

Vortex formation and quantum turbulence with rotating paddle potentials in a two-dimensional binary Bose-Einstein condensate

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We conduct a theoretical study of the creation and dynamics of vortices in a two-dimensional binary Bose-Einstein condensate with a mass imbalance between the species. To initiate the dynamics, we use one or two rotating paddle potentials in one species, while the other species is influenced only via the interspecies interaction. In both species, the number and the dominant sign of the vortices are determined by the rotation frequency of the paddle potential. Clusters of positive and negative vortices form at a low rotation frequency comparable to that of the trap when using the single paddle potential. In contrast, vortices of the same sign tend to dominate as the rotation frequency of the paddle increases, and the angular momentum reaches a maximum value at a paddle frequency where the paddle velocity becomes equal to the sound velocity of the condensate. When the rotation frequency is sufficiently high, the rapid annihilation of vortex-antivortex pairs significantly reduces the number of vortices and antivortices in the system. For two paddle potentials rotating in the same direction, the vortex dynamics phenomenon is similar to that of a single paddle. However, when the paddle potentials are rotated in the opposite direction, both positive- and negative-signed vortices occur at all rotational frequencies. At the low rotation frequencies, the cluster of like-signed vortices produces the $k^{-5/3}$ and k^{-3} power laws in the incompressible kinetic energy spectrum at low and high wave numbers, respectively, a hallmark of the quantum turbulent flows.

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I. INTRODUCTION

Vortices [1] render a Bose-Einstein condensate (BEC) an excellent platform for examining various scaling aspects of quantum turbulence [2–4] which are quantum counterparts of classical turbulence [5–7]. The renowned Kolmogorov 5/3 law is one of the most well known of these scaling laws [8,9]. Several strategies are available from the current state BEC experiments [10] to generate nonlinear defects such as vortices and solitons [11–16]. These include laser stirring [17–19], rotating the confining potential [20–22], interaction with the optical vortex [23–26], quenching through the phase transition [27], and counterflow dynamics [28–31], just to name a few [32–34]. Theoretically, numerous intriguing aspects of three-dimensional [8,35–38] and two-dimensional (2D) quantum turbulence (QT) [39–50] have been examined. Moreover, very recently developed machine learning techniques can be utilized to detect and classify quantum vortices [51,52]. The incredible tunability of the atom-atom interaction via Feshbach resonance [53,54] and the outstanding maneuverability of dimension [55] have also resulted in the significant development of the QT experiment in BECs. In that regard, Ref. [39] shows a turbulent tangle of vortices formed by oscillating perturbation. Spontaneous clustering of the same circulation vortices has also been demonstrated experimentally [56,57]. It is worth noting that clustering of vortices [58,59] implies the transfer of energy from small to

large length scales, illustrating the so-called inverse energy cascade [36,60], a well-known phenomenon that occurs in classical 2D turbulence [61,62]. The experiment in [57], for instance, employs a paddle that swiftly moves through the bulk of the BEC, causing randomly distributed vortices that quickly assemble into Onsager point vortex clusters, a notion that has also been studied theoretically by White *et al.* [63].

Given that the optical paddle potential is a dependable way to create 2D QT, we attempt to conduct a detailed theoretical examination of the production of vortex complexes, the behavior of angular momentum, and the onset of quantum turbulence in a two-component system by utilizing rotating paddle potentials. Furthermore, we use a more complicated system with 2D binary BECs [64–66], where only one species is exposed to the rotating paddle. We specifically identify the frequency regimes of the rotating paddle where the maximum angular momentum can be imparted to the condensates, as well as systemically investigate the distinct behavior emerging from single and double paddle potentials. The system of 2D binary BECs, which exhibits a variety of instability phenomena [67–71] and nonlinear structures [72–75], is intriguing in its own right. Using the so-called tune-out technique [76], the previously mentioned species-selective interaction resulting in the formation of optical paddle potential can be experimentally performed. In this tune-out method, when one species interacts with the laser light, the other remains unaffected. Furthermore, we investigate a wide range of rotating frequencies of the paddle potential, allowing us to pinpoint the domain in which clustering of the same circulation vortices arises, exhibiting the well-known scaling rule of 2D QT, i.e., Kolmogorov's $-5/3$ scaling law [43,48,77].

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Although different stirring configurations are available in the literature [56,63,78–80], the main objective of employing a rotating paddle potential in this paper is to impart a finite net angular momentum to one of the binary species within a specific frequency regime and transfer this momentum to the other species. We also look at a region dominated solely by identical signed multiple vortices. Furthermore, when the paddle rotates more vigorously, the vortical content of the system drops due to the generation of a large number of sound waves [40,79,81,82]. When there is finite interspecies contact interaction, vortex formation can occur even in the second component of the condensate. Most importantly, the vortex in one component is connected by a complementary structure, referred to as a vortex-bright soliton [48,83], in the other. In addition, we demonstrate the effect of double paddle potentials, in which paddles can rotate in either the same or opposite directions.

This article is arranged as follows. Section II describes our setup and delves into the Gross-Pitaevskii (GP) equations. In Sec. III we investigate the nonequilibrium dynamics of a binary system consisting of a mass-imbalanced system using both single (Sec. III A) and double paddle potentials (Sec. III B). Section IV examines the incompressible and compressible kinetic energy spectra. Finally, we summarize our findings and discuss potential future extensions in Sec. V. Appendix A briefly describes the creation of vortices and their dynamics in a binary BEC with equal mass. In Appendix B we demonstrate vortex creation using the negative paddle potential.

II. GROSS-PITAEVSKII EQUATION

We consider binary BECs, referred to as species A and B, that are confined in 2D harmonic trapping potentials [84]. The species consists of a number of atoms N_i of mass m_i ($i = A, B$). The form of the trapping potentials reads $V_{\text{trap}} = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$, where ω_x , ω_y , and ω_z are trapping frequencies in the x , y , and z directions, respectively. To implement a quasi-2D BEC in the x - y plane, we consider the following criterion for the trap frequencies: $\omega_x = \omega_y = \omega \ll \omega_z$. We apply single or double stirring potential V_p generated by a far-off-resonance blue-detuned laser beam shaped into an elliptic paddle in species A to induce vortices in the condensate [56]. The potential V_{p_α} , with $\alpha \in \{1, 2\}$, can be expressed as [63]

$$V_{p_\alpha}(x, y, t) = V_0 \exp \left(- \frac{\eta^2 [\tilde{x}_\alpha \cos(\omega_p t) - \tilde{y}_\alpha \sin(\omega_p t)]^2}{d^2} - \frac{[\tilde{y}_\alpha \cos(\omega_p t) + \tilde{x}_\alpha \sin(\omega_p t)]^2}{d^2} \right), \quad (1)$$

where $\tilde{x}_\alpha = x - x_{p,\alpha}$ and $\tilde{y}_\alpha = y - y_{p,\alpha}$, considering the center of the paddle potential at $(x_{p,\alpha}, y_{p,\alpha})$ for the α paddle. Here V_0 is the peak strength of the potential, ω_p is the rotation frequency of the paddle, and η and d determine the paddle elongation and width, respectively.

In the quasi-2D regime, the motion of atoms in the z direction becomes insensitive and the wave functions $\Psi_{A(B)}$ can be expressed as $\psi_{A(B)}(x, y)\zeta(z)$, where $\zeta(z) = (\lambda/\pi)^{1/4} \exp(-\lambda z^2/2)$ is the ground state in the z direction

and $\lambda = \omega_z/\omega$ is the trap aspect ratio. After integrating out the variable z , the 2D dimensionless time-dependent GP equation that governs the dynamics of a BEC is given by [85,86]

$$i \frac{\partial \psi_i}{\partial t} = \left[- \frac{1}{2} \frac{m_B}{m_i} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \frac{m_i}{m_B} (x^2 + y^2) + \sum_{j=A,B} g_{ij} N_j |\psi_j|^2 + \delta_{Ai} (V_{P_1} + V_{P_2}) \right] \psi_i, \quad (2)$$

where $i = A, B$. Here the effective 2D nonlinear interaction coefficient is determined by the term $g_{ij} = \sqrt{\lambda/2\pi} 2\pi a_{ij} m_B/m_{ij}$, with a_{ij} the scattering length; $l = \sqrt{\hbar/m_B \omega}$ is the oscillator length; and $m_{ij} = m_i m_j / (m_i + m_j)$ denotes the reduced mass. The dimensionless Eq. (2) is written in terms of length scale l , timescale $1/\omega$, and energy scale $\hbar\omega$. The i th species wave function is normalized to $\int |\psi_i|^2 d^2 r = 1$.

In this paper we explore the turbulent phenomena that arise from the potentials formed by the rotating single paddle V_{P_1} and the double paddles $V_{P_1} + V_{P_2}$. The paddle potentials are maintained in the condensate for the time $0 \leq t \leq \tau$. Afterward, the paddle is linearly ramped off to zero over a time $t = \Delta\tau$, during which the relation

$$V_{P_{1(2)}} \rightarrow V_{P_{1(2)}} \left(1 - \frac{t - \tau}{\Delta\tau} \right)$$

holds in Eq. (1). Here we consider a binary BEC of ^{133}Cs (species A) and ^{87}Rb (species B) elements having different masses [66]. The numbers of atoms in species A and B are equal and we take $N_A = N_B = 60000$. The harmonic trap potential is designed to have a frequency of $\omega = 2\pi \times 30.832$ rad/s and the aspect ratio $\lambda = 100$. The intraspecies scattering lengths are $a_{AA} = 280a_0$ and $a_{BB} = 100.4a_0$, where a_0 is the Bohr radius [66]. The interspecies scattering length a_{AB} is chosen to reside in the miscible regime, as the relation of the miscibility, i.e., $a_{AB}^2 \leq a_{AA} a_{BB}$ [87], is maintained by the scattering lengths. We numerically solve the GP equation using the split-step Crank-Nicolson method [88]. The ground state of the system is generated by propagating the wave functions of the BEC in imaginary time. In order to inspect the dynamical evolution of the condensate, we utilize the ground state generated in imaginary time as the initial state and solve Eq. (2) in real time. Moreover, the system's initial state is prepared by placing a paddle-shaped stationary obstacle, as expressed in Eq. (1), in the component A. Our simulation runs on the spatial extent from $-20.48l$ to $20.46l$ in both the x and y directions with 2D 2048×2048 grid points.

III. CREATION OF VORTICES USING PADDLE POTENTIAL

As discussed in Ref. [63], an optical paddle potential can generate vortex in a BEC in a variety of ways, which include (i) rotating the paddle about a fixed center, (ii) moving the paddle about a fixed center, and (iii) both rotating and moving the paddle simultaneously in the BEC. Though we have considered only the rotation of the paddle potential to generate vortices in this work, we have employed both a single paddle and a double paddle potential to generate vortices. In particular, while the single paddle potential rotates in species

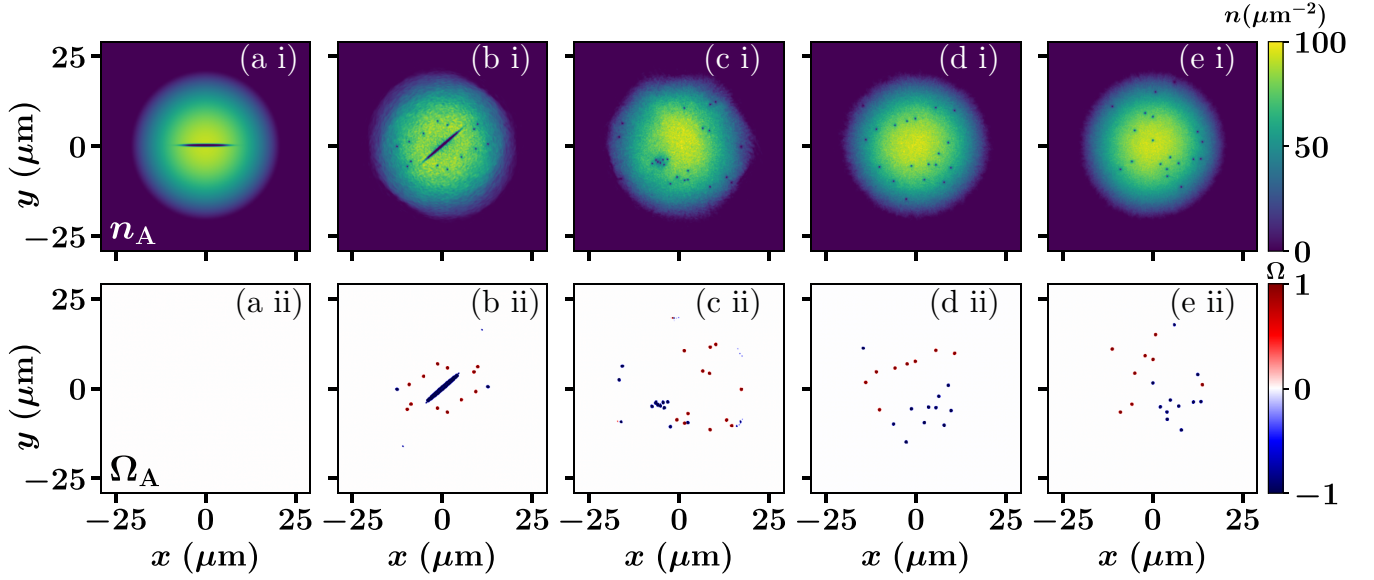


FIG. 1. Snapshot of (a i)–(e i) density n_A and (a ii)–(e ii) vorticity Ω_A profiles of species A at different instants of time: (a) $t = 0$ s, (b) $t = 0.08$ s, (c) $t = 0.43$ s, (d) $t = 2.01$ s, and (e) $t = 3.48$ s. The binary BECs are made of ^{133}Cs – ^{87}Rb atoms. An elliptical paddle potential characterized by the parameters $\eta = 0.05$ and $d = 0.1l$ is rotated with the angular frequency $\omega_p = \omega$ within species A (^{133}Cs) in order to trigger the dynamics. The color bars of the top and bottom rows represent the number density n in μm^{-2} and the vorticity Ω , respectively. The binary BECs are initialized in a two-dimensional harmonic potential with frequency $\omega/2\pi = 30.832$ Hz, $\lambda = 100$ and having intra- and interspecies scattering lengths $a_{AA} = 280a_0$, $a_{BB} = 100.4a_0$, and $a_{AB} = 0$. The number of atoms for both species are $N_A = N_B = 60\,000$.

A with the paddle center being located at $(x_p, y_p) = (0, 0)$, the double paddle potentials can rotate in either the same or opposite directions about their center at $(x_p, y_p) = (\pm r_A/4, 0)$, respectively, where $r_A = 6.1l$ is the root-mean-square radius of species A (for $a_{AB} = 0$). The parameters for the single paddle are $\eta = 0.05$ and $d = 0.1l$ and for the double paddle $\eta = 0.1$ and $d = 0.1l$ for both. These values determine the elliptical shape of the paddle according to Eq. (1). Moreover, we choose the peak strength of the paddles to be $V_0 = 10\mu_A$, where μ_A is the chemical potential of species A. As previously stated, after establishing the initial state with a stationary paddle, at $t = 0$, the paddle is rotated at a frequency of ω_p with full amplitude until the time $\tau = 40\omega^{-1} = 206$ ms and then ramped off to zero within $\Delta\tau = 10\omega^{-1}$. With the parameters ω_p , η , d , and V_0 , the paddle potentials may be externally regulated, allowing for control of the creation of a vortex or an antivortex in the BEC. In the BEC the presence of a vortex or an antivortex yields a finite amount of angular momentum, which can be expressed, for the i th species, as

$$L_z^i = -i \int \psi_i^* \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \psi_i dx dy. \quad (3)$$

To study the dynamical formation of the vortices, we measure the density-weighted vorticity of condensates as [31,89]

$$\Omega_i = \nabla \times \mathbf{J}_i \quad (4)$$

for a better spatially resolved measurement, with $\mathbf{J}_i = \frac{\hbar}{2m} (\psi_i \nabla \psi_i^* - \psi_i^* \nabla \psi_i)$ the probability current density. We remark that by using the Madelung transformation [90] $\psi_i = \sqrt{n_i} e^{i\phi_i}$, Eq. (4) can be cast into the form $\Omega_i = \nabla \times n_i \mathbf{u}_i$. Notably, the multiplication of condensate velocity \mathbf{u}_i with the density n_i ensures that we compute the vorticity of the i th species only in the region where the condensate is located.

A. Single paddle

This section examines the implications of a single paddle potential rotating with frequency ω_p about the center of species A. Although rotation orientation can be clockwise or counterclockwise, we focus on a paddle rotating in the clockwise direction. We note that the results obtained for the counterclockwise direction will be essentially identical to those obtained for the clockwise direction. First, we demonstrate the behavior of the BEC without interspecies interactions, the paddle potential does not influence species B and therefore species B remains unaltered during the dynamics. When the paddle rotates in species A, vortices and antivortices form around it. The number of vortices and antivortices in particular is strongly dependent on the rotation frequency. Figures 1(a) and 1(b) show the time evolution of the density and vorticity of species A at the paddle frequency $\omega_p = \omega$, the trap frequency.

The initial state of species A, with the paddle potential being elongated along the x axis, is shown in Fig. 1(a i). At $t = 0.08$ s, after the rotation of the paddle has been established [Fig. 1(b ii)], both vortices (red) and antivortices (blue) are generated in species A. In fact, a close inspection of Fig. 1(b ii) reveals that the vortex-antivortex structures are located symmetrically with respect to the paddle. Additionally, the number of vortices exceeds that of antivortices [Fig. 1(b ii)].

The generation of vortices and antivortices continues until 0.258 s, at which point the paddle potential vanishes. It is worth noting that the numbers of vortices and antivortices are nearly equal around this time. Following that, a considerable number of vortex-antivortex pairs decay due

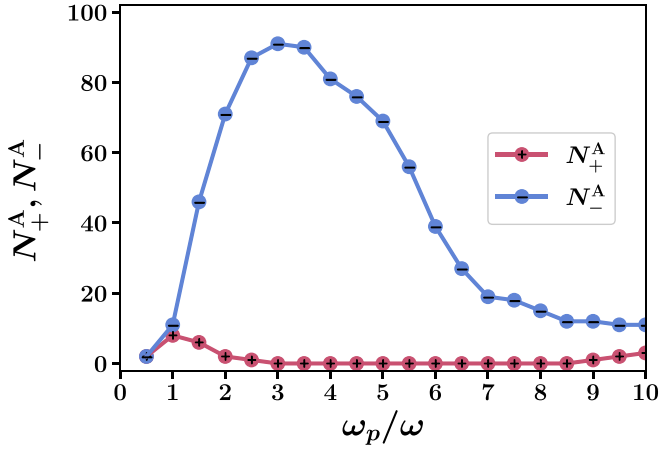


FIG. 2. Variation of the number of vortices N_+^A and antivortices N_-^A of species A at steady state as a function of the paddle frequency ω_p . The other parameters are the same as in Fig. 1.

to self-annihilation or drifting out of the condensate [see Fig. 1(d ii)]. However, some of the vortices and antivortices form vortex dipoles (vortex-antivortex pairs) or vortex pairs (pairs of identical charges) or vortex clusters, as depicted in Figs. 1(c ii)– 1(e ii). Without being annihilated, alongside the lone vortices and antivortices, these vortex dipoles, vortex pairs, and vortex or antivortex structures remain in the BEC for an extended period.

When the paddle frequency ω_p increases, the vortex complexes exhibit a distinct behavior. At steady state, the number of antivortices vastly exceeds that of vortices for a clockwise rotation of the paddle potential with frequencies $\omega < \omega_p < 7\omega$. After removing the paddle potential, vortex-antivortex annihilation begins, finally eliminating all vortices from the condensate. In Fig. 2 we show the number of vortices (N_+^A) and antivortices (N_-^A) as a function of the rotation frequency ω_p . The imbalance $|N_+^A - N_-^A|$ is almost zero until $\omega_p = \omega$, indicating that an equal number of vortex-antivortex pairs are generated. Afterward, such an imbalance gradually increases and becomes maximum at $\omega_p \approx 3\omega$ when only the antivortices (having negative circulation) exist in the system. For $\omega_p > 3\omega$, as it is evident from Fig. 2, both the total number of vortices $N_+^A + N_-^A$ and the imbalance decrease with ω_p .

Figures 3(a) and 3(b) show vorticity profiles Ω_A of species A for $\omega_p = 3\omega$ and 10ω , respectively, at $t = 3.5$ s. Notably, the largest number of antivortices survives for $\omega_p = 3\omega$ [Fig. 3(a)] and this number falls as ω_p increases. As ω_p increases beyond $\omega_p > 7\omega$, only a few of both vortices and antivortices survive due to a higher annihilation rate (per unit number of vortices and antivortices) of vortex-antivortex pairs [Fig. 3(b)]. As a result, the system has almost no vortex or antivortex structure in the long-time dynamics (density profiles not shown here for brevity), imparting much less angular momentum to the condensate, as shown in Figs. 3(c) and 5.

The above scenario of nonlinear structure formations in species A can further be elucidated by invoking the angular momentum of species A, L_z^A . The time evolutions of $L_z^A(t)$ for various ω_p are displayed in Fig. 3(c). The $L_z^A(t)$ remains negative throughout the time evolution, indicating the surplus

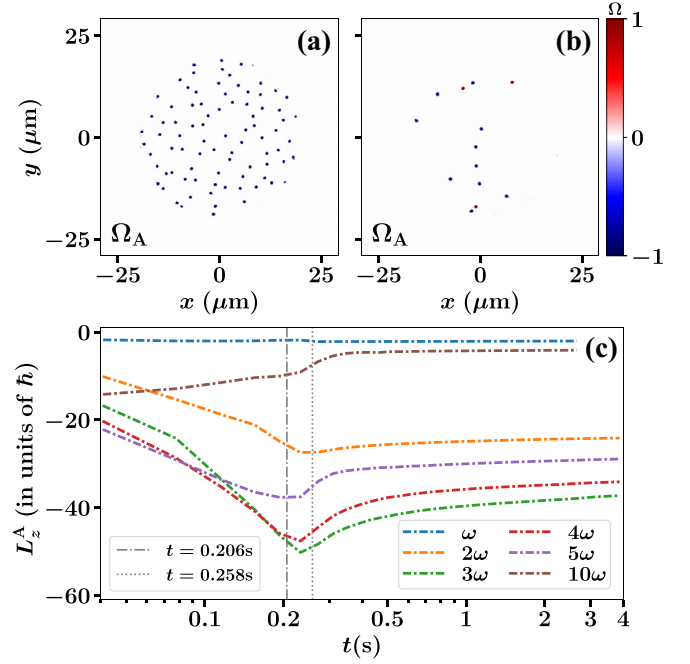


FIG. 3. Snapshots of the vorticity profiles of species A (Ω_A) taken at $t = 3.5$ s for two different paddle frequencies (a) $\omega_p = 3\omega$ and (b) $\omega_p = 10\omega$ of rotation of the paddle potential. Shown also is (c) the time evolution (logarithmic scale) of angular momentum (L_z^A) for different values of ω_p (see the legends). The vertical lines in (c) represent the times when the amplitude of the paddle started to ramp off and become zero, respectively. The color bar of (a) and (b) represents the vorticity Ω . The other parameters are the same as in Fig. 1.

of antivortices. For $\omega_p = \omega$, $L_z^A(t)$ remains nearly constant. The $|L_z^A(t)|$ monotonically increases at the early stage of the dynamics, reaches a maximum at a time within the time interval τ , and then decreases to reach a stationary value in the long-time dynamics. The maximum value of $|L_z^A(t)|$ is the largest for $\omega_p = 3\omega$, a result which emanates from the maximum number of antivortices displayed in Fig. 3(a). For larger ω_p , the net angular momentum imparted to the condensate by the generated vortex-antivortex drastically diminishes, indicating a smaller imbalance between vortex and antivortex numbers [Fig. 3(b)].

The existence of paddle potential in species A affects species B for nonzero interspecies interactions a_{AB} . For a strong enough interaction, the repulsive paddle potential on species A effectively acts as an attractive potential on species B. Due to rotation of this attractive potential, vortices and antivortices are generated in species B [91] (see Appendix B). In particular, vortices and antivortices are created in species B and their number can be controlled by a_{AB} . Additionally, the null density region at the vortex or antivortex site in one species is filled by the other species' localized density hump. Figures 4(a) and 4(b) show the density pattern at $t = 3.5$ s for the interspecies interaction with $a_{AB} = 80a_0$ and $\omega_p = 3\omega$. The other parameters, i.e., $\eta = 0.05$, $d = 0.1l$, and $V_0 = 10\mu_A$, are similar to the $a_{AB} = 0$ case. Notably, the scattering lengths explored here ensure that the condensates are miscible, allowing us to directly analyze the role of

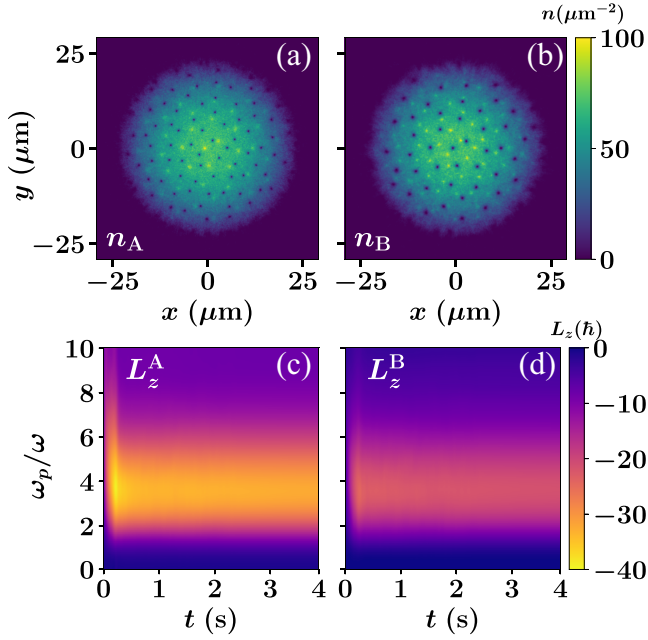


FIG. 4. Snapshots of density profiles of (a) species A (n_A) and (b) species B (n_B), with interspecies scattering length $a_{AB} = 80a_0$ and $\omega_p = 3\omega$ at $t = 3.5$ s. Also shown are the variations of angular momentum (c) L_z^A and (d) L_z^B varying with paddle frequency ω_p and time t . The color bars represent the number densities (a) n_A and (b) n_B in μm^{-2} and (c) and (d) the angular momentum in units of \hbar .

mean-field coupling. Moreover, the paddle potential in species A performs clockwise rotation. Both species accommodate only antivortices solely in the long-time dynamics, which is similar to the noninteracting scenario. This behavior implies that within a particular frequency range, a cluster of identical vortices forms that is entirely determined by the direction of paddle rotation, regardless of the species interaction. Furthermore, it is worth mentioning that species A possesses a smaller healing length due to the larger mass and the intraspecies interaction. This makes the vortices of species A smaller in size compared to those of species B.

The creation and stability of vortex complexes in the presence of nonzero interspecies interaction can be further comprehended by evaluating the angular momentum L_z^i of both species. The time evolutions of L_z^A and L_z^B as a function of ω_p are shown in Figs. 4(c) and 4(d), respectively. A close inspection indicates that the angular momenta of both species are maximum at $\omega_p \approx 3.5\omega$, similar to that in the $a_{AB} = 0$ case [Figs. 4(c), 4(d), and 5]. For $\omega_p > 7\omega$, L_z^i becomes very small due to the higher annihilation rate of the vortex-antivortex pairs. The L_z^A is more pronounced than L_z^B , indicating that the antivortex number is always high in species A. Most significantly, we find that the frequency response to the angular momentum follows a skewed normal distribution,¹ as depicted in Fig. 5, with the maximum of the distribution

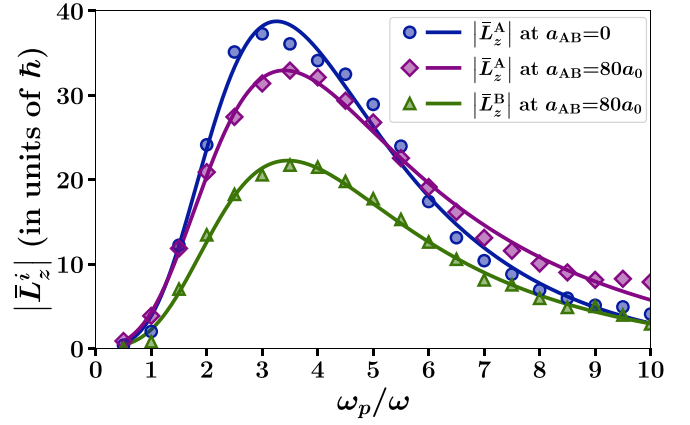


FIG. 5. Variation of the absolute value of the time-averaged angular momentum $|\bar{L}_z^i|$ of species A at $a_{AB} = 0$ and $80a_0$, and $|\bar{L}_z^B|$ of species B at $a_{AB} = 80a_0$ as a function of paddle frequency (see the legend). The markers and solid curves show the values of angular momentum from the simulations and the fittings with skewed normal distribution, respectively. Here the paddle configuration is the same as that described in Fig. 1.

occurring at $\omega_p \approx 3.2\omega$ for $a_{AB} = 0$ and at $\omega_p \approx 3.45\omega$ for $a_{AB} = 80a_0$. Given that, at higher paddle frequencies ($\omega_p \gtrsim 7\omega$), the annihilation process does not completely remove the vortices and antivortices from the condensates, leaving a small but finite angular momentum that leads to the long tail on the side of $\omega_p > 3\omega$, this distribution is quite expected. Finally, let us comment that both species can end up with nearly equal angular momenta in the strong-interaction limit $a_{AB} \simeq 150a_0$. However, a detailed study of this regime is beyond the scope of the present paper.

The angular momentum achieves its maximum value for $\omega_p \approx 3\omega - 4\omega$, which can be explained by examining the sound velocity of the condensates. There are two distinct sound velocities for a binary BEC, namely, c_+ and c_- , representing the density and spin sound velocity, respectively [92,93]. These two sound velocities can be expressed as

$$c_{\pm}^2 = \frac{1}{2} [c_A^2 + c_B^2 \pm \sqrt{(c_A^2 - c_B^2)^2 + 4c_{AB}^4}], \quad (5)$$

where $c_i = \sqrt{g_{ii}n_i/m_i}$ and $c_{AB}^4 = g_{AB}^2 n_A n_B / m_A m_B$, with n_i representing the peak density of the i th condensate. For the noninteracting case with the peak density of $2.88 \times 10^{14} \text{ cm}^{-3}$, we determine the sound velocity of species A (c_s) to be $c_A = 2.36 \text{ mm/s}$ based on the averaged peak density $n_A/2$ (see also Refs. [93,94]). The velocity of rotating paddle is $v = a\omega_p$, where $a = d/\eta$ is the semimajor axis of the paddle. With the values of parameters d , η , and $\omega_p = 3\omega$ considered herein, we find that the paddle velocity amounts to 2.26 mm/s , which is very close to c_A . This close proximity of paddle and sound velocities results in the maximum amount of angular momentum near the paddle frequency $\omega_p = 3\omega$. For the interacting case with $a_{AB} = 80a_0$ we find the density

¹The skewed normal probability density function is given by $f(x) = \frac{2}{\sigma} \phi(\frac{x-\mu}{\sigma}) \Phi(\alpha \frac{x-\mu}{\sigma})$, where $\phi(x) = e^{-x^2/2}/\sqrt{2\pi}$ is the standard normal probability density function and $\Phi(x) = \int_{-\infty}^x \phi(t) dt$ is

the cumulative distribution function. In addition, σ , μ , and α are the standard deviation, mean, and skewed parameter, respectively. For $\alpha = 0$, $f(x)$ becomes a normal distribution.

sound velocity to be $c_+ = 2.56$ mm/s with the peak densities $n_A = 2.21 \times 10^{14}$ cm $^{-3}$ and $n_B = 2.27 \times 10^{14}$ cm $^{-3}$. The value of c_+ is very close the paddle velocity, which amounts to 2.63 mm/s, corresponding to $\omega_p = 3.5\omega$, where the absolute angular momenta of species A and B take maximum values (Fig. 5). Note that vortex generation starts when the paddle rotation velocity v surpasses a critical velocity, which in our case is around $0.25c$, where c is the sound velocity. As already discussed, with the increase of paddle frequency, and hence v , a vortex-antivortex imbalance is created, increasing the angular momentum. When v exceeds the sound velocity c the drag force becomes very pronounced, resulting in stronger dissipation in the condensates [91,95–97]. This dissipation causes the total vortex number and the vortex-antivortex imbalance to decrease, thus creating a peak of $|L_z^{A(B)}|$ at $v \approx c$.

B. Double paddle

After discussing the impact of a single paddle potential, we now look at a more complex scenario involving two paddle potentials. In this situation one can have two distinct scenarios depending upon the relative orientation of the paddle potentials. Here we attempt to answer the question of how the addition of a second paddle potential and its relative rotational orientation relative to the first one alters the vortex structures and angular momentum of the system when compared to the case of a single paddle. With that intent, we consider two paddles rotating in species A and having a center at $(\pm r_A/4, 0)$. Moreover, we choose $\eta = 0.1$ and keep d the same as in the single paddle case. Depending on the relative rotational orientation of the two potentials, different dynamical behavior can emerge. When both paddles rotate in the same direction, the effects are similar to those mentioned previously for a single paddle. To substantiate the above statement, we demonstrate the variation of angular momentum with time (logarithmic scale) in Fig. 6 for clockwise rotation of the paddle potential with interspecies scattering length $a_{AB} = 80a_0$. For paddle frequency close to $\omega_p = 4\omega$, L_z^A and L_z^B are most prominent; the corresponding antivortex structures generated in species A and species B are shown in Figs. 6(b) and 6(c), respectively. However, we should note that as we halved the paddle length with respect to the single paddle case and increased the paddle number to two, the maximum angular momentum generated in the system is reduced for the double corotating paddle than for the single paddle case. For example, $L_z^A \approx -40\hbar\omega$ at $\omega_p = 3\omega$ (Fig. 3), whereas $L_z^A \approx -9\hbar\omega$ at the same ω_p (Fig. 6) for the double paddle potentials. Surprisingly, a more interesting case occurs when one paddle rotates in the clockwise direction and the other in the counterclockwise way [Fig. 7(a i) and 7(b i)]. Because the rotational directions of the paddles are opposite, each paddle contributes an equal number of vortices of the opposite sign [see Figs. 7(a iii) and 7(b iii)]. In the long-term dynamics of both species, this equal distribution of vortices and antivortices leads to a high rate of annihilation, meaning that just a few vortices and antivortices survive in the long-time dynamics [Figs. 7(a iii)-(a v) and 7(b iii)-(b v)]. To further appreciate the previous argument, we calculate the time average of the angular momentum defined as $\bar{L}_z^i = \int L_z^i dt / \int dt$ for different rotation frequencies ω_p of the double paddle poten-

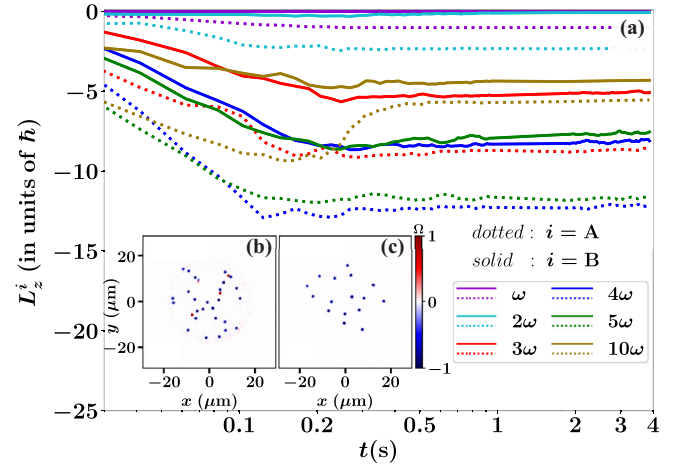


FIG. 6. (a) Time evolution (logarithmic scale) of angular momentum L_z^i for species $i = A, B$ at interspecies interaction $a_{AB} = 80a_0$ for different paddle frequencies $\omega_p = \omega, 2\omega, 3\omega, 4\omega$, and 10ω . Here two identical paddles rotate in species A in the clockwise direction. The insets depict the snapshot of vorticity profiles of (b) species A (Ω_A) and (c) species B (Ω_B) at $t = 3.5$ s with $\omega_p = 4\omega$.

tials [see Fig. 8(a)]. For $\omega_p < \omega$, the \bar{L}_z^i remains zero. Within $2\omega \lesssim \omega_p \lesssim 5\omega$, both L_z^A and L_z^B shows extremely fluctuating behavior with respect to ω_p . Recall that this is also the frequency region where a maximum number of vortex-antivortex creations occur. The vortex and antivortex either annihilate each other or one of them drifts away from the condensate, leading to a finite imbalance of the vortex-antivortex number. The finite imbalance between the vortex-antivortex numbers in the dynamics can result in a finite angular momentum of either positive or negative sign, a behavior which is highly fluctuating with respect to ω_p . The fluctuation is somewhat reduced in the frequency range $\omega_p > 5\omega$. Here, for increasing ω_p , the annihilation mechanism becomes the dominant mechanism responsible for reducing both vortices and antivortices, which exist in nearly equal numbers. Consequently, the imbalance between the vortex and antivortex numbers decreases, leading to a relatively small fluctuation in $\bar{L}_z^{A(B)}$.

Additionally, we calculate the standard deviation of time-averaged angular momentum $\bar{L}_z^{A(B)}$ using data from five different runs with added noise for each run [see Fig. 8(b)]. The corresponding standard deviation $\sigma_{A(B)}$ for species A (species B) is defined as

$$\sigma_{A(B)}^2 = \frac{\sum_j (\bar{L}_z^{A(B),j} - \bar{L}_{z,\text{mean}}^{A(B)})^2}{N_s}, \quad (6)$$

where $\bar{L}_{z,\text{mean}}^{A(B)} = \sum_j \bar{L}_z^{A(B),j} / N_s$ and N_s is the number of data sets, each with different initial random noise. Figure 8(b) depicts that the fluctuations are high in the frequency range $2\omega < \omega_p < 5\omega$. Furthermore, the fact that the fluctuation in $\bar{L}_z^{A(B)}$ is indeed due to the fluctuation in the vortex-antivortex imbalance $|N_+^{A(B)} - N_-^{A(B)}|$ can be evinced from Fig. 8(c). Finally, before closing this section let us also remark on another interesting observation from our study that the $|N_+^{A(B)} - N_-^{A(B)}|$ scales as $\sqrt{|\Gamma_{A(B)}|}$, where the quantity $\Gamma_{A(B)} = \int \Omega_{A(B)} dx dy$ represents the net circulation of the

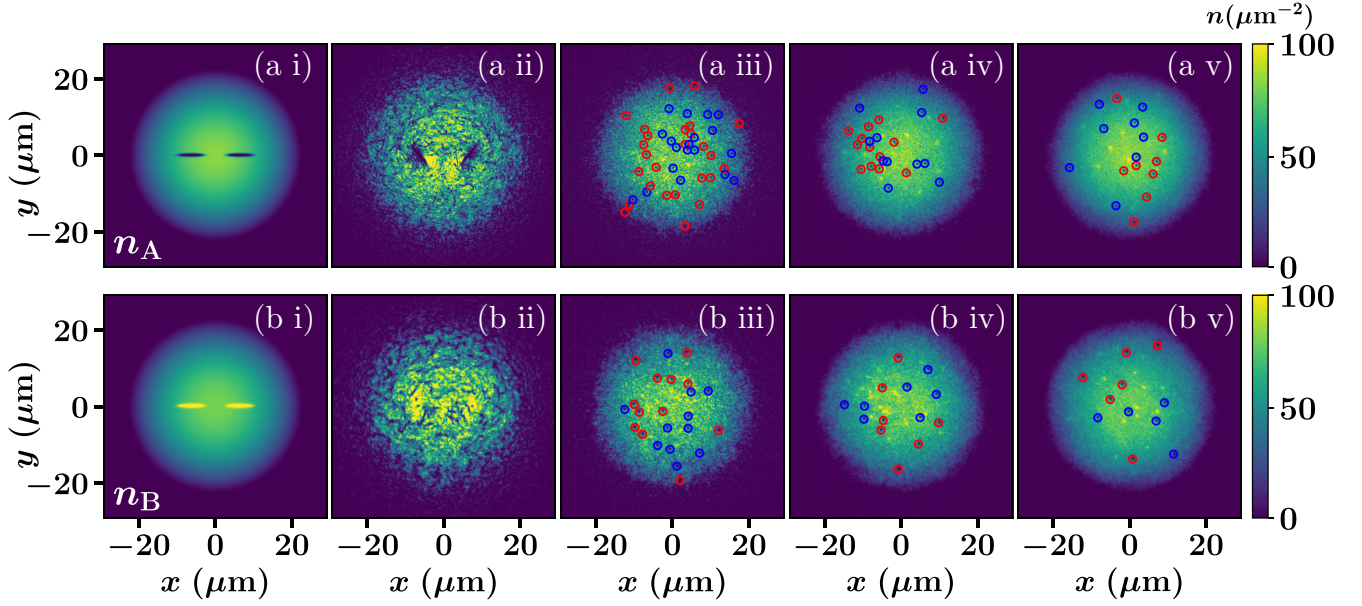


FIG. 7. Snapshot of the densities of (a) species A, n_A , and (b) species B, n_B , with interspecies interaction $a_{AB} = 80a_0$ at different instants of time: (a i) and (b i) $t = 0$ s, (a ii) and (b ii) $t = 0.04$ s, (a iii) and (b iii) $t = 0.4$ s, (a iv) and (b iv) $t = 1.0$ s, and (a v) and (b v) $t = 2.0$ s. Two elliptic paddles characterized by the parameters $\eta = 0.1$ and $d = 0.1l$ and rotating oppositely to each other with frequency $\omega_p = 5\omega$ within species A (^{133}Cs) are used to trigger the dynamics. The color bars represent the number density n in μm^{-2} . The vortices (antivortices) are marked with red (blue) in (a iii)–(a v) and (b iii)–(b v) [vortex identification is not done in (a ii) and (b ii) due to the large number].

vortex clusters [see Fig. 8(d), where we demonstrate $\sqrt{|\Gamma_{A(B)}|}$ as a function of ω_p].

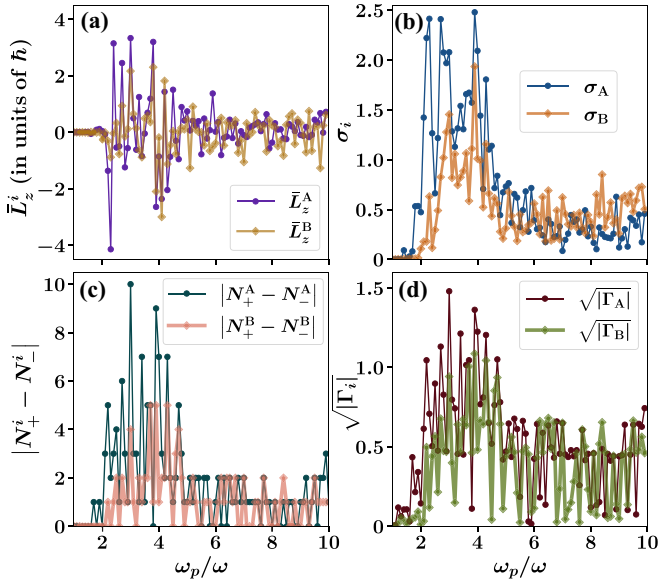


FIG. 8. Variation of (a) time-averaged angular momentum \bar{L}_z^i , (b) its standard deviation σ_i , (c) vortex imbalance $|N_+^i - N_-^i|$ at steady state, and (d) $\sqrt{|\Gamma_i|}$ (Γ_i is the circulation at steady state), as a function of ω_p for the i th species ($i = A, B$). The scattering lengths are given by $a_{AA} = 280a_0$, $a_{BB} = 100a_0$, and $a_{AB} = 80a_0$. The dynamics has been triggered by employing two paddle potentials rotating counterclockwise. The paddle configuration is described in Fig. 7.

IV. ENERGY SPECTRA

To better understand the system when it is subjected to a paddle potential, we compute its kinetic energy spectrum, whose scaling laws provide insight into the development of quantum turbulence in the system. Note that these scaling laws have already been reported in the literature [3,9,77]. However, the primary objective here is to determine how the onset of turbulence depends on paddle frequencies or under what parameter regime the binary condensate system should develop turbulent features.

In order to do so we decompose the kinetic energy into compressible and incompressible parts associated with sound waves and vortices, respectively [8,98]. The energy decomposition is performed by defining a density-weighted velocity field, which reads $\sqrt{n_i}\mathbf{u}_i$, with $\mathbf{u}_i = \frac{\hbar}{m}\nabla\theta_i$, where n_i and θ_i are the position-dependent condensate density and phase of the i th species, respectively. The velocity field is separated into a solenoidal (incompressible) part \mathbf{u}_i^{ic} and an irrotational (compressible) part \mathbf{u}_i^{c} such that $\mathbf{u}_i = \mathbf{u}_i^{\text{ic}} + \mathbf{u}_i^{\text{c}}$, obeying $\nabla \cdot \mathbf{u}_i^{\text{ic}} = 0$ and $\nabla \times \mathbf{u}_i^{\text{c}} = 0$. Once these velocity fields are calculated following Refs. [8,31,40,89], we can calculate the incompressible energy $\mathcal{E}_i^{\text{ic}}$ and compressible energy \mathcal{E}_i^{c} as

$$\mathcal{E}_i^{\text{ic(c)}} = \frac{1}{2} \int n_i |\mathbf{u}_i^{\text{ic(c)}}|^2 dx dy. \quad (7)$$

Afterward, the compressible and incompressible energy spectra for the i th species can be calculated as

$$E_i^{\text{ic(c)}}(k) = \frac{k}{2} \sum_{q=x,y} \int_0^{2\pi} |F_q(\sqrt{n_i}\mathbf{u}_{q,i}^{\text{ic(c)}})|^2 d\phi, \quad (8)$$

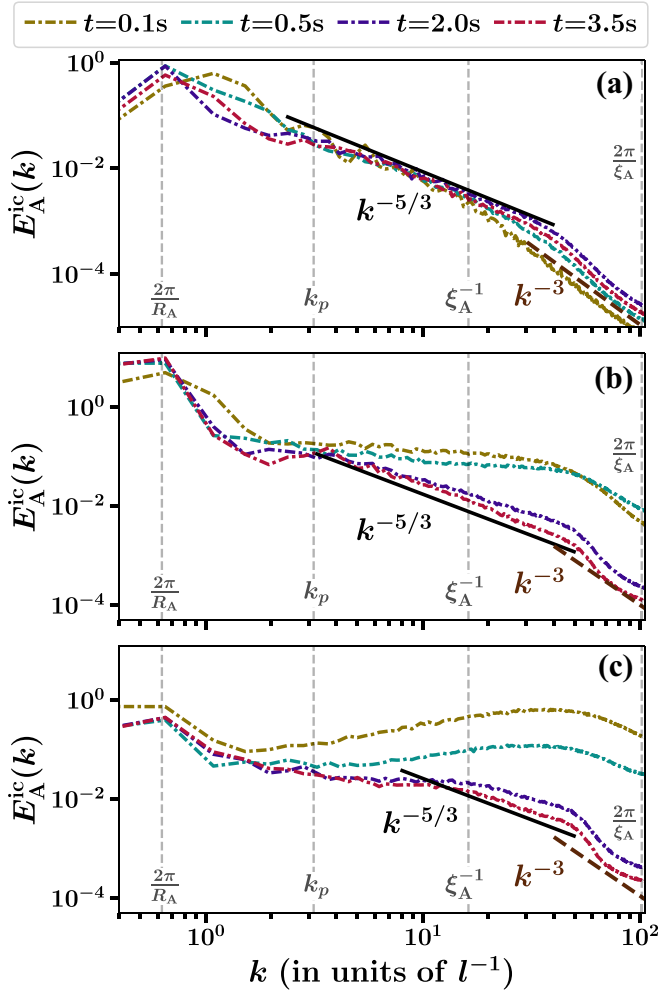


FIG. 9. Incompressible kinetic energy spectra of species A, $E_A^{\text{ic}}(k)$, at different time instants (see the legend) for different paddle frequencies (a) $\omega_p = \omega$, (b) $\omega_p = 3\omega$, and (c) $\omega_p = 8\omega$. The solid and dashed lines represent the slopes of $k^{-5/3}$ and k^{-3} , respectively. The interspecies scattering length $a_{AB} = 0$. The dashed vertical lines define the positions at $k = 2\pi/R_A$ ($R_A = 10l$ is the Thomas-Fermi radius), k_p ($2\pi/a$, with a the semimajor axis of the paddle), ξ_A^{-1} , and $2\pi/\xi_A$, respectively. Here the healing length ξ_A of species A is $0.062l$. Except for ω_p , all other parameters are the same as in Fig. 1.

where $F_q(\sqrt{n_i}\mathbf{u}_i^{\text{ic}(\text{c})})$ denotes the Fourier transformation of $\sqrt{n_i}\mathbf{u}_{q,i}^{\text{ic}(\text{c})}$, corresponding to the q th component of $\mathbf{u}_i = (u_{x,i}, u_{y,i})$.

We present incompressible energy spectra $E_A^{\text{ic}}(k)$ of species A in Fig. 9 at various time instants and frequencies ω_p corresponding to the single paddle case at $a_{AB} = 0$. Due to no interspecies interaction, species B is not impacted by the paddle potential, which allows us to focus on species A. For $\omega_p = \omega$, $E_A^{\text{ic}}(k)$ attains a stationary state at early time ($t = 0.1$ s) and maintains it until $t = 3.5$ s, as evidenced from Fig. 9(a). Moreover, $E_A^{\text{ic}}(k)$ exhibits a k^{-3} power law in the region $30 \lesssim k \lesssim 100$ and a $k^{-5/3}$ power law in the region $2 \lesssim k \lesssim 30$. The $k^{-5/3}$ and k^{-3} power laws are associated with the inertial range of energy cascade and internal structure of the vortex core, respectively [43,48]. Note that for $\omega_p = \omega$ vortex pairs and a vortex dipole are visible in Figs. 1(c ii)–

1(e ii) [44,56,82,99]. Surprisingly, for the frequency $\omega_p = 3\omega$, where only the same-sign vortices dominate, we notice that the k^{-3} spectrum develops for a very narrow k range in our system [see Fig. 9(b)] and after that ($\omega_p > 3\omega$) the spectra deviate from the -3 scaling law [see Fig. 9(c)]. However, while the $k^{-5/3}$ spectrum develops in the long-time dynamics for a wide k range, it does not emerge in the early-time dynamics [Fig. 9(b)]. Finally, we notice that the k ranges where the spectra follow $-5/3$ scaling become narrower with an increase of ω_p [see Fig. 9(c)]. This behavior is expected since the system at $\omega_p \gtrsim 7\omega$ is primarily governed by the generation of a huge number of sound waves caused by the rapid annihilation of the vortices and antivortices. Another interesting observation from our study is that the most extended inertial range of the energy cascade occurs at the paddle frequency $\omega_p \approx 3\omega$, where both species hold the maximum amount of angular momentum. The positions of the inertial ranges vary depending on ω_p . For low $\omega_p (\simeq \omega)$ and high $\omega_p (> 5\omega)$ the inertial ranges occur at lower and higher wave numbers than the inverse of the healing length ξ_A^{-1} , respectively. For the intermediate frequencies it occurs on both sides of $k = \xi_A^{-1}$ (see Fig. 9).

Next we turn to the scenario of finite interspecies interaction characterized by $a_{AB} = 80a_0$ and investigate whether species B produces the power-law spectra in the incompressible sector of its energy (see Fig. 10). We note that $k^{-5/3}$ and k^{-3} power laws are manifested in a similar manner within the ranges $1 \lesssim k \lesssim 10$ and $20 \lesssim k \lesssim 100$, respectively, for $\omega_p = \omega$. Like in species A, the ranges of the $-5/3$ scaling law in species B become narrower with the increase of paddle frequency after $\omega_p \approx 3\omega$ and the positions of the inertial ranges change with ω_p . Moreover, species B contains vortices with larger cores than those of species A. At large ω_p the high-momentum acoustic waves are less in species B than in species A because of the reduced strength of paddle potential at lower interspecies interaction. This makes incompressible kinetic energy at high momentum more discernible in species B than species A. Consequently, the k^{-3} scaling law, which is related to the vortex core structure, appears in $E_B^{\text{ic}}(k)$ for the frequency range $\omega \lesssim \omega_p \lesssim 10\omega$. We note that for this condition ($a_{AB} = 80a_0$) the $E_A^{\text{ic}}(k)$ does not demonstrate different behavior with regard to ω_p when compared to that of the $a_{AB} = 0$ case (hence not shown here for brevity).

We now explain the compressible kinetic energy spectra [48,89,100] $E_A^{\text{c}}(k)$ of species A and $E_B^{\text{c}}(k)$ of species B for a few representative cases subjected to the single paddle potential, shown in Fig. 11. To begin, in the case of $\omega_p = \omega$, we notice that a power-law region with $E_A^{\text{c}}(k) \propto k$ develops in the low- k region of the spectrum, a relation that expresses the frequencies of Bogoliubov's elementary excitations at low wave number [Fig. 10(a)]. The spectrum reaches a maximum near k ranging from 20 to 40 (the peak positions differ for different time instants until the system reaches an equilibrium) before rapidly dropping. As the paddle frequency increases ($\omega_p \gtrsim 7\omega$), the spectrum $E_A^{\text{c}}(k)$ follows a power-law exponent of $-7/2$ at large k , as shown in Fig. 11(b) for a specific $\omega_p = 8\omega$. Notably, this scaling is associated with superfluid turbulence of equilibrium sound waves, which has also been reported in Refs. [48,77,101]. Interestingly enough, for $a_{AB} = 80a_0$, we observe the scaling law $k^{-3/2}$ in the intermediate- k range for the frequency $\omega_p \gtrsim 5\omega$ as shown in Fig. 11(c) for $\omega_p = 6\omega$.

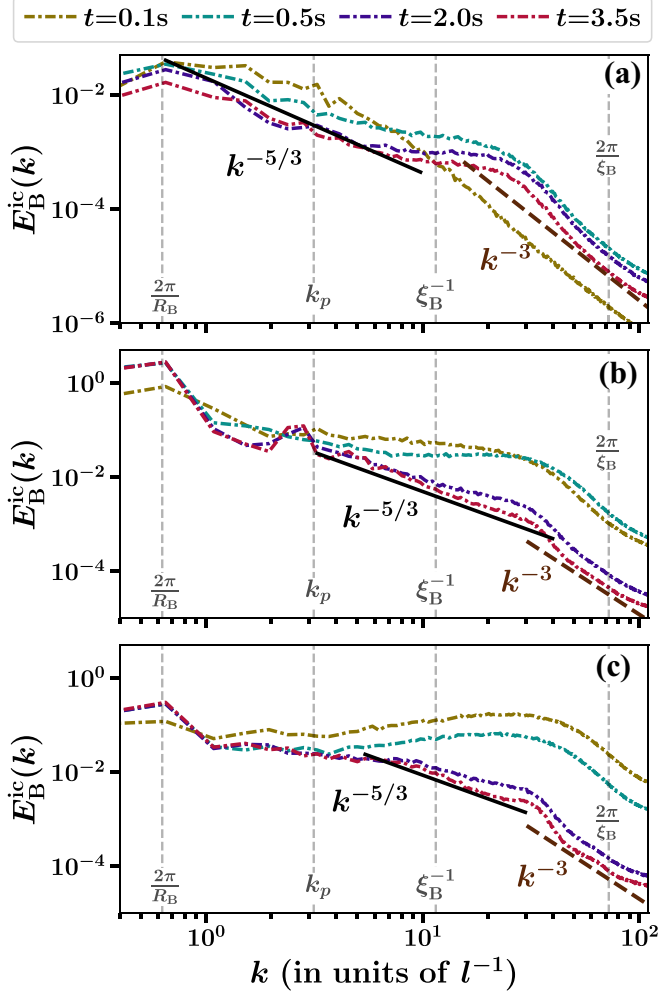


FIG. 10. Incompressible kinetic energy spectra of species B, $E_B^{\text{ic}}(k)$, at different time instants (see the legend) for different paddle frequencies (a) $\omega_p = \omega$, (b) $\omega_p = 3\omega$, and (c) $\omega_p = 8\omega$. The solid and dashed lines represent the slopes of $k^{-5/3}$ and k^{-3} , respectively. The interspecies scattering length $a_{AB} = 80a_0$. The dashed vertical lines define the positions at $k = 2\pi/R_B$ ($R_B \approx 10l$ is the Thomas-Fermi radius), k_p ($2\pi/a$, with a the semimajor axis of the paddle), ξ_B^{-1} , and $2\pi/\xi_B$. Here the healing length ξ_A of species B is $0.099l$. Except for ω_p , all other parameters are the same as in Fig. 4.

This power law which appears at k higher than the driving wave number k_p reveals signatures of weak wave turbulence [77,102]. Let us note that the acoustic disturbance must not be strong for the manifestation of this scaling [102]; hence, it is more apparent in species B under weaker-interspecies-interaction regimes, while huge acoustic disturbances prevent the development of the same scaling in species A. We observed for strong enough interspecies interactions (e.g., $a_{AB} = 140a_0$) that the $-3/2$ scaling law disappears from species B (not shown). However, a detailed discussion of this is beyond the scope of the present paper.

V. CONCLUSION

We have explored the phenomenon of nonlinear structure formations and their dynamics using optical paddle potential

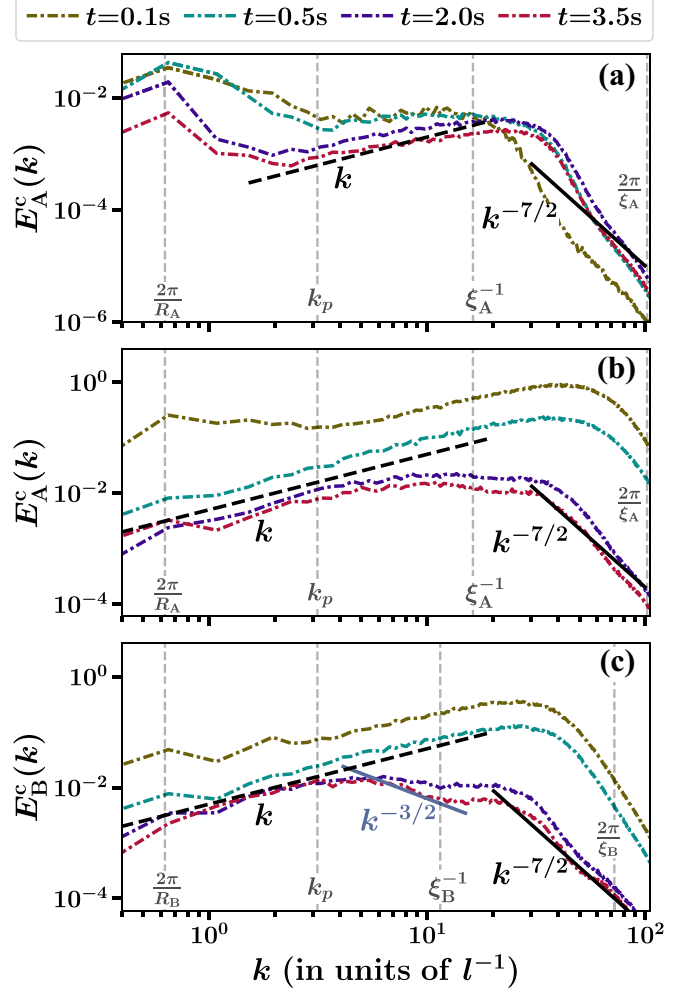


FIG. 11. Compressible kinetic energy spectra of species A, $E_A^c(k)$, at (a) $\omega_p = \omega$ and (b) $\omega_p = 8\omega$, without interspecies interaction $a_{AB} = 0$ and of species B, $E_B^c(k)$, at (c) $\omega_p = 6\omega$ with $a_{AB} = 80a_0$ at different time instants (see the legend). Black solid and dashed lines represent the slopes of $k^{-7/2}$ and k , respectively, and the blue solid line in (c) represents the slope of $k^{-3/2}$. The dashed vertical lines of (a) and (b) and of (c) are described in Figs. 9 and 10, respectively. All the parameters are the same as discussed in Sec. III A.

in a binary BEC composed of two distinct atomic elements. One of the species (species A) experiences rotating single or double paddle potentials, while the other species (species B) is only influenced by the interspecies contact interaction. The paddles are rotated for a finite amount of time, resulting in the creation of vortices. In the long-time dynamics, the sign and number of vortices are dependent on the frequency and orientation of paddle rotation. Additionally, we discussed the effect of paddle rotation on other species. We observed many diagnostics to obtain insight into the dynamics, including the density, the vorticity, the z component of the angular momentum, and the species' compressible and incompressible energy spectra.

Clusters of positive and negative vortices emerge within the system when a single paddle potential is rotated with a low rotational frequency. Interestingly, when the frequency

was gradually increased, we observed a transition to a regime dominated by same-sign vortices, with species A gaining the maximum angular momentum. At larger paddle frequencies, the annihilation of vortex-antivortex pairs becomes considerable, reducing the system's total vortical content. The behavior mentioned above holds for species A in both the absence and presence of interspecies interaction. Interestingly enough, when interspecies contact is enabled, species B exhibits similar dynamical behavior. However, species B has a substantially lower vortex and angular momentum than species A in the miscible regime. When two paddle potentials are employed, their relative orientations become crucial in determining the vortical content of species A. For the paddles rotating with the same orientation, the behavior is almost identical to the single paddle applied to species A. However, when the two paddles rotate oppositely to each other, due to the almost equal number of vortex-antivortex structures formed regardless of the rotation frequency of the paddles, the net angular momentum imparted to the system during the long-time dynamics fluctuates about zero.

Following that, we explored the system's dynamics by invoking the compressible and incompressible kinetic energy spectra. However, a key highlight of this work is its examination of various power-law scalings of the kinetic energy spectra. We observed $-5/3$ and -3 power-law scaling in the low- and high-wave-number regimes of the incompressible energy spectrum, respectively, in the low-rotation-frequency regime, where we saw clusters of identical-sign vortices. These scalings provided evidence of the development of quantum turbulence in our system at low frequencies. However, analogous scaling was not apparent in the incompressible energy spectrum as the rotation frequency increases.

There are many research directions to be pursued as a future research endeavor. One would be to extend the present work in the presence of finite temperature [103]. Extending the present work to the three-dimensional setup and exploring the corresponding nonlinear defect formations would be equally interesting [38,104–106]. Another vital prospect would be to employ dipolar BECs to inspect the impact of the long-range interaction [107]. Finally, the investigations discussed previously would be equally fascinating at the beyond-mean-field level, where significant correlations between particles exist [108].

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APPENDIX A: MASS-BALANCED BINARY BOSE-EINSTEIN CONDENSATE

In the main text we focused our discussion on the mass-imbalanced binary BECs since such a system is the most suitable for creating species-selective potential by the tune-out approach. To examine to what extent the phenomenology dif-

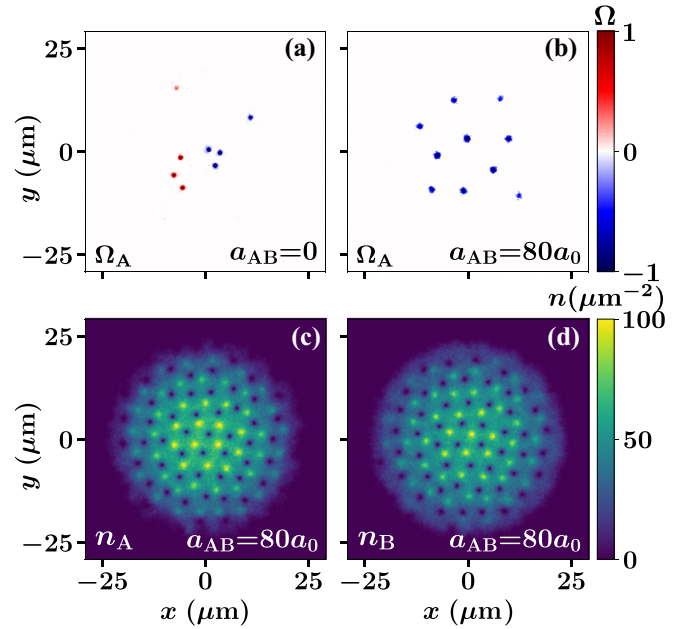


FIG. 12. Vorticity Ω_A of species A at scattering lengths (a) $a_{AB} = 0$ and (b) $a_{AB} = 80a_0$ at $\omega_p = \omega$. Also shown are the density profiles (c) n_A and (d) n_B of species A and B, respectively, for $a_{AB} = 80a_0$ and $\omega_p = 3\omega$. All the snapshots shown are at $t = 3.5$ s. The binary BEC is realized at two hyperfine levels of ^{87}Rb atoms. To trigger the dynamics, an optical paddle potential is rotated in species A. The color bars represent (a) and (b) the vorticity Ω_A and the number densities (c) n_A and (d) n_B in μm^{-2} .

fers from a system of mass-balanced system, here we consider a binary BEC composed of ^{87}Rb atoms with two different hyperfine levels [72,109]. We take an equal number of atoms of both species, namely, $N_A = N_B = 60\,000$. The intraspecies scattering lengths are $a_{AA} = 95.4a_0$ and $a_{BB} = 100.4a_0$ [110]. All other parameters, such as the trapping configuration and paddle configuration, are the same as in the single paddle case of Sec. III. We examine the creation of vortices using a single rotating paddle with $a_{AB} = 0$ and $80a_0$. For paddle frequency $\omega_p = \omega$ we observe clustering of opposite-sign vortices at $a_{AB} = 0$ [see Fig. 12(a)]. However, at higher interspecies scattering length $a_{AB} = 80a_0$, the clustering is not visible [Fig. 12(b)]; instead we observe a sparse cluster composed of same-signed vortices. The number of vortices organized into lattice structure increases as we increase the paddle frequency ($2\omega \lesssim \omega_p \lesssim 5\omega$) in species A due to the direct impact of paddle rotation. In addition, as an effect of interspecies interaction, a vortex lattice is also formed in species B [Figs. 12(c) and 12(d)].

Since the interactions between two species in ^{87}Rb - ^{87}Rb are very similar, we find that the latticelike structure that appears here is more organized [72] than that of ^{133}Cs - ^{87}Rb binary BECs [compare Figs. 12(c) and 12(d) with Figs. 4(a) and 4(b)]. Finally, we comment that we could not find any significant difference in incompressible and compressible kinetic energy spectra as a function of ω_p between the mass-balanced and mass-imbalanced systems [Fig. 9(c)].

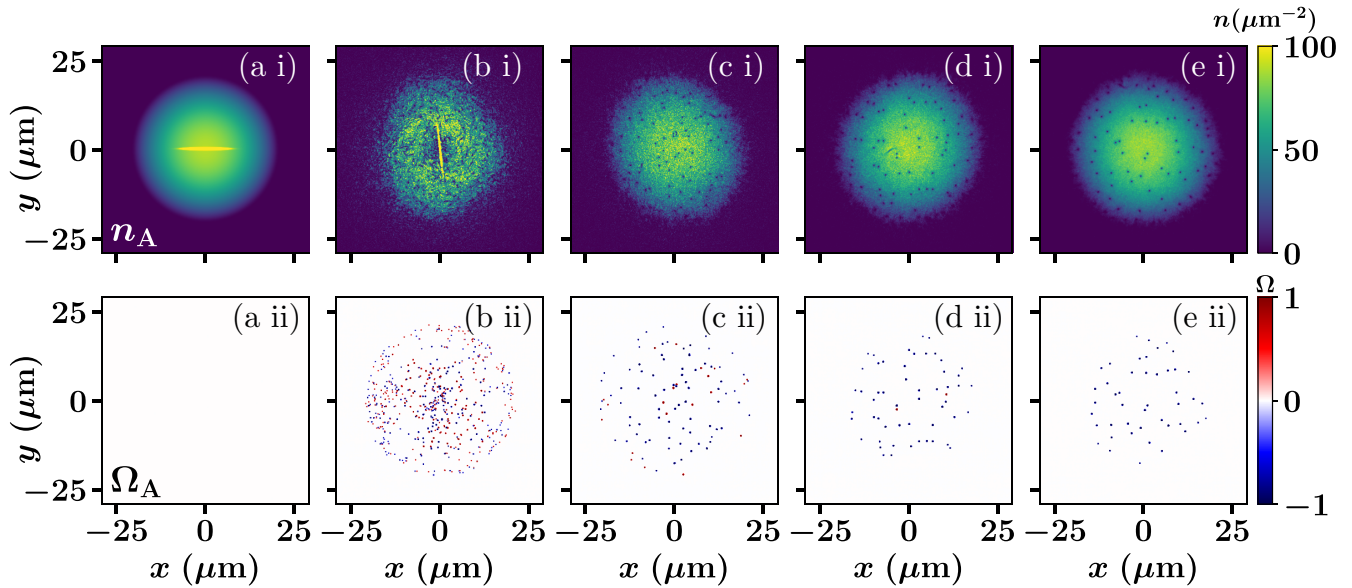


FIG. 13. Snapshot of (a i)–(e i) density (n_A) species and (a ii)–(e ii) vorticity (Ω_A) profiles of ^{133}Cs at different instants of time: (a) $t = 0$ s, (b) $t = 0.1$ s, (c) $t = 0.5$ s, (d) $t = 1.0$ s, and (e) $t = 2.1$ s. An elliptical paddle potential with amplitude $V_0 = -10\mu_A$ and characterized by the parameters $\eta = 0.05$ and $d = 0.1l$ is rotated with the angular frequency $\omega_p = 4\omega$ within species A. The color bars represent (a i)–(e i) the number density n in μm^{-2} and (a ii)–(e ii) the vorticity Ω . The binary BECs are initialized in a two-dimensional harmonic potential with frequency $\omega/2\pi = 30.832$ Hz, $\lambda = 100$, and having intra- and interspecies scattering lengths $a_{AA} = 280a_0$, $a_{BB} = 100.4a_0$, and $a_{AB} = 0$. The numbers of atoms for both species are $N_A = N_B = 60\,000$.

APPENDIX B: VORTEX CREATION USING NEGATIVE PADDLE POTENTIAL

In the main text we focused our discussion on the paddle potential with $V_0 = 10\mu_A$. This results in the density-depleted region in the condensate and creates vortex-antivortex structures when set into rotation. We remark that a rotating negative paddle potential would result in similar dynamics generating vortex-antivortex structures during the dynamics. In order to demonstrate that, we have considered a ^{133}Cs - ^{87}Rb condensate of $N_A = N_B = 60\,000$ particles confined in the harmonic trap with the frequency $\omega/2\pi = 30.832$ Hz and the anisotropy parameter $\lambda = 100$, with $a_{AA} = 280a_0$, $a_{BB} =$

$100.4a_0$, and $a_{AB} = 0$ (Fig. 13). Species A is subjected to the paddle potential with $V_0 = -10\mu_A$, creating the density hump at its center, whereas species B is unaffected [Fig. 13(a i)]. To trigger the dynamics, the paddle is rotated at the frequency $\omega_p = 4\omega$. A huge number of vortices and antivortices can be noticed at $t = 0.1$ s [Fig. 13(b)]. Then the number of vortices significantly decreases as time progresses [Figs. 13(c) and 13(d)]. Finally, in the long-time dynamics antivortices dominate in the system [Fig. 13(e)]. Note that similar behavior has been observed for the positive paddle potential at $\omega_p = 3\omega$, as discussed in the main text. This suggests that the phenomenon takes place irrespective of the attractive or repulsive paddle potential.

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