Magic-state resource theory for the ground state of the transverse-field Ising model

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Ground states of quantum many-body systems are both entangled and possess a kind of quantum complexity, as their preparation requires universal resources that go beyond the Clifford group and stabilizer states. These resources—sometimes described as *magic*—are also the crucial ingredient for quantum advantage. We study the behavior of the stabilizer Rényi entropy in the integrable transverse field Ising spin chain. We show that the locality of interactions results in a localized stabilizer Rényi entropy in the gapped phase, thus making this quantity computable in terms of local quantities in the gapped phase, while measurements involving L spins are necessary at the critical point to obtain an error scaling with $O(L^{-1})$.

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I. INTRODUCTION

Quantum mechanics is different from classical physics in two ways: First, composite quantum systems can exhibit correlations stronger than any classical correlation, i.e., entanglement. Second, because quantum states and operations constitute the bedrock for computation that goes beyond the classical Turing machine model and can outperform classical algorithms [1–6]. The resource useful for such a quantum advantage consists of those states and operations that go beyond the stabilizer formalism and the Clifford group [7–18].

Entanglement has been widely studied in the context of quantum many-body systems [19] from its role in quantum phase transitions [20–24] to issues of simulability [25–33] to the onset or thermalization and chaos in closed quantum systems [34–41], the structure of exotic quantum phases of matter [42–57], and black-hole dynamics [34,41,58]. On the other hand, magic state resource theory has only very recently been the object of investigation in the field of quantum systems with many particles [59,60]. This is mainly due to the difficulty of computing nonstabilizerness for high-dimensional spaces [61]. Recently, though, the authors of this paper have proposed the stabilizer Rényi entropy as a more amenable way of computing nonstabilizerness based on the Rényi entropy associated to the decomposition of a state in the Pauli basis [62], which has also led to its experimental measurement [63–65].

In this paper, we set out to show the role that magic state resource theory plays in the ground state of local integrable quantum many-body systems. The model studied here is the transverse field Ising model for a spin one-half chain with N sites. We show how to compute the stabilizer Rényi entropy in terms of the ground-state correlation functions. In this way, we see how the decay of correlation functions influences the many-body nonstabilizerness. Away from the critical point, where the ground state is weakly entangled and two-point correlation functions decay exponentially, it is possible to estimate the stabilizer Rényi entropy by single spin measurements reliably. At the critical point, on the other hand, one needs to measure an entire block of spins to obtain a reliable estimate, with an error scaling with a characteristic power-law $O(L^{-1})$. The result is of notable importance for experimental measurements of nonstabilizerness in a quantum many-body system, as in a gapped phase this can be performed by few spin measurements (even just a single spin).

As a last comment, our findings can be relevant for the investigation of the emergence of quantum spacetime in the context of AdS + CFT correspondence: In a recent paper [61], the authors speculated on the role of nonstabilizerness in AdS + CFT, and argue that it is a key ingredient to fill the complex structure of the AdS black hole interior, dual to a CFT state. Magic-state resource theory indeed reveals itself as an important piece of information that cannot be detected by only looking at the entanglement. In this context, it is well-known that a quantum many-body system at the criticality is described by a CFT [20,66]. Our results thus give insights regarding the role played by nonstabilizerness in AdS + CFT correspondence: This resource is delocalized in spatial degrees of freedom as, at criticality only, it can be extracted by a system containing L spins with an error decaying only polynomially in L. From this result, it can be reasonably argued that delocalization of nonstabilizerness is a universal property in CFT quantum states—being the correlation functions decaying polynomially—thus revealing fascinating perspectives in AdS + CFT correspondence.

II. SETUP AND MODEL

Let us start by briefly reviewing the stabilizer Rényi entropy [62]. Consider an N-qubit system and the decomposition of state ρ in the Pauli basis given by $\rho = \frac{1}{2^N}\sum_{P\in\mathbb{P}(N)} \mathrm{tr}(P\rho)P$, with $\mathbb{P}(N)$ being the Pauli group. The

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2−stabilizer Rényi entropy $M_2(\rho)$ is then defined as

$$M_2(\rho) := -\log_2 \mathbb{E}_{\mathcal{P}}[\mathrm{tr}^2(P\rho)], \quad (1)$$

i.e., as the average of $\mathrm{tr}^2(P\rho)$ on a state-dependent probability distribution defined as $\mathcal{P}(\rho) := \{2^{-N}\mathrm{tr}^2(P\rho)\mathrm{tr}^{-1}(\rho^2)\}$. It is interesting to note that for ρ pure, $M_2(\rho)$ reduces to the two-Rényi entropy of the classical probability distribution $\mathcal{P}(\rho)$ (modulo an offset of $-N$).

We study the behavior of M_2 in the ground state of the transverse field Ising model for a spin one-half N-site chain with Hamiltonian

$$H(\lambda) = -\sum_{i=1}^{N} \left(\sigma_i^x\sigma_{i+1}^x + \lambda\sigma_i^z\right), \quad (2)$$

where σ_i^α, for $\alpha = x, y, z$, are Pauli matrices defined on the ith site. The model displays a quantum phase transition at $\lambda = 1$ between a disordered and a symmetry-breaking phase. The critical point corresponds to a conformal field theory with $c = \frac{1}{2}$ [67]. For $\lambda \to \infty$ and $\lambda = 0$, the Hamiltonian reduces to a stabilizer Hamiltonian [68] with stabilizer groups $\mathbb{Z} \rhd \mathbb{P}$ and $\mathbb{X} \rhd \mathbb{P}$, respectively. The model $H(\lambda)$ is integrable through standard techniques [69,70]. First, by a Jordan-Wigner transformation introducing fermionic modes and subsequently by Fourier and Bogoliubov transformations [71]. Following these techniques, let us introduce the Majorana operators A_l and B_l:

$$A_l := \bigotimes_{i<l} \sigma_i^z \otimes \sigma_l^x; \quad B_l := \bigotimes_{i<l} \sigma_i^z \otimes \sigma_l^y. \quad (3)$$

These operators obey the anticommutation relations $\{A_l, A_{l'}\} = \{B_l, B_{l'}\} = 2\delta_{ll'}$ and $\{A_l, B_{l'}\} = 0$.

The computation of $M_2(\lambda)$ for the ground state $|G(\lambda)\rangle$ of such a class of Hamiltonians relies on the fact that the ground state can be fully characterized by just the two-point correlation functions by virtue of the Wick theorem: One can compute all the correlation functions of an arbitrary product of Majorana fermions by just knowing the 2−point correlation functions $\langle A_lA_{l'}\rangle = \langle B_lB_{l'}\rangle = \delta_{ll'}$ and [71] $\langle A_lB_{l'}\rangle \equiv \langle A_lB_{l+r}\rangle \equiv G_r(\lambda)$, where

$$G_r(\lambda) = -\frac{i}{\pi}\int_0^\pi \frac{\sin\theta\sin\theta r - (\lambda - \cos\theta)\cos\theta r}{\sqrt{\sin^2\theta + (\lambda-\cos\theta)^2}}. \quad (4)$$

Indeed, let $\mathcal{C}(\{i\}_k, \{j\}_l) := \langle A_{i_1}\cdots A_{i_k}B_{j_1}\cdots B_{j_l}\rangle$ be the expectation value on the ground state $|G(\lambda)\rangle$ of an arbitrary ordered product of Majorana fermions, where $\{i\}_k := \{i_1, \ldots, i_k \,|\, N \geqslant i_1 > \ldots > i_k \geqslant 1\}$ is a set of ordered indexes ranging over all the sites. The computation of $C(\{i\}_k, \{j\}_l)$ can be done through the Pfaffian technique [72], which leads to $\mathcal{C}(\{i\}_k, \{j\}_l) = 0$ unless $k = l$ and

$$\mathcal{C}(\{i\}_k, \{j\}_k) = \begin{vmatrix} \langle A_{i_1}B_{j_1}\rangle & \langle A_{i_1}B_{j_2}\rangle & \cdots & \langle A_{i_1}B_{j_k}\rangle \\ \langle A_{i_2}B_{j_1}\rangle & \langle A_{i_2}B_{j_2}\rangle & \cdots & \langle A_{i_2}B_{j_k}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle A_{i_k}B_{j_1}\rangle & \langle A_{i_k}B_{j_2}\rangle & \cdots & \langle A_{i_k}B_{j_k}\rangle \end{vmatrix}, \quad (5)$$

i.e., to compute the generic $2k$-point correlators of Majorana fermions, it is sufficient to compute the determinant of a $k \times k$ matrix, which can be efficiently done numerically by a poly(k) algorithm.

All the $2k$-point correlations functions can be also obtained by considering the maximum rank $2N$-point correlation function of Majorana fermions $C(\{i\}_N, \{j\}_N) = \langle A_1A_2\cdots A_NB_1B_2\cdots B_N\rangle$; indeed, it is easy to see that one can obtain any correlation function of order $2k$ by considering any minor of $C(\{i\}_N, \{j\}_N)$ of lower rank k. Since a $N \times N$ matrix contain $\binom{N}{k}^2$ minors of order k, there are $\sum_{k=0}^{N}\binom{N}{k}^2 = \binom{2N}{N} \simeq \frac{4^N}{\sqrt{N}}$ nonzero correlation functions of Majorana fermions.

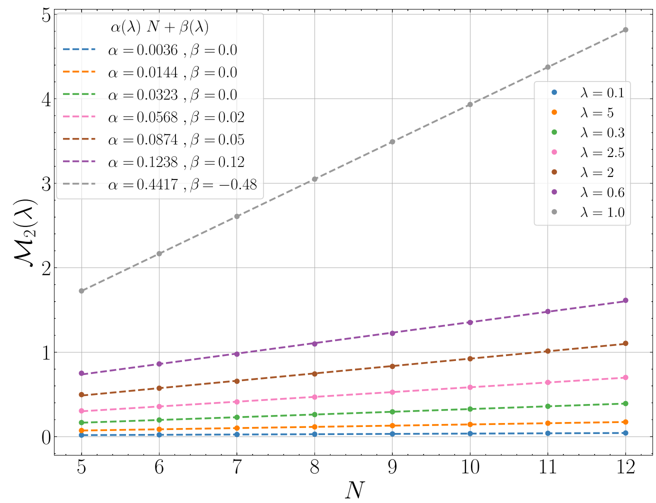

FIG. 1. Numerical simulations of the stabilizer Rényi entropy of the ground state $|G(\lambda)\rangle$ of the Hamiltonian in Eq. (2) for $\lambda = 0.1, 0.3, 0.6$, $\lambda = 1$ and $\lambda = 2, 2.5, 5$ as a function of the length of the chain $N \in [5, 12]$. The curves are fitted to be straight lines for any λs, with slopes $\alpha(\lambda)$ and intercepts $\beta(\lambda)$ fitted in the top-left corner.

III. GROUND-STATE NONSTABILIZERNESS

In this section, we compute M_2 in the ground state $|G(\lambda)\rangle$ and discuss some of its properties. To this end, we need the knowledge of all the 4^N expectation values of Pauli strings $P \in \mathbb{P}(N)$ on the ground state $|G(\lambda)\rangle$. Except for $\lambda = 0$, $\lambda \to \infty$, all the other points feature a nontrivial value for the stabilizer Rényi entropy because the state cannot be factorized.

It is easy to see that any $P \in \mathcal{P}(N)$ can be written (up to a global phase) as an ordered product of Majorana fermions, as $P \propto A_{i_1}\cdots A_{i_k}B_{j_1}\cdots B_{j_l}$ for some $\{i\}_k, \{j\}_l$, which means that we can write the two-stabilizer Rényi entropy for $|G(\lambda)\rangle$ as

$$M_2(\lambda) := M_2(|G(\lambda)\rangle) = -\log_2 \frac{1}{2^N}\sum_{\{i\}_k,\{j\}_k \leqslant N} C(\{i\}_k, \{j\}_k)^4. \quad (6)$$

As the above formula shows, the computation of the nonstabilizerness requires $\sim 4^N$ determinants, which makes the computation exponentially hard in N. Let us provide an upper bound to the two-stabilizer entropy given by the zero-stabilizer entropy $M_0(\lambda) \geqslant M_2(\lambda)$ [62], which essentially counts the number of nonzero entries card($|\psi\rangle$) in the probability distribution $\mathcal{P}(|\psi\rangle\langle\psi|)$ as $M_0(|\psi\rangle) := \log_2\mathrm{card}(|\psi\rangle) - N$. As explained above, there are $\binom{2N}{N}$ nonzero Majorana correlations functions and thus we can upper bound the two-stabilizer Rényi entropy as $M_2(\lambda) \lesssim N - \frac{1}{2}\log_2 N$.

We evaluate numerically formula Eq. (6) for $N = 5, \ldots, 12$, see Fig. 1. The calculations clearly show a linear behavior of the stabilizer Rényi entropy for any $\lambda \neq 0, \infty$:

$$M_2(\lambda) = \alpha(\lambda)N + \beta(\lambda), \tag{7}$$

with both slope $\alpha(\lambda)$ and intercept $\beta(\lambda)$ depending on intensity λ of the external magnetic field. In particular, we observe an increasing slope $\alpha(\lambda)$ from $\lambda = 0$ toward the criticality at $\lambda = 1$, where $\alpha(\lambda)$ approaches its maximum $\alpha(1) \approx 0.44$, and then it starts decreasing again in the disordered phase, $\lambda > 1$. We thus find agreement with the result in Ref. [13]: the ground state at the critical point, and the corresponding $\frac{1}{2}$ CFT, achieves the highest value of nonstabilizerness among the λs. However, this result does not tell us the full story, as the behavior of nonstabilizerness with λ is quite smooth and is $O(N)$ for every value of λ. As we show in the following section, the locality of the interactions together with a gap implies that nonstabilizerness is localized, whereas at the critical point nonstabilizerness cannot be resolved by local measurements.

IV. ACCESS NONSTABILIZERNESS BY LOCAL MEASUREMENTS

Although more amenable than a minimization procedure [73], computing the stabilizer entropy is an exponentially difficult task. However, the locality of the interactions in the Hamiltonian and the presence of a gap results in a fast decay of correlation functions in the ground state, while a power law characterizes the critical point. One thus wonders if one can exploit this locality to access the stabilizer Rényi entropy by local quantities. This results both in the possibility of a realistic experimental measurement of nonstabilizerness in the ground state of quantum many-body systems and a computational advantage.

Let us focus on asymptotic behavior in N, so $M_2(\lambda) \approx \alpha(\lambda)N$. We refer to $\alpha(\lambda)$ as the *density of nonstabilizerness*. In the above, $\approx$ stands for up to an order N^{-1}. Now, it is clear that if one is able to measure the density $\alpha(\lambda)$, then one accesses the nonstabilizerness of the ground state. Can we measure the density of nonstabilizerness $\alpha(\lambda)$ by just looking at the local properties of the reduced density matrix of L spins? To answer the question, we first divide the chain of N sites into N/L subchains of L first-neighbor sites. Consider the following quantum map $\mathcal{L}(|G(\lambda)\rangle\langle G(\lambda)|^{\otimes N/L}) = \bigotimes_{s=0}^{N/L-1} \rho_{L_i}$, where $\rho_{L_s} := \mathrm{tr}_{N-L_s}(|G(\lambda)\rangle\langle G(\lambda)|)$, where $L_s = (sL+1, \dots, (s+1)L)$. To estimate the density of nonstabilizerness $\alpha(\lambda)$ of the ground state, we thus measure the density $\alpha_L(\lambda)$ present in a subsystem of size L. Thanks to the translational invariance of the Hamiltonian in Eq. (2), all the reduced density matrices are equal to $\rho_L \equiv \mathrm{tr}_{N-L_0}(|G(\lambda)\rangle\langle G(\lambda)|)$, and thus the local density of nonstabilizerness $\alpha_L(\lambda)$ depends on the number of sites L of the subchains and not on their locations. Define the L density of nonstabilizerness as

$$\alpha_L(\lambda) := \frac{1}{L} M_2(\rho_L), \tag{8}$$

where $M_2(\rho_L)$ is the stabilizer Rényi entropy of the mixed state ρ_L [see Eq. (1)] which in terms of Majorana correlation functions reads

$$M_2(\rho_L) = -\log_2 \frac{\sum_{\{i\}_k,\{j\}_k \leqslant L} \mathcal{C}(\{i\}_k, \{j\}_k)^4}{\sum_{\{i\}_k,\{j\}_k \leqslant L} \mathcal{C}(\{i\}_k, \{j\}_k)^2}. \tag{9}$$

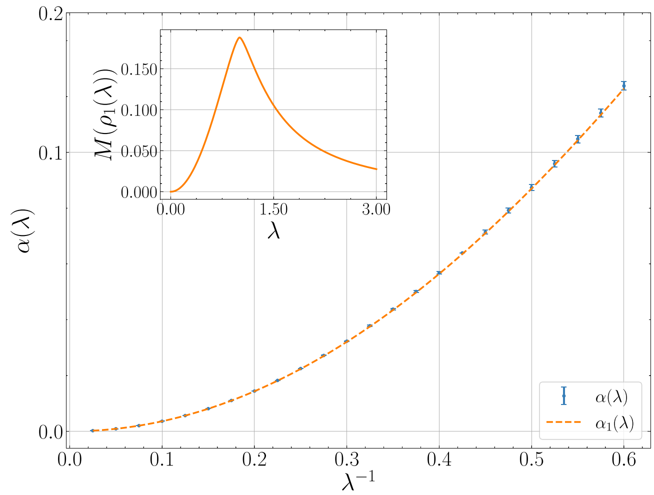

FIG. 2. Plot of the single spin density of nonstabilizerness $\alpha_1(\lambda)$ for $\lambda^{-1} \leqslant 0.6$, computed in Eq. (10), versus the density of nonstabilizerness $\alpha(\lambda)$ extracted through the fits in Fig. 1.

The latter equation, unlike Eq. (6), contains only correlation functions on at most L sites, thus it does not involve global measurements, rather it involves just measurements on the local observable via the reduced density matrix ρ_L, which makes it analytically computable for a reasonable L. First note that for $L \to N$, one has $\alpha_L(\lambda) \to \alpha(\lambda)$. Then, how good is the approximation for a finite L and how does it depend on λ? Let us look at the accuracy of the measurement of the L density of nonstabilizerness by looking at the percent error $\epsilon_L(\lambda) := \frac{|\alpha(\lambda)-\alpha_L(\lambda)|}{\alpha(\lambda)}$ we make by measuring the density of nonstabilizerness via local measurements. We find that, away from the criticality, i.e., in regions $\lambda \ll 1$ and $\lambda \gg 1$, $\epsilon_\lambda(L) < 0.001$ for any L. We thus conclude that, away from the critical point, one can access the nonstabilizerness of the ground state by just measuring the nonstabilizerness of the density matrix of an $O(1)$ of spins, in fact, even a single qubit density matrix ρ_1. We show the agreement between the the 1−density of nonstabilizerness $\alpha_1(\lambda)$ and the density of nonstabilizerness $\alpha(\lambda)$ in Fig. 2 for $\lambda > 1$. The region $\lambda < 1$ features the same behavior, indicating that the nonstabilizerness does not reveal the symmetry of the ground state.

The situation changes at the critical point, i.e., $\lambda = 1$: one finds $\epsilon_L(\lambda) = O(L^{-1})$, cf. Fig. 3. The different behaviors of the error, i.e., $O(1)$ vs $O(L^{-1})$, are reminiscent of different behaviors of the entanglement entropy, displaying an area law everywhere, but at the critical point where the entanglement entropy of a density matrix of L spin scales as $\sim \log_2 L$.

Thus, away from the critical point, the approximation works great also for $L = 1$, which can be computed by hand: The single site density matrix reads [23] $\rho_1(\lambda) = \frac{1}{2}(I + \langle\sigma^z\rangle\sigma^z)$, whose stabilizer Rényi entropy

$$M(\rho_1(\lambda)) = \log_2 \frac{1 + \langle\sigma^z\rangle^2}{1 + \langle\sigma^z\rangle^4}, \tag{10}$$

where $|\langle\sigma^z\rangle| = G_0(\lambda)$, cf. Eq. (4); see the inset in Fig. 2 for a plot.

In the following, we lay down a theoretical argument supporting the fact that measuring the single spin density of

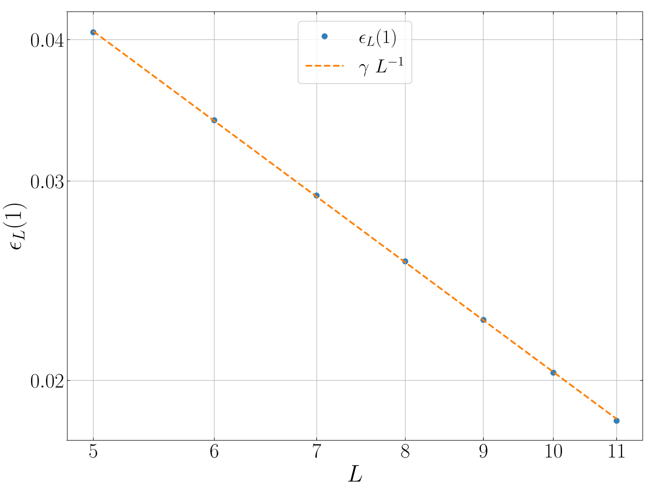

FIG. 3. Comparison of the error $\epsilon_L(1)$ for $L \in \{5, 11\}$ with the fit γL^{-1} with $\gamma = 0.2034 \pm 0.0002$.

nonstabilizerness is already sufficient away from the critical point $\lambda = 1$. It is well-known that [71], away from the criticality (without loss of generality (w.l.o.g.) let us say $\lambda \gg 1$), the two-point correlation functions in Eq. (4) decay faster than exponentially with r. By making the first order expansion $G_r(\lambda) \simeq G_0(\lambda)\delta_{r,0}$, one gets a fair approximation of $G_r(\lambda)$ as long as the higher terms in $r \neq 0$ are exponentially suppressed. By using the above form of the two-correlation functions to compute higher-order functions as in Eq. (5), one gets $|\mathcal{C}(\{i\}_k, \{j_k\})| = |G_0(\lambda)|^k \delta_{\{i\}_k}^{\{j\}_k}$. This means that the only nonzero correlation functions correspond to Pauli operators belonging to the subgroup $\mathbb{Z} \leqslant \mathbb{P}(N)$ containing all the σ^z Pauli strings. The fact that the Pauli strings that count are those belonging to $\mathbb{Z}$ can be also understood by looking to the Hamiltonian in Eq. (2): For $\lambda \gg 1$, the dominant term is $\lambda \sum_i \sigma_i^z$ whose eigenstates are stabilizer states belonging to the stabilizer group $\mathbb{Z}$. In other words, we are estimating the average in Eq. (6) by (importance) sampling the probability distribution with Pauli strings $P \in \mathbb{Z}$. Thus, the estimated density of nonstabilizerness can be computed as

$$\alpha(\lambda) \simeq -\frac{1}{N}\log_2 \frac{\sum_{\{i\}_k,\{j\}_k \leqslant N} G_0(\lambda)^4 \delta_{\{i\}_k}^{\{j\}_k}}{\sum_{\{i\}_k,\{j\}_k \leqslant N} G_0(\lambda)^2 \delta_{\{i\}_k}^{\{j\}_k}}, \quad (11)$$

where we introduced a normalization over the sampling given by $\sum_{\{i\}_k,\{j\}_k \leqslant N} \mathcal{C}(\{i\}_k, \{j\}_k)^2$, cf. Eqs. (1) and (6). The straightforward computation of Eq. (11), together with the fact that $G_0(\lambda)^2 = \langle \sigma^z \rangle^2$ leads to Eq. (10). Thus, the density of nonstabilizerness estimated by importance sampling does coincide with the $L-$density of non-stabilizerness with $L = 1$.

The fact that one can access nonstabilizerness from local measurements is nontrivial and, in general, is not true. We can show it by considering a simpler example: Suppose having a bipartite system AB, a random pure state $|\Psi_{AB}\rangle$, and consider the percent different in nonstabilizerness $\epsilon_{AB} = (M_{AB} - M_A - M_B)/M_{AB}$; here M_{AB}, M_A, M_B are the stabilizer Rényi entropies of $|\Psi_{AB}\rangle$ and $\rho_A = \mathrm{tr}_B(|\Psi_{AB}\rangle\langle\Psi_{AB}|)$ and ρ_B, respectively. Thanks to the typicality of the stabilizer Rényi entropy [62] and the two-Rényi entropy [36] over the set of Haar-random states, one gets $\epsilon_{AB} \approx 1$ (up to an exponentially small error in $\dim(AB)$), which means that the nonstabilizerness cannot be accessed locally for the majority of states in the Hilbert space. The above argument can be straightforwardly generalized to the case of the multipartite system $A_1A_2 \cdots A_h$.

V. CONCLUSIONS AND OUTLOOK

The complex pattern of the ground-state wave function of a quantum many-body system depends on the interplay between its entanglement and the non-Clifford resources, or nonstabilizerness, that it contains. Although both in the gapped phase and at the critical point the ground state of the transverse field Ising model contains an extensive amount of nonstabilizerness, away from criticality this is localized. On the other hand, at the critical point, its nonstabilizerness is delocalized and described by a power law.

These results raise a number of questions. First, one could extend these methods to models featuring localization hrough disorder or frustration. One expects that any form of localization would result in being able to evaluate non-stabilizerness by few-site quantities. Second, the same methods can be used to study the dynamics of a quantum many-body system after a quench. It would be interesting to see whether nonstabilizerness delocalizes as the system evolves in time and if equilibration ensues. Moreover, it is very intriguing to study the behavior of nonstabilizerness in such systems when integrability is broken. The role of quantum complexity implied in the conjunction of nonstabilizerness and entanglement for the onset of thermalization and nonintegrable behavior has been recently studied in the context of doped quantum circuits [74–76] and Hamiltonians [77,78], but a local quantum many-body system is its most natural setting. The main result of this paper opens the way to the experimental measurement of nonstabilizerness by local measurements, for instance, in ultracold atom gases realizing the Bose-Hubbard model. Finally, although further investigation is necessary, we can argue that the delocalization of nonstabilizerness at the critical point suggests that the CFT theory, underlying critical many body systems, enjoys delocalization of nonstabilizerness as well.

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