Nonreciprocal propagation of surface electromagnetic waves in structures comprising magneto-optical materials

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This paper investigates theoretically the effect of nonreciprocity on surface electromagnetic waves (SEWs) propagating in magneto-optical structures in mutually opposite directions, i.e., on waves with tangential wave numbers k of opposite sign. Namely, of our interest is a correlation between the numbers of forward ($k > 0$) and backward ($k < 0$) propagating SEWs, which manifests itself as a limit on the total number of SEWs. The peculiarity of this limit is that the maximum total number of forward- and backward-propagating SEWs is less than the sum of the maximum numbers of SEWs for each of the opposite directions taken individually. We derive a relation between the electromagnetic surface impedances relevant to forward- and backward-propagating waves. Afterward, by using general properties of these impedances valid regardless of the crystallographic symmetry, we analyze the roots of dispersion equations without solving them explicitly. Various types of magneto-optical structures are considered. In particular, it is shown that, in the bicrystal formed of two half-infinite magneto-optical media, the maximum total number of forward- and backward-propagating SEWs is two at a given value of $|k|$, whereas in each of the mutually opposite directions two SEWs can exist. For example, if two forward-propagating SEWs are known to exist, then no backward-propagating SEW exists. In the case where a metal film is inserted between two different half-infinite magneto-optical media, the maximum number of SEWs in one direction is four but the maximum of the total number of forward- and backward-propagating SEWs is six rather than eight.

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I. INTRODUCTION

Basic phenomena characterizing the propagation of electromagnetic waves in magneto-optically active materials, such as the Faraday rotation of polarization and the Kerr effect, have been known since long ago but they still attract researchers' attention because of the wide application of these materials to modern photonics and optoelectronics [1–3]. Phenomenologically, magneto-optical activity is explained by specific properties of the dielectric permittivity and magnetic permeability, namely, in nonabsorbing media they are complex Hermitian tensors. As a result, magneto-optical materials turn out to be nonreciprocal since, according to the Onsager reciprocity relations, the reciprocity occurs when the dielectric permittivity and magnetic permeability are symmetric tensors [4–6]. Correspondingly, the received electromagnetic signals will be different if the position of the radiation and observation points are interchanged [4–6]. In practice, the nonreciprocity underlies the operation principle of magneto-optical isolators, circulators, and switches governing the propagation of bulk and waveguide electromagnetic modes [7–14].

Apart from bulk and waveguide electromagnetic waves there also exist surface electromagnetic waves (SEWs). They emerge in dielectric-metal structures [15–25], in dielectric superlattices [26–31], and at the interface between two half-infinite dielectrics [32–39]. SEWs in magneto-optical media have been studied theoretically and experimentally in Refs. [40–48]. It has been shown that the dispersion curves of forward- and backward-propagating SEWs can be different and in addition SEWs can propagate unidirectionally, namely, a SEW propagates along a given direction but there is no wave in the opposite direction. These results have been obtained by solving the dispersion equations either explicitly analytically, when possible, or numerically, so they are relevant only to SEWs in those specific cases which were under consideration. General symmetry conditions securing the nonreciprocal electromagnetic wave propagation in magnetic crystals have been discussed in Ref. [49].

Note that nonreciprocity also shows up in the spin-wave propagation in magnetic crystals [50–54]. Nonreciprocity of spin waves can be utilized for the creation of energy-efficient magnonic logic devices [54–57].

In the present paper we analyze correlations between the existence of forward- and backward-propagating SEWs in magneto-optically active structures without any reference to particular crystallographic symmetry and without solving the boundary-value problem explicitly either analytically or numerically. To be more specific, keeping in mind the fact that the number of SEWs along a fixed direction can differ from the number of SEWs in the reverse direction, we establish the permissible maximum of the total number of SEWs propagating forward and backward.

Our theory will be based on the general properties of surface electromagnetic impedances. We have already used this method to study the SEW propagation in anisotropic

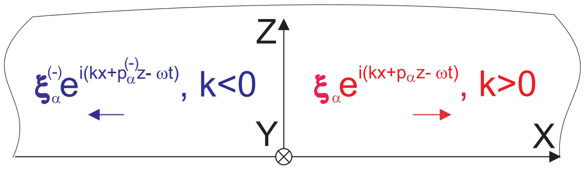

FIG. 1. Geometry of propagation. The medium occupies the half space $z > 0$. Waves with $k > 0$ and $k < 0$ propagates to the right and to the left, respectively. The red and blue arrows indicate the directions of propagation.

and bianisotropic periodic superlattices as well as at the interface between two homogeneous half-infinite media in Refs. [58–60]. These papers analyze the maximum permissible number of SEWs only along one direction in some of the structures considered in the present paper and do not address the SEW propagation in mutually opposite directions. It is worth noting that the idea of exploiting properties of the impedance matrices, which follow from fundamental physical principles, in the analysis of the existence of surface waves has been put forward in Refs. [61,62] as applied to surface acoustic waves in half-infinite homogeneous anisotropic solids. Subsequently, this idea has been used in Ref. [63] for the analysis of the existence of SEWs in a half-infinite homogeneous magneto-optically inactive material in contact with an isotropic dielectric or a superconductor.

We assume no absorption since otherwise the impedances are non-Hermitian matrices, so that our method turns out to be inapplicable. Semiquantitative arguments given in Refs. [58–60] allowed us to conclude that sufficiently weak absorption cannot increase the permissible number of SEWs established under the assumption of no absorption. In addition, we consider only nonradiative SEWs, i.e., SEWs with frequencies smaller than a limiting frequency above which bulk modes appear, since the impedances are also non-Hermitian matrices in the presence of bulk waves.

Our paper is organized as follows. In Sec. II we establish relations between the characteristics of forward- and backward-propagating SEWs as well as between the surface impedances associated with these waves. In Sec. III, with the aid of these relations, we derive the permissible maximum number of forward- and backward-propagating SEWs in total in different magneto-optical structures. Section IV summarizes our results. In Appendix A a number of general relations is derived. Some general aspects of our approach is briefly described in Appendix B. Appendix C discusses the SEW propagation in materials possessing natural optical activity. Appendixes D, E, and F contain details of the proof of statements given in Secs. III A and III E as well as results of numerical computations. The influence of absorption of electromagnetic waves is discussed in Appendix G.

II. SURFACE IMPEDANCES OF MAGNETO-OPTICAL MEDIA

We consider SEWs in materials bounded by $z = \text{const.}$ planes (Fig. 1) and under boundary conditions requiring the continuity of the tangential components $E_{x,y}$ and $H_{x,y}$ of the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields. With this in mind, we seek partial solutions of the Maxwell equations propagating in the positive direction of the axis X with frequency ω and wave number k in the form

$$\tilde{\boldsymbol{\xi}}_\alpha(\mathbf{r}, t) = \boldsymbol{\xi}_\alpha e^{i[kx+p_\alpha z-\omega t]}, \quad k > 0, \quad \alpha = 1, \ldots, 4, \tag{1}$$

where the four-component vector $\boldsymbol{\xi}_\alpha$ is composed of $E_{x,y}$ and $H_{x,y}$,

$$\boldsymbol{\xi}_\alpha = \begin{pmatrix} \mathbf{U}_\alpha \\ \mathbf{V}_\alpha \end{pmatrix}, \quad \mathbf{U}_\alpha = \begin{pmatrix} -E_{\alpha y} \\ H_{\alpha y} \end{pmatrix}, \quad \mathbf{V}_\alpha = \begin{pmatrix} H_{\alpha x} \\ E_{\alpha x} \end{pmatrix}, \tag{2}$$

$\mathbf{r} = (x\ z)^t$ is the radius vector, and the superscript ^{t} denotes the transposition.

Assume for a moment that the medium is not only magneto-optically active but also bianisotropic, so that the constitutive connections read as follows:

$$\mathbf{D} = \hat{\boldsymbol{\varepsilon}}\mathbf{E} + \hat{\boldsymbol{\kappa}}\mathbf{H}, \quad \mathbf{B} = \hat{\boldsymbol{\kappa}}^\dagger\mathbf{E} + \hat{\boldsymbol{\mu}}\mathbf{H}, \tag{3}$$

where $\mathbf{D}$ is the electric displacement, $\mathbf{B}$ is the magnetic induction, $\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{\varepsilon}}^\dagger$ and $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}^\dagger$ are the complex Hermitian tensors of dielectric permittivity and magnetic permeability, respectively, and $\hat{\boldsymbol{\kappa}}$ is a complex pseudotensor characterizing the bianisotropic coupling [5,6,64,65].

By excluding the z components of $\mathbf{E}$ and $\mathbf{H}$ as well as the electric displacement $\mathbf{D}$ and the magnetic induction $\mathbf{B}$ with the help of (3) from the Maxwell equations, we arrive at the eigenvalue problem for a 4×4 matrix $\hat{\mathbf{N}}$,

$$\hat{\mathbf{N}}\boldsymbol{\xi}_\alpha = p_\alpha\boldsymbol{\xi}_\alpha, \quad \alpha = 1, \ldots, 4, \tag{4}$$

of which the eigenvectors and eigenvalues are, respectively, the vectors $\boldsymbol{\xi}_\alpha$ and the normal wave numbers p_α of modes (1) (see Appendix A).

The explicit expression of $\hat{\mathbf{N}}$ actually depends on the order of $E_{x,y}$ and $H_{x,y}$ in $\boldsymbol{\xi}_\alpha$ and material properties of media. In practice, different definitions of $\boldsymbol{\xi}_\alpha$ are used, yielding different expressions of the matrix, see, e.g., Refs. [66–68]. In our case $\hat{\mathbf{N}}$ can be written in the form

$$\hat{\mathbf{N}} = \omega\hat{\mathbf{A}}_T - k\hat{\mathbf{B}}_T - \frac{k^2}{\omega}\hat{\mathbf{C}}_T, \tag{5}$$

where

$$\hat{\mathbf{A}}_T = \hat{\mathbf{T}}\hat{\mathbf{A}}, \quad \hat{\mathbf{B}}_T = \hat{\mathbf{T}}\hat{\mathbf{B}}, \quad \hat{\mathbf{C}}_T = \hat{\mathbf{T}}\hat{\mathbf{C}}, \tag{6}$$

$\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ are 4×4 matrices constructed of combinations of the material constants [59],

$$\hat{\mathbf{T}} = \begin{pmatrix} \hat{\mathbf{0}} & \hat{\mathbf{I}} \\ \hat{\mathbf{I}} & \hat{\mathbf{0}} \end{pmatrix}, \tag{7}$$

and $\hat{\mathbf{0}}$ and $\hat{\mathbf{I}}$ are 2×2 zero and identity matrices.

The matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ do not involve k and depend on frequency only through material constants. In addition, $\hat{\mathbf{A}} = \hat{\mathbf{A}}^\dagger$, $\hat{\mathbf{B}} = \hat{\mathbf{B}}^\dagger$, and $\hat{\mathbf{C}} = \hat{\mathbf{C}}^\dagger$, so

$$(\hat{\mathbf{T}}\hat{\mathbf{N}})^\dagger = (\hat{\mathbf{T}}\hat{\mathbf{N}}). \tag{8}$$

Since $\hat{\mathbf{N}}$ is a 4×4 matrix, one has four modes (1) with normal wave numbers p_α, $\alpha = 1, \ldots, 4$, for fixed values of k and ω. In view of (8), if p_α is an eigenvalue of $\hat{\mathbf{N}}$, then the complex conjugate p_α^* is also an eigenvalue. Hence, amid four eigenvalues there is either a pair of complex-conjugate eigenvalues or a pair of purely real eigenvalues.

We are interested in the frequency interval $0 < \omega < \omega_L$ where all four modes (1) are inhomogeneous for a given k, that is, where all four p_α are complex. Thus there are two pairs of complex conjugate p_α in this interval. Assign the index α in such a way that

$$p_{\alpha+2} = p_\alpha^*, \quad \mathrm{Im}(p_\alpha) > 0, \quad \alpha = 1, 2 \tag{9}$$

(but $\boldsymbol{\xi}_{\alpha+2} \neq \boldsymbol{\xi}_\alpha^*$ since $\hat{\mathbf{N}} \neq \hat{\mathbf{N}}^*$). Note that ω_L can be called the limiting frequency of bulk modes because for $\omega > \omega_L$ such modes emerge, i.e., there is at least one pair of purely real eigenvalues. At $\omega = \omega_L$ two eigenvalues, which constitute a complex-conjugate pair in the interval $\omega < \omega_L$, coalesce into one real eigenvalue and $\hat{\mathbf{N}}$ does not diagonalize. Keeping to the terminology used when studying physical phenomena governed by non-Hermitian matrices (see, e.g., Ref. [69]), one can also call ω_L an exceptional point of $\hat{\mathbf{N}}$.

In accordance with (9) the wave fields

$$\tilde{\boldsymbol{\xi}}_+ = \sum_{\alpha=1}^{2} b_\alpha \tilde{\boldsymbol{\xi}}_\alpha(\mathbf{r}, t), \quad \tilde{\boldsymbol{\xi}}_- = \sum_{\alpha=3}^{4} b_\alpha \tilde{\boldsymbol{\xi}}_\alpha(\mathbf{r}, t), \tag{10}$$

where b_α are constants, decay to zero when $z \to +\infty$ and $z \to -\infty$, respectively. With this in mind, we introduce 2×2 impedance matrices $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ relating the vectors $\mathbf{U}_\alpha$ and $\mathbf{V}_\alpha$ (2),

$$\mathbf{V}_\alpha = i\hat{\mathbf{Z}}\mathbf{U}_\alpha, \quad \mathbf{V}_{\alpha+2} = -i\hat{\mathbf{Z}}'\mathbf{U}_{\alpha+2}. \ \alpha = 1, 2, \tag{11}$$

so that

$$\hat{\mathbf{Z}} = -i\hat{\mathbf{V}}\hat{\mathbf{U}}^{-1}, \quad \hat{\mathbf{Z}}' = i\hat{\mathbf{V}}'\hat{\mathbf{U}}'^{-1}, \tag{12}$$

where

$$\begin{aligned} \hat{\mathbf{U}} &= (\mathbf{U}_1, \mathbf{U}_2), \quad \hat{\mathbf{V}} = (\mathbf{V}_1, \mathbf{V}_2), \\ \hat{\mathbf{U}}' &= (\mathbf{U}_3, \mathbf{U}_4), \quad \hat{\mathbf{V}}' = (\mathbf{V}_3, \mathbf{V}_4), \end{aligned} \tag{13}$$

are 2×2 matrices composed of the vectors $\mathbf{U}_\alpha$ and $\mathbf{V}_\alpha$.

The properties of the impedances defined by (11) are the same as those of their counterparts introduced in Refs. [58–60], where we analyze the propagation of SEWs only in one direction in bianisotropic bicrystals and superlattices, and in Refs. [70–74], which study the propagation of surface acoustic waves in superlattices. For convenience, we list the necessary properties of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$:

$$\hat{\mathbf{Z}} = \hat{\mathbf{Z}}^\dagger, \quad \hat{\mathbf{Z}}' = \hat{\mathbf{Z}}'^\dagger \text{ for } \omega < \omega_L, \tag{14}$$

$$\hat{\mathbf{Z}} \text{ and } \hat{\mathbf{Z}}' \text{ are positive definite matrices at } \omega \to 0, \tag{15}$$

$$\hat{\mathbf{Z}} + \hat{\mathbf{Z}}' \text{ is a positive definite matrix for } \omega < \omega_L, \tag{16}$$

$$\text{the eigenvalues of } \hat{\mathbf{Z}} \text{ and } \hat{\mathbf{Z}}' \text{ are finite in the interval } 0 < \omega < \omega_L, \tag{17}$$

$$\frac{\partial \hat{\mathbf{Z}}}{\partial \omega} \text{ and } \frac{\partial \hat{\mathbf{Z}}'}{\partial \omega} \text{ are negative definite matrices for } \omega < \omega_L. \tag{18}$$

Hermiticity of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$, i.e., property (14), follows from the absence of the energy flux of localized wave fields along the normal to the boundary (Appendix A). In view of (14) all eigenvalues of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ are purely real for $\omega < \omega_L$. Property (15) is due to the positiveness of the energy of static electric and magnetic fields. Owing to (15), all eigenvalues of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ are positive in the vicinity of $\omega = 0$. Property (16) is a consequence of (15) and the fact that $\det(\hat{\mathbf{Z}} + \hat{\mathbf{Z}}') \neq 0$. The vanishing of this determinant would mean the existence of a localized wave just inside an infinite homogeneous medium [see Eq. (31) after replacement of $\hat{\mathbf{Z}}_2'$ by $\hat{\mathbf{Z}}_1'$] but such a wave cannot emerge since there is no boundary to localize an electromagnetic field. Properties (16) and (18) yield (17).

Property (18) holds true regardless of the frequency dispersion of material constants. It follows from a relation between the frequency derivative of the impedances and the integral W of the time-averaged energy over the depth of the half-infinite medium [59]. For example, in the half space $z > 0$, where a localized wave $\tilde{\boldsymbol{\xi}}_+$ is a linear combination (10) of modes $\alpha = 1$ and $\alpha = 2$,

$$4W = i\left(\mathbf{V}^\dagger \frac{\partial \mathbf{U}}{\partial \omega} + \mathbf{U}^\dagger \frac{\partial \mathbf{V}}{\partial \omega}\right)\bigg|_{z=0} = -\mathbf{U}^\dagger \frac{\partial \hat{\mathbf{Z}}}{\partial \omega}\mathbf{U} > 0, \tag{19}$$

where $\mathbf{U}$ and $\mathbf{V} = i\hat{\mathbf{Z}}\mathbf{U}$ are the vectors forming the vector $\tilde{\boldsymbol{\xi}}_+ = (\mathbf{U}\ \mathbf{V})^t$ at $z = 0$, the vector $\mathbf{U}$ is assumed arbitrary and, by virtue of (14), $\mathbf{V}^\dagger = -i\mathbf{U}^\dagger\hat{\mathbf{Z}}$. Through (18) and the spectral decomposition of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ one can prove that the frequency derivatives of the eigenvalues of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ are negative.

A more comprehensive discussion of the properties of impedances and details of their formal derivation can be found in Refs. [58,59,70–74].

Let us call waves (1) forward propagating and waves

$$\tilde{\boldsymbol{\xi}}_\alpha^{(-)}(\mathbf{r}, t) = \boldsymbol{\xi}_\alpha^{(-)} e^{i[kx + p_\alpha^{(-)}z - \omega t]}, \quad k < 0, \tag{20}$$

which propagate in the negative direction of the axis X, backward propagating. The vectors $\boldsymbol{\xi}_\alpha^{(-)}$ and wave numbers $p_\alpha^{(-)}$ are found by solving the eigenvalue problem

$$\hat{\mathbf{N}}^{(-)}\boldsymbol{\xi}_\alpha^{(-)} = p_\alpha^{(-)}\boldsymbol{\xi}_\alpha^{(-)}, \quad \alpha = 1, \dots, 4, \tag{21}$$

where $\hat{\mathbf{N}}^{(-)} = \omega\hat{\mathbf{A}}_T + |k|\hat{\mathbf{B}}_T - \frac{k^2}{\omega}\hat{\mathbf{C}}_T$.

Adopting the same rule of labeling backward-propagating modes as of forward-propagating ones, viz.

$$p_{\alpha+2}^{(-)} = p_\alpha^{(-)*}, \quad \mathrm{Im}(p_\alpha^{(-)}) > 0, \quad \alpha = 1, 2, \tag{22}$$

[cf. Eq. (9)] we introduce the impedances $\hat{\mathbf{Z}}^{(-)}$ and $\hat{\mathbf{Z}}'^{(-)}$ via the relations

$$\mathbf{V}_\alpha^{(-)} = i\hat{\mathbf{Z}}^{(-)}\mathbf{U}_\alpha^{(-)}, \quad \mathbf{V}_{\alpha+2}^{(-)} = -i\hat{\mathbf{Z}}^{(-)\prime}\mathbf{U}_{\alpha+2}^{(-)}, \tag{23}$$

$\alpha = 1, 2$, where $\mathbf{U}_\alpha^{(-)}$ and $\mathbf{V}_\alpha^{(-)}$ are counterparts of $\mathbf{U}_\alpha$ and $\mathbf{V}_\alpha$ (2), respectively [cf. Eq. (11)]. The impedances $\hat{\mathbf{Z}}^{(-)}$ and $\hat{\mathbf{Z}}'^{(-)}$ possess properties (14)–(18) because the sign of k has no effect on these properties.

Relations and properties (4)–(23) hold true regardless of the bianisotropic coupling. Let us now turn to nonbianisotropic nonabsorbing magneto-optical materials. In this case the bianisotropic coupling vanishes, i.e., $\hat{\boldsymbol{\kappa}} = 0$ in (3), but $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are still Hermitian tensors. We put $\hat{\boldsymbol{\kappa}} = 0$ in $\hat{\mathbf{A}}_T$, $\hat{\mathbf{B}}_T$, and $\hat{\mathbf{C}}_T$ and explicitly compute these matrices. Comparing $\hat{\mathbf{N}}$ with $\hat{\mathbf{N}}^{(-)}$ reveals that, in magneto-optical media

$$\hat{\mathbf{N}}^{(-)} = -\hat{\mathbf{S}}\hat{\mathbf{N}}\hat{\mathbf{S}}, \tag{24}$$

where $\hat{\mathbf{S}}$ is a 4×4 matrix,

$$\hat{\mathbf{S}} = \begin{pmatrix} \hat{\mathbf{P}} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & -\hat{\mathbf{P}} \end{pmatrix}, \quad \hat{\mathbf{P}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{25}$$

Owing to (9) and (22), from (24) it follows that

$$p_\alpha^{(-)} = -p_{\alpha+2}, \quad p_{\alpha+2}^{(-)} = -p_\alpha, \quad \alpha = 1, 2, \tag{26}$$

$$\boldsymbol{\xi}_\alpha^{(-)} = \hat{\mathbf{S}}\boldsymbol{\xi}_{\alpha+2}, \quad \boldsymbol{\xi}_{\alpha+2}^{(-)} = \hat{\mathbf{S}}\boldsymbol{\xi}_\alpha, \ \alpha = 1, 2. \tag{27}$$

From (26) it is seen that the frequency $\omega_L^{(-)}$ below which all eigenvalues $p_\alpha^{(-)}$ are complex equals ω_L.

Combining (11), (23), and (27) leads to

$$\hat{\mathbf{Z}}^{(-)} = \hat{\mathbf{P}}\hat{\mathbf{Z}}'\hat{\mathbf{P}}, \quad \hat{\mathbf{Z}}^{(-)\prime} = \hat{\mathbf{P}}\hat{\mathbf{Z}}\hat{\mathbf{P}}. \tag{28}$$

These equalities allow us to analyze the permissible number of forward- and backward-propagating SEWs.

III. SURFACE WAVES

By using (28) we will establish the maximum total number M_{tot} of forward- and backward-propagating SEWs in different magneto-optical structures. It will be shown that, in most structures, M_{tot} is less than twice the maximum number M of SEWs propagating in one direction. The number M is the same for opposite directions. Note that $M_{\text{tot}} = 2M$ would look to be more natural.

The general idea of our approach is briefly discussed in Appendix B. In addition, Fig. 2 in Appendix B shows schematically the frequency dependence of the eigenvalues of matrices involved in our considerations.

Structures considered below will be indicated as "Case I," "Case II," etc. Such an indicator is at the end of the statement concerning the maximum number of SEWs in the corresponding structure.

It is worth noting that $\hat{\boldsymbol{\kappa}} \neq 0$ modifies the explicit expressions of $\hat{\mathbf{A}}_T$, $\hat{\mathbf{B}}_T$, and $\hat{\mathbf{C}}_T$ in such a way that direct links of type (24)–(28) do not exist in the general case. Therefore one can anticipate that, in bianisotropic materials, $M_{\text{tot}} = 2M$. The case where the bianisotropy is due to natural optical activity is discussed in Appendix C.

A. Bicrystal

Let a bicrystal be composed of magneto-optical media 1 and 2 occupying the half spaces $z > 0$ and $z < 0$, respectively. By using vectors (2) and taking into account (9) and (22) one can write the boundary conditions on the interface $z = 0$ in the form

$$\sum_{\alpha=1}^{2} b_\alpha \boldsymbol{\xi}_{1,\alpha} = \sum_{\alpha=1}^{2} b_{\alpha+2}\boldsymbol{\xi}_{2,\alpha+2}, \tag{29}$$

$$\sum_{\alpha=1}^{2} c_\alpha \boldsymbol{\xi}_{1,\alpha}^{(-)} = \sum_{\alpha=1}^{2} c_{\alpha+2}\boldsymbol{\xi}_{2,\alpha+2}^{(-)}, \tag{30}$$

for forward- and backward-propagating SEWs, respectively. The subscript J in $\boldsymbol{\xi}_{J,\alpha}$ and $\boldsymbol{\xi}_{J,\alpha}^{(-)}$ labels the eigenvectors of the matrices $\hat{\mathbf{N}}$ and $\hat{\mathbf{N}}^{(-)}$ in the upper ($J = 1$) and lower ($J = 2$) parts of the bicrystal, b_α and c_α, $\alpha = 1, \ldots, 4$, are unknown constants.

Assign the same subscripts $J = 1$ and $J = 2$ to the impedances of media 1 and 2, respectively. Due to (11), the dispersion equation for forward-propagating SEWs can be cast into the form

$$\det(\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}_2') \equiv \det \hat{\mathbf{Z}}_b = 0, \tag{31}$$

whereas, by virtue of (23) and (28), the dispersion equation for backward-propagating SEWs reduces to

$$\det(\hat{\mathbf{Z}}_1^{(-)} + \hat{\mathbf{Z}}_2^{(-)\prime}) = \det(\hat{\mathbf{Z}}_1' + \hat{\mathbf{Z}}_2) \equiv \det \hat{\mathbf{Z}}_b' = 0. \tag{32}$$

The subscript b means "bicrystal."

It occurs that each of equations (31) and (32) can have at most two roots at a given value of $|k|$ in the interval $\omega < \Omega_L = \min(\omega_{L,1}, \omega_{L,2})$, where $\omega_{L,J}$, $J = 1, 2$, are the limiting frequencies in media 1 and 2. However, in total these equations can have at most two, rather than four, roots (see Appendix D). As a result,

> given $|k|$, up to two SEWs can exist in each of the mutually opposite directions on the interface between two half-infinite magneto-optical media in the interval $\omega < \Omega_L$ but the total number of forward- and backward-propagating SEWs cannot be greater than two (Case I).

In particular, if a SEW emerges in the forward direction then only one backward-propagating SEW can exist. If two forward SEWs emerge, then no backward SEW exists. An example of the bicrystal supporting two SEWs in one direction is given in Ref. [59] in the context of the study of SEWs in a pair of the so-called complementary bicrystals. Computations reveal that no SEWs exist in the opposite direction in this bicrystal.

The above statement is valid when one of the magneto-optical media is replaced by an optically inactive nonbianisotropic medium. However, the maximum total number turns out to be four if the new medium is reciprocal but bianisotropic, i.e., when the medium features natural optical activity (see Appendix C).

B. Magneto-optical medium–half-infinite metal

We assume the relative dielectric permittivity of a metal ε_m isotropic, purely real and negative in a frequency interval $\omega_a < \omega < \omega_p$, where $\omega_p > \omega_L$ is a bulk plasma frequency. It is also assumed that the relative magnetic permeability μ_m is real, isotropic, and positive. The frequency dependence of ε_m and μ_m is restricted only by the inequalities

$$\frac{\partial(\omega\varepsilon_m)}{\partial\omega} > 0, \quad \frac{\partial(\omega\mu_m)}{\partial\omega} > 0, \tag{33}$$

securing the positiveness of the averaged energy [75,76].

All the four partial modes in the film are inhomogeneous with

$$p_\alpha = -p_{\alpha+2} = ip, \quad p = \sqrt{k^2 + \frac{\omega^2|\varepsilon_m|\mu_m}{c^2}}, \tag{34}$$

where $\alpha = 1, 2$ and c is the light velocity in the vacuum. The impedances $\hat{\mathbf{Z}}_m$ and $\hat{\mathbf{Z}}_m'$ defined in accordance with (11) or

(23) are diagonal real matrices,

$$\hat{\mathbf{Z}}_m = \hat{\mathbf{Z}}'_m = \begin{pmatrix} \dfrac{p}{\omega\mu_0\mu} & 0 \\ 0 & -\dfrac{p}{\omega\varepsilon_0|\varepsilon_m|} \end{pmatrix}, \tag{35}$$

where μ_0 and ε_0 are the magnetic and dielectric constants.

Let a magneto-optical medium occupy the half space $z > 0$. In this instance, due to Eqs. (11), (23), (28), and (35), the dispersion equations for forward- and backward-propagating SEWs read as follows:

$$\det(\hat{\mathbf{Z}} + \hat{\mathbf{Z}}_m) \equiv \det \hat{\mathbf{Z}}_{\rm dm} = 0, \tag{36}$$

$$\det(\hat{\mathbf{Z}}' + \hat{\mathbf{Z}}_m) \equiv \det \hat{\mathbf{Z}}'_{\rm dm} = 0, \tag{37}$$

respectively, where the matrices $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ are the impedances of the magneto-optical medium and the subscript "dm" means "dielectric metal."

In view of (33), the frequency derivatives of the elements of $\hat{\mathbf{Z}}_m$ prove to be negative and hence $\partial\hat{\mathbf{Z}}_m/\partial\omega$ is a negative definite matrix. Due to this fact as well as (17) and (18) the eigenvalues of $\hat{\mathbf{Z}}_{\rm dm}$ and $\hat{\mathbf{Z}}'_{\rm dm}$ monotonically decrease with increasing frequency, so each eigenvalue can vanish at most once in the interval $\omega_a < \omega < \Omega_{LM} = \min(\omega_L, \omega_p)$. The eigenvalues do not need to be positive at ω_a, but if all the four eigenvalues are positive at that frequency, then each of equations (36) and (37) can have two roots.

Contracting the vector $\mathbf{t} = (1\ 0)^t$ with the matrix

$$\hat{G}_{\rm dm} = \hat{\mathbf{Z}}_{\rm dm} + \hat{\mathbf{Z}}'_{\rm dm} = \sum_{\alpha=1}^{2}(\lambda_\alpha \mathbf{q}_\alpha \otimes \mathbf{q}^*_\alpha + \lambda'_\alpha \mathbf{q}'_\alpha \otimes \mathbf{q}'^*_\alpha), \tag{38}$$

where λ_α and λ'_α are the eigenvalues of $\hat{\mathbf{Z}}_{\rm dm}$ and $\hat{\mathbf{Z}}'_{\rm dm}$, respectively, and $\mathbf{q}_\alpha$ and $\mathbf{q}_\alpha$, $\alpha = 1, 2$, are their orthonormalized eigenvectors, we obtain that

$$\sum_{\alpha=1}^{2}(\lambda_\alpha|\mathbf{t}^\dagger\mathbf{q}_\alpha|^2 + \lambda'_\alpha|\mathbf{t}^\dagger\mathbf{q}'_\alpha|^2) = Z_{\rm dm,11} + Z'_{\rm dm,11} + \frac{p}{\omega\mu_0\mu} > 0, \tag{39}$$

since, due to (16), the sum of the diagonal elements $Z_{\rm dm,11}$ and $Z'_{\rm dm,11}$ of the matrices $\hat{\mathbf{Z}}_{\rm dm}$ and $\hat{\mathbf{Z}}'_{\rm dm}$ is positive. Inequality (40) holds true provided that at least one of the four eigenvalues λ_α and λ'_α, $\alpha = 1, 2$, is positive in the interval $\omega_a < \omega < \Omega_{LM}$. Therefore at most three eigenvalues can vanish; that is, equations (36) and (37) have at most three roots in total. Thus,

> given $|k|$, up to two SEWs can exist in each of the two mutually opposite directions at the boundary of magneto-optical and metal half-infinite media for $\omega < \Omega_{LM}$, but the total number of forward- and backward-propagating SEWs cannot be greater than three (Case II).

An example of the bicrystal consisting of a magneto-optical medium and a metal where two SEWs exist for one direction of propagation is given in Ref. [60]. Additional computations reveal that only one SEW exists in the opposite direction which matches the above statement.

C. Metal film

We suppose that a metal film of thickness h occupies the layer $0 < z < h$ and magneto-optical media 1 and 2 occupy the regions $z > h$ and $z < 0$, respectively. The material properties of the metal are given above and we consider the frequency interval $\omega_a < \omega < \Omega_{LM} = \min(\omega_{L,1}, \omega_{L,2}, \omega_p)$.

The tangential components of a forward-propagating field at $z = 0$ and $z = h$ can be related through a 4×4 matrix

$$\hat{\mathbf{Z}}_{\rm mf} = \begin{pmatrix} Z^{\rm (TE)}_{11} & 0 & Z^{\rm (TE)}_{12} & 0 \\ 0 & Z^{\rm (TM)}_{11} & 0 & Z^{\rm (TM)}_{12} \\ Z^{\rm (TE)}_{12} & 0 & Z^{\rm (TE)}_{11} & 0 \\ 0 & Z^{\rm (TM)}_{12} & 0 & Z^{\rm (TM)}_{11} \end{pmatrix}, \tag{40}$$

where $Z^{\rm (TE)}_{ij}$ and $Z^{\rm (TM)}_{ij}$ are ij elements of the 2×2 matrices

$$\hat{\mathbf{Z}}^{\rm (TE)} = \frac{p}{\omega\mu_0\mu_m}\hat{\mathbb{Z}}, \quad \hat{\mathbf{Z}}^{\rm (TM)} = -\frac{p}{\omega\varepsilon_0|\varepsilon_m|}\hat{\mathbb{Z}}, \tag{41}$$

$$\hat{\mathbb{Z}} = \begin{pmatrix} \coth(ph) & -\mathrm{csch}(ph) \\ -\mathrm{csch}(ph) & \coth(ph) \end{pmatrix}, \tag{42}$$

which separately relate TE fields and TM fields in the film at $z = h$ and $z = 0$, see Ref. [60] for more details. Namely,

$$\begin{pmatrix} \mathbf{V}^{(h)} \\ -\mathbf{V}^{(0)} \end{pmatrix} = -i\hat{\mathbf{Z}}_{\rm mf}\begin{pmatrix} \mathbf{U}^{(h)} \\ \mathbf{U}^{(0)} \end{pmatrix}, \tag{43}$$

where the indices h and 0 label fields at $z = h$ and $z = 0$, and the vectors $\mathbf{U}^{(h),(0)}$ and $\mathbf{V}^{(h),(0)}$ incorporate $E^{(h),(0)}_{x,y}$ and $H^{(h),(0)}_{x,y}$ in accordance with (2). The subscript "mf" means "metal film."

Due to Eq. (11),

$$\mathbf{V}^{(h)} = i\hat{\mathbf{Z}}_1\mathbf{U}^{(h)}, \quad \mathbf{V}^{(0)} = -i\hat{\mathbf{Z}}'_2\mathbf{U}^{(0)}, \tag{44}$$

where $\hat{\mathbf{Z}}_J$ and $\hat{\mathbf{Z}}'_J$, $J = 1, 2$, are, respectively, the impedances of media 1 and 2, so that the boundary conditions at $z = 0$ and $z = h$ will be obeyed provided that

$$(\hat{\mathbf{Z}}_{\rm d} + \hat{\mathbf{Z}}_{\rm mf})\begin{pmatrix} \mathbf{U}^{(h)} \\ \mathbf{U}^{(0)} \end{pmatrix} = \mathbf{0}, \tag{45}$$

where $\hat{\mathbf{Z}}_{\rm d}$ is a 4×4 block matrix,

$$\hat{\mathbf{Z}}_{\rm d} = \begin{pmatrix} \hat{\mathbf{Z}}_1 & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{Z}}'_2 \end{pmatrix}, \tag{46}$$

the index d comes from "dielectric."

By analogy, as applied to backward-propagating waves, we obtain the relation

$$(\hat{\mathbf{Z}}^{(-)}_{\rm d} + \hat{\mathbf{Z}}^{(-)}_{\rm mf})\begin{pmatrix} \mathbf{U}^{(-)(h)} \\ \mathbf{U}^{(-)(0)} \end{pmatrix} = \mathbf{0}, \tag{47}$$

where

$$\hat{\mathbf{Z}}^{(-)}_{\rm d} = \begin{pmatrix} \hat{\mathbf{Z}}^{(-)}_1 & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{Z}}^{(-)\prime}_2 \end{pmatrix}, \tag{48}$$

and the matrix $\hat{\mathbf{Z}}^{(-)}_{\rm mf}$ is the counterpart of $\hat{\mathbf{Z}}_{\rm mf}$ (40).

Owing to (27), (28), and (40), we have

$$\hat{\mathbf{Z}}^{(-)}_{\rm d} = \hat{\mathbf{Q}}\hat{\mathbf{Z}}'_{\rm d}\hat{\mathbf{Q}}, \quad \hat{\mathbf{Z}}^{(-)}_{\rm mf} = \hat{\mathbf{Z}}_{\rm mf} = \hat{\mathbf{Q}}\hat{\mathbf{Z}}_{\rm mf}\hat{\mathbf{Q}}, \tag{49}$$

where

$$\hat{\mathbf{Z}}'_{\rm d} = \begin{pmatrix} \hat{\mathbf{Z}}'_1 & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{Z}}_2 \end{pmatrix}, \quad \hat{\mathbf{Q}} = \begin{pmatrix} \hat{\mathbf{P}} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{P}} \end{pmatrix}. \tag{50}$$

Hence the dispersion equations for forward- and backward-propagating SEWs can be written in the form

$$\det(\hat{\mathbf{Z}}_{\rm d} + \hat{\mathbf{Z}}_{\rm mf}) \equiv \det \hat{\mathbf{Z}}_{\rm st} = 0, \tag{51}$$

$$\det(\hat{\mathbf{Z}}'_{\rm d} + \hat{\mathbf{Z}}_{\rm mf}) \equiv \det \hat{\mathbf{Z}}'_{\rm st} = 0, \tag{52}$$

respectively, where the index "st" means "structure." (Note that the idea of representing the dispersion equation of waves guided by a film or plate in terms of a Hermitian matrix relating the fields at the opposite faces has been put forward in Ref. [77] as applied to acoustic waves.)

The number of roots of equation (51) has been established [60] where we investigated the one direction propagation of SEWs in a bianisotropic bicrystal containing a metal insertion. According to Ref. [60], equation (51) can have at most four roots. By analogy, equation (52) has the same maximum number of roots. These conclusions follow from the fact that the eigenvalues λ_α and λ'_α, $\alpha = 1, \dots, 4$, of the matrices $\hat{\mathbf{Z}}_{\rm st}$ and $\hat{\mathbf{Z}}'_{\rm st}$, respectively, decrease monotonically with increasing frequency. Such a frequency dependence is a consequence of property (18) of the impedances of half-infinite media and the fact that

$$\frac{\partial \hat{\mathbf{Z}}_{\rm mf}}{\partial \omega} \text{ is a negative definite matrix.} \tag{53}$$

The latter can be verified by proceeding similarly to the derivation of (18) but integrating inequality (13) of Ref. [59] over the film thickness. Statement (53) can also be proved by explicitly computing the derivatives of $\hat{\mathbf{Z}}^{(\rm TE)}$ and $\hat{\mathbf{Z}}^{(\rm TM)}$ (41) and taking into account inequalities (33).

In this paper we are interest in the permissible total number of roots of equations (51) and (52). To find this number, we add up $\hat{\mathbf{Z}}_{\rm st}$ and $\hat{\mathbf{Z}}'_{\rm st}$ and replace $\hat{\mathbf{Z}}_{\rm st}$ and $\hat{\mathbf{Z}}'_{\rm st}$ by their spectral decompositions,

$$\hat{G}_{\rm st} = \hat{\mathbf{Z}}_{\rm st} + \hat{\mathbf{Z}}'_{\rm st} = \sum_{\alpha=1}^{4} (\lambda_\alpha \mathbf{e}_\alpha \otimes \mathbf{e}^*_\alpha + \lambda'_\alpha \mathbf{e}'_\alpha \otimes \mathbf{e}'^*_\alpha), \tag{54}$$

where $\mathbf{e}_\alpha$ and $\mathbf{e}'_\alpha$, $\alpha = 1, \dots, 4$, are the orthonormalized eigenvectors of $\hat{\mathbf{Z}}_{\rm st}$ and $\hat{\mathbf{Z}}'_{\rm st}$. In view of (16) $G_{st,11} = Z_{1,11} + Z'_{1,11} > 0$ and $G_{st,33} = Z_{2,11} + Z'_{2,11} > 0$, where $Z_{J,11}$ and $Z'_{J,11}$ are the elements 11 of $\hat{\mathbf{Z}}_J$ and $\hat{\mathbf{Z}}'_J$, $J = 1, 2$, respectively, and $G_{st,jj}$, $j = 1, 3$, is the diagonal elements jj of $\hat{G}_{\rm st}$.

It can also be verified that $\hat{\mathbf{Z}}^{(\rm TE)}$ (41) is a positive-definite matrix. Therefore, by contracting $\hat{G}_{\rm st}$ with the vector $\mathbf{t} = (t_1\ 0\ t_3\ 0)^t$ orthogonal to $\mathbf{e}_1$, i.e., $\mathbf{e}^\dagger_1 \mathbf{t} = e^*_{1,1} t_1 + e^*_{1,3} t_3 = 0$, we obtain that

$$G_{st,11}|t_1|^2 + G_{st,33}|t_3|^2 + \mathbf{t}'^\dagger \hat{\mathbf{Z}}^{(\rm TE)} \mathbf{t}' = \sum_{\alpha=2}^{4} \lambda_\alpha |\mathbf{e}^\dagger_\alpha \mathbf{t}|^2 + \sum_{\alpha=1}^{4} \lambda'_\alpha |\mathbf{e}^\dagger_\alpha \mathbf{t}|^2 > 0, \tag{55}$$

where $\mathbf{t}' = (t_1\ \ t_3)^t$. From (55) it follows that, of the seven eigenvalues remaining in (55), at least one must be positive in the whole range $\omega < \Omega_{LM}$. Hence assuming $\lambda_1 > 0$ we see that at most six of eight λ_α and λ'_α, $\alpha = 1, \dots, 4$, can turn negative, so, in view of the monotonic decrease of λ_α and λ'_α, equations (51) and (52) can have at most six roots in total at $|k|$ given. Thus,

> given $|k|$, in the structure formed of two different magneto-optical media separated by a metal film, there can be at most six forward- and backward-propagating SEWs in total in the interval $\omega_a < \omega < \Omega_{LM}$, although in each of the mutually opposite directions up to four SEWs are allowed (Case III).

Consider the case where one of the two media (e.g., medium 1) is nonbianisotropic and magneto-optically inactive. One should distinguish between two options: (1) Medium 1 is optically anisotropic and oriented arbitrarily; (2) medium 1 is isotropic or anisotropic but oriented in such a way that the plane XZ is either the plane of symmetry or perpendicular to a symmetry axis. We call a medium of the first type an anisotropic "ordinary" dielectric. In a medium of the second type the electromagnetic modes propagating in the plane XZ are split into TE polarized modes and TM polarized modes. We conventionally call such a medium an isotropic "ordinary" dielectric since it appears that the anisotropy is unimportant in this case. It is only important that in the plane XZ partial modes are either TE or TM polarized.

The impedances of an anisotropic "ordinary" dielectric are nondiagonal matrices. The difference from the impedances of magneto-optical media is that the nondashed and dashed impedances prove to be directly related, viz. $\hat{\mathbf{Z}}'_1 = \hat{\mathbf{Z}}^t_1$ (see Appendix C). In consequence, as it has been shown in Ref. [60], equation (51), and hence equation (52) as well, can have not more than three roots because, owing to (14)–(18) and (C4), at least one of the eigenvalues λ_α, $\alpha = 1, \dots, 4$, and one of the eigenvalues λ'_α, $\alpha = 1, \dots, 4$ must stay positive. The analysis of the behavior of the other eigenvalues reveals that all of them can vanish. Therefore,

> given $|k|$, in the magneto-optical medium–metal film–anisotropic "ordinary" dielectric structure, up to three SEWs can emerge in each of the mutually opposite directions in the interval $\omega_a < \omega < \Omega_{LM}$ and the total number of forward- and backward-propagating SEWs cannot be greater than six (Case IV).

The impedances of an isotropic "ordinary" dielectric are diagonal matrices,

$$\hat{\mathbf{Z}}_1 = \hat{\mathbf{Z}}'_1 = \begin{pmatrix} Z_{\rm TE} & 0 \\ 0 & Z_{\rm TM} \end{pmatrix} \tag{56}$$

in the interval $\omega < \omega_{L1} = \min(\omega_E, \omega_M)$, where $\omega_E = v_E k$ and $\omega_M = v_M k$ are the limiting frequencies of TE and TM bulk waves, respectively,

$$Z_{\rm TE} = \frac{p_E}{\omega \mu_0 \mu}, \quad Z_{\rm TM} = \frac{p_M}{\omega \varepsilon_0 \sqrt{\varepsilon_{11}\varepsilon_{33}}}, \tag{57}$$

$$p_E = \sqrt{k^2 - \omega^2/v_E^2}, \quad p_M = \sqrt{k^2 - \omega^2/v_M^2}, \tag{58}$$

$$v_E = \frac{c}{\sqrt{\varepsilon_{22}\mu}}, \quad v_M = \frac{c}{\sqrt{(\varepsilon_{11}\sin^2\theta + \varepsilon_{33}\cos^2\theta)\mu}}, \tag{59}$$

the magnetic permeability $\mu > 0$ is supposed isotropic, $\varepsilon_{ii} > 0$, $i = 1, 2, 3$, are the components of the tensor $\hat{\boldsymbol{\varepsilon}}$ in the crystal physics coordinate system $X_oY_oZ_o$. The current coordinate system XYZ is rotated relative to $X_oY_oZ_o$ by an angle θ around the axis $Y_o||Y$ (i.e., the planes XZ and X_oZ_o coincide). Note that p_E and p_M (58) are not normal wave numbers of TE and TM modes unless $\theta = 0$, or $\pi/2$, or $\varepsilon_{11} = \varepsilon_{33}$.

Equations (51) and (52) can still individually have up to three roots when medium 1 is an isotropic "ordinary" dielectric. However, these equations cannot have more than five roots in total. To prove it we will use the fact that $\hat{\mathbf{Z}}_1$ is a diagonal matrix and inequalities

$$Z_{\rm TE} > 0, \quad Z_{\rm TM} > 0, \quad \frac{\partial Z_{\rm TE}}{\partial \omega} < 0, \quad \frac{\partial Z_{\rm TM}}{\partial \omega} < 0. \tag{60}$$

We suppose that each of the equations has three roots, so that there are six roots in total. Accordingly, in the vicinity of Ω_{LM} three eigenvalues of $\hat{\mathbf{Z}}_{\rm st}$ and three eigenvalues of $\hat{\mathbf{Z}}'_{\rm st}$ are negative, e.g., $\lambda_\alpha < 0$ and $\lambda'_\alpha < 0, \alpha = 1, 2, 3$, whereas λ_4 and λ'_4 stay positive up to Ω_{LM}. By contracting the matrices $\hat{\mathbf{Z}}_{\rm st}$ and $\hat{\mathbf{Z}}'_{\rm st}$, respectively, with the vectors $\mathbf{t} = (t_1\ 0\ t_3\ 0)^t$ and $\mathbf{s} = (s_1\ 0\ s_3\ 0)^t$ such that $\mathbf{e}_4^\dagger \mathbf{t} = 0$ and $\mathbf{e}_4'^\dagger \mathbf{s} = 0$, we obtain the inequalities

$$\begin{aligned}\mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm st}\mathbf{t} &= Z_{\rm TE}|t_1|^2 + Z'_{2,11}|t_2|^2 + \mathbf{t}'^\dagger \hat{\mathbf{Z}}^{\rm (TE)}\mathbf{t}' \\ &= \sum_{\alpha=1}^{3} \lambda_\alpha |\mathbf{e}_\alpha^\dagger \mathbf{t}|^2 < 0, \\ \mathbf{s}^\dagger \hat{\mathbf{Z}}'_{\rm st}\mathbf{s} &= Z_{\rm TE}|s_1|^2 + Z_{2,11}|s_2|^2 + \mathbf{s}'^\dagger \hat{\mathbf{Z}}^{\rm (TE)}\mathbf{s}' \\ &= \sum_{\alpha=1}^{3} \lambda'_\alpha |\mathbf{e}_\alpha'^\dagger \mathbf{s}|^2 < 0,\end{aligned} \tag{61}$$

where $\mathbf{t}' = (t_1\ t_3)^t$, $\mathbf{s}' = (s_1\ s_3)^t$, $Z_{J,11}$ and $Z'_{J,11}$, $J = 1, 2$, are the 11 elements of the matrices $\hat{\mathbf{Z}}_J$ and $\hat{\mathbf{Z}}'_J$, respectively.

Since $\hat{\mathbf{Z}}^{\rm (TE)}$ (41) is a positive-definite matrix, $\mathbf{t}'^\dagger \hat{\mathbf{Z}}^{\rm (TE)}\mathbf{t}' > 0$ and $\mathbf{s}'^\dagger \hat{\mathbf{Z}}^{\rm (TE)}\mathbf{s}' > 0$. In addition $Z_{\rm TE} > 0$. In summary, from inequalities (61) it follows that $Z_{2,11} < 0$ and $Z'_{2,11} < 0$. However, $Z_{2,11}$ and $Z'_{2,11}$ cannot be both negative because, according to (16), the matrix $\hat{\mathbf{Z}}_2 + \hat{\mathbf{Z}}'_2$ is positive definite and hence its diagonal element $Z_{2,11} + Z'_{2,11}$ is positive. Thus inequalities (61) cannot hold true simultaneously, so our assumption about the existence of six roots of equations (51) and (52) is incorrect because it yields incompatible relations. Therefore,

> given $|k|$, in the magneto-optical medium–metal film–isotropic "ordinary" dielectric structure, up to three SEWs can emerge in each of the mutually opposite directions in the interval $\omega_a < \omega < \Omega_{LM}$, but the total number of forward- and backward-propagating SEWs cannot be greater than five (Case V).

Let a metal film be inserted into a homogeneous magneto-optical medium. In this case $\hat{\mathbf{Z}}_1 = \hat{\mathbf{Z}}_2$ and $\hat{\mathbf{Z}}'_1 = \hat{\mathbf{Z}}'_2$ since materials 1 and 2 are identical. It can be observed that then

$$\hat{\mathbf{Z}}'_{\rm st} = \hat{\mathbf{T}}\hat{\mathbf{Z}}_{\rm st}\hat{\mathbf{T}}, \tag{62}$$

where $\hat{\mathbf{T}}$ is matrix (7). Hence det $\hat{\mathbf{Z}}'_{\rm st}$ = det $\hat{\mathbf{Z}}_{\rm st}$, wherefrom it follows that forward- and backward-propagating SEWs occur pairwise at the same frequency. Owing to (14)–(18) and (53), the determinant of $\hat{\mathbf{Z}}_{\rm st}$ can vanish at most thrice [60]. As a result,

> given $|k|$, forward- and backward-propagating SEWs guided by a metal film embedded in a homogeneous magneto-optical medium emerge pairwise, the frequencies of both SEWs are equal, and the maximum total number of SEWs is six in the interval $\omega_a < \omega < \Omega_{LM}$ (Case VI).

Reference [60] gives examples of the existence of the maximum number of SEWs in one direction (e.g., forward) in the structures where a metal film is between two different magneto-optical media (structure 1), between isotropic "ordinary" dielectric and magneto-optical medium (structure 2), and embedded in a homogeneous magneto-optical medium (structure 3). Namely, four SEWs have been found in structure 1 and three SEWs in structure 2 and structure 3. Computations reveal that two backward-propagating SEWs exist in each of structures 1 and 2, so that the total number of SEWs in these structures reaches its maximum six and five, respectively. An example of six branches of forward- and backward-propagating SEWs in structure 1 is given in Fig. 3 (see Appendix E).

In structure 3 we find three backward-propagating SEWs and frequencies coincide with those of their forward-propagating counterparts (the corresponding figure just coincides with Fig. 2 of Ref. [60]).

D. Dielectric film

Let an isotropic dielectric film of width h with dielectric permittivity $\varepsilon_{\rm df} > 0$ and magnetic permeability $\mu_{\rm df} > 0$ be between magneto-optical media 1 and 2. SEWs guided by dielectric films can be considered through the same relations which were applied to the case of metal films but it is necessary to replace μ_m and $-|\varepsilon_m|$ by $\mu_{\rm df}$ and $\varepsilon_{\rm df}$ in p [see second of Eqs. (34)] and in the factors at the matrices $\hat{\mathbf{Z}}^{\rm (TE)}$ and $\hat{\mathbf{Z}}^{\rm (TM)}$ (41).

We confine ourselves to the interval $\omega < \Omega_{LD} = \min(\omega_{L,1}, \omega_{L,1}, \omega_{\rm df})$, where the limiting frequency $\omega_{\rm df}$ of bulk waves in the film is determined through the condition $\omega_{\rm df} = v_{\rm df}k$ and $v_{\rm df} = c/\sqrt{\mu_{\rm df}\varepsilon_{\rm df}}$. The point is that the interval $\omega < \Omega_{LD}$ can be viewed as a counterpart of the interval $\omega < \Omega_{LM}$ where SEWs guided by the metal film are analyzed since at $\omega < \Omega_{LD}$, like in the metal film at $\omega < \Omega_{LM}$, there are no bulk waves and hence the appearance of waveguide solutions is excluded. Therefore one can find out how the sign of the dielectric permittivity affects the existence of SEWs.

The dispersion equations for forward- and backward-propagating SEWs are similar to equations (51) and (52),

$$\det(\hat{\mathbf{Z}}_{\rm d} + \hat{\mathbf{Z}}_{\rm df}) \equiv \det \hat{\mathbf{Z}}_{\rm sd} = 0, \tag{63}$$

$$\det(\hat{\mathbf{Z}}'_{\rm d} + \hat{\mathbf{Z}}_{\rm df}) \equiv \det \hat{\mathbf{Z}}'_{\rm sd} = 0, \tag{64}$$

respectively, where the impedances $\hat{\mathbf{Z}}_{\rm d}$ and $\hat{\mathbf{Z}}'_{\rm d}$ are the same as in (51) and (52), the matrix $\hat{\mathbf{Z}}_{\rm df}$ is arranged exactly like $\hat{\mathbf{Z}}_{\rm mf}$ (40), the subscript "sd" means "structure dielectric." The difference is that both $\hat{\mathbf{Z}}^{TE}$ and $\hat{\mathbf{Z}}^{TM}$ forming $\hat{\mathbf{Z}}_{\rm df}$ prove to be positive-definite matrices because $\varepsilon_{\rm df} > 0$ and $\mu_{\rm df} > 0$. In consequence,

$$\hat{\mathbf{Z}}_{\rm df} \text{ is a positive definite matrix at } \omega < \Omega_{LD}, \tag{65}$$

unlike $\hat{\mathbf{Z}}_{\rm mf}$ which is sign indefinite. At the same time the matrix $\partial\hat{\mathbf{Z}}_{\rm df}/\partial\omega$ is negative definite, as is $\partial\hat{\mathbf{Z}}_{\rm mf}/\partial\omega$.

By virtue of the properties of $\hat{\mathbf{Z}}_{\rm d}$, $\hat{\mathbf{Z}}'_{\rm d}$, and $\hat{\mathbf{Z}}_{\rm df}$ each of the eigenvalues λ_α and λ'_α, $\alpha = 1, \ldots, 4$, of the matrices $\hat{\mathbf{Z}}_{\rm sd}$ and $\hat{\mathbf{Z}}'_{\rm sd}$ can vanish once, so each of equations (63) and (64) considered individually, like equations (51) and (52), can individually have up to four roots for $|k|$ given.

Let us find the permissible total number of roots of (63) and (64). In view of (16) and (65) the matrix $\hat{\mathbf{G}}_{\rm sd} = \hat{\mathbf{Z}}_{\rm sd} + \hat{\mathbf{Z}}'_{\rm sd}$ is positive definite unlike its counterpart $\hat{\mathbf{G}}_{\rm st}$ (54), so the contraction of $\hat{\mathbf{G}}_{\rm sd}$ with an arbitrary vector is positive. Assume $\lambda_\alpha > 0$, $\alpha = 1, 2, 3$, and contract $\hat{\mathbf{G}}_{\rm sd}$ with the vector $\mathbf{t}$ of which the components are $t_i = \delta_{ijkl} e^*_{1,j} e^*_{2,k} e^*_{3,l}$, $i, j, k, l = 1, \ldots, 4$, δ_{ijkl} is the antisymmetric unit tensor, $e_{\alpha,j}$, $\alpha = 1, 2, 3$, are components of the orthonormalized eigenvectors $\mathbf{e}_\alpha$ of $\hat{\mathbf{Z}}_{\rm sd}$, i.e., $\mathbf{t}$ is such that $\mathbf{e}^\dagger_\alpha \mathbf{t} = 0$, $\alpha = 1, 2, 3$, and $|\mathbf{e}^\dagger_4 \mathbf{t}| = 1$. We obtain

$$\mathbf{t}^\dagger \hat{\mathbf{G}}_{\rm sd} \mathbf{t} = \lambda_4 + \sum_{\alpha=1}^{4} \lambda'_\alpha |\mathbf{e}'^\dagger_\alpha \mathbf{t}|^2 > 0 \tag{66}$$

[cf. Eq. (55)], so the option where more than four eigenvalues of $\hat{\mathbf{Z}}_{\rm sd}$ and $\hat{\mathbf{Z}}'_{\rm sd}$ would be negative is impossible because any three eigenvalues can be excluded from $\mathbf{t}^\dagger \hat{\mathbf{G}}_{\rm sd} \mathbf{t}$ by the relevant choice of $\mathbf{t}$. Hence, (63) and (64) cannot have more than four roots in total, and we conclude that

> given $|k|$, in the structure formed of two different magneto-optical media separated by a dielectric film, there can be at most four forward- and backward-propagating SEWs in total in the interval $\omega < \Omega_{LD}$, although in each of the two mutually opposite directions up to four SEWs can exist (Case VII).

Appendix E gives an example of the existence of four SEWs for one direction (Fig. 4). Correspondingly SEWs do not exist in the opposite direction.

When one of the half-infinite media is an anisotropic "ordinary" dielectric, the maximum total number of roots of equations (63) and (64) is still four but each of these equations can have only three roots, like in the case of the metal film, i.e.,

> given $|k|$, in the magneto-optical medium–dielectric film–anisotropic "ordinary" dielectric structure, up to three SEWs can emerge in each of the mutually opposite directions in the interval $\omega < \Omega_{LD}$, but the total number of forward- and backward-propagating SEWs cannot be greater than four (Case VIII).

The maximum total number of SEWs changes when medium 1 is isotropic "ordinary" dielectric. It can be proved that in this instance equations (63) and (64) can have at most two roots in total, although each of them, if taken individually, can have two roots (see Appendix F). In summary,

> given $|k|$, in the magneto-optical medium–dielectric film–isotropic "ordinary" dielectric structure, up to two SEWs can emerge in each of the mutually opposite directions in the interval $\omega < \Omega_{LD}$, but the total number of forward- and backward-propagating SEWs cannot be greater than two (Case IX).

An example of the existence of two SEWs in one direction is given in Appendix E (Fig. 5). Accordingly, none SEW emerges in the opposite direction.

Let us bisect a homogeneous magneto-optical medium and insert a dielectric film between the halves. In this case the matrices $\hat{\mathbf{Z}}_{\rm sd}$ and $\hat{\mathbf{Z}}'_{\rm sd}$ satisfy relation (62), so equations (63) and (64) have the same roots. Since the maximum total number of their roots remains four, we conclude that

> given $|k|$, forward- and backward-propagating SEWs guided by a dielectric film embedded in a homogeneous magneto-optical medium emerge pairwise, the frequencies of both SEWs are equal, and the maximum total number of SEWs is four in the interval $\omega < \Omega_{LD}$ (Case X).

There can exist up to two forward-propagating SEWs and two backward-propagating SEWs and they emerge as if the medium were reciprocal. An example of the structure supporting two pairs of SEWs is given in Appendix E (Example 4).

The above statements about the number of SEWs also hold true when the film is an optically anisotropic dielectric oriented in such a way that the plane XZ supports TE and TM modes. The explicit expressions of the matrices $\hat{\mathbf{Z}}^{\rm (TE)}$ and $\hat{\mathbf{Z}}^{\rm (TM)}$ forming $\hat{\mathbf{Z}}_{\rm df}$ change compared with the isotropic version, but these changes do not affect those their properties, which are used in the analysis of the roots of dispersion equations.

TABLE I. Maximum number of SEWs in different magneto-optical structures (columns I–X correspond to Cases I–X considered in Sec. III). The maximum number of SEWs propagating in each of two mutually opposite directions (forward and backward) is given in the row "f/b SEWs." The maximum total number of forward- and backward-propagating SEWs is given in the row "f+b SEWs."

	Maximum number of SEWs									
	I	II	III	IV	V	VI	VII	VIII	IX	X
f/b SEWs	2	2	4	3	3	3	4	3	2	2
f+b SEWs	2	3	6	6	5	6	4	4	2	4

IV. CONCLUDING REMARKS

We have established the maximum total number $M_{\rm tot}$ of forward- and backward-propagating SEWs in structures containing magneto-optical materials. In other words, given the value $|k|$ of the tangential wave number, the number of SEWs propagating in one direction plus the number of SEWs propagating in the opposite direction cannot exceed $M_{\rm tot}$. The number $M_{\rm tot}$ can be viewed as a characteristics of the structure because it is dependent on the type of the structure but not on specific values of material constants. For the same reason the permissible maximum number M of SEWs propagating in each of mutually opposite directions is also a characteristics of the structure (M is the same for the forward and backward directions).

It would be natural if $M_{\rm tot} = 2M$ was always fulfilled. However, the nonreciprocity of magneto-optical materials results in a specific correlation between the permissible total number of SEWs propagating in the mutually opposite directions. Namely, it has been found that, at a given a value $|k|$ of the tangential wave number, in most cases $M_{\rm tot}$ is less than the sum of the permissible maxima of SEWs which can propagate in each of the two directions, i.e., the typical option is that $M_{\rm tot} < 2M$.

Table I visually demonstrates the above peculiarity of $M_{\rm tot}$. In most cases the number in the row "f+b SEWs" is less than twice the number above it in the row "f/b SEWs." As follows from our analysis, this manifestation of the nonreciprocity is of a general nature in the sense that it arises in magneto-optical media of arbitrary crystallographic symmetry, regardless of the specific values and frequency dependence of material constants.

The maximum total number depends on where SEWs propagate. For example, it has been shown that the maximum number of SEWs propagating in each of the two opposite directions in a magneto-optical medium–metal film–magneto-optical medium structure is four if the magneto-optical media are different but the maximum total number of forward- and backward-propagating SEWs is six rather than eight (Case III). The replacement of the metal film by dielectric one does not change the maximum number of SEWs propagating in one direction at frequencies less than the limiting frequency of bulk modes in the film. There can still exist up to four SEWs in each of the opposite directions but now the maximum total number of SEWs propagating forward and backward decreases to four (Case VII). This difference is a consequence of different signs of the dielectric permittivity of films which yields a difference in the properties of their impedances.

It has been proved that SEWs supported by a film embedded in a homogeneous magneto-optical medium emerge in forward- and backward-propagating pairs and the frequencies of the waves in a pair are equal (Cases VI and X). Thus the nonreciprocity does not show up and SEWs occur in the same way as in reciprocal materials.

The total number of forward- and backward-propagating SEWs and the number of SEWs in one direction in a particular structure can be less than their maximal numbers. The permissible maximum number of SEWs emerges provided that the magneto-optical effect is strong enough for the number of SEWs to overcome the limit imposed by the real dielectric permittivity and magnetic permeability of a magneto-optically inactive dielectric. For instance, at most two SEWs, one in each of the opposite direction, can exist on the interface between such a dielectric and metal whereas the magneto-optical dielectric–metal interface supports up to three SEWs (Case II), so that the magneto-optical effect has to give rise to an additional wave. Computations for materials with specially selected constants which were used in our previous papers [59,60] as well as examples given in Appendix E confirm that the total number of SEWs can reach the permissible maxima. In particular, if $M_{\text{tot}} = M$, then M_{tot} SEWs can exist in one direction and there is no SEW propagating in the opposite direction (see Examples 2 and 3 in Appendix E).

We assumed that materials do not absorb electromagnetic waves. Taking into account absorption makes the impedances non-Hermitian, so a rigorous general analysis of the existence problem for SEWs requires a different approach.

At the same time some conclusions about the influence of absorption on the existence of SEWs can be drawn assuming that absorption is weak. First of all we note that disregarding absorption is a typical approximation in dielectrics where absorption is really extremely small at frequencies below certain critical threshold. The effect of absorption is more significant in metals where losses are not extremely small. However, if absorption is fairly weak, then it cannot increase the maximum number of SEWs (see Appendix G). Apart from this fact, through perturbation theory one can show that the imaginary correction to the SEW frequency is of the first order of smallness whereas the real correction is of second order, so the shift of the SEW dispersion curve will be smaller than its width.

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APPENDIX A

Let us insert an electromagnetic field $\{\mathbf{E}(z), \mathbf{H}(z)\}e^{i(kx-\omega t)}$ in the Maxwell equations. The result can be represented in the form

$$\frac{1}{i}\frac{d\boldsymbol{\xi}}{dz} = \hat{\mathbf{T}}(\omega\boldsymbol{\psi} + k\hat{\mathbf{J}}\boldsymbol{\phi}), \tag{A1}$$

$$-k\hat{\mathbf{J}}^t\boldsymbol{\xi} = \omega\boldsymbol{\nu}, \tag{A2}$$

where the vector $\boldsymbol{\xi}$ and the matrix $\hat{\mathbf{T}}$ are defined by Eqs. (2) and (7), respectively, $\boldsymbol{\psi} = (-D_y\ B_y\ B_x\ D_x)^t$,

$$\boldsymbol{\phi} = \begin{pmatrix} \mathbb{H}_z \\ \mathbb{E}_z \end{pmatrix}, \quad \boldsymbol{\nu} = \begin{pmatrix} \mathbb{B}_z \\ \mathbb{D}_z \end{pmatrix}, \quad \hat{\mathbf{J}} = \begin{pmatrix} \hat{\mathbf{I}} \\ \hat{\mathbf{0}} \end{pmatrix}. \tag{A3}$$

By expressing $\boldsymbol{\psi}$, $\boldsymbol{\phi}$, and $\boldsymbol{\nu}$ in terms of $\boldsymbol{\xi}$ with the help of (A2) and (3) we reduce (A1) to $d\boldsymbol{\xi}/dz = i\hat{\mathbf{N}}\boldsymbol{\xi}$, where $\hat{\mathbf{N}}$ is matrix (7), see the Appendix of Ref. [59] for more details. Substituting (1) in these equations yields (4).

Expressions of the matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ entering (6) are given in the Appendix of Ref. [59]. These matrices and hence the matrix $\hat{\mathbf{T}}\hat{\mathbf{N}}$ are Hermitian for any real frequencies. In contrast, the impedances $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ defined by relations (11) are Hermitian only in the interval $\omega < \omega_L$ [property (14)]. Indeed, for $\omega < \omega_L$ all four p_α are complex, so, due to the law of energy conservation, the time-averaged normal component $\mathbb{P}_z = 0.5\text{Re}([\mathbf{E}^* \times \mathbf{H}]_z)$ of the energy flux of modes (1) vanishes necessarily for purely real k and ω. In particular, by virtue of (2) and (10), $\mathbb{P}_z = 0.5(\mathbf{U}^\dagger\mathbf{V} + \mathbf{V}^\dagger\mathbf{U}) = 0$ for a wave field $\tilde{\boldsymbol{\xi}}_+ = (\mathbf{U}\ \mathbf{V})^t$ at $z = 0$. By expressing $\mathbf{V}$ in terms of $\mathbf{U}$ we find that $\mathbf{U}^\dagger(\hat{\mathbf{Z}} - \hat{\mathbf{Z}}^\dagger)\mathbf{U} = 0$, so $\hat{\mathbf{Z}} = \hat{\mathbf{Z}}^\dagger$.

APPENDIX B

According to Sec. III, dispersion equations for SEWs can be cast into the form of the equalities to zero of the determinants of $n \times n$ matrices. Let us denote here these matrices by the symbols $\hat{\mathbf{Z}}_F$ and $\hat{\mathbf{Z}}_B$ regardless of structures. The matrices $\hat{\mathbf{Z}}_F$ and $\hat{\mathbf{Z}}_B$ enter the dispersion equations for forward- and backward-propagating SEWs, respectively, and are constructed from the impedances of the elements of the structure. The value of n depends on the number of the boundary conditions which, in turn, is directly proportional to the number of interfaces in the structure under consideration (see Sec. III).

The analysis of roots of the equations $\det\hat{\mathbf{Z}}_F = 0$ and $\det\hat{\mathbf{Z}}_B = 0$ is equivalent to the analysis of the vanishing of the eigenvalues λ_i and λ_{i+n}, $i = 1, \ldots, n$ of $\hat{\mathbf{Z}}_F$ and $\hat{\mathbf{Z}}_B$,

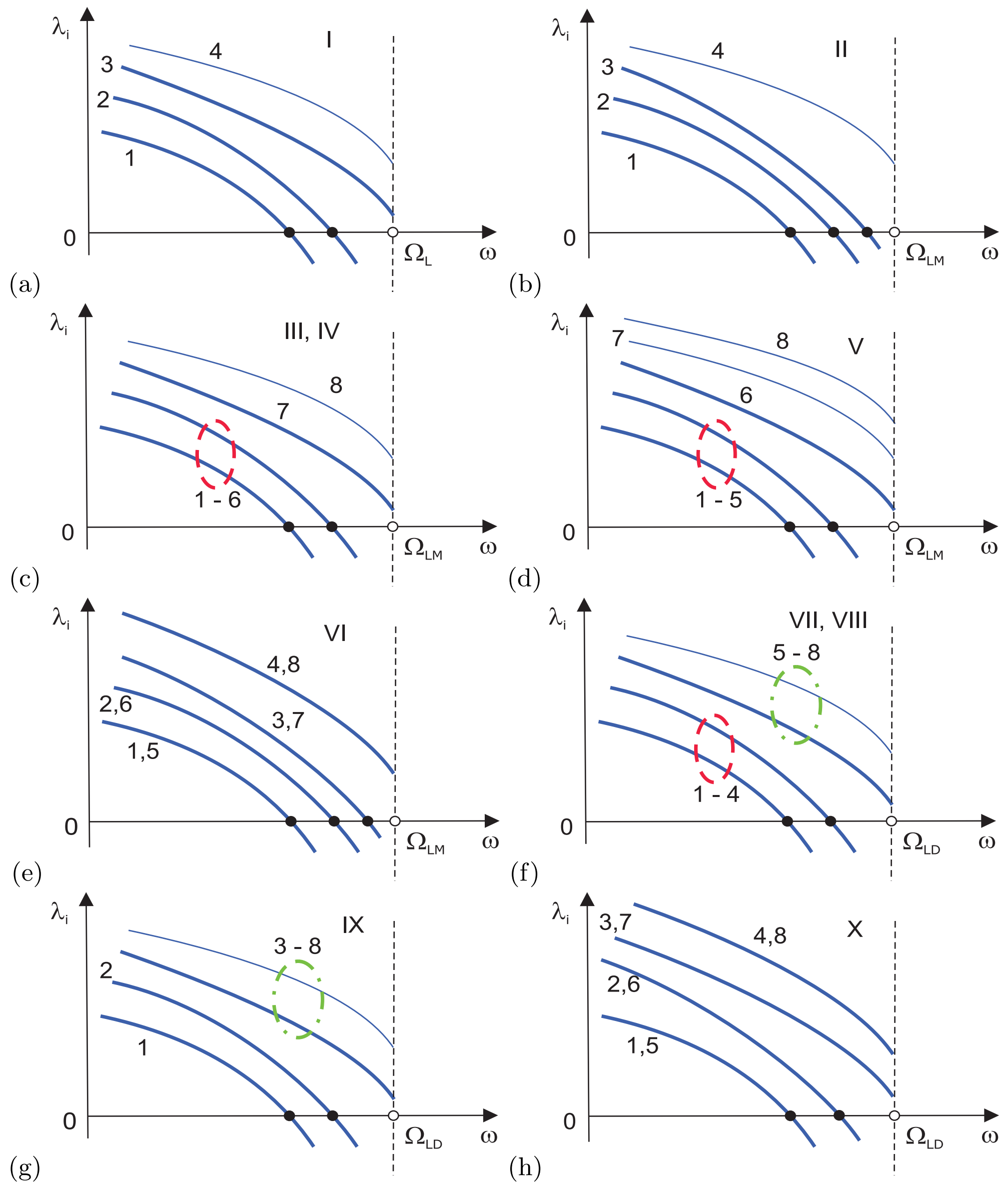

FIG. 2. Schematic frequency dependence of the eigenvalues $\lambda_i(\omega)$, $i = 1, \ldots, 2n$ of the impedances corresponding to forward ($i = 1, \ldots, n$) and backward ($i = n+1, \ldots, 2n$) propagating SEWs, where n is the number of eigenvalues of the impedance corresponding to SEWs propagating in one direction (forward or backward). The eigenvalues and frequencies are assumed normalized, so that they are dimensionless. Structures are marked by Roman numerals I–X following Sec. III and Table I. A dashed line ellipse with index of the type "$1 - m$" embraces m curves $\lambda_i(\omega)$, $i = 1, \ldots, m$ passing through zero. A dash-dotted line ellipse with index of the type "$q - 2n$" embraces $2n + 1 - q$ curves $\lambda_i(\omega)$, $i = q, \ldots, 2n$ not passing through zero. A label of the type "$m, m+n$" means that the eigenvalues $\lambda_m(\omega)$ and $\lambda_{m+n}(\omega)$ associated with forward- and backward-propagating SEWs, respectively, coincide.

respectively. The properties of λ_i, $i = 1, \ldots, 2n$ stem from those of the impedances discussed in Secs. II and III. Namely, the eigenvalues are real and decrease monotonically with increasing frequency in the interval confined from above by the limiting frequency either Ω_L or Ω_{LM} or Ω_{LD} (see Sec. III). All $\lambda_i > 0$ in dielectrics as $\omega \to 0$. Some of the eigenvalues can be negative in metals at the lower limit ω_a of the frequency interval (see Sec. III B) but we assume all them positive at $\omega = \omega_a$ since this condition favors the existence of the maximum of roots.

Figures 2(a)–2(h) depict schematically the behavior of $\lambda_i(\omega)$ in the case where the total number of forward- and backward-propagating SEWs is maximal. In Cases I and II [Figs. 1(a) and 1(b)] $n = 2$ and the total number of eigenvalues is four. In the other Cases $n = 4$, so the total number of eigenvalues is eight. Since the sequence of vanishing λ_i, $i = 1, \ldots, 2n$ is arbitrary, for definiteness we assume that the eigenvalues of $\hat{\mathbf{Z}}_F$ ($i = 1, \ldots, n$) pass through zero first. The eigenvalues of $\hat{\mathbf{Z}}_B$ ($i = n+1, \ldots, 2n$) vanish when the permissible total maximum number of SEWs $M_{\mathrm{tot}} \leqslant 2n$ is greater than the permissible maximum of SEWs $M \leqslant n$ in one direction [Figs. 2(b)–2(e) and 1(h)].

The maximum number of vanishing eigenvalues is the same in Cases III and IV (see Sec. III C), so the frequency dependence of eigenvalues for these two cases are shown in the same figure [Fig. 2(c)]. For analogous reason, Fig. 2(f) corresponds to Cases VII and VIII (see Sec. III D).

APPENDIX C

In bianisotropic materials the Onsager reciprocity relations reduce to $\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{\varepsilon}}^t$, $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}^t$, and $\hat{\boldsymbol{\kappa}} = -\hat{\boldsymbol{\kappa}}^*$, see Eq. (3) and Refs. [5,6]. They are fulfilled provided that the medium features natural optical activity but is not magneto-optically active and do not possess the magnetoelectric effect [5,6,64,65,78]. Having computed explicitly the matrix $\hat{\mathbf{N}}$ and taken into account the fact that $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are purely real and $\hat{\boldsymbol{\kappa}}$ is purely imaginary, we find that

$$\hat{\mathbf{N}}^{(-)} = -\hat{\mathbf{S}}\hat{\mathbf{N}}^*\hat{\mathbf{S}}, \tag{C1}$$

where the matrix $\hat{\mathbf{S}}$ is given by the first of Eqs. (25). Hence,

$$p_\alpha^{(-)} = -p_\alpha^*, \quad \boldsymbol{\xi}_\alpha^{(-)} = \hat{\mathbf{S}}\boldsymbol{\xi}_\alpha^*, \quad \alpha = 1, \dots, 4, \tag{C2}$$

so that, by virtue of (9) and (14),

$$\hat{\mathbf{Z}}^{(-)} = \hat{\mathbf{P}}\hat{\mathbf{Z}}'^t\hat{\mathbf{P}}, \quad \hat{\mathbf{Z}}^{(-)\prime} = \hat{\mathbf{P}}\hat{\mathbf{Z}}^{\prime\prime}\hat{\mathbf{P}}, \tag{C3}$$

where $\hat{\mathbf{P}}$ is given by the second of Eqs. (25) [cf. Eqs. (24) and (26)–(28)].

In view of (C2) and (C3) the reciprocity shows up in the fact that forward- and backward-propagating SEWs have alike frequencies at a given $|k|$. For instance, in bicrystals formed of two materials possessing natural optical activity, the dispersion equations (31) and (32) turn out to be identical because, due to (C3), $\hat{\mathbf{Z}}'_b = \hat{\mathbf{Z}}^t_b$.

If the bicrystal consists of a naturally optically active medium 1 and magneto-optically active medium 2, then, due to (28) and (C3), $\hat{\mathbf{Z}}'_b = \hat{\mathbf{Z}}^t_1 + \hat{\mathbf{Z}}_2$ [cf. $\hat{\mathbf{Z}}'_b$ in Eq. (32)] and our analysis of equations (31) and (32) reveal that they can have in total up to four roots.

In optically anisotropic "ordinary" media ($\hat{\boldsymbol{\kappa}} = 0$) the matrix $\hat{\mathbf{N}}$ is purely real, so that (C1) reduces to $\hat{\mathbf{N}}^{(-)} = -\hat{\mathbf{S}}\hat{\mathbf{N}}\hat{\mathbf{S}}$. This equality looks precisely like (24) but $\hat{\mathbf{N}}$ in (24) is a complex-valued matrix and this fact ultimately leads to the difference in the SEW propagation in magneto-optical materials and "ordinary" materials. Indeed, since $\hat{\mathbf{N}}$ is purely real, in view of (9) we have $\boldsymbol{\xi}_{\alpha+2} = \boldsymbol{\xi}_\alpha^*$, $\alpha = 1, 2$, and therefore

$$\hat{\mathbf{Z}}' = \hat{\mathbf{Z}}^t. \tag{C4}$$

When a medium with natural optical activity is joined to an "ordinary" dielectric a bicrystal is formed in which the Onsager reciprocity relations hold true. Equalities (C3) and (C4) allow one to verify that in this case SEWs indeed propagate identically forward and backward.

APPENDIX D

Let us analyze the roots of Eqs. (31) and (32) at a given value of $|k|$ in the interval $\omega < \Omega_L = \min(\omega_{L,1}, \omega_{L,2})$. First of all, we note that $\hat{\mathbf{Z}}_b$ and $\hat{\mathbf{Z}}'_b$ as well as

$$\hat{\mathbf{G}}_b = \hat{\mathbf{Z}}_b + \hat{\mathbf{Z}}'_b = \hat{\mathbf{G}}_1 + \hat{\mathbf{G}}_2, \tag{D1}$$

where

$$\hat{\mathbf{G}}_J = \hat{\mathbf{Z}}_J + \hat{\mathbf{Z}}'_J, \quad J = 1, 2, \tag{D2}$$

possess properties (14)–(18). Correspondingly, the eigenvalues λ_α, $\alpha = 1, 2$, of $\hat{\mathbf{Z}}_b$ and the eigenvalues λ'_α, $\alpha = 1, 2$, of $\hat{\mathbf{Z}}'_b$ behave in the range $\omega < \Omega_L$ as follows:

$$\mathrm{Im}(\lambda_\alpha) = \mathrm{Im}(\lambda'_\alpha) = 0, \quad \alpha = 1, 2, \tag{D3}$$

$$\lambda_\alpha > 0, \quad \lambda'_\alpha > 0, \quad \alpha = 1, 2 \text{ at } \omega \to 0, \tag{D4}$$

$$\lambda_\alpha \text{ and } \lambda'_\alpha, \quad \alpha = 1, 2, \text{ monotonically decrease with increasing } \omega. \tag{D5}$$

Hence, an eigenvalue can vanish only once, so each of equations (31) and (32) has at most two roots.

Represent $\hat{\mathbf{G}}_b$ (D1) in the form

$$\hat{\mathbf{G}}_b = \sum_{\alpha=1}^{2} (\lambda_\alpha \mathbf{e}_\alpha \otimes \mathbf{e}_\alpha^* + \lambda'_\alpha \mathbf{e}'_\alpha \otimes \mathbf{e}_\alpha^{\prime *}), \tag{D6}$$

where $\mathbf{e}_\alpha$ and $\mathbf{e}_\alpha$, $\alpha = 1, 2$, are the orthonormalized eigenvectors of $\hat{\mathbf{Z}}_b$ and $\hat{\mathbf{Z}}'_b$, respectively, and the symbol $\otimes$ stands for the dyadic product. In view of (16) the matrix $\hat{\mathbf{G}}_b$ is positive definite at $\omega < \Omega_L$. Therefore at least two eigenvalues of four λ_α and λ'_α must stay positive at $\omega < \Omega_L$. Indeed, the positive definiteness of $\hat{\mathbf{G}}_b$ implies that the contraction of $\hat{\mathbf{G}}_b$ with any vector is positive. Assuming $\lambda_2 > 0$ and multiplying $\hat{\mathbf{G}}_b$ from both sides by the eigenvector $\mathbf{e}_1$ of $\hat{\mathbf{Z}}_b$ we obtain that

$$\mathbf{e}_1^\dagger \hat{\mathbf{G}}_b \mathbf{e}_1 = \lambda_1 + \sum_{\alpha=1}^{2} \lambda'_\alpha |\mathbf{e}_1^\dagger \mathbf{e}'_\alpha|^2 > 0. \tag{D7}$$

This inequality cannot hold true unless at least one eigenvalue of λ_1, λ'_1 and λ'_2 is positive.

The fact that at most two of four eigenvalues λ_α and λ'_α can be negative means that equations (31) and (32) can have at most two roots in total. For instance, if (31) has two roots, then (32) has no roots. If (31) is known to have one root, then (32) can have not more than one root.

APPENDIX E

In examples demonstrating the occurrence of the permissible maximum number of forward- and backward-propagating SEWs we can use "model" materials provided that their material constants secure the validity of properties of impedances. These properties are satisfied provided that in dielectrics $\partial(\omega\hat{\boldsymbol{\varepsilon}})/\partial\omega$ and $\partial(\omega\hat{\boldsymbol{\mu}})/\partial\omega$ are positive-definite matrices. The tensors $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are Hermitian in magneto-optical materials and purely real in magneto-optically inactive ones ("ordinary" dielectrics). The dielectric permittivity and magnetic permeability of metals must fulfill the conditions listed in the beginning of Sec. III B.

Below we assume that the geometry of propagation relative to the coordinate system XYZ is as described in Sec. II. The quantities in Figs. 3–5 are normalized to the frequency $\omega_0 = 1.2\pi \times 10^{15}$ Hz, which corresponds to the wavelength $\lambda_0 = 0.5$ μm in the vacuum, and to $k_0 = 2\pi/\lambda_0$.

Example 1. Let a magneto-optical material be characterized in the coordinate system XYZ by the dielectric permittivity with nonzero components $\varepsilon_{11} = \varepsilon_{22} = 5$, $\varepsilon_{33} = 4$, $\varepsilon_{12} = -\varepsilon_{21} = 4.2i$. The magnetic permeability is $\mu_{jj} = 1$, $j = 1, 2, 3$. Samples 1 and 2 of this material are rotated through the

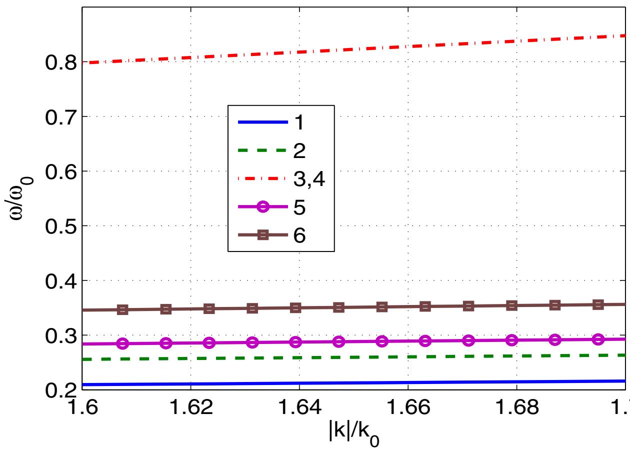

FIG. 3. Four branches of forward-propagating SEWs (lines 1–4) and two branches of backward-propagating SEW (lines 5 and 6) guided by a metal film inserted between two different magneto-optical media. Frequencies of two fast forward SEW (lines 3 and 4) practically merge with each other and also with line $\Omega_{LM}(|k|)/\omega_0$.

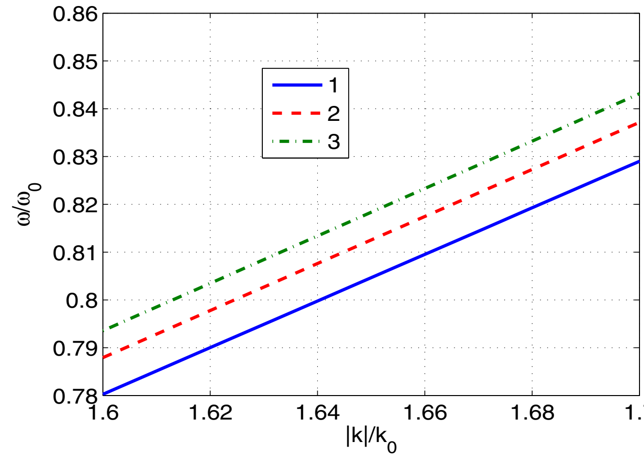

FIG. 5. Two branches of SEWs (lines 1 and 2) guided in one direction by an isotropic dielectric film inserted between in a magneto-optical medium and an isotropic "ordinary" dielectric. There are no SEWs in the opposite direction. Line 3 is $\Omega_{LD}(|k|)/\omega_0$.

angle 20° anticlockwise and clockwise, respectively, around the axis X. As a result, their dielectric permittivities turn out to be different in the XYZ frame. Afterwards we bisect them along the plane perpendicular to the axis Z and put a metallic film between the upper half of medium 1 and the lower part of medium 2. The thickness of the film is 30 nm. The dielectric permittivity of the film is described by the Drude formula $\varepsilon_m = 1 - \omega_p^2/\omega^2$, where $\omega_p = 1.15\omega_0$, and its relative magnetic permeability equals 25.

In this structure we find four forward-propagating SEWs and two backward ones (Case III, see Fig. 3. The frequencies of two fast forward SEWs practically merge, so they are shown as one dot-dashed line (line 3, 4).

Example 2. Let a magneto-optical material be characterized in the coordinate system XYZ by the dielectric permittivity and magnetic permeability with nonzero components $\varepsilon_{11} =$

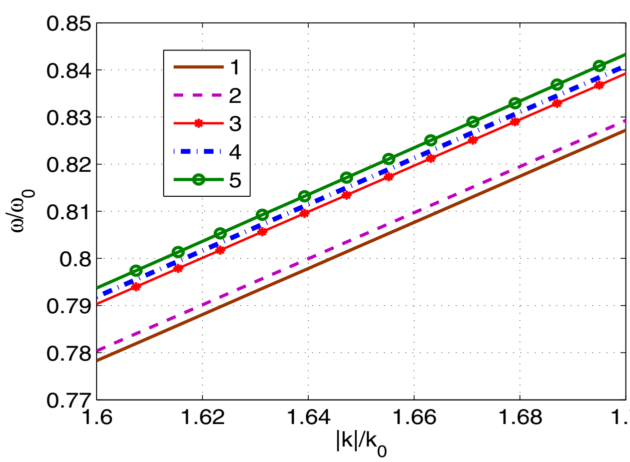

FIG. 4. Four branches of SEWs guided in one direction by an isotropic dielectric film inserted between different magneto-optical media (lines 1–4). Line 5 shows the limiting frequency $\Omega_{LD} = \omega_{L1,L2}$ as a function of k.

$\varepsilon_{22} = 5$, $\varepsilon_{33} = 4$, $\varepsilon_{12} = -\varepsilon_{21} = 4.2i$ and $\mu_{jj} = 1$, $j = 1, 2, 3$, $\mu_{12} = -\mu_{21} = 0.85i$. Two samples 1 and 2 of this material are rotated about the X axis through an angle of 20° anticlockwise and clockwise, respectively, so that they have different material constants with respect to the coordinate system XYZ. Afterwards we bisect each of the samples along a plane perpendicular to the Z axis, take the upper half of sample 1 and the lower half of sample 2 and insert between them the 0.5-μm-thick dielectric film with dielectric permittivity $\varepsilon_{\rm df} = 4.053$. The value of $\varepsilon_{\rm df}$ is chosen such that the limiting frequency $\omega_{\rm df} = 0.79ck$ of bulk waves in the film is slightly greater than the limiting frequency $\omega_{L1} = \omega_{L2} = 0.7895ck$ of bulk waves in the magneto-optical medium for the direction along the axis X.

Our computations reveal that four SEWs exist in the positive direction of the X axis (Fig. 4) whereas, in accordance with a statement proved in Sec. III D (Case VII), no SEWs emerge in the negative direction.

Example 3. Let us replace the lower part of the structure used in Example 2 by the "ordinary" dielectric with dielectric permittivity $\varepsilon_{11} = 5$, $\varepsilon_{22} = 4$, $\varepsilon_{33} = 4$. The dielectric film of thickness 1 μm is of the same material as in Example 2. In this case the maximum total number of SEWs is two, since the dielectric supports TE and TM modes in the plane XZ (Case IX). We find that these two SEWs exist only for the forward direction (Fig. 5).

Example 4. A sample of the magneto-optical material described in Example 2 is rotated around the axis X and bisected along a plane perpendicular to the axis Z. The dielectric film, the same one as in Example 2, is inserted between the two halves, so one has a structure where a dielectric is embedded in a homogeneous magneto-optical material. Computations reveal that in total four SEWs exist, i.e., two forward- and two backward-propagating ones, as it should be in accordance with Sec. III D (Case X). We omit the corresponding figure because it would be very similar to Fig. 5. Each of lines 1 and 2 in Fig. 5 reproduces fairly accurately the dispersion dependence of a pair of SEWs propagating in mutually opposite directions. Line 3 in Fig. 5 reproduces accurately the dependence $\Omega_{LD}(k)/\omega_0$ since in both cases $\Omega_{LD}(k)$ is the limiting frequency in the magneto-optical medium.

APPENDIX F

If medium 1 is an isotropic "ordinary" dielectric, then each of equations (63) and (64) can have at most two roots. Indeed, assuming, e.g., $\lambda_1 > 0$ and contracting $\hat{\mathbf{Z}}_{\rm sd}$ with the vector $\mathbf{t} = (t_1\ t_2\ 0\ 0)^t$ orthogonal to $\mathbf{e}_1$, we obtain the inequality

$$\mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm sd}\mathbf{t} = \sum_{\alpha=2}^{4} \lambda_\alpha |\mathbf{e}_\alpha^\dagger \mathbf{t}|^2 = \mathbf{t}'^\dagger(\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}'_{\rm df})\mathbf{t}' > 0, \tag{F1}$$

where $\mathbf{t}' = (t_1\ t_2)^t$. The contraction $\mathbf{t}^\dagger\hat{\mathbf{Z}}_{\rm sd}\mathbf{t}$ is positive because $\hat{\mathbf{Z}}_1$ (56) is a positive-definite matrix and $\hat{\mathbf{Z}}'_{\rm df}$ is diagonal 2×2 matrix with elements $Z_{11}^{TE} > 0$ and $Z_{11}^{TM} > 0$. From (35) it follows that at most two eigenvalues of $\hat{\mathbf{Z}}_{\rm sd}$ can be negative, so (63) can have at most two roots, which completes the proof.

It can be shown that one of the eigenvalues λ'_α, $\alpha = 1, 2$, of the block $\hat{\mathbf{Z}}'_2$ of $\hat{\mathbf{Z}}_{\rm d}$ (48) is necessary negative in the vicinity of Ω_{LD} if (63) has one root. If (63) has two roots, then both eigenvalues of $\hat{\mathbf{Z}}'_2$ are necessary negative in the vicinity of Ω_{LD}.

The first statement is proved as follows. If (63) has one root ω_1, then, e.g., $\lambda_4 < 0$ at $\omega > \omega_1$. Representing $\hat{\mathbf{Z}}'_2$ in the form $\hat{\mathbf{Z}}'_2 = \sum_1^2 \lambda'_\alpha \mathbf{q}'_\alpha \otimes \mathbf{q}'^*_\alpha$, where $\mathbf{q}'_\alpha$, $\alpha = 1, 2$, are the eigenvectors of $\hat{\mathbf{Z}}'_2$, and contracting $\hat{\mathbf{Z}}_{\rm sd}$ with a vector $\mathbf{t}$ orthogonal to $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ we arrive at the inequality

$$\begin{aligned}\mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm sd}\mathbf{t} &= \mathbf{t}'^\dagger \hat{\mathbf{Z}}_1 \mathbf{t}' + \mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm df}\mathbf{t} + \sum_{\alpha=1}^{2} \lambda'_\alpha |\mathbf{q}'^\dagger_\alpha \mathbf{t}''|^2 \\ &= \lambda_4 |\mathbf{e}_4^\dagger \mathbf{t}|^2 < 0,\end{aligned} \tag{F2}$$

where $\mathbf{t}' = (t_1\ t_2)^t$ and $\mathbf{t}'' = (t_3\ t_4)^t$. The contractions $\mathbf{t}'^\dagger\hat{\mathbf{Z}}_1\mathbf{t}'$ and $\mathbf{t}^\dagger\hat{\mathbf{Z}}_{\rm df}\mathbf{t}$ are positive because $\hat{\mathbf{Z}}_1$ and $\hat{\mathbf{Z}}_{\rm df}$ are positive-definite matrices for $\omega < \Omega_{LD}$. Hence at least one of two λ'_α must be negative when $\omega > \omega_1$ since otherwise the sum in (F2) could not be negative.

Let (63) have two roots ω_1 and $\omega_2 > \omega_1$. In consequence, two of four λ_α and one of two λ'_α are negative for $\omega > \omega_2$, e.g., $\lambda_3, \lambda_4, \lambda'_1 < 0$. The contraction of $\hat{\mathbf{Z}}_{\rm sd}$ with a vector $\mathbf{t}$ orthogonal to $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{q} = (0\ 0\ q'_{1,1}\ q'_{1,2})^t$, where $q'_{1,i}$, $i = 1, 2$, are the components of the eigenvector $\mathbf{q}'_1$, yields

$$\begin{aligned}\mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm sd}\mathbf{t} &= \mathbf{t}'^\dagger \hat{\mathbf{Z}}_1 \mathbf{t}' + \mathbf{t}^\dagger \hat{\mathbf{Z}}_{\rm df}\mathbf{t} + \lambda'_2 |\mathbf{q}'^\dagger_2 \mathbf{t}''|^2 \\ &= \sum_{\alpha=3}^{4} \lambda_\alpha |\mathbf{e}_\alpha^\dagger \mathbf{t}|^2 < 0,\end{aligned} \tag{F3}$$

so we conclude that in fact both eigenvalues of $\hat{\mathbf{Z}}'_2$ must be negative at $\omega > \omega_2$.

Analogous conditions on the eigenvalues of the impedance $\hat{\mathbf{Z}}_2$ in $\hat{\mathbf{Z}}'_{\rm d}$ (50) must be fulfilled in order for equation (64) to have one or two roots, so equations (63) and (64) can have in total three or four roots provided that more than two eigenvalues of $\hat{\mathbf{Z}}_2$ and $\hat{\mathbf{Z}}'_2$ are negative in the interval $\omega < \Omega_{LD}$. However, at most two of four eigenvalues of $\hat{\mathbf{Z}}_2$ and $\hat{\mathbf{Z}}'_2$ can be negative because $\hat{\mathbf{Z}}_2 + \hat{\mathbf{Z}}'_2$ is a positive-definite matrix (the proof is similar to that given in Sec. III A regarding the eigenvalues of the matrix $\hat{\mathbf{Z}}_b$ and $\hat{\mathbf{Z}}'_b$). Therefore (63) and (64) can have in total at most two roots, which completes the proof.

APPENDIX G

Weak absorption in metals cannot increase the maximum number of SEWs. For definiteness, let us consider a magneto-optical dielectric metal structure (Case II). The permissible maximum could be increased by absorption provided that three SEWs exist under the no-loss approximation, i.e., the eigenvalues λ_1, λ_2, λ_3 vanish [see lines 1–3 in Fig. 2(b)], and the eigenvalue λ_4 (line 4) is only slightly greater than zero in the neighborhood of Ω_{LM}.

By differentiating $\hat{\mathbf{Z}}_m$ (35) as well as $\hat{\mathbf{Z}}_{\rm dm}$ and $\hat{\mathbf{Z}}'_{\rm dm}$ in Eqs. (36) and (37) with respect to the real dielectric permittivity ε_m of the metal we find that

$$\frac{\partial \hat{\mathbf{Z}}_{\rm dm}}{\partial \varepsilon_m} \text{ and } \frac{\partial \hat{\mathbf{Z}}'_{\rm dm}}{\partial \varepsilon_m} \text{ are negative definite matrices,} \tag{G1}$$

so, using spectral decompositions of $\hat{\mathbf{Z}}_{\rm dm}$ and $\hat{\mathbf{Z}}'_{\rm dm}$, one can prove that $\partial\lambda_i/\partial\varepsilon_m \neq 0$, $i = 1, \dots, 4$ (to be more precise, $\partial\lambda_i/\partial\varepsilon_m < 0$).

Hence we could make λ_4 vanish at $\omega < \Omega_{LM}$ by changing just the real ε_m, obtaining thereby four SEWs in a nonabsorbing structure rather than three. Therefore, a new SEW can emerge thanks to weak absorption only if the number of SEWs under no-loss conditions is less than the maximum.

Note that the derivative of $\hat{\mathbf{Z}}_f$ (40) with respect to ε_m and hence the derivatives of $\hat{\mathbf{Z}}_{\rm st}$ (51) and $\hat{\mathbf{Z}}'_{\rm st}$ (52) (see Sec. III C) are negative definite matrix. Therefore the above arguments also apply to structures containing metal films.

[1] A. K. Zvezdin and V. A. Kotov, *Modern Magnetooptics and Magnetooptical Materials* (Institute of Physics Publishing, Bristol and Philadelphia, 1997).

[2] T. Haider, A Review of Magneto-Optic Effects and Its Application, Int. J. Electromagn. Appl. **7**, 17 (2017).

[3] P. Fumagalli and J. Schoenes, *Magneto-optics* (Walter de Gruyter GmbH, Berlin, Boston, 2022).

[4] R. J. Potton, Reciprocity in optics, Rep. Prog. Phys. **67**, 717 (2004).

[5] C. Caloz, A. Alù, S. Tretyakov, D. Sounas, K. Achouri, and Z.-L. Deck-Léger, Electromagnetic Nonreciprocity, Phys. Rev. Appl. **10**, 047001 (2018).

[6] V. S. Asadchy, M. S. Mirmoosa, A. Díaz-Rubio, S. Fan, and S. A. Tretyakov, Tutorial on electromagnetic nonreciprocity and its origins, Proc. IEEE **108**, 1684 (2020).

[7] Horst Dötsch, N. Bahlmann, O. Zhuromskyy, M. Hammer, L. Wilkens, R. Gerhardt, P. Hertel, and A. F. Popkov, Applications of magneto-optical waveguides in

integrated optics: Review, J. Opt. Soc. Am. B **22**, 240 (2005).
[8] L. Bi1, J. Hu, P. Jiang, D. H. Kim, G. F. Dionne, L. C. Kimerling, and C. A. Ross, On-chip optical isolation in monolithically integrated non-reciprocal optical resonators, Nat. Photonics **5**, 758 (2011).
[9] Y. Shoji and T. Mizumoto, Magneto-optical non-reciprocal devices in silicon photonics, Sci. Tech. Adv. Mater. **15**, 014602 (2014).
[10] Y. Shoji, K. Miura, and T. Mizumoto, Optical nonreciprocal devices based on magneto-optical phase shift in silicon photonics, J. Opt. (Bristol, U. K.) **18**, 013001 (2016).
[11] K. Srinivasan and B. J. H. Stadler, Magneto-optical materials and designs for integrated TE- and TM-mode planar waveguide isolators: A review, Opt. Mater. Express **8**, 3307 (2018).
[12] Y. Zhang, Q. Du, Ch. Wang, T. Fakhrul, S. Liu, L. Deng, D. Huang, P. Pintus, J. Bowers, C. A. Ross, J. Hu, and L. Bi, Monolithic integration of broadband optical isolators for polarization-diverse silicon photonics, Optica **6**, 473 (2019).
[13] W. Yan, Y. Yang, Sh. Liu, Y. Zhang, Sh. Xia, T. Kang, W. Yang, J. Qin, L. Deng, and L. Bi, Waveguide-integrated high-performance magneto-optical isolators and circulators on silicon nitride platforms, Optica **7**, 1555 (2020).
[14] S. Liu, Y. Shoji, and T. Mizumoto, TE-mode magneto-optical isolator based on an asymmetric microring resonator under a unidirectional magnetic field, Opt. Express **30**, 9934 (2022).
[15] H. Raether, *Surface Plasmons on Smooth and Rough Surfaces and on Gratings*, Springer Tracts in Modern Physics (Springer, Berlin, 1988).
[16] S. A. Maier, *Plasmonics: Fundamentals and Applications* (Springer, New York, 2007).
[17] *Plasmonic Nanoguides and Circuits*, edited by S. I. Bozhevolnyi (Pan Stanford Publishing Pte. Ltd., 2009).
[18] R. Li, C. Cheng, F. F. Ren, J. Chen, Y. X. Fan, J. Ding, and H. T. Wang, Hybridized surface plasmon polaritons at an interface between a metal and a uniaxial crystal, Appl. Phys. Lett. **92**, 141115 (2008).
[19] V. I. Alshits and V. N. Lyubimov, Resonance excitation of polaritons and plasmons at the interface between a uniaxial crystal and a metal, J. Exp. Teor. Fiz. **138**, 669 (2010) [J. Exp. Theor. Phys. **111**, 591 (2010)].
[20] M. Liscidini and J. E. Sipe, Quasiguided surface plasmon excitations in anisotropic materials, Phys. Rev. B **81**, 115335 (2010).
[21] C. Zhou, T. G. Mackay, and A. Lakhtakia, Surface-plasmon-polariton wave propagation supported by anisotropic materials: Multiple modes and mixed exponential and linear localization characteristics, Phys. Rev. A **100**, 033809 (2019).
[22] Y. Chen, A. Norrman, S. A. Ponomarenko, and A. T. Friberg, Partially coherent surface plasmon polariton vortex fields, Phys. Rev. A **100**, 053833 (2019).
[23] A. Iurov, D. Huang, G. Gumbs, W. Pan, and A. A. Maradudin, Effects of optical polarization on hybridization of radiative and evanescent field modes, Phys. Rev. B **96**, 081408(R) (2017).
[24] V. I. Alshits and V. N. Lyubimov, Plasmon polaritons with uniquely long free path, Pis'ma v JETF **112**, 127 (2020) [JETP Lett. **112**, 127 (2020)].
[25] A. Passian, R. H. Ritchie, A. L. Lereu, T. Thundat, and T. L. Ferrell, Curvature effects in surface plasmon dispersion and coupling, Phys. Rev. B **71**, 115425 (2005).
[26] E. L. Albuquerque and M. G. Cottam, Superlattice plasmon polaritons, Phys. Rep. **233**, 67 (1993).
[27] R. N. Costa Filho, M. G. Cottam, E. L. Albuquerque, and G. A. Farias, Surface polaritons in layered structures of anisotropic media, Phys. Rev. B **54**, 2949 (1996).
[28] M. L. Bah, A. Akjouj, E. H. El Boudouti, B. Djafari-Rouhani, and L. Dobrzynski, Surface and interface optical waves in superlattices: Transverse electric localized and resonant modes, J. Phys.: Condens. Matter **8**, 4171 (1996).
[29] A. Yariv and P. Yeh, *Photonics: Optical Electronics in Modern Communications*, 6th ed., Oxford Series in Electrical and Computer Engineering (Oxford University, New York, 2007).
[30] M. Kaliteevski, I. Iorsh, S. Brand, R. A. Abram, J. M. Chamberlain, A. V. Kavokin, and I. A. Shelykh, Tamm plasmon-polaritons: Possible electromagnetic states at the interface of a metal and a dielectric Bragg mirror, Phys. Rev. B **76**, 165415 (2007).
[31] D. Aurelio and M. Liscidini, Electromagnetic field enhancement in Bloch surface waves, Phys. Rev. B **96**, 045308 (2017).
[32] M. I. D'yakonov, New type of electromagnetic wave propagating at an interface, Sov. Phys.-JETP **67**, 714 (1988) [Zh. Eksp. Teor. Fiz. **94**, 119 (1988)].
[33] A. N. Darinskii, Dispersionless polaritons on a twist boundary in optically uniaxial crystals, Kristallografiya **46**, 916 (2001) [Crystallogr. Rep. **46**, 842 (2001)].
[34] V. I. Alshits and V. N. Lyubimov, Dispersionless polaritons at symmetrically oriented surfaces of biaxial crystals, Fiz. Tverd. Tela **44**, 1895 (2002) [Phys. Solid State **44**, 1988 (2002)].
[35] A. N. Furs, V. M. Galynsky, and L. M. Barkovsky, Surface polaritons in symmetry planes of biaxial crystals, J. Phys. A: Math. Gen. **38**, 8083 (2005).
[36] S. R. Nelatury, J. A. Polo, and A. Lakhtakia, Surface waves with simple exponential transverse decay at a biaxial bicrystalline interface, J. Opt. Soc. Am. A **24**, 856 (2007).
[37] M. Faryad and A. Lakhtakia, Grating-coupled excitation of multiple surface plasmon-polariton waves, Phys. Rev. A **84**, 033852 (2011).
[38] J. Polo, T. Mackay, and A. Lakhtakia, *Electromagnetic Surface Waves: A Modern Perspective* (Elsevier, London, 2013).
[39] E. E. Narimanov, Dyakonov waves in biaxial anisotropic crystals, Phys. Rev. A **98**, 013818 (2018).
[40] J. J. Brion, R. F. Wallis, A. Hartstein, and E. Burstein, Theory of Surface Magnetoplasmons in Semiconductors, Phys. Rev. Lett. **28**, 1455 (1972).
[41] R. E. Camley, Nonreciprocal surface waves, Surf. Sci. Rep. **7**, 103 (1987).
[42] B. L. Johnson and R. E. Camley, Nonreciprocal propagation of surface waves in quasiperiodic superlattices, Phys. Rev. B **44**, 1225 (1991).
[43] A. N. Furs and L. M. Barkovsky, Surface electromagnetic waves in Faraday media, Zh. Tekhn. Fiz. **73**, 9 (2003) [Tech. Phys. **48**, 385 (2003)].
[44] T. Goto, A. V. Baryshev, M. Inoue, A. V. Dorofeenko, A. M. Merzlikin, A. P. Vinogradov, A. A. Lisyansky, and A. B. Granovsky, Tailoring surfaces of one-dimensional magnetophotonic crystals: Optical Tamm state and Faraday rotation, Phys. Rev. B **79**, 125103 (2009).
[45] K. Liu, A. Torki, and S. He, One-way surface magnetoplasmon cavity and its application for nonreciprocal devices, Opt. Lett. **41**, 800 (2016).

[46] F. Abbasi, A. R. Davoyan, and N. Engheta, One-way surface states due to nonreciprocal light-line crossing, New J. Phys. **17**, 063014 (2015).

[47] S. A. Hassani Gangaraj, G. W. Hanson, M. G. Silveirinha, K. Shastri, and M. Antezza, Unidirectional and diffractionless surface plasmon polaritons on three-dimensional nonreciprocal plasmonic platforms, Phys. Rev. B **99**, 245414 (2019).

[48] M. N. Romodina, I. V. Soboleva, and A. A. Fedyanin, Magneto-optical switching of Bloch surface waves in magnetophotonic crystals, J. Magn. Magn. Mater. **415**, 82 (2016).

[49] D. Szaller, S. Bordács, and I. Kézsmárki, Symmetry conditions for nonreciprocal light propagation in magnetic crystals, Phys. Rev. B **87**, 014421 (2013).

[50] M. Mruczkiewicz, M. Krawczyk, G. Gubbiotti, S. Tacchi, Y. A. Filimonov, D. V. Kalyabin, I. V. Lisenkov, and S. A. Nikitov, Nonreciprocity of spin waves in metallized magnonic crystal, New J. Phys. **15**, 113023 (2013).

[51] S. Tacchi, R. E. Troncoso, M. Ahlberg, G. Gubbiotti, M. Madami, J. Akerman, and P. Landeros, Interfacial Dzyaloshinskii-Moriya Interaction in Pt/CoFeB Films: Effect of the Heavy-Metal Thickness, Phys. Rev. Lett. **118**, 147201 (2017).

[52] M. Madami, Y. Khivintsev, G. Gubbiotti, G. Dudko, A. Kozhevnikov, V. Sakharov, A. Stal'makhov, A. Khitun, and Y. Filimonov, Nonreciprocity of backward volume spin wave beams excited by the curved focusing transducer, Appl. Phys. Lett. **113**, 152403 (2018).

[53] H. Wang, J. Chen, T. Liu, J. Zhang, K. Baumgaertl, Ch. Guo, Y. Li, Ch. Liu, P. Che, S. Tu, S. Liu, P. Gao, X. Han, D. Yu, M. Wu, D. Grundler, and H. Yu, Chiral Spin-Wave Velocities Induced by All-Garnet Interfacial Dzyaloshinskii-Moriya Interaction in Ultrathin Yttrium Iron Garnet Films, Phys. Rev. Lett. **124**, 027203 (2020).

[54] J. Chen, H. Yu, and G. Gubbiotti, Unidirectional spin-wave propagation and devices, J. Phys. D: Appl. Phys. **55**, 123001 (2022).

[55] K. Szulc, P. Graczyk, M. Mruczkiewicz, G. Gubbiotti, and M. Krawczyk, Spin-Wave Diode and Circulator Based on Unidirectional Coupling, Phys. Rev. Appl. **14**, 034063 (2020).

[56] J. Chen, J. Hu, and H. Yu, Chiral magnonics: Reprogrammable nanoscale spin wave networks based on chiral domain walls, iScience **23**, 101153 (2020).

[57] L. Sheng, J. Chen, H. Wang, and H. Yu, Magnonics based on thin-film iron garnets, J. Phys. Soc. Jpn. **90**, 081005 (2021).

[58] A. N. Darinskii and A. L. Shuvalov, Surface electromagnetic waves in anisotropic superlattices, Phys. Rev. A **102**, 033515 (2020).

[59] A. N. Darinskii, Surface electromagnetic waves in bianisotropic superlattices and homogeneous media, Phys. Rev. A **103**, 033501 (2021).

[60] A. N. Darinskii, Surface plasmon polaritons in metal films on anisotropic and bianisotropic substrates, Phys. Rev. A **104**, 023507 (2021).

[61] K. A. Ingebrigtsen and A. Tonning, Elastic surface waves in crystal, Phys. Rev. **184**, 942 (1969).

[62] J. Lothe and D. M. Barnett, On the existence of surface wave solutions for anisotropic half-spaces with free surface, J. Appl. Phys. **47**, 428 (1976).

[63] A. N. Furs and L. M. Barkovsky, General existence conditions of polaritons in anisotropic, superconductive and isotropic systems, J. Opt. A **1**, 109 (1999).

[64] A. Serdyukov, I. Semchenko, S. Tretyakov, and A. Sihvola, *Electromagnetics of Bi-Anisotropic Materials: Theory and Applications* (Gordon and Breach Science Publishers, London, 2001).

[65] T. Mackay and A. Lakhtakia, *Electromagnetic Anisotropy and Bianisotrophy: A Field Guide* (World Scientific, Singapore, 2010).

[66] D. W. Berreman, Optics in stratified and anisotropic media: 4×4-matrix formulation, J. Opt. Soc. Am. **62**, 502 (1972).

[67] J. Ning and E. L. Tan, Generalized eigenproblem of hybrid matrix for Bloch-Floquet waves in one-dimensional photonic crystals, J. Opt. Soc. Am. B **26**, 676 (2009).

[68] T. G. Mackay and A. Lakhtakia, *The Transfer-Matrix Method in Electromagnetics and Optics* (Morgan & Claypool, 2020).

[69] M.-A. Miri and A. Alù, Exceptional points in optics and photonics, Science **363**, eaar7709 (2019).

[70] A. N. Darinskii and A. L. Shuvalov, Surface acoustic waves on one-dimensional phononic crystals of general anisotropy: Existence considerations, Phys. Rev. B **98**, 024309 (2018).

[71] A. N. Darinskii and A. L. Shuvalov, Existence of surface acoustic waves on half-infinite one-dimensional piezoelectric phononic crystals of general anisotropy, Phys. Rev. B **99**, 174305 (2019).

[72] A. N. Darinskii and A. L. Shuvalov, Surface acoustic waves in one-dimensional piezoelectric phononic crystals with symmetric unit cell, Phys. Rev. B **100**, 184303 (2019).

[73] A. N. Darinskii and A. L. Shuvalov, Interfacial acoustic waves in one-dimensional anisotropic phononic bicrystals with symmetric unit cell, Proc. R. Soc. London, Ser. A **475**, 20190371 (2019).

[74] A. N. Darinskii and A. L. Shuvalov, Stoneley-type waves in anisotropic periodic superlattices, Ultrasonics **109**, 106237 (2021).

[75] L. D. Landau, L. P. Pitaevskii, and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Butterworths, London, 1984).

[76] A. Zangwill, *Modern Electrodynamics* (Cambridge University, Cambridge, England, 2013).

[77] A. L. Shuvalov, On the theory of wave propagation in anisotropic plates, Proc. R. Soc. London, Ser. A **456**, 2197 (2000).

[78] F. I. Fedorov, The theory of the optical activity of crystals, Sov. Phys. Usp. **15**, 849 (1973); Uspekhi Fiz. Nauk. **108**, 762 (1972).