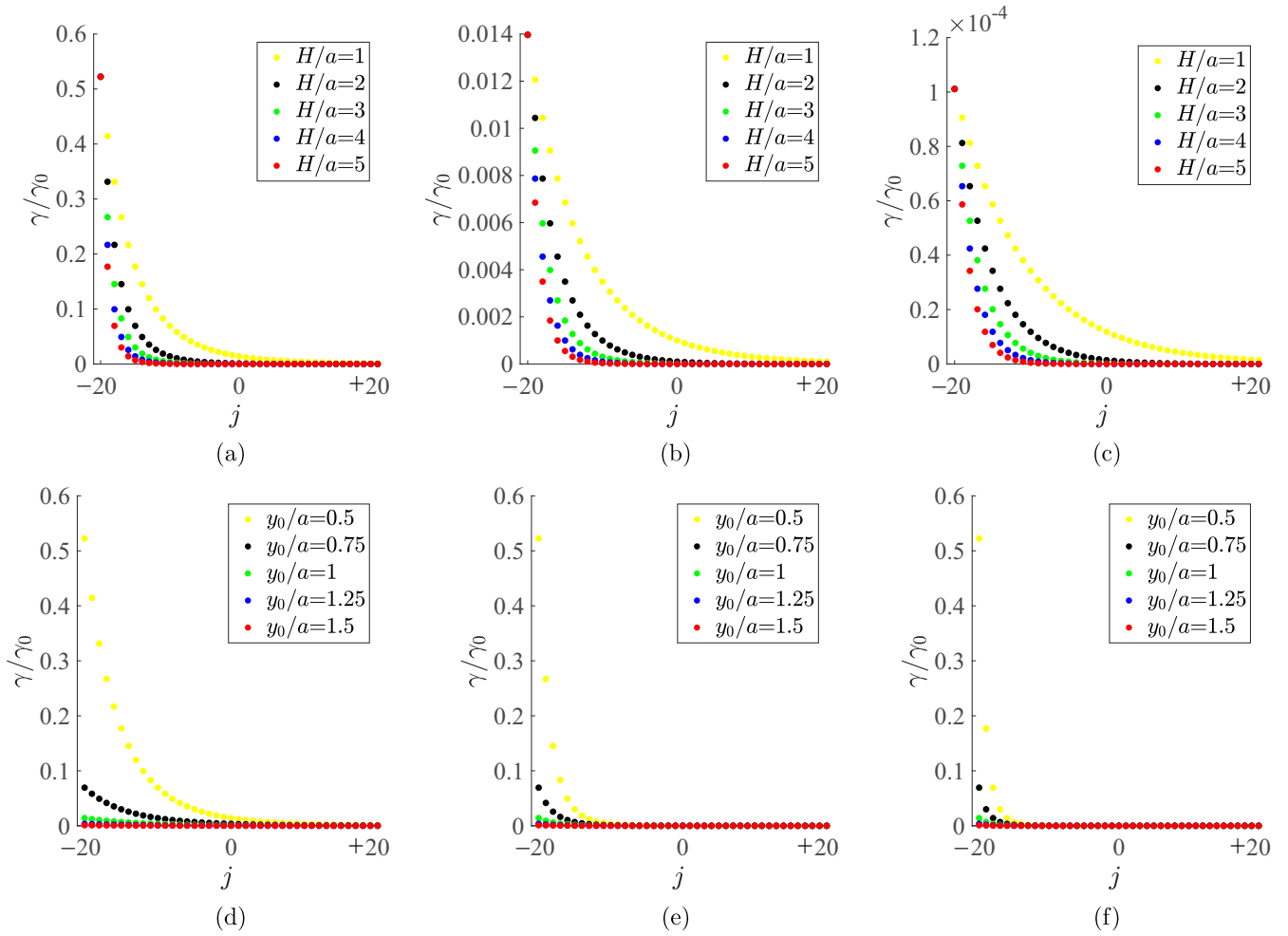


FIG. 11. The numerical results of the spontaneous decay rate of guided modes for a single waveguide at the position of each atom. Each decay is normalized by the spontaneous decay rate in free space. (a)–(c) The decay rate with varying $H/a = 1, 2, 3, 4,$ and 5 , and constant (a) $y_0/a = 0.5$, (b) $y_0/a = 1$, and (c) $y_0/a = 2$, as labeled in yellow, cyan, green, blue, and red dots, respectively. (d)–(f) The decay rate with varying $y_0/a = 0.5, 0.75, 1, 1.25,$ and 1.5 , and constant (d) $H/a = 1$, (e) $H/a = 3$, and (f) $H/a = 5$, as labeled in yellow, cyan, green, blue, and red dots, respectively.

first calculating eigenstates, and then conducting the Gaussian fitting. The unit is the atom site number.

(iii) The average FWHM of all subradiant states, which provides the relevant information of the concentration of

all subradiant states, is labeled as $\langle \text{FWHM} \rangle$. This quantity describes the average level of concentration for this system. We obtain the $\langle \text{FWHM} \rangle$ by the following steps: We first numerically calculate the eigenstates, then conduct the Gaussian

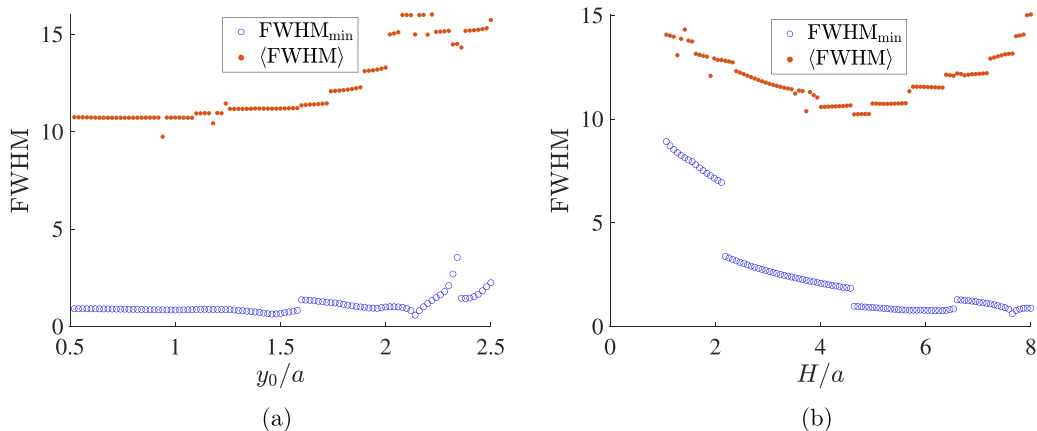


FIG. 12. (a) FWHM_{\min} and $\langle \text{FWHM} \rangle$ vs y_0/a when $H/a = 5$. (b) FWHM_{\min} and $\langle \text{FWHM} \rangle$ vs H/a when $y_0 = 0.5$.

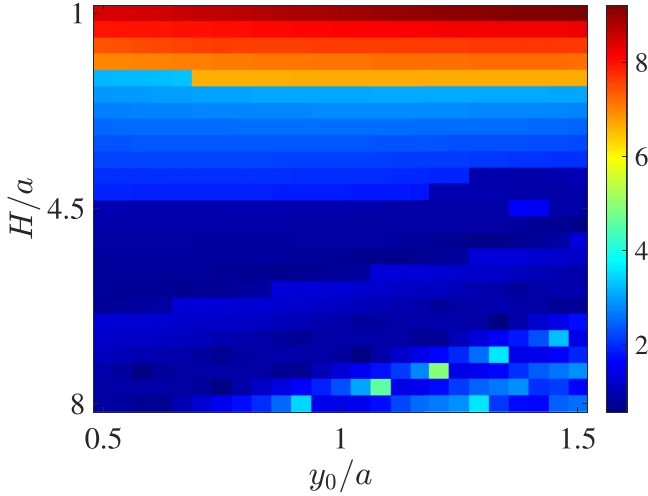


FIG. 13. The heat map of FWHM_{\min} . The colors indicate the amplitude of FWHM_{\min} .

fitting on all subradiant states to get the FWHM for each subradiant state. At last, we do the average operation. The unit is the atom site number.

We then study the effects of y_0 , H , d , and N on our proposed system, with details as follows:

1. Effects of parameters H and y_0

In our system, H and y_0 reflect how quickly the single- and cross-atom couplings vary and how strong they are. We plot the numerical results of the spontaneous decay rate determined by the distance $r - a$ from the atom to the top waveguide surface in Fig. 11. When y_0 is a constant, larger H indicates that the decrease rate of coupling strengths γ is faster, as exhibited in Figs. 11(a)–11(c). Similarly, when H is constant, larger y_0 means lower couplings, as exhibited in Figs. 11(d)–11(f). One can notice that y_0 determines where the value of γ starts at $j = -20$. H/a and y_0/a together determine the varying rate of γ on the atomic positions, while here we find the varying rate of H/a matters more than y_0/a 's, as exhibited in Figs. 11(d)–11(f). Here we choose $d/\lambda = 10.65$, $N = 41$ in the simulations.

We also calculate the FWHM with varying y_0/a and H/a , as illustrated in Fig. 12. According to Fig. 12(a), as y_0/a increases, the FWHM roughly stays constant until y_0/a reaches 2. As to Fig. 12(b), as H/a increases, FWHM_{\min} decreases, which means the concentration increases and roughly stays constant after $H/a = 4.6$, whereas $\langle \text{FWHM} \rangle$ increase at first and then decreases after H/a reaches 4.6.

Moreover, we make a heat map on FWHM_{\min} , as exhibited in Fig. 13. The cool colors in Fig. 13 indicate better concentrations than the warm colors. One can explicitly see the transition in Fig. 13. For example, when $H/a \lesssim 2$, it shows that eigenstates are not concentrated, while for $H/a > 2$, it indicates that the eigenstates exhibit smaller FWHM_{\min} , meaning the concentration. Therefore, we find that the decrease of the vertical width of the atomic array, H , brings the transition from concentrated states to nonconcentrated states.

We further plot $\langle \tau \rangle$, as exhibited in Fig. 14. As one can see in Fig. 14, $\langle \tau \rangle$ gets longer as y_0/a increases, and $\langle \tau \rangle$ gets longer as well when H/a increases.

Here we select parameters ($y_0/a = 0.5$, $H/a = 1$), ($y_0/a = 0.5$, $H/a = 5$), ($y_0/a = 2.2$, $H/a = 1$), and ($y_0/a = 2.2$, $H/a = 5$) to show the corresponding localization, which are demonstrated in Fig. 15. From Figs. 15(a) and 15(c), one can clearly see that when $H/a = 1$, these eigenstates exhibit properties as bulk states. Though the minimal loss states, i.e., mode $m = 1$, is still concentrated, FWHM_{\min} is very big so the concentration feature disappears. In Figs. 15(b) and 15(d), one can see that the models exhibit suggested phenomena, including well-concentrated states. One can clearly see from Figs. 11(a)–11(c), no matter what y_0/a is, the shape of the γ curve for the same H/a is roughly the same, and this is why Figs. 15(a) and 15(c) behave similarly in terms of the localization, while the lifetime of these states is different. Similar arguments also work for Figs. 15(b) and 15(d).

To better illustrate the effect caused by H/a , we further plot Fig. 16. Here we take $y_0/a = 0.5$ as a constant since y_0/a does not change the tendencies of couplings, but only changes the amplitudes. As shown in Fig. 16, the localization gradually changes from bulk states to the suggested phenomena including well-concentrated states as H/a increases.

From what has been discussed above, we come to the following conclusion: there is no phase transition in our system as H/a and y_0/a change. The occurrence of the suggested phenomena including well-concentrated states mainly depends on the decrease rate of γ , which is closely related to H/a . Specifically, if γ decreases too slowly, the eigenstates behave as bulk states; on the other hand, the phenomena of the concentrated states will emerge. y_0/a mainly tunes the lifetime of the eigenstates.

2. Effects of parameter d

We set $d > 10\lambda = 8520$ nm such that $d \gg \lambda = 852$ nm to reduce the impact from the environment as much as possible. Here we choose $H/a = 3$, $y_0/a = 0.5$, $N = 41$ in the calculations.

We plot $\langle \tau \rangle$, shown in Fig. 17(a), and also make plots of FWHM_{\min} and $\langle \text{FWHM} \rangle$, exhibited in Fig. 17(b). We find the periodicity to be roughly $0.5d/\lambda$. In our proposed model, the Hamiltonian is written as

$$\hat{H}_{\text{eff}} = -\frac{i}{2} \sum_j (\gamma_{Lj} + \gamma_{Rj}) \hat{\sigma}_j^\dagger \hat{\sigma}_j - i \sum_{j>l} \sqrt{\gamma_{Ll}\gamma_{Lj}} \hat{\sigma}_l^\dagger \hat{\sigma}_j e^{ik(x_j-x_l)} - i \sum_{j>l} \sqrt{\gamma_{Rl}\gamma_{Rj}} \hat{\sigma}_j^\dagger \hat{\sigma}_l e^{ik(x_j-x_l)}, \quad (\text{E1})$$

where $k(x_j - x_l) = 2\pi(j - l)d/\lambda$, so d/λ is the phase factor of the cross-atom couplings.

According to Fig. 17(b), $\langle \text{FWHM} \rangle$ and FWHM_{\min} are in the range of [9,14] and [1,7], respectively, which indicates a good concentration when varying d . We further choose two spacings $d/\lambda = 10.25$, $d/\lambda = 10.72$ and $d/\lambda = 10.73$, which correspond to the widest $\langle \text{FWHM} \rangle = 13.98$ and $\text{FWHM}_{\min} = 7.06$, and the narrowest $\text{FWHM}_{\min} = 1.83$, respectively, as plotted in Fig. 18. In Fig. 18, we find that the main feature of the concentrated subradiant states and extended super-

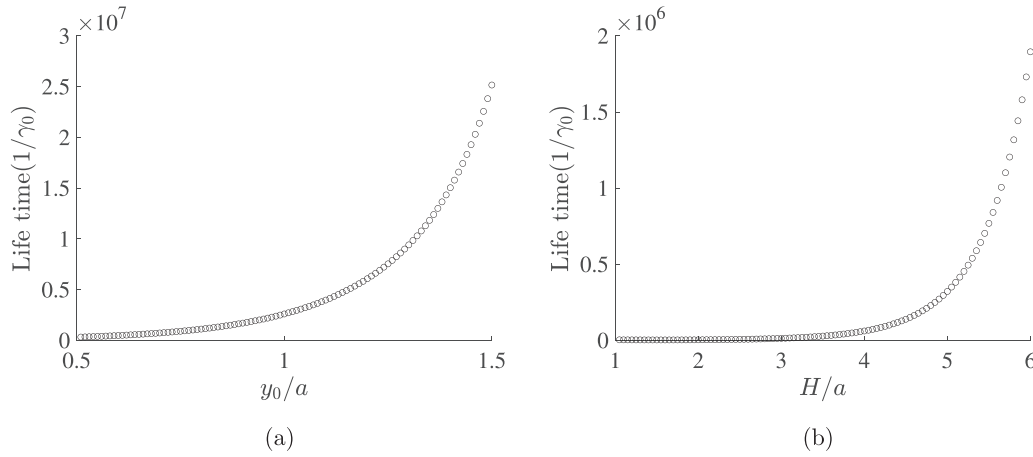


FIG. 14. (a) The lifetime $\langle \tau \rangle$ vs y_0/a , when $H/a = 5$. (b) The lifetime $\langle \tau \rangle$ vs H/a , when $y_0/a = 0.5$.

radiant states persists, but the details of the distributions' localization inevitably changes due to the varying phase of the interactions. In Figs. 18(b) and 18(c), the corresponding atoms' spacing, which are $d/\lambda = 10.72$ and $d/\lambda = 10.73$, respectively, are very close, while both of their FWHM_{\min} and localizations are so different and hence indicate a transition at $d/\lambda \sim 10.72$ – 10.73 . Additionally, we can find this kind of transition periodically.

3. Effects of parameter N

Here we choose $H/a = 3$, $y_0/a = 0.5$, and $d/\lambda = 10.65$ and perform simulations with different N . $\langle \text{FWHM} \rangle$ and FWHM_{\min} and their corresponding linearly fitted curves are

plotted in Fig. 19. One can see the goodness of linear fitting, i.e., $R^2 = 0.9461$ and $0.9989 \rightarrow 1$, indicating that the change of N does not change the main features of our system. In particular, we further plot localizations of eigenstates for four different N , including $N = 41, 101, 151$, and 201 , as illustrated in Fig. 20.

Additionally, in our system, we have $(N - 1)\theta d = H$ in geometry. When N is too large, the tilted angle θ will be extremely small, which is potentially beyond the experimental limit. Therefore, we choose $N = 41$ to show the main feature of the system in our paper.

In summary, we study the model over a broad range of parameters. We find that the existence that is found of con-

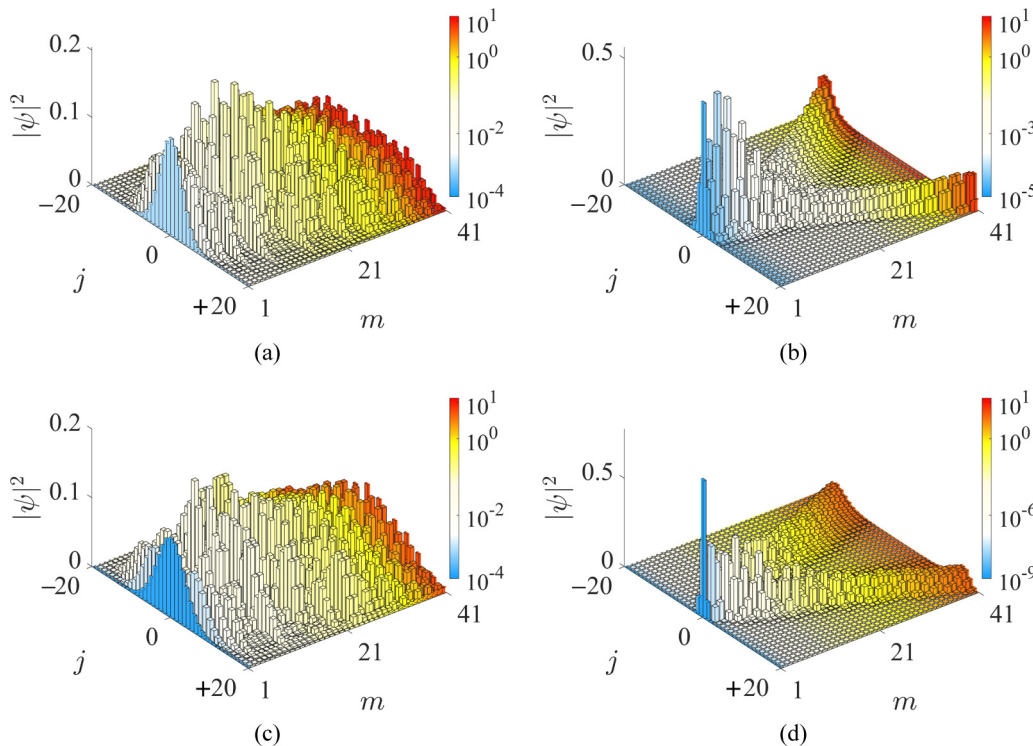


FIG. 15. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ at (a) $y_0/a = 0.5, H/a = 1$, (b) $y_0/a = 0.5, H/a = 5$, (c) $y_0/a = 2.2, H/a = 1$, and (d) $y_0/a = 2.2, H/a = 5$. Colors for each eigenstate indicate the collective decay rates.

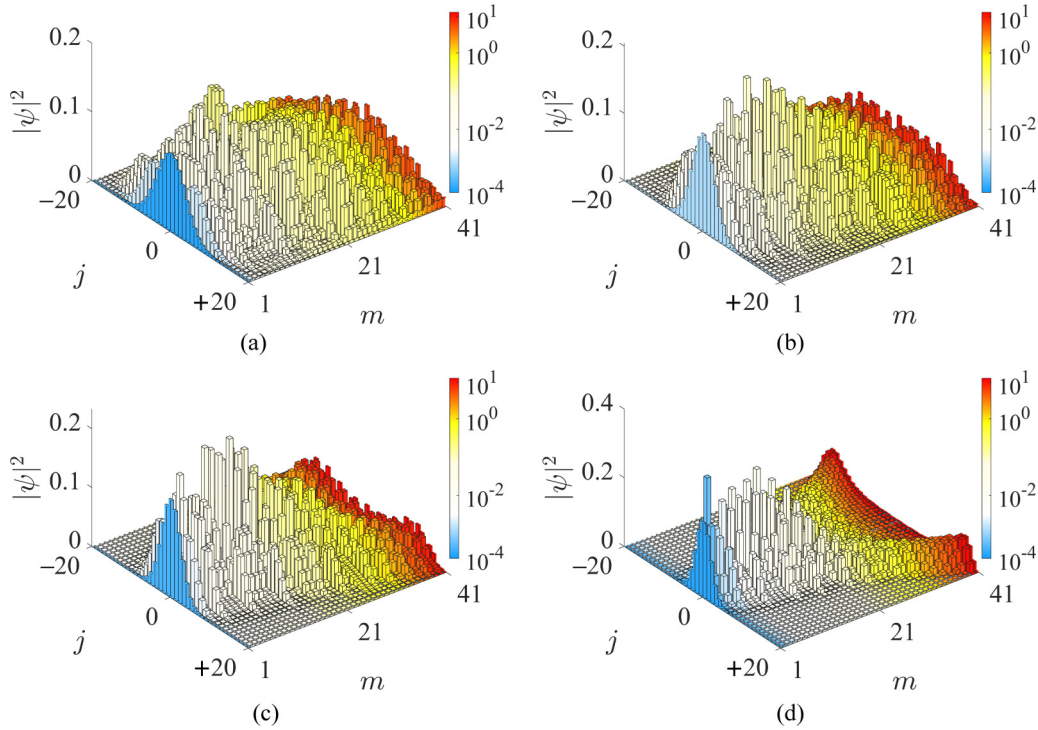


FIG. 16. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ at $y_0/a = 0.5$ and (a) $H/a = 0.5$, (b) $H/a = 1$, (c) $H/a = 1.5$, and (d) $H/a = 3$. Colors for each eigenstate indicate the collective decay rates.

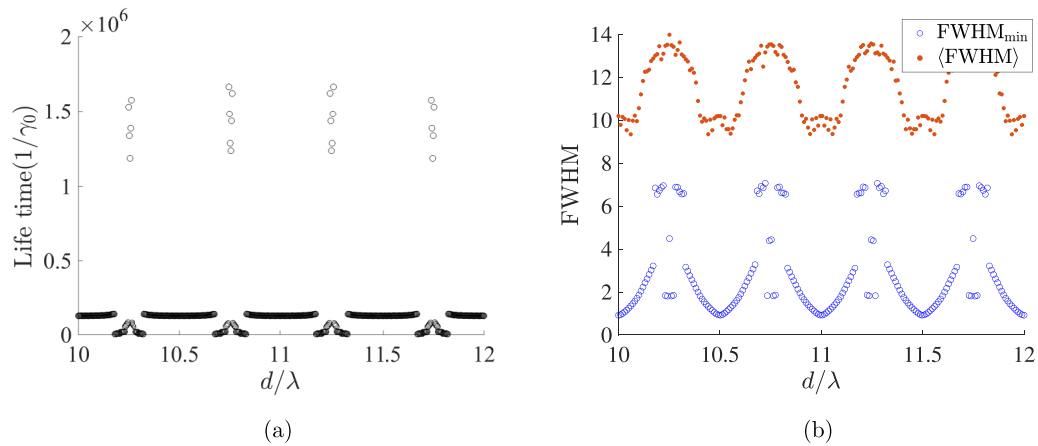


FIG. 17. (a) $\langle \tau \rangle$ vs d/λ . (b) FWHM_{\min} and $\langle \text{FWHM} \rangle$ vs d/λ , as indicated by blue dots and red dots, respectively.

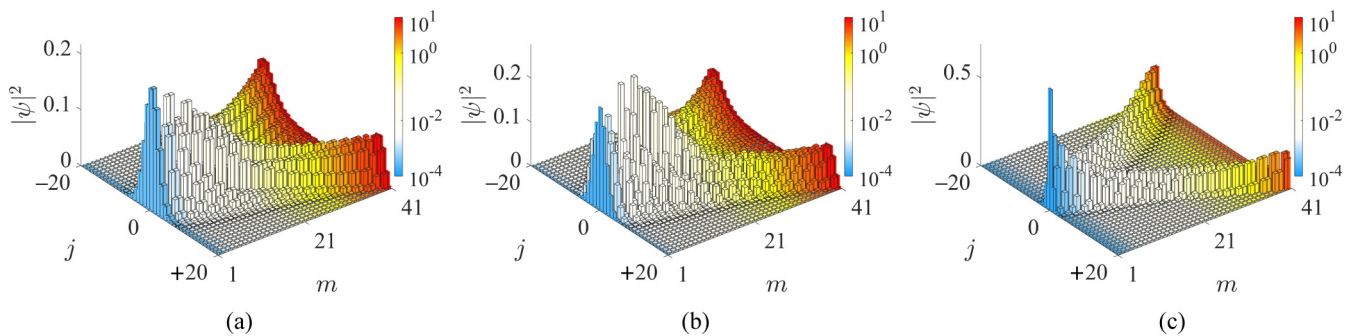


FIG. 18. Intensity distributions vs atom site j for all eigenstates with $m = 1, 2, \dots, 41$ at (a) $d/\lambda = 10.25$, $\langle \text{FWHM} \rangle = 13.98$, (b) $d/\lambda = 10.72$, $\text{FWHM}_{\min} = 7.06$, and (c) $d/\lambda = 10.73$, $\text{FWHM}_{\min} = 1.83$. Colors for each eigenstate indicate the collective decay rates.

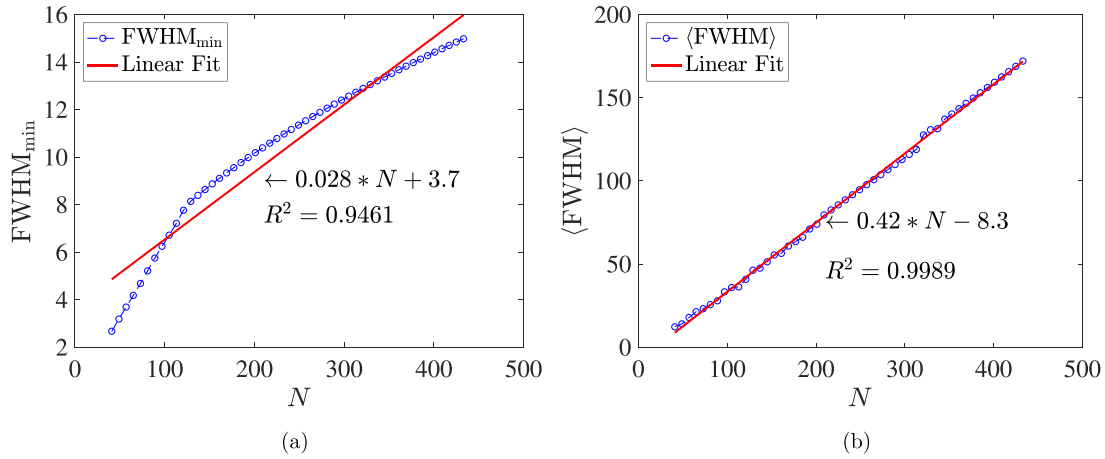


FIG. 19. (a) FWHM_{\min} and the corresponding linearly fitted curves, as labeled by the blue dotted line and red line, respectively. The R -square of the linear fitting is 0.9461. (b) $\langle \text{FWHM} \rangle$ and the corresponding linearly fitted curves, as labeled by the blue dotted line and red line, respectively. The R -square of the linear fitting is 0.9989.

concentrated states with subradiant features persists for the scale of the system, $N \rightarrow \infty$, which provides further evidence of connecting to NHSE [84]. Moreover, the occurrence of the interesting phenomena that are found mainly depends on the varying rate of $\gamma_{R(L)j}$ on the atomic positions, which is a result of the geometry of the atomic array. In other words, if $\gamma_{R(L)j}$ varies largely [such as the yellow dotted lines in Figs. 11(e) and 11(f)], then the distributions of eigenstates show the

phenomena of concentrated subradiant states and extended superradiant states [see Figs. 15(b) and 15(d), and Fig. 16(d)]. However, when the varying rate of $\gamma_{R(L)j}$ decreases [such as the yellow dotted lines in Figs. 11(a)–11(d)], a transition occurs and the system exhibits bulk states [see Figs. 15(a) and 15(c), and Figs. 16(a) and 16(b)]. In addition, the faster the decrease is, the better the concentration of the system will perform.

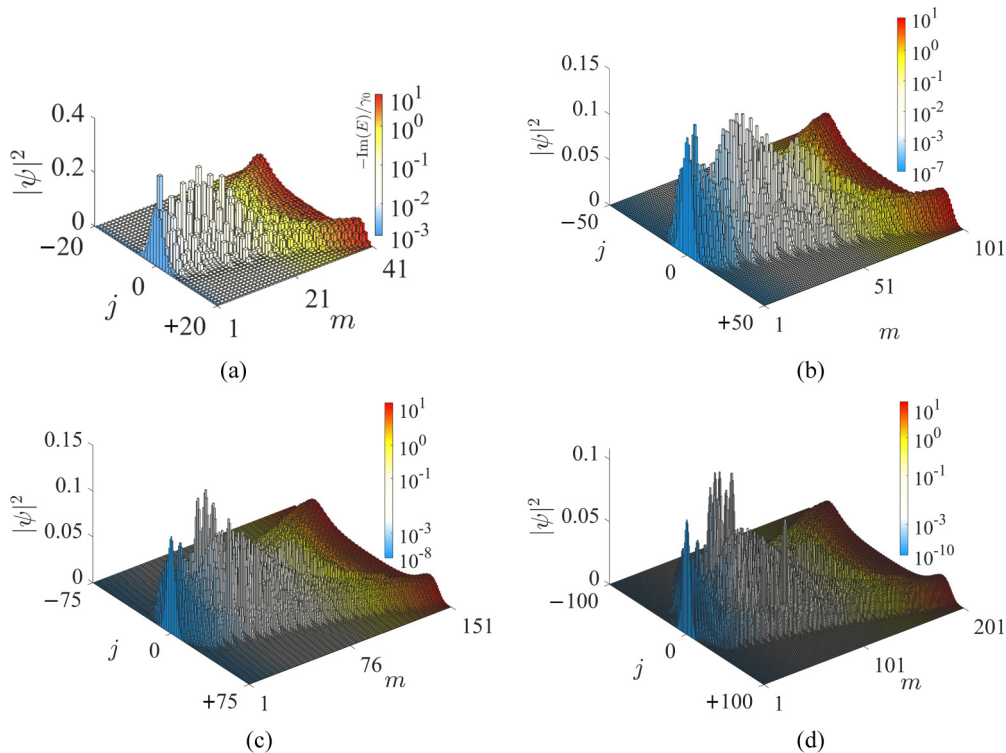


FIG. 20. Intensity distributions vs atom site j for all eigenstates at (a) $N = 41$, (b) $N = 101$, (c) $N = 151$ and (d) $N = 201$. Colors for each eigenstate indicate the collective decay rates.

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