Markovian and non-Markovian dynamics of quantum coherence in the extended XX chain

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(Received 14 May 2022; accepted 14 September 2022; published 27 September 2022)

The Markovian and non-Markovian dynamics of quantum coherence, including single-spin coherence, two-spin coherence, and its distribution (collective and localized coherence), are investigated in the XX spin chain with three-spin interaction. We find that the single-spin and localized coherence do not decrease but increase with time, which makes the dynamic evolution of the two-spin coherence more robust than entanglement and quantum discord. The three-spin interaction can lower the values of quantum coherence in Markovian evolution and heighten the oscillation frequency of quantum coherence in non-Markovian evolution because it can speed up the flow of coherence information. Finally, the trade-off relation between the collective and the localized coherence is clearly shown in their dynamic evolution. Thanks to the coherence trade-off, we find the substitution of localized coherence for collective coherence in dominating the dynamics of the two-spin coherence.

DOI: 10.1103/PhysRevA.106.032220

I. INTRODUCTION

Quantum coherence, characterizing the superposition properties of quantum states, is an essential feature of quantum mechanics. It is also a cornerstone of quantum information science [1–3]. Searching its physically meaningful and mathematically rigorous quantifiers is of fundamental and practical significance for many research fields related to coherence. A rigorous framework to quantify quantum coherence was formally introduced by Baumgratz *et al.* recently. They proposed a set of axioms which need to be satisfied by any coherence measure [4]. This work gave rise to a series of theoretical and experimental investigations [5,6]. Theoretical investigations mainly focus on the quantification of coherence based on different physical thoughts [7–13], their critical properties in quantum spin chains [14–24], and so on [25–29].

In contrast to other quantum quantifiers, such as entanglement and quantum discord, quantum coherence has several unique features [3,29–31]. For example, in a bipartite system, its coherence can be localized in an individual qubit, or be manifested as correlations between the two qubits. Thus, researchers generally decompose coherence into two parts based on the different physical conceptions, such as intrinsic and local coherences [30], localized and collective coherences [31], and the local and global coherences [32,33]. It is crucial but time consuming for the decomposition of the intrinsic and local coherences to find out the minimized state $\sigma_S^{\min}$ from all separable states. Based on the quantum version of Jensen-Shannon divergence, Radhakrishnan *et al.* introduced a basis-independent coherence measure by using the closest product state to replace the minimized state $\sigma_S^{\min}$ [31]. Then two parts of the total coherence include the collective and localized coherences, which are easy to obtain. It is also found that quantum coherence and its distribution obey the triangle inequality and trade-off relation [30,31]. The coherence distribution and its fundamental properties are theoretically and experimentally investigated in quantum spin systems [34–37].

Stable quantum resources are vital for quantum information processing tasks, but the quantum coherence of an open quantum system is fragile and may eventually vanish because of the environmental perturbations and quantum fluctuations. This is a great obstacle to the applications of quantum coherence in quantum technology. In order to find some solutions to the tough issue, the coherence dynamics of an open quantum system has been extensively studied recently [38–43]. However, these works mainly focus on the dynamics of total coherence not the coherence decomposition. It is well known that the quantum coherence and its decomposition are closely correlated but substantially different. So it is very necessary to study the dynamic evolution of coherence decomposition and check whether some physical properties still remain, such as the trade-off relation. Moreover, the dynamics of an open quantum system is generally categorized as the Markovian and non-Markovian evolution [44,45]. The unidirectional flow of the information from the open system to the environment is known as Markovian evolution, which is bound to cause a loss of quantum information embodied in open quantum systems. A flow of information attached by memory effects from the environment back to the open system represents the key property of the non-Markovian evolution, and the memory effects have a strong impact on the behaviors of the system. Therefore, for the open quantum systems, the Markovian and non-Markovian dynamics are indispensable to explore the physical properties of quantum coherence and its decomposition.

However, most recently, the dynamics of coherence decomposition in Markovian and non-Markovian environment is scarcely investigated, such as collective and localized

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coherences, which is obtained by the square root of quantum version of the Jensen-Shannon divergence, and extensively used in various of investigations [30,31,35–37]. Quantum spin systems represent the ideal platform to study the physical properties and meaning of the quantum coherence. In this paper, we investigate the Markovian and non-Markovian dynamics of the quantum coherence and its distribution (collective and localized coherence) in the XX spin chain with three-site interaction. It is interesting that the single-spin and localized coherence increase not decrease with the time, which makes the two-spin coherence more robust than entanglement and quantum discord. Three-spin interaction strengthens the correlation between the system and the environment and brings about some important impacts on the coherence dynamics. We also find that the triangle inequality and trade-off relations among the two-spin coherence, collective coherence, and localized coherence are maintained in dynamic evolution.

The outline of this paper is as follows. In Sec. II, an overview of the XX model with three-spin interaction is first given. We also introduce two coherence measures based on the quantum skew information, quantum Jensen-Shannon divergence, and an alternative coherence distribution. In Sec. III, we investigate the Markovian and non-Markovian dynamics of single-spin coherence, two-spin coherence, and its distribution. The static behaviors of two-spin coherence as a function of the three-spin interaction are also studied. Our conclusions are given in Sec. IV.

II. PHYSICAL MODEL AND COHERENCE MEASURES

A. Description of the physical model

The Hamiltonian of the XX spin model with three-spin interaction is [46–48]

$$H = J\sum_{n=1}^{N}\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right) - J'\sum_{n=1}^{N}\left(S_n^x S_{n+1}^z S_{n+2}^x + S_n^y S_{n+1}^z S_{n+2}^y\right), \quad (1)$$

where S_n^μ ($\mu = x, y, z$) are the spin-$\frac{1}{2}$ operators on the nth site. J denotes the two-spin interaction arising from the nearest-neighbor qubits, and J' denotes the three-spin interaction arising from the next-nearest-neighbor qubits, respectively. N is the total number of the spins in the chain, and the periodic boundary condition is assumed. In the following, the spin chain model is considered in the thermodynamic limit $N \to \infty$. By performing the Jordan-Wigner and Fourier transformations in sequence (see the Appendix), the Hamiltonian (1) can be exactly diagonalized in the following form:

$$H = \sum_k \varepsilon(k) c_k^\dagger c_k, \quad (2)$$

where $\varepsilon(k)$ is the energy spectrum, its expression is $\varepsilon(k) = \cos(k) + \frac{\alpha}{2}\cos(2k)$ with $\alpha = \frac{J'}{J}$.

Here, we concentrate on the quantum coherence of the nearest-neighbor spin-pair located at sites m and $m+1$ in the spin chain, and its initial state is assumed to be maximally entangled. Then, the rest of the chain is the spin-chain environment. The reduced density matrix of two spins at sites m and $m+1$ can be written in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as (see the Appendix)

$$\rho_{m,m+1} = \begin{pmatrix} x^+ & 0 & 0 & 0 \\ 0 & y^+ & z^* & 0 \\ 0 & z & y^- & 0 \\ 0 & 0 & 0 & x^- \end{pmatrix}, \quad (3)$$

where $z = \langle c_m^\dagger c_{m+1}\rangle$, $x^+ = \langle n_m n_{m+1}\rangle (n_m = c_m^\dagger c_m)$, $y^+ = \langle n_m(1 - n_{m+1})\rangle$, $x^- = \langle 1 - n_m - n_{m+1} + n_m n_{m+1}\rangle$, and $y^- = \langle n_{m+1}(1 - n_m)\rangle$. The analytic expressions of the matrix elements z and x^+ are given by

$$z = \frac{1}{8\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f^*(k) f(k') e^{i(k-k')m - ik'} dk\, dk', \quad (4)$$

$$x^+ = \langle n_m\rangle\langle n_{m+1}\rangle - zz^*, \quad (5)$$

with

$$\langle n_m\rangle = \frac{1}{8\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f^*(k) f(k') e^{i(k-k')m} dk\, dk', \quad (6)$$

$$f(k) = e^{i[km - \varepsilon(k)t]}(1 + e^{i(k+\phi)}). \quad (7)$$

Since the coherence can exist within an individual qubit, we also investigate the coherence dynamics of the single-spin system. All single-spin density matrices are the same due to the translation invariance. The single-spin density matrix ρ_m can be obtained by trace out site $m+1$ from Eq. (3), its specific form is written as

$$\rho_m = \begin{pmatrix} x^+ + y^+ & 0 \\ 0 & x^- + y^- \end{pmatrix}. \quad (8)$$

B. Measurements of quantum coherence

In this section, we will introduce two kinds of coherence measure, which are based on the quantum skew information [7] and quantum Jensen-Shannon divergence [30,31]. It will ensure the accuracy of our results to a certain extent.

First, Girolami introduced an observable measure of quantum coherence for states of finite-dimensional systems, which is based on the quantum skew information and can be written as [7]

$$QC_{\mathrm{QSI}}(\rho, K) = -\tfrac{1}{2}\mathrm{Tr}\{[\sqrt{\rho}, K]^2\}, \quad (9)$$

where ρ represents the density matrix of a quantum state and K denotes an observable. The coherence information embodied in a quantum state is usually skewed to an observable. The square-root terms sometimes prevent us from recasting the skew information as a function of observables. Therefore, a simplified version is introduced by Girolami [7],

$$QC_{\mathrm{QSI}}^{\mathrm{S}}(\rho, K) = -\tfrac{1}{4}\mathrm{Tr}\{[\rho, K]^2\}. \quad (10)$$

It can be measured in an interferometric setup only by performing two programmable measurements, regardless of the dimension of the quantum system, so it is a meaningful and an experimentally friendly coherence measure. This coherence measure is basis dependent since it is dependent on the observable. For the single-spin system, after a derivation according to Eq. (9), we find that the analytical expressions of the σ_x and σ_y coherence (coherence carried by ρ_m when measuring σ_x or σ_y) are the same and the σ_z coherence is equal to zero. For the two-spin system, the coherence measure can be written as $QC_{\rm QSI}(\rho_{m,m+1}, K_m \otimes I_{m+1})$ [7,14]. Coincidentally, after a calculation according to Eq. (10), we find that the analytical expressions of the σ_x and σ_y coherence are also the same and the analytical expression of the σ_z coherence only contains the matrix element z. Therefore, we will consider the σ_x (σ_y) coherence of the single-spin and two-spin system in the following investigation.

Second, Radhakrishnan and co-workers introduced a basis-independent coherence measure by means of the quantum version of Jensen-Shannon divergence and the maximally mixed state, which can be expressed as [30,31]

$$QC_{\rm QSJD}(\rho) = \sqrt{S\left(\frac{\rho+\rho_I}{2}\right) - \frac{S(\rho)+S(\rho_I)}{2}}, \qquad (11)$$

where $\rho_I \equiv \frac{1}{d}\sum_{i=1}^{d} |i\rangle\langle i|$ is the maximally mixed state in a d-dimensional Hilbert space. It is a basis invariant state. The maximally mixed state will become a 4×4 identity matrix for a bipartite system. S denotes the von Neumann entropy. The basis-independent coherence is also called the total coherence in the following paragraphs.

The total coherence in multipartite systems, originates from the collective participation of several subsystems and the individual subsystem. In the two-spin system, the collective coherence may exist due to the collective participation of two subsystems. It is defined as the distance from a bipartite state to the closest product state [31],

$$QC_{\rm C}(\rho) \equiv \mathcal{D}(\rho, \pi_\rho) = \sqrt{S\left(\frac{\rho+\pi_\rho}{2}\right) - \frac{S(\rho)+S(\pi_\rho)}{2}}, \qquad (12)$$

where $\pi_\rho = \rho_m \otimes \rho_{m+1}$ is the closest product state. The localized coherence can be attributed to coherence located within the subsystem. It is defined as the distance from the closest product state to the maximally mixed state [31],

$$QC_{\rm L}(\rho) \equiv \mathcal{D}(\pi_\rho, \rho_I) = \sqrt{S\left(\frac{\pi_\rho+\rho_I}{2}\right) - \frac{S(\pi_\rho)+S(\rho_I)}{2}}, \qquad (13)$$

III. QUANTUM COHERENCE OF THE CENTRAL SPIN SYSTEMS

Based on the trace distance of quantum states, Mahmoudi *et al.* have studied the non-Markovianity of the XX spin chain with three-spin interaction [47]. They found that there was a critical point $\alpha_c = \frac{J'}{J} \simeq 0.5$ between the Markovian and the non-Markovian regime. The properties of the system must be affected by the unidirectional or bidirectional flow of information between the system and environment, so the Markovian and non-Markovian dynamics are indispensable for a further understanding of open quantum system physics. Therefore, we mainly investigate the coherence dynamics of the central spin systems in the Markovian and the non-Markovian regimes, and the impact of three-spin interaction on the static behaviors of quantum coherence is also discussed in the following section.

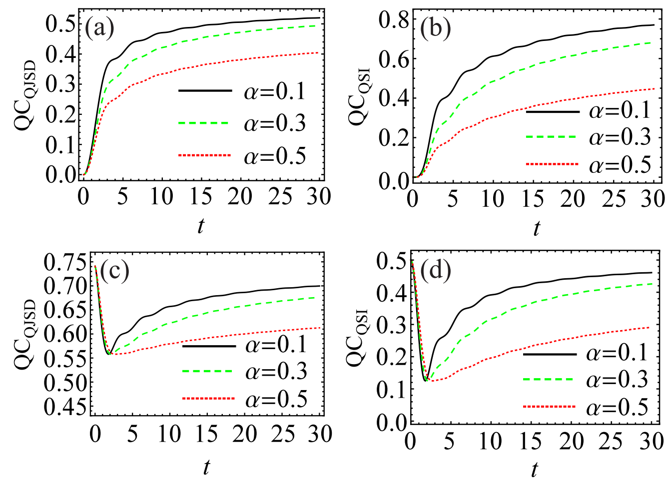

FIG. 1. The single-spin coherence (a and (b) and two-spin coherence (c) and (d), quantified, respectively, by quantum Jensen-Shannon divergence ($QC_{\rm QJSD}$) and quantum skew information ($QC_{\rm QSI}$) as a function of the time t are given in (a)–(d) under the different values of α.

A. Markovian dynamics of quantum coherence

The physical model possesses Markovian features as $\alpha = \frac{J'}{J} \leqslant \alpha_c \simeq 0.5$. For the single-spin and two-spin systems in the Markovian environment, we can obtain their coherences by use of corresponding coherence measures [Eqs. (9)–(13)]. Thus, the Markovian dynamics of the single-spin coherence, two-spin coherence, and its distribution (collective and localized coherence) will be studied in this section. For the single-spin system, we can calculate the quantum coherences based on the quantum Jensen-Shannon divergence and the quantum skew information. Figures 1(a) and 1(b) shows the Markovian dynamics of single-spin coherences with respect to the time under the different values of α. First, two kinds of quantum coherence with different values of α are zero at the beginning. Then, they increase sharply in a short timeand reach the maximum values in a slow way. This is an interesting phenomenon that the single-spin coherence does not decrease but increases with the time and finally reaches its maximum value. For an open quantum system, it is important and convenient for us to perform quantum information processing. Furthermore, we also find that the quantum coherence becomes smaller for a larger value of α. The larger value of α means the stronger next-nearest-neighbor interaction, which may speed up the information flow from the system to surrounding environment and lower the single-spin coherence.

For the two-spin system, we also calculate the quantum coherence based on the quantum Jensen-Shannon divergence and the quantum skew information. By the way, in order to get a compact solution, we use the simplified version of quantum skew information to calculate the quantum coherence of two-spin system. The Markovian dynamics of two-spin coherence with the different values of α is shown in Figs. 1(c) and 1(d). It is found that two kinds of two-spin coherence decrease sharply in a short time ($t \leqslant 1.8$), then, they gradually increase with time. They all have the minima at time $t \simeq 1.8$. Mahmoudi *et al.* have demonstrated that the quantum correlations

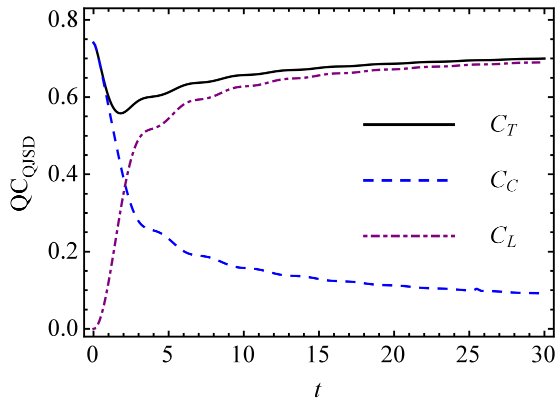

FIG. 2. The total coherence (C_T), collective coherence (C_C), and localized coherence (C_L) as a function of the time t. The ratio of next-nearest-neighbor interaction to nearest-neighbor interaction is set to $\alpha = \frac{J'}{J} = 0.1$.

of the two-spin system decrease with the time and finally disappear in the Markovian environment [47]. Therefore, we can conclude that the two-spin coherence (total coherence) has a better capability of withstanding the decoherence mechanism of the Markovian environment. Furthermore, we also find that the two-spin coherence becomes smaller with a larger value of α. It means that a stronger three-spin interaction can make the system environment more connected, and the faster information flow from system to environment may lead to a loss of quantum coherence. Whereas, the concurrence becomes smaller with the weaker three-spin interaction [47]. According to above investigations, we find that the two-spin coherence is more robust than quantum correlations (entanglement and quantum discord) in Markovian dynamics. Motivated by the Markovian dynamics of the single-spin coherence, we attempt to explain the dynamic performances of two-spin coherence by discussing the Markovian dynamics of coherence distribution (collective and localized coherence). The two-spin coherence (total coherence), collective coherence, and localized coherence can be calculated by the Eqs. (11)–(13), respectively. We display the total (C_T), collective (C_C), and localized coherence (C_L) as a function of the time in Fig. 2. It is found that the localized coherence first undergoes a rapid rise in a short time and increases in a slow way. It is not surprising that the dynamics of localized coherence is the same as the single-spin coherence because both coherences exist in the individual spin. Second, we find that the collective coherence always decreases with the time, and the rate of decay goes from high to low. It is well known that the collective coherence is the distance from a quantum state to the closest product state, and is equal to the total mutual information of the systems, including both quantum and classical correlation between the two subsystems [49]. Therefore, the dynamic performances of collective coherence are similar with the mutual information studied by Mahmoudi *et al.* [47]. Lastly, it is found in Fig. 2 that three kinds of quantum coherence obey two fundamental relations during their Markovian dynamics. They are the triangle inequality $C_C(\rho) + C_L(\rho) \geqslant C_T(\rho)$ and the trade-off relation (the collective coherence decreases, but the localized coherence increases with respect to the time). Therefore, the trade-off relation between the collective and the localized coherence can explain fully the reason for the dynamic performances of two-spin coherence. In other words, the dynamic performance of two-spin coherence is dominated by the collective coherence at the beginning, and it is soon replaced by localized coherence.

In a word, the two-spin coherence is more robust than quantum correlations in Markovian dynamics due to the contribution from localized coherence. Here, we note that there are two factors through which the localized coherence (or single-spin coherence) becomes larger: One is by interacting with the spin-chain environment, and the other is through a transformation of the collective coherence into localized coherence. Second, the three-spin interaction may strengthen the relationship between the central spin system and spin-chain environment, accelerate the flow of coherence information from the system to surrounding environment, and finally lead to the loss of the single-spin and two-spin coherence. Furthermore, we also find that the two-spin coherence and its distribution (collective and localized coherence) obey the triangle inequality and the trade-off relation in Markovian dynamics. Lastly, the quantum coherences quantified by two coherence measures show the same Markovian dynamics regardless of the single-spin and two-spin system, which guarantees the accuracy of our results. Whereas, the coherence measure based on the quantum Jensen-Shannon divergence reveals more properties of the Markovian dynamics due to the coherence decomposition.

B. Non-Markovian dynamics of quantum coherence

The physical model possesses non-Markovian features as $\alpha = \frac{J'}{J} > \alpha_c \simeq 0.5$. Based on the corresponding coherence measure from Eqs. (9)–(13), the single-spin coherence, two-spin coherence, and its distribution can be obtained, respectively. We will investigate the non-Markovian dynamics of the quantum coherence in the following section.

The non-Markovian dynamics of the single-spin coherence under different values of α is displayed in Figs. 3(a) and 3(b). It is found that the single-spin coherences are initially zero for all values of α, but they increase to their maximum values in a short time and begin to change in an oscillating way. Although the dynamic performances of single-spin coherence are the damped oscillation, the maximum values of coherence in each period remain unchanged. In addition, we also find that the single-spin coherence under a larger value of α increases to the maximum value at an earlier time, and its oscillation frequency becomes higher. The oscillating behavior of coherence dynamics depends on the memory effects of the information flow from the environment back to the single-spin system. The stronger three-spin interaction (larger value of α) makes system environment more connected, speeds up the information exchange between system and environment, and finally heightens the oscillation frequency of non-Markovian dynamics. The non-Markovian dynamics of the two-spin coherence under different values of α is displayed in Figs. 3(c) and 3(d). Here, we use the simplified version of quantum skew information to obtain the compact solution of coherence (QC_{QSI}). The two-spin coherences with different values of α are initially at the same maxima ($QC_{QJSD} = 0.7408$, $QC_{QSI} = 0.5$), and

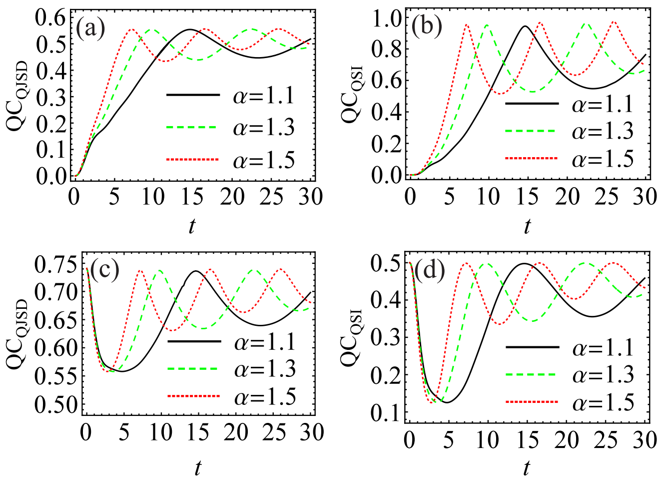

FIG. 3. The single-spin coherence (a) and (b) and two-spin coherence (c) and (d), quantified, respectively, by quantum Jensen-Shannon divergence (QC_{QJSD}) and quantum skew information (QC_{QSI}) as a function of the time t are given in (a)–(d) under the different values of α.

oscillate with different frequencies and amplitudes, respectively. The dynamic performances of two-spin coherence are the damped oscillation, but the maximum values of coherence in each period remain unchanged. Again, we find that the coherence with the larger values of α has the higher frequency due to faster information exchange between the system and environment. In contrast to the time evolution of quantum correlations (concurrence and quantum discord) in Ref. [47], the two-spin coherence is more robust because the quantum correlations oscillate between the zero and a lower value (i.e., the revival of the quantum correlations).

In order to better understand the non-Markovian dynamics of two-spin coherence, we investigate the dynamic evolution of the coherence distribution (collective and localized coherences). The two-spin coherence (total coherence $C_{\rm T}$), collective coherence ($C_{\rm C}$), and localized coherence ($C_{\rm L}$) as a function of the time are displayed in Fig. 4. It is found that the collective coherence first drops sharply with time and

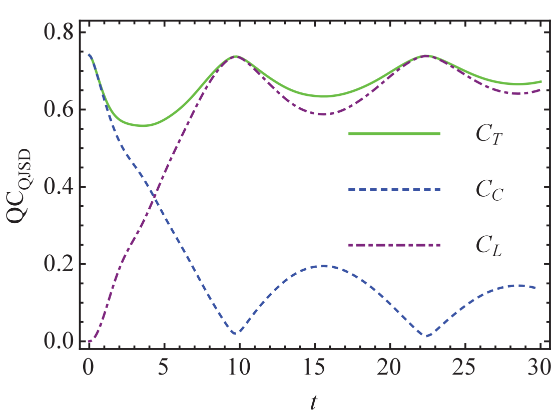

FIG. 4. The total coherence ($C_{\rm T}$), collective coherence ($C_{\rm C}$), and localized coherence ($C_{\rm L}$) as a function of the time t. The ratio of next-nearest-neighbor interaction to nearest-neighbor interaction is set to $\alpha = \frac{J'}{J} = 1.3$.

almost reduced to zero, and later on the collective coherence undergoes a damped oscillation in time. The mentioned behaviors are similar to the mutual information, concurrence, and quantum discord studied by Mahmoudi *et al.* [47] because the collective coherence mainly contains the contributions from the classical and quantum correlations [47,49]. Both the localized coherence and single-spin coherence depict the coherence properties of the individual qubit, so they have the similar non-Markovian dynamics shown in Figs. 3(a), 3(b) and 4. Again, two fundamental properties about the relations among the total, collective, and localized coherence are found. They are the triangle inequality $C_{\rm C}(\rho) + C_{\rm L}(\rho) \geqslant C_{\rm T}(\rho)$ and the trade-off relations (the localized coherence increases when the collective coherence decreases with the time and vice versa). Furthermore, we find that the dynamic evolution of the total coherence initially follows the collective coherence, and is quickly contrary to it. Then, the localized coherence takes the dominant role in the following evolution of the total coherence.

In brief, we find that the contribution from localized coherence prompts the two-spin coherence to become more robust in non-Markovian dynamics. The stronger three-spin interaction can speed up the information exchange of the system environment and increase the oscillation frequency of the single-spin and two-spin coherence in non-Markovian evolution. Moreover, the coherence measure, based on the quantum Jensen-Shannon divergence, displays richer dynamic characteristics due to its coherence decomposition. Thanks to the coherence trade-off, we find that the leading role in dominating the dynamics of the two-spin coherence is taken from collective coherence to localized coherence.

C. Impact of three-spin interaction on static behaviors of two-spin coherence

In this paper, the larger value of $\alpha = \frac{J'}{J}$ means the stronger next-nearest-neighbor interaction with respect to the nearest-neighbor interaction. When the two-spin system evolves from the initial state ($t = 0$) to the target state at a fixed time, i.e., $t = 1$, $t = 2$, $t = 4$, or $t = 10$, we study the impact of three-spin interaction on static behaviors of two-spin coherence. The two-spin coherence based on the quantum Jensen-Shannon divergence and quantum skew information as a function of α under different values of time are displayed in Fig. 5. It is found that two-spin coherence possesses a maximum around $\alpha = 1.0$ as the time $t = 1$. The static behavior of quantum coherence is the same as entanglement studied by Mahmoudi *et al.* [47]. However, with the increase in time, the maximal coherence becomes the minimum, and the position of the minimum gradually moves to a smaller value of α in the left side of the Fig. 5 ($0 < \alpha \lesssim 1.3$). When $\alpha \gtrsim 1.3$, the static behavior of two-spin coherence gradually is changed from a monotonic decrease to a damp oscillation with respect to α. The premise behind the discussion about the static behaviors of entanglement and quantum discord is that the time is less than disentangled time t_r in Ref. [47]. Whereas, the two-spin coherence never disappear in the Markovian or non-Markovian dynamics due to the contribution from the localized coherence. Therefore, the dynamic properties

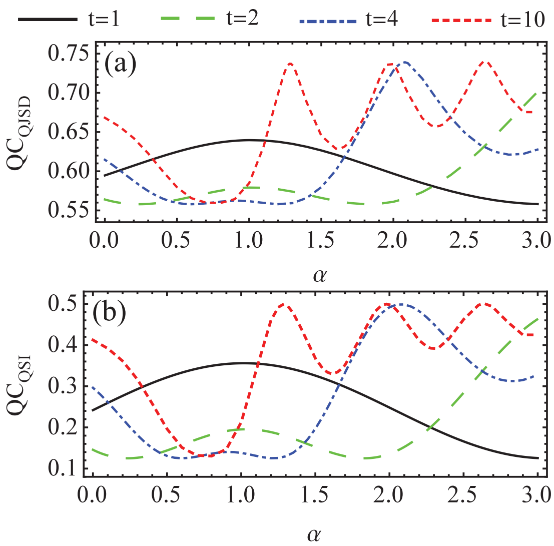

FIG. 5. The two-spin coherences quantified by quantum Jensen-Shannon divergence (QC_{QJSD}) and quantum skew information (QC_{QSI}) as a function of the parameter α are given in (a) and (b) under the four fixed times, respectively.

of localized coherence may lead to these different behaviors between the quantum coherence and the quantum correlations.

The parameter α denotes the ratio of three-spin interaction to two-spin interaction. The three-spin interaction is assumed to be stronger than the two-spin interaction when we discuss the non-Markovian dynamics of the quantum coherence. Even $0 < \alpha < 3$ when we investigate the static behaviors of quantum coherence as a function of α. Some researchers may cast doubt on the parameter setting, i.e, the next-nearest-neighbor interaction is larger than nearest-neighbor interaction. Actually, this parameter configuration is often used in experimental and theoretical researches recently. Peng *et al.* has investigated the three-spin Ising model without two-spin interaction in a transverse magnetic fieldand predict a novel transition. With the help of a NMR quantum simulator, they simulated such a system and observe the quantum phase transition [50]. People have also theoretically studied the topological quantum phase transition in spin chains with such a parameter configuration [24,34,35,51]. Therefore, the parameter setting, three-spin interaction is stronger than the two-spin interaction, has both theoretical and practical significances.

IV. CONCLUSIONS

Based on the coherence measures of quantum Jensen-Shannon divergence and quantum skew information, the Markovian and non-Markovian dynamics of quantum coherence, including single-spin coherence, two-spin coherence, and its distribution, are investigated in the XX spin chain with three-spin interaction. It is interesting that the single-spin coherence and localized coherence do not decrease but increase with the time at the beginning. They finally reach the maxima in Markovian dynamics and show the damp oscillation (but the maximum coherence in each period remains unchanged) in non-Markovian dynamics. It is for this reason that the Markovian and non-Markovian dynamics of the two-spin coherence (total coherence) are more robust than the quantum discord and entanglement, and the static behaviors of two-spin coherence with respect to $\alpha = \frac{J'}{J}$ are different from the concurrence.

Second, the stronger three-spin interaction can make system environment more connected. It can speed up the flow of coherence information from the system to surrounding environment in Markovian dynamics and force the single-spin and two-spin coherence to become smaller. Meanwhile, it can speed up the information exchange between system and environment and heighten the oscillation frequencies of the single-spin and two-spin coherence in non-Markovian dynamics. Lastly, the two-spin coherence C_{T}, and its distribution (collective coherence C_{C}, and localized coherence C_{L}) still obey the triangle inequality $C_{\mathrm{C}}(\rho) + C(\rho) \geqslant C_{\mathrm{T}}(\rho)$ and the trade-off relations in the Markovian and non-Markovian dynamics. Thanks to the coherence trade-off, we find that the dynamic performances of two-spin coherence are initially dominated by the collective coherence, and it is soon replaced by localized coherence. In a word, our research results make us have deeper insights into the coherence dynamics of the open quantum system.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants No. 61575055, No. 11874132, and No. 11675046, Doctoral Research Start-up Foundation of Harbin Normal University (Grant No. XKB202112), and the Fundamental Research Funds for the Universities of Heilongjiang Province of China (Grant No. 2021-KYYWF-0188).

APPENDIX: THEORETICAL DERIVATION OF THE PHYSICAL MODEL

In order to get a better understanding of the physical model, we divide the total Hamiltonian [Eq. (1)] into three parts, which are the two-spin system H_S, the external environment H_E, and the interaction term H_I. They can be written as [47,48]

$$H_S = J\left(S_m^x S_{m+1}^x + S_m^y S_{m+1}^y\right), \tag{A1}$$

$$\begin{aligned} H_E = & -J' \sum_{n \neq m-2, m-1, m, m+1}^{N} \left(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y\right) S_{n+1}^z \\ & + J \sum_{n \neq m-1, m, m+1}^{N} \left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right), \end{aligned} \tag{A2}$$

and

$$\begin{aligned} H_I = & J\left(S_{m-1}^x S_m^x + S_{m-1}^y S_m^y + S_{m+1}^x S_{m+2}^x + S_{m+1}^y S_{m+2}^y\right) \\ & - J' \sum_{n=m-2}^{m+1} \left(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y\right) S_{n+1}^z. \end{aligned} \tag{A3}$$

This model is exactly solvable. We first use the Jordan-Wigner transformation,

$$S_n^z = c_n^\dagger c_n - \frac{1}{2},$$
$$S_n^+ = c_n^\dagger \exp\left(i\pi \sum_{l<n} c_l^\dagger c_l\right),$$
$$S_n^- = c_n \exp\left(-i\pi \sum_{l<n} c_l^\dagger c_l\right), \tag{A4}$$

to map spins to one-dimensional spinless fermions with creation operators $c_l^\dagger$ and annihilation operators c_l. After a straightforward deduction, the total Hamiltonian becomes

$$H = \frac{J}{2}\sum_n (c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n) - \frac{J'}{4}\sum_n (c_n^\dagger c_{n+2} + c_{n+2}^\dagger c_n). \tag{A5}$$

Now by introducing the Fourier transformation,

$$c_n = \frac{1}{\sqrt{N}}\sum_k c_k \exp(-ikn),$$
$$c_n^\dagger = \frac{1}{\sqrt{N}}\sum_k c_k^\dagger \exp(ikn), \tag{A6}$$

where c_k is the momentum eigenstates. Then the fermion operators are transformed to the momentum space. The total Hamiltonian can be also diagonalized.

We assume that the central two-spin system is initially disentangled with the spin-chain environment, i.e., at $t = 0$, the two-spin system and the chain environment are supposed to be described by the product state [47,48],

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + e^{i\phi}|\uparrow\downarrow\rangle)_{\rm S} \otimes (|\downarrow\downarrow\downarrow\cdots\downarrow\rangle)_{\rm E}, \tag{A7}$$

and the initial state can be rewritten as $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(c_m^\dagger|0\rangle + c_{m+1}^\dagger e^{i\phi}|0\rangle)$ by means of the fermion operators. $|0\rangle$ denotes the vacuum state, and ϕ is a phase factor. Using the time-evolution operator, $U(t) = \exp(-iHt)$ ($\hbar$ is taken to be 1), the time evolution of the total system is determined by $\rho_{\rm tot}(t) = U(t)\rho_{\rm tot}(0)U(t)^\dagger$, where $\rho_{\rm tot}(0) = \rho_{m,m+1}(0) \otimes \rho_E(0)$. Then, one can obtain the evolved reduced density matrix of the two-spin system by tracing over the environment, denoted by $\rho_{m,m+1}(t) = {\rm Tr}_{\rm E}[\rho_{\rm tot}(t)]$. Due to the translation invariance of spin-chain system, the reduced density $\rho_{m,m+1}(t)$ matrix is always the same whatever m takes.

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