Quantum phase measurement of two-qubit states in an open waveguide

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We present a method for quantum state tomography within a single-excitation subspace of two-qubit states in an open waveguide. The system under investigation consists of three qubits in an open waveguide, separated by a distance comparable to the wavelength of the electromagnetic field. We show that the modulation of the frequency of the central ancillary qubit allows us to obtain unambiguous information about the initial phase difference $\varphi_1 - \varphi_3$ of the edge qubits via the measurement of the evolution of their probability amplitudes.

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I. INTRODUCTION

Extracting information about the quantum state is an essential task in the benchmarks of quantum devices or quantum information algorithms. This is referred to as quantum state tomography (QST). As in the classical tomography, when we reconstruct a three dimensional image of the object by the use of its various projections on a two-dimensional plane, QST reconstructs the state by the use of sequences of quantum gates and projective measurements [1]. A consequence of projective measurements is that the state is destroyed, therefore these sequences should be implemented onto a set of identical quantum systems or onto the same system prepared repeatedly in the same state [2]. In circuit Quantum Electrodynamics (cQED) one can perform directly measurements in the energy basis of qubits, or equivalently, measurement of the z-projection on the Bloch sphere. These measurements are typically dispersive-shift based, where the resonance frequency of the readout resonator is qubit-state dependent [3]. To obtain the two remaining projections, one implements X and Y gates prior to the measurement [4]. To reconstruct the state of a single qubit at least three different gates are needed, and the density matrix has three independent elements that can be reconstructed using the measurement results. For two qubits the problem is already considerably more resource-demanding, as the number of gates increases to 9 for a two qubit state, and the full density matrix has 15 independent elements that have to be determined [5].

Open quantum systems without additional resonators are of the special interest both experimentally [6–9] and theoretically [10–17].. In these systems interference effects appear when the distance between qubits is comparable to a characteristic wavelength. The interference is caused by the effective interaction between the qubits via virtual photons. There are several theoretical works devoted to mentioned interference effects [18,19], synchronization and superradiance [20] as well as experimental realizations of long-distance interacting superconducting qubits [21,22].

Here we investigate an open quantum system consisting of an open waveguide, two main qubits and one ancillary central qubit, and we restrict the Hilbert space to a single-excitation subspace. By employing frequency modulation of the ancillary qubit [23] we obtain a one-to-one mapping between the phase of the two qubit off-diagonal density-matrix element in the single-excitation subspace and the measurement result in the energy basis. Thus, the quantum state could be reconstructed by two measurements: the σ_z components of the two qubits without modulation, to get the absolute values of the amplitude probabilities; and the σ_z components of two qubits with modulation, to get the phases of the amplitude probabilities.

In contrast to a common practice where for tomography reconstruction the gate pulses are applied to the measured qubits, in our method, the measurement pulse is applied to the ancillary qubit. Until the projective measurements two qubits do not undergo any external influence.

The paper is structured as follows.

In Sec. II we obtain the time-dependent differential equations for the probability amplitudes $\beta_{1-3}(t)$ of the three qubits, which account for the modulation of the frequency of the central qubit.

The main results of the paper are described in Sec III. In Sec. III A we consider the free evolution of three-qubit system. We show that the free evolution probabilities $|\beta_1(t)|^2$ and $|\beta_3(t)|^2$ depend on the phase difference $\varphi_1 - \varphi_3$. However, the population difference $|\beta_1(t)|^2 - |\beta_3(t)|^2$ is phase independent. It is shown that from free evolution measurements we can find both the initial values of probability amplitudes $\beta_{1,3}(0)$ and the quantity $\cos(\varphi_1 - \varphi_3)$. In Sect. III B we consider the solution of the equations obtained in Sec. II under frequency modulation, $f(t) \neq 0$ with the initial conditions $\beta_1(t)|_{t=0} = \beta_1(0)$; $\beta_2(t)|_{t=0} = 0$; $\beta_3(t)|_{t=0} = \beta_3(0)$. From the results obtained in this section, we may conclude that modulating the frequency of the second qubit allows us to obtain unambiguous information about the initial phase difference $\varphi_1 - \varphi_3$ via the measurement of the evolution of the probability amplitudes $|\beta_1(t)|^2$, $|\beta_3(t)|^2$.

II. FORMULATION OF THE PROBLEM

We consider a linear chain of three equally spaced qubits which are coupled to the photon field in an open waveguide (see Fig. 1).

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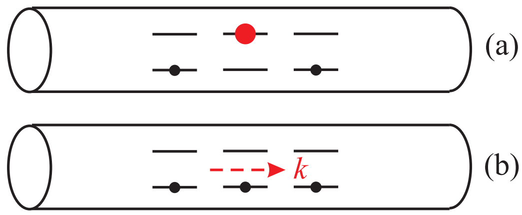

FIG. 1. Schematic of the single-excitation subspace for a three-qubit chain in an open waveguide. (a) One qubit is excited, whereas the other two qubits are in the ground state. (b) All three qubits are in the ground state, and a single photon propagates in the waveguide.

The distance between neighboring qubits is equal to d. The Hilbert space of each qubit is spanned by the excited-state vector $|e\rangle$ and the ground-state vector $|g\rangle$. The Hamiltonian that accounts for the interaction between qubits, and the electromagnetic field is as follows (we use $\hbar = 1$ throughout the paper),

$$H = H_0 + \sum_k \omega_k a_k^\dagger a_k + H_{\text{int}}, \tag{1}$$

where H_0 is the Hamiltonian of the bare qubits and H_{int} is the interaction Hamiltonian between the qubits and the photons in the waveguide,

$$H_0 = \frac{1}{2}\Omega \sum_{n=1}^{3}\left(1 + \sigma_z^{(n)}\right) + \frac{1}{2} f(t)\left(1 + \sigma_z^{(2)}\right), \tag{2}$$

$$H_{\text{int}} = \sum_{n=1}^{3}\sum_k g_k^{(n)} e^{-ikx_n}\sigma_-^{(n)} a_k^{+} + \text{H.c.} \tag{3}$$

In Eq. (2), the two edge qubits have equal frequencies, Ω, whereas the frequency of a central qubit, $\Omega_C(t)$ may be time dependent: $\Omega_C(t) = f(t) + \Omega$, i.e., detuned by $f(t)$ from the edge qubits. The quantity $g_k^{(n)}$ in Eq. (3) denotes the coupling between the nth qubit and the photon field, whereas x_n is the position of nth qubit.

Below we consider a single-excitation subspace with either a single photon in the waveguide and all qubits in the ground state or with no photons in a waveguide and only one qubit in the chain being excited. The Hamiltonian Eq. (3) conserves the number of excitations (number of excited qubits + number of photons). Therefore, at any instant of time the system will remain within the single-excitation subspace. The wave function of an arbitrary single-excitation state can then be written in the form

$$|\Psi(t)\rangle = \sum_{n=1}^{3}\beta_n(t)e^{-i\Omega t}|n, 0_k\rangle + \sum_k \gamma_k(t)e^{-i\omega_k t}|G, 1_k\rangle, \tag{4}$$

where $\beta_n(t)$ is the amplitude of the nth qubit, $|G, 1_k\rangle = |g_1, g_2, g_3\rangle \otimes |1_k\rangle$, $|1, 0_k\rangle = |e_1, g_2, g_3\rangle \otimes |0_k\rangle$, $|2, 0_k\rangle = |g_1, e_2, g_3\rangle \otimes |0_k\rangle$, $|3, 0_k\rangle = |g_1, g_2, e_3\rangle \otimes |0_k\rangle$, and $\gamma_k(t)$ is a single-photon probability amplitude which is related to a spectral density of spontaneous emission.

The equations for the amplitudes $\beta_n(t)$ and $\gamma_k(t)$ in Eq. (4) can be found from the time-dependent Schrödinger equation $id|\Psi\rangle/dt = H|\Psi\rangle$. For the probability amplitudes $\beta_n(t)$ we obtain the following equations (the details of the derivation are given in Appendix A):

$$\begin{aligned}
\frac{d\beta_1}{dt} &= -\frac{\Gamma}{2}\left(\beta_1 + \beta_2 e^{ikd} + \beta_3 e^{i2kd}\right),\\
\frac{d\beta_2}{dt} &= -if(t)\beta_2(t) - \frac{\Gamma}{2}\left(\beta_1 e^{ikd} + \beta_2 + \beta_3 e^{ikd}\right),\\
\frac{d\beta_3}{dt} &= -\frac{\Gamma}{2}\left(\beta_1 e^{i2kd} + \beta_2 e^{ikd} + \beta_3\right),
\end{aligned} \tag{5}$$

where $k = \Omega/v_g$ and Γ is the rate of spontaneous emission of qubit into the waveguide mode.

The wave function which describing the dynamic evolution of the $\beta_n(t)'$s is the projection of the single- excitation wavefunction Eq. (4) on the vacuum photon state,

$$|\Psi(t)\rangle_0 = \langle 0_k|\Psi(t)\rangle = \sum_{n=1}^{3}\beta_n(t)|n\rangle, \tag{6}$$

where $|n\rangle = \langle 0_k|n, 0_k\rangle$ describes the state with the nth qubit excited.

We consider the initial state in the following form:

$$|\Psi(0)\rangle_0 = |\beta_1(0)|e^{i\varphi_1}|1\rangle + |\beta_3(0)|e^{i\varphi_3}|3\rangle, \tag{7}$$

therefore, the second (central) qubit is initially not excited.

In Eq. (7) $|\beta_1(0)|$, $|\beta_3(0)|$ determine the probability to find the first and third qubits, respectively, in an excited state, and φ_1, φ_3 are the phases of the amplitude probabilities of these qubits. By definition, this two-qubit state is described by the density matrix,

$$\rho(0) = \begin{pmatrix} |\beta_1(0)|^2 & |\beta_1(0)||\beta_3(0)|e^{i(\varphi_1-\varphi_3)} \\ |\beta_1(0)||\beta_3(0)|e^{-i(\varphi_1-\varphi_3)} & |\beta_3(0)|^2 \end{pmatrix}. \tag{8}$$

The aim of tomography is to obtain all the elements of the density matrix. Here we suppose that one can measure $|\beta_1(0)|$, $|\beta_3(0)|$, i.e., z component for each qubit. The only left component is the phase difference $\varphi_1 - \varphi_3$, and finding it is the centerpiece of our proposal.

In what follows we show that modulating the frequency of the second qubit [23] allows for the extraction of the information about the initial values $|\beta_1(0)|$, $|\beta_3(0)|$, and about the phase difference $\varphi_1 - \varphi_3$ via the measurement of the probability amplitudes $|\beta_1(t)|^2$, $|\beta_3(t)|^2$. In a typical cQED setup, the frequency modulation is realized by varying the current through a line used to produce a bias magnetic field.

III. TOMOGRAPHY OF THE TWO-QUBIT STATE

A. Free evolution of the three-qubit system

We consider first the solution of Eqs. (5) in the absence of a modulation signal $f(t)=0$ with the initial conditions $\beta_1(t)|_{t=0}=\beta_1(0)$; $\beta_2(t)|_{t=0}=0$; $\beta_3(t)|_{t=0}=\beta_3(0)$. For this case, we obtain for $kd=2\pi$ the following solution:

$$\beta_1(t)=\tfrac{1}{3}[\beta_1(0)+\beta_3(0)]e^{-(3\Gamma/2)t}+\tfrac{2}{3}\beta_1(0)-\tfrac{1}{3}\beta_3(0), \tag{9}$$

$$\beta_2(t)=\tfrac{1}{3}[\beta_1(0)+\beta_3(0)]e^{-(3\Gamma/2)t}-\tfrac{1}{3}\beta_1(0)-\tfrac{1}{3}\beta_3(0), \tag{10}$$

$$\beta_3(t)=\tfrac{1}{3}[\beta_1(0)+\beta_3(0)]e^{-(3\Gamma/2)t}-\tfrac{1}{3}\beta_1(0)+\tfrac{2}{3}\beta_3(0). \tag{11}$$

Neglecting the first decaying terms on the right-hand side of Eqs. (9)–(11) for the time $t>t_0$ where $\Gamma t_0\gg 1$, we obtain

$$\begin{aligned}|\beta_1(t)|^2&=\tfrac{1}{9}(4|\beta_1(0)|^2+|\beta_3(0)|^2)\\&\quad-\tfrac{4}{9}|\beta_1(0)\|\beta_3(0)|\cos(\varphi_1-\varphi_3),\\|\beta_2(t)|^2&=\tfrac{1}{9}(|\beta_3(0)|^2+|\beta_1(0)|^2)\\&\quad+\tfrac{2}{9}|\beta_1(0)\|\beta_3(0)|\cos(\varphi_1-\varphi_3),\end{aligned} \tag{12}$$

$$\begin{aligned}|\beta_3(t)|^2&=\tfrac{1}{9}(4|\beta_3(0)|^2+|\beta_1(0)|^2)\\&\quad-\tfrac{4}{9}|\beta_1(0)\|\beta_3(0)|\cos(\varphi_1-\varphi_3),\\|\beta_1(t)|^2-|\beta_3(t)|^2&=\tfrac{1}{3}(|\beta_1(0)|^2-|\beta_3(0)|^2).\end{aligned} \tag{13}$$

It follows from Eq. (13) that if initially $|\beta_1(0)|=|\beta_3(0)|=1/\sqrt{2}$, then, at any time $|\beta_1(t)|=|\beta_3(t)|$.

Whereas the evolution of $|\beta_1(t)|^2$ and $|\beta_3(t)|^2$ each depend on the phase difference $\varphi_1-\varphi_3$, their difference is phase independent as seen from Eq. (13).

Therefore, from the normalization condition $|\beta_1(0)|^2+|\beta_3(0)|^2=1$, we obtain from Eq. (13) $|\beta_1(0)|^2=\frac{1}{2}[1+3d(t_0)]$, $|\beta_3(0)|^2=[1-3d(t_0)]$, where the measured quantity $d(t_0)=|\beta_1(t_0)|^2-|\beta_3(t_0)|^2$. Then, from any of Eqs. (12) we can obtain $\cos(\varphi_1-\varphi_3)$.

However, unambiguous knowledge of the phase difference would require some additional information, for example, the value of $\sin(\varphi_1-\varphi_3)$. In the following subsection, we show that this quantity can be obtained by the frequency modulation of initially not excited central qubit.

B. Measurement of the phase difference by frequency modulation

Next we consider the solution of Eqs. (5) under frequency modulation $f(t)\neq 0$ with the initial conditions $\beta_1(t)|_{t=0}=\beta_1(0)$; $\beta_2(t)|_{t=0}=0$; $\beta_3(t)|_{t=0}=\beta_3(0)$.

Solving Eqs. (5) for $kd=2\pi$ yields the following results (the details of the derivation are given in Appendix B):

$$\begin{aligned}|\beta_1(t)|^2-|\beta_3(t)|^2&=\tfrac{1}{3}e^{-\Lambda(t)}[(|\beta_1(0)|^2-|\beta_3(0)|^2)\cos u(t)\\&\quad+2|\beta_1(0)||\beta_3(0)|\sin(\phi_1-\phi_3)\sin u(t)]\end{aligned} \tag{14}$$

$$\begin{aligned}|\beta_1(t)|^2+|\beta_3(t)|^2&=\tfrac{1}{18}(e^{-2\Lambda(t)}+9)\\&\quad+\tfrac{1}{18}[\beta_1^*(0)\beta_3(0)+\beta_1(0)\beta_3^*(0)]\\&\quad\times(e^{-2\Lambda(t)}-9)\end{aligned} \tag{15}$$

$$|\beta_2(t)|^2=\tfrac{1}{9}e^{-2\Lambda(t)}[1+2|\beta_1(0)||\beta_3(0)|\cos(\varphi_1-\varphi_3)], \tag{16}$$

where

$$u(t)=\frac{2}{3}\int_0^t f(\tau)d\tau, \tag{17}$$

$$\Lambda(t)=\frac{4}{27\Gamma t}\left(\int_0^t f(\tau)d\tau\right)^2=\frac{1}{3\Gamma t}u^2(t). \tag{18}$$

It worth noting that Eqs. (14) and (15) are found for $\Gamma t\gg|F(t)|$ (where $F(t)$ is defined in (B12)) or, equivalently, $\Gamma\gg\Delta\Omega$, where $\Delta\Omega$ is the deviation of the frequency of a second qubit from that of the edge qubits. From the formal point of view, it means that the quantity $\Lambda(t)\ll 1$ and in (B21) we neglect the decaying exponent $e^{\lambda_2}\approx e^{-(3/2)\Gamma t}$. Also, from Eqs. (17) and (18), one sees that the dynamics is defined only by the area under the time-function $f(\tau)$. When $f(\tau)\neq 0$, periodic oscillations exist, see Eq. (14), with a time-dependent decay rate $\Lambda(t)\ll 1$. As soon as the detuning between the central qubit and the side qubits goes to zero $[f(\tau)=0]$ the integral value becomes constant and the oscillatory dynamics stops. In this sense, $f(\tau)$ could be any arbitrary nonbreaking function.

In principle, Eqs. (14) and (15) allow us to obtain both the initial probability amplitudes $\beta_1(0)$, $\beta_3(0)$, and the phase difference $\varphi_1-\varphi_3$. For a π pulse $[u(t_\pi)=\pi]$ we obtain from (14),

$$d(t_\pi)=-\tfrac{1}{3}(|\beta_1(0)|^2-|\beta_3(0)|^2), \tag{19}$$

where the measured quantity is the population difference $d(t_\pi)=|\beta_1(t_\pi)|^2-|\beta_3(t_\pi)|^2$. Together with the normalizing condition $|\beta_1(0)|^2+|\beta_3(0)|^2=1$ we obtain from Eq. (19) $|\beta_1(0)|^2=\frac{1}{2}[1-3d(t_\pi)]$, $|\beta_3(0)|^2=\frac{1}{2}[1+3d(t_\pi)]$. We then repeat the measurements for the same initial conditions by applying a $\pi/2$ pulse $[u(t_{\pi/2})=\pi/2]$. We obtain

$$d(t_{\pi/2})=\tfrac{2}{3}|\beta_1(0)||\beta_3(0)|\sin(\varphi_1-\varphi_3). \tag{20}$$

In Eq. (20) the amplitudes $\beta_1(0)$, $\beta_3(0)$ can be obtained either from the free evolution (Sec. III A) or from the π-pulse measurements in Eq. (14). Therefore, the quantity $\sin(\varphi_1-\varphi_3)$ is obtained from Eq. (20). In order to obtain the phase difference $\varphi_1-\varphi_3$ unambiguously we may use Eq. (15) which, under the assumption $\Lambda\ll 1$, can be written as

$$S(t_{\pi/2})=\tfrac{5}{9}-\tfrac{8}{9}|\beta_1(0)||\beta_3(0)|\cos(\varphi_1-\varphi_3), \tag{21}$$

where $S(t_{\pi/2})=|\beta_1(t_p)|^2+|\beta_3(t_p)|^2$.

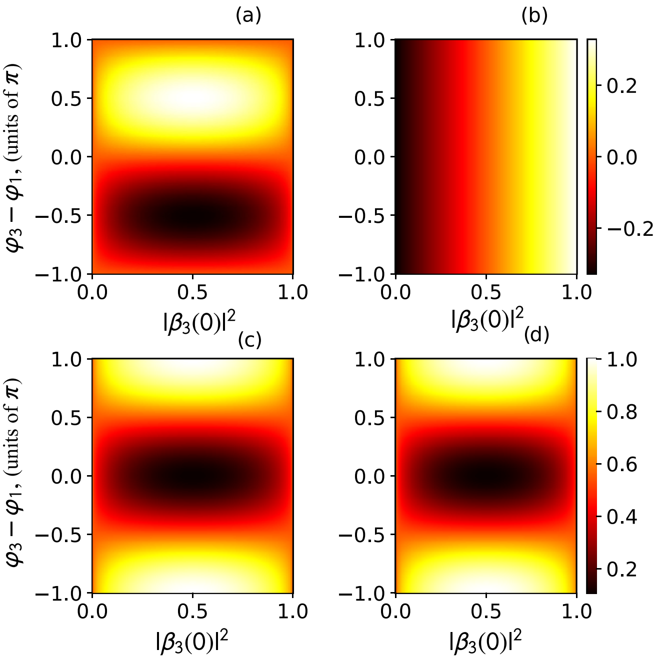

FIG. 2. Population difference (a) and (b) and sum (c) and (d) dependence on the initial-state parameters (populations and phase difference) after modulation pulses columnwise corresponding to $u(t) = \pi/2$ (a) and (c) and $u(t) = \pi$ (b) and (d).

From Eq. (14) we see that the measurable value $|\beta_1(t)|^2 - |\beta_3(t)|^2$ presents a mix of two types of information. The first term depends only on the initial population difference, whereas the phase information is contained in the second term. Moreover, Λ characterizes the information leak rate from the system to the measurable value. So, at $t = 0$ the exponent $\Lambda(0)$ is infinite, $u(0)$ tends to zero, and no information can be obtained. This rate depends naturally on coupling between the qubits and the open waveguide, as well as on strength of the modulation.

The interplay between phase and amplitude information in Eq. (14) is shown in Fig. 2, where the difference $|\beta_1(t)|^2 - |\beta_3(t)|^2$ is taken in the limit $e^{-\Lambda(t)} \to 1$. We suppress the first term by choosing $u(t) = \pi/2$, and from Fig. 2 one sees that for any initial phase difference between qubit states $|1\rangle$ and $|3\rangle$ there is a unique value of the population differences. We also note that in the limit when $|\beta_3(t)|^2 = 0, 1$ the measurable value equals 0, which becomes clear from Eq. (8) where off-diagonal elements vanish and the phases are totally uncertain.

As a demonstration of our method, we verified the validity of Eq. (14) by numerical simulation for initially equal probability amplitudes $|\beta_1(0)| = |\beta_3(0)| = 1/\sqrt{2}$, $\beta_2(0) = 0$, and $\varphi_3 - \varphi_1 = 0.4\pi$. In this case, the only nonzero term on the right-hand side of Eq. (14) is proportional to $\sin u(t)$. For a $\pi/2$ modulation $[u(t) = \pi/2]$ the population difference at the end of the pulse is proportional to $\sin(\varphi_1 - \varphi_3)$ as it follows from (20). This behavior is shown in Fig. 3.

Alternatively, for a π-modulation pulse $[u(t) = \pi]$ the population difference $|\beta_1(t)|^2 - |\beta_3(t)|^2$ after the end of the pulse becomes equal to zero which is shown in Fig. 4.

Also, it is worth mentioning that after the modulation pulse $|\beta_1(t)|^2 + |\beta_3(t)|^2 \neq 1$ because the central qubit

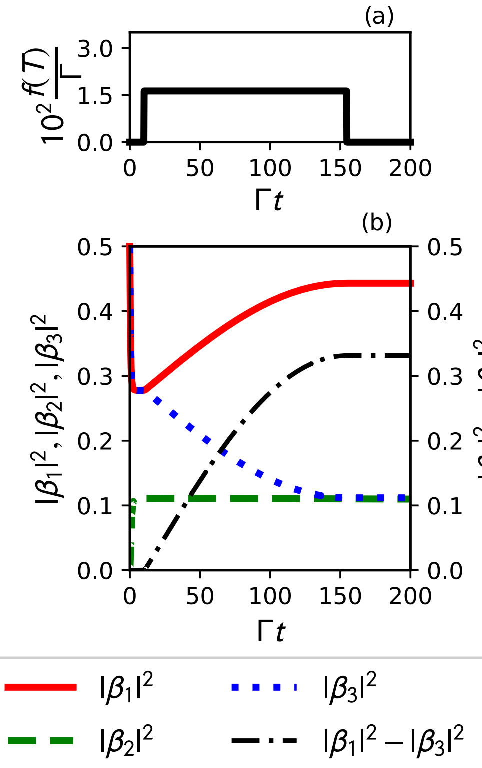

FIG. 3. Evolution of probabilities under modulation of the frequency of the central qubit with parameters $kd = 2\pi$, $u(t) = \pi/2$, $|\beta_1(0)| = |\beta_3(0)| = 1/\sqrt{2}$, $|\beta_2(0)| = 0$, $\varphi_3 - \varphi_1 = 0.4\pi$. (a) The modulation is realized as a pulse, starting at $t_0\Gamma = 10$ and ending at $t_{\mathrm{end}}\Gamma = 151$. (b) Probabilities $|\beta_1(t)|^2$ (solid red line), $|\beta_3(t)|^2$ (dotted blue line), $|\beta_2(t)|^2$ (dashed green line), and population difference $|\beta_1(t)|^2 - |\beta_3(t)|^2$ (dashed-dot black line).

becomes partially excited. Nevertheless, we are interested only in a combination of measured populations. To summarize to this section, we may conclude that modulating the frequency of the second qubit allows us to obtain the unambiguous information about the phase difference $\varphi_1 - \varphi_3$ via the measurement of the evolution of the probability amplitudes $|\beta_1(t)|^2$, $|\beta_3(t)|^2$.

To emulate the reconstruction procedure we take the state with an unknown phase difference $\varphi_1 - \varphi_3$ in the range of $-\pi \ldots +\pi$ and with unknown populations $\beta_{1,3}$. Then, we simulate the dynamics after the $u = \pi$ pulse and get the difference of populations $d(t_\pi) = |\beta_1|^2 - |\beta_3|^2$. The estimation of the populations from Eq. (19) is as follows:

$$|\beta_1^{\mathrm{est}}(0)| = \sqrt{\frac{1}{2}[1 - 3d(t_\pi)]},$$
$$|\beta_3^{\mathrm{est}}(0)| = \sqrt{\frac{1}{2}[1 + 3d(t_\pi)]}. \qquad (22)$$

At the next step we simulate the dynamics after a $u = \pi/2$ pulse and take the populations $|\beta_1|^2$ and $|\beta_3|^2$ after the pulse. Then, following equations Eqs. (20) and (21), where S and $d(t_{\pi/2})$ are, in fact, the measured values, we find the sin and

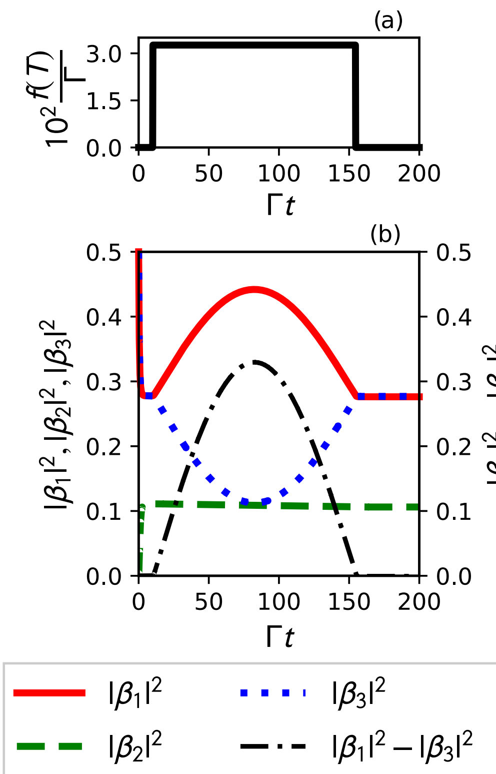

FIG. 4. Evolution of probabilities under modulation of the frequency of the central qubit with parameters $kd = 2\pi$, $u(t) = \pi$, $|\beta_1(0)| = |\beta_3(0)| = 1/\sqrt{2}$, $|\beta_2(0)| = 0$, $\varphi_3 - \varphi_1 = 0.4\pi$. (a) The modulation is realized as a pulse, starting at $t_0\Gamma = 10$ and ending at $t_{\rm end}\Gamma = 151$. (b) Probabilities $|\beta_1(t)|^2$ (solid red line), $|\beta_3(t)|^2$ (dotted blue line), $|\beta_2(t)|^2$ (dashed green line), and population difference $|\beta_1(t)|^2 - |\beta_3(t)|^2$ (dashed-dot black line).

cos values of estimated phase $\varphi_{\rm est}$,

$$\sin(\varphi_{\rm est}) = \frac{3}{2}\frac{d(t_{\pi/2})}{|\beta_1^{\rm est}(0)||\beta_3^{\rm est}(0)|},$$
$$\cos(\varphi_{\rm est}) = -\left(S - \frac{5}{9}\right)\frac{9}{8|\beta_1^{\rm est}(0)||\beta_3^{\rm est}(0)|}, \tag{23}$$

which allows to explicitly get $\varphi_{\rm est}$ through arctangent. These two steps are enough to reconstruct the state in the form of Eq. (8).

IV. CONCLUSION

In this paper we have considered three noninteracting qubits embedded in an open waveguide. For this system we have described experimentally accessible method for the reconstruction within a single-excitation subspace of arbitrary two-qubit state. The method is based on the modulation of the frequency of a central ancillary qubit which allows us to determine the elements of reduced density matrix for two edge qubits.

In contrast to a common quantum tomography reconstruction where the gate pulses are applied to the measured qubits, in our method the measurement pulse is applied to the ancillary qubit. Until the projective measurements two edge qubits do not undergo any external influence.

In our treatment we explicitly account for the radiative dissipation channel which is described by the coupling rate Γ. We neglect the rate of nonradiative intrinsic losses, Γ_{nr} and pure dephasing Γ_φ. The total decoherence rate of a qubit $\gamma = \Gamma/2 + \Gamma'$ where $\Gamma' = \Gamma_{nr}/2 + \Gamma_\varphi$. In superconducting qubits which can be strongly coupled to the one-dimensional (1D) mode continuum of a waveguide the coupling rate Γ dominates over all other decoherence channels. Typically, the quantity Γ' is, at least,ten times less than the coupling rate Γ [6,28–30]. The corresponding lifetimes set the bounds for the duration of the measurement pulse Δt: $1/\Gamma \ll \Delta t \ll 1/\Gamma'$. The rates Γ and Γ' are independent quantities. The coupling rate Γ that describes the interaction of a qubit with fundamental modes of a waveguide, depends on specific on-chip circuitry, whereas Γ_{nr} depends on the proper shielding against external noise, and Γ_φ depends on the substrate properties. Therefore, the above double inequality can, in principle, be satisfied with state-of-the-art superconducting qubit technology. Therefore, we strongly believe that under above conditions the ignoring the nonradiative losses and pure dephasing is justified and cannot have a significant effect on the main reported results.

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APPENDIX A: DERIVATION OF THE EQUATIONS FOR THE PROBABILITY AMPLITUDES $\beta_n(t)$

The equations for probability amplitudes $\beta_n(t)$ of the qubits and that of the photon $\gamma_k(t)$ from Eq. (4) can be found from the time-dependent Schrödinger equation $id|\Psi\rangle/dt = H|\Psi\rangle$. For the amplitudes we obtain

$$\frac{d\beta_1}{dt} = -i\sum_k g_k e^{-ikd}\gamma_k(t)e^{-i(\omega_k-\Omega)t}, \tag{A1}$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - i\sum_k g_k\gamma_k(t)e^{-i(\omega_k-\Omega)t}, \tag{A2}$$
$$\frac{d\beta_3}{dt} = -i\sum_k g_k e^{ikd}\gamma_k(t)e^{-i(\omega_k-\Omega)t}, \tag{A3}$$
$$\frac{d\gamma_k(t)}{dt} = -ig_kF_k(t)e^{i(\omega_k-\Omega)t}, \tag{A4}$$

where

$$F_k(t) = \beta_1(t)e^{ikd} + \beta_2(t) + \beta_3(t)e^{-ikd}. \tag{A5}$$

From (A4) we obtain

$$\gamma_k(t) = -ig_k\int_0^t F_k(t')e^{i(\omega_k-\Omega)t'}dt'. \tag{A6}$$

The expression (A6) allows us to remove the photon amplitude $\gamma_k(t)$ from the equations for the qubits' amplitudes

(A1)–(A3). The result is as follows:

$$\frac{d\beta_1}{dt} = -\sum_k g_k^2 e^{-ikd} \int F_k(t') e^{-i(\omega_k-\Omega)(t-t')} dt',$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - \sum_k g_k^2 \int_0^t F_k(t') e^{-i(\omega_k-\Omega)(t-t')} dt',$$
$$\frac{d\beta_3}{dt} = -\sum_k g_k^2 e^{ikd} \int_0^t F_k(t') e^{-i(\omega_k-\Omega)(t-t')} dt', \quad \text{(A7)}$$

In accordance with the Wigner-Weiskopff approximation we take the quantity $F_k(t)$ out of the integrands,

$$\frac{d\beta_1}{dt} = -\sum_k g_k^2 e^{-ikd} F_k(t) I_k(\Omega, t),$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - \sum_k g_k^2 F_k(t) I_k(\Omega, t),$$
$$\frac{d\beta_3}{dt} = -\sum_k g_k^2 e^{ikd} F_k(t) I_k(\Omega, t), \quad \text{(A8)}$$

where

$$I_k(\Omega, t) = \int_0^t e^{-i(\omega_k-\Omega)(t-t')} dt' = \int_0^t e^{-i(\omega_k-\Omega)\tau} d\tau$$
$$\approx \int_0^\infty e^{-i(\omega_k-\Omega)\tau} d\tau = \pi\delta(\omega_k - \Omega) - i\,\mathrm{P}\left(\frac{1}{\omega_k - \Omega}\right). \quad \text{(A9)}$$

We assume $g_{-k} = g_k, I_{-k}(\Omega, t) = I_k(\Omega, t)$ and leave the summation in (A8) over positive values of k (positive frequencies),

$$\frac{d\beta_1}{dt} = -\sum_{k>0} g_k^2 [e^{-ikd} F_k(t) + e^{ikd} F_{-k}(t] I_k(\Omega, t),$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - \sum_{k>0} g_k^2 [F_k(t) + F_{-k}(t)] I_k(\Omega, t),$$
$$\frac{d\beta_3}{dt} = -\sum_{k>0} g_k^2 [e^{ikd} F_k(t) + e^{-ikd} F_{-k}(t)] I_k(\Omega, t). \quad \text{(A10)}$$

Inserting the explicit form of $F_k(t)$ (A5) in (A10) results in the following expressions:

$$\frac{d\beta_1}{dt} = -2\sum_{k>0} g_k^2 (\beta_1 + \beta_2 \cos kd + \beta_3 \cos 2kd) I_k(\Omega, t),$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - 2\sum_{k>0} g_k^2 (\beta_1 \cos kd + \beta_2 + \beta_3 \cos kd) I_k(\Omega, t),$$
$$\frac{d\beta_3}{dt} = -2\sum_{k>0} g_k^2 (\beta_1 \cos 2kd + \beta_2 \cos kd + \beta_3) I_k(\Omega, t). \quad \text{(A11)}$$

The next step is to relate the coupling constants g_k to the qubit decay rate of spontaneous emission into the waveguide mode. In accordance with Fermi golden rule we define the qubit decay rates by the following expressions:

$$\Gamma = 2\pi \sum_k g_k^2 \delta(\omega_k - \Omega). \quad \text{(A12)}$$

For the 1D case, a summation over k is replaced by an integration over ω in accordance with the prescription,

$$2\sum_{k>0} \Rightarrow \frac{L}{2\pi} 2\int_0^\infty d|k| = \frac{L}{\pi v_g} \int_0^\infty d\omega_k, \quad \text{(A13)}$$

where L is a length of the waveguide, and we assumed a linear dispersion law $\omega_k = v_g|k|$. The application of (A13) to (A12), allows to derive a relation between the coupling constant g_k and the decay rate Γ,

$$g_k = \left(\frac{v_g \Gamma}{2L}\right)^{1/2}. \quad \text{(A14)}$$

Now we can calculate the different terms in (A11),

$$\sum_k g_k^2 I_k(\Omega, t) = \sum_k g_k^2 \left[\pi\delta(\omega_k - \Omega) - i\,\mathrm{P}\left(\frac{1}{\omega_k - \Omega}\right)\right]$$
$$= \frac{\Gamma}{2} - i\,\mathrm{P}\left(\frac{g_k^2}{\omega_k - \Omega}\right) \approx \frac{\Gamma}{2}, \quad \text{(A15)}$$

$$2\sum_{k>0} g_k^2 \cos(kd) I_k(\Omega, t)$$
$$= \frac{L}{v_g}\int_0^\infty g_k^2 \cos(kd)\delta(\omega_k - \Omega) d\omega_k$$
$$- 2i \sum_{k>0} \mathrm{P}\left(\frac{g_k^2 \cos(kd)}{\omega_k - \Omega}\right)$$
$$= \frac{L}{v_g} g_\Omega^2 \cos(k_\Omega d) - i\frac{L}{v_g \pi} g_\Omega^2\, \mathrm{P}\int_0^\infty \frac{\cos\left(\frac{\omega}{v_g} d\right)}{\omega - \Omega}. \quad \text{(A16)}$$

For the principal value integral in (A16) we obtain

$$\mathrm{P}\int_0^\infty d\omega \frac{\cos\left(\frac{\omega}{v_g} d\right)}{\omega - \Omega} = -\pi\, \sin(k_\Omega d), \quad \text{(A17)}$$

where $k_\Omega = \Omega/v_g$.

The expression (A17) is exact if counter-rotating terms in the qubit-field interaction are taken into account (Supplemental Material in Ref. [24]). Nevertheless, within a rotating-wave approximation Eq. (A17) provides good accuracy for $d > \lambda/4$ [25],

$$2\sum_{k>0} g_k^2 \cos(kd) I_k(\Omega, t) = \frac{L}{v_g} g_\Omega^2 e^{ik_\Omega d} = \frac{\Gamma}{2} e^{ik_\Omega d}. \quad \text{(A18)}$$

Similar calculations also give for the sum in (A11),

$$2\sum_{k>0} g_k^2 \cos(2kd) I_k(\Omega, t) = \frac{\Gamma}{2} e^{2ikd}. \quad \text{(A19)}$$

In (A15) the decay rate Γ is defined by (A12). The principal value in (A15) gives rise to the shift of the qubit frequency. Therefore, we incorporate it in the renormalized qubit frequency and will not write it explicitly anymore.

Collecting together (A15), (A18), and (A19) we write the final form of the equations (A11),

$$\frac{d\beta_1}{dt} = -\frac{\Gamma}{2}\left(\beta_1 + \beta_2 e^{ikd} + \beta_3 e^{i2kd}\right),$$
$$\frac{d\beta_2}{dt} = -if(t)\beta_2(t) - \frac{\Gamma}{2}\left(\beta_1 e^{ikd} + \beta_2 + \beta_3 e^{ikd}\right),$$
$$\frac{d\beta_3}{dt} = -\frac{\Gamma}{2}\left(\beta_1 e^{i2kd} + \beta_2 e^{ikd} + \beta_3\right), \tag{A20}$$

APPENDIX B: DERIVATION OF EQ. (14)

Equations (5) can be written in the matrix form

$$\frac{d\widehat{\beta}}{dt} = A(t)\widehat{\beta}(t), \tag{B1}$$

where

$$\widehat{\beta}(t)=\begin{pmatrix} \beta_1(t) \\ \beta_2(t) \\ \beta_3(t) \end{pmatrix}, \tag{B2}$$

$$A(t) = -\frac{\Gamma}{2}\begin{pmatrix} 1 & e^{ikd} & e^{2ikd} \\ e^{ikd} & 1+if(t)\frac{2}{\Gamma} & e^{ikd} \\ e^{2ikd} & e^{ikd} & 1 \end{pmatrix}. \tag{B3}$$

It is easy to verify that the matrices $A(t)$ do not commute at different times $[A(t_1), A(t_2)] \neq 0$. In this case the solution of (B1) can be obtained in the form

$$\widehat{\beta}(t) = e^{M(t)}\widehat{\beta}(0), \tag{B4}$$

where the Magnus operator $M(t)$ can be written as infinite series expansion [26],

$$M(t) = \sum_{n=1}^{\infty} M_n(t). \tag{B5}$$

The first two terms in (B5) are as follows:

$$M_1(t) = \int_0^t dt_1 A(t_1),\; M_2(t) = \frac{1}{2}\int_0^t dt_2 \int_0^{t_2} dt_1 [A(t_1)A(t_2)]. \tag{B6}$$

According to Silvester's matrix theorem (named after Sylvester) any analytic function $z(M)$ of a quadratic nn matrix M can be expressed as a polynomial in M, in terms of the eigenvalues and eigenvectors of M [27]. Specifically, the theorem states that

$$z(M) = \sum_{i=1}^{n} z(\lambda_i)B_i, \tag{B7}$$

where λ_i's are the characteristic roots of the equation,

$$\det|M - \lambda I| = 0, \tag{B8}$$

and

$$B_i = \prod_{j=1,\, i\neq j}^{n} \frac{M - \lambda_j I}{\lambda_i - \lambda_j}, \tag{B9}$$

where I is the identity matrix.

The Silvester's formula (B7) holds for any quadratic diagonalizable matrix all roots of which are different.

In the sum of (B5) we neglect all terms except for the first one, $M(t) = M_1(t)$,

$$M_1(t) = \int_0^t dt_1 A(t_1) = -\frac{\Gamma t}{2}\begin{pmatrix} 1 & e^{ikd} & e^{2ikd} \\ e^{ikd} & \Omega(t) & e^{ikd} \\ e^{2ikd} & e^{ikd} & 1 \end{pmatrix}, \tag{B10}$$

where

$$\Omega(t) = 1 + i\frac{2}{\Gamma t}\int_0^t f(t_1)dt_1 = 1 - \frac{2}{\Gamma t}F(t), \tag{B11}$$
$$F(t) = -i\int_0^t f(t_1)dt_1. \tag{B12}$$

Next, we find the characteristic roots $\lambda_i(t)$ of the matrix $M_1(t)$, which are the roots of the equation,

$$\det|M_1(t) - \lambda(t)I| = 0. \tag{B13}$$

Equation (B13) is a cubic equation,

$$(1-\lambda)^2(\Omega-\lambda) + 2e^{4ikd} - e^{4ikd}(\Omega-\lambda) - 2e^{2ikd}(1-\lambda) = 0, \tag{B14}$$

with the following three roots,

$$\lambda_{1,2}(t) = -\frac{\Gamma t}{2}\left(1 + \frac{1}{2}e^{2ikd}\right) + \frac{1}{2}F(t) \pm \frac{1}{4}e^{ikd}\sqrt{(8+e^{2ikd})\Gamma^2 t^2 + 4F^2(t)e^{-2ikd} + 4F(t)\Gamma t}, \tag{B15}$$

$$\lambda_3(t) = \frac{\Gamma t}{2}(e^{2ikd} - 1). \tag{B16}$$

In Eq. (B15) the roots λ_1, λ_2 correspond to $+, -$ signs, respectively.

The application of (B7) to $z(M) = e^M$ gives rise to the following equation:

$$e^{M_1(t)} = B_1 e^{\lambda_1} + B_2 e^{\lambda_2} + B_3 e^{\lambda_3}, \tag{B17}$$

where

$$B_1 = \left(\frac{M_1 - \lambda_2 I}{\lambda_1 - \lambda_2}\right)\left(\frac{M_1 - \lambda_3 I}{\lambda_1 - \lambda_3}\right) = \frac{M_1^2 - (\lambda_2+\lambda_3)M_1 + \lambda_2\lambda_3 I}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}, \tag{B18}$$

$$B_2 = \left(\frac{M_1 - \lambda_1 I}{\lambda_2 - \lambda_1}\right)\left(\frac{M_1 - \lambda_3 I}{\lambda_2 - \lambda_3}\right) = \frac{M_1^2 - (\lambda_1+\lambda_3)M_1 + \lambda_1\lambda_3 I}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}, \tag{B19}$$

$$B_3 = \left(\frac{M_1 - \lambda_1 I}{\lambda_3 - \lambda_1}\right)\left(\frac{M_1 - \lambda_2 I}{\lambda_3 - \lambda_2}\right) = \frac{M_1^2 - (\lambda_1+\lambda_2)M_1 + \lambda_2\lambda_1 I}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}. \tag{B20}$$

Equations (B17)–(B20) are valid for any value of kd.

Below we assume $kd = \pi n$, where n is a positive integer. For this case $\lambda_3 = 0$ and we obtain

$$e^{M_1(t)} = B_1 e^{\lambda_1} + B_2 e^{\lambda_2} + B_3, \tag{B21}$$

$$B_1 = \frac{M_1^2 - \lambda_2 M_1}{(\lambda_1 - \lambda_2)\lambda_1},$$
$$B_2 = \frac{M_1^2 - \lambda_1 M_1}{(\lambda_2 - \lambda_1)\lambda_2},\quad B_3 = \frac{M_1^2 - (\lambda_1 + \lambda_2)M_1}{\lambda_1 \lambda_2} + I, \tag{B22}$$

where

$$\lambda_{1,2}(t) = -\frac{3\Gamma t}{4} + \frac{1}{2}F(t) \pm (-1)^n \tfrac{1}{4}\sqrt{9\Gamma^2 t^2 + 4F^2(t) + 4F(t)\Gamma t}. \tag{B23}$$

Below we perform the calculations for $kd = 2\pi$. Using Eq. (B4) and the explicit expression (B10) for the matrix $M_1(t)$ we obtain from (B21)–(B23) the expressions for qubits amplitudes $\beta_1(t)$, $\beta_3(t)$,

$$\beta_1(t, 2\pi) = \frac{\Gamma t}{R}[\beta_1(0) + \beta_3(0)]\left(\frac{\left(\frac{3\Gamma t}{2} + F - \frac{R}{2}\right)}{\left(-\frac{3\Gamma t}{2} + F + \frac{1}{2}R\right)}e^{\lambda_1} - \frac{\left(\frac{3}{2}\Gamma t + F + \frac{R}{2}\right)}{\left(-\frac{3\Gamma t}{2} + F - \frac{1}{2}R\right)}e^{\lambda_2}\right) + \frac{1}{2}[\beta_1(0) - \beta_3(0)], \tag{B24}$$

$$\beta_3(t, 2\pi) = \frac{\Gamma t}{R}[\beta_1(0) + \beta_3(0)]\left(\frac{\left(\frac{3\Gamma t}{2} + F - \frac{R}{2}\right)}{\left(-\frac{3\Gamma t}{2} + F + \frac{1}{2}R\right)}e^{\lambda_1} - \frac{\left(\frac{3}{2}\Gamma t + F + \frac{R}{2}\right)}{\left(-\frac{3\Gamma t}{2} + F - \frac{1}{2}R\right)}e^{\lambda_2}\right) - \frac{1}{2}[\beta_1(0) - \beta_3(0)], \tag{B25}$$

where

$$R = \sqrt{9\Gamma^2 t^2 + 4F^2(t) + 4F(t)\Gamma t}. \tag{B26}$$

Now we analyze the quantities R, λ_1, and λ_2 for $\Gamma t \gg |F|$. We obtain

$$R \approx 3\Gamma t + \frac{2}{3}F + \frac{16}{27}\frac{F^2}{\Gamma t}, \tag{B27}$$

$$\lambda_1(t) = -\frac{3\Gamma t}{4} + \frac{1}{2}F(t) + \frac{1}{4}R \approx \frac{2}{3}F + \frac{4}{27}\frac{F^2}{\Gamma t}, \tag{B28}$$

$$\lambda_2(t) = -\frac{3\Gamma t}{4} + \frac{1}{2}F(t) - \frac{1}{4}R \approx -\frac{3}{2}\Gamma t + \frac{1}{3}F - \frac{4}{27}\frac{F^2}{\Gamma t}. \tag{B29}$$

Therefore, in (B24) and (B25) we neglect the decaying exponent $e^{\lambda_2} \approx e^{-(3/2)\Gamma t}$. The quantity e^{λ_1} we write in the following form:

$$e^{\lambda_1} \approx \exp\left(\frac{2}{3}F + \frac{4}{27}\frac{F^2}{\Gamma t}\right) \equiv e^{-iu(t)}e^{-\Lambda(t)}, \tag{B30}$$

where

$$u(t) = \frac{2}{3}\int_0^t f(\tau)d\tau, \tag{B31}$$

$$\Lambda(t) = \frac{4}{27\Gamma t}\left(\int_0^t f(\tau)d\tau\right)^2. \tag{B32}$$

For this approximation Eqs. (B24) and (B25) take the form

$$\beta_1(t, 2\pi) = \tfrac{1}{6}[\beta_1(0) + \beta_3(0)]e^{-iu(t)}e^{-\Lambda(t)} + \tfrac{1}{2}[\beta_1(0) - \beta_3(0)], \tag{B33}$$

$$\beta_3(t, 2\pi) = \tfrac{1}{6}(\beta_1(0) + \beta_3(0))e^{-iu(t)}e^{-\Lambda(t)} - \tfrac{1}{2}(\beta_1(0) - \beta_3(0)), \tag{B34}$$

$$\beta_2(t) = -\frac{(\beta_1(0) + \beta_3(0))}{3}e^{-iu(t)}e^{-\Lambda(t)}. \tag{B35}$$

From (B33), (B34) we finally obtain for $kd = 2\pi$

$$|\beta_1(t)|^2 - |\beta_3(t)|^2 = \tfrac{1}{3}[|\beta_1(0)|^2 - |\beta_3(0)|^2]e^{-\Lambda(t)}\cos u(t) - \tfrac{1}{3}i[\beta_1^*(0)\beta_3(0) - \beta_1(0)\beta_3^*(0)]e^{-\Lambda(t)}\sin u(t), \tag{B36}$$

$$|\beta_1(t)|^2 + |\beta_3(t)|^2 = \tfrac{1}{18}(e^{-\Lambda(t)} + 9) + \tfrac{1}{9}[\beta_1^*(0)\beta_3(0) + \beta_1(0)\beta_3^*(0)](e^{-\Lambda(t)} - 9), \tag{B37}$$

$$|\beta_2(t)|^2 = \tfrac{1}{9}e^{-2\Lambda(t)}[1 + 2|\beta_1(0\|\beta_3(0)|\cos(\varphi_1 - \varphi_3)], \tag{B38}$$

which are Eqs. (14), (19), and (16) from the main text.

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