Scattering times of quantum particles from the gravitational potential and equivalence principle violation

Durmuş Demir
Sabancı University, Faculty of Engineering and Natural Sciences, 34956 Tuzla, İstanbul, Turkey

(Received 1 February 2022; revised 5 July 2022; accepted 10 August 2022; published 23 August 2022)

Universality of motion under gravity, the equivalence principle, is violated for quantum particles. Here, we study the time it takes for a quantum particle to scatter from the gravitational potential and show that the scattering time, formulated here using the opportune Bohmian formulation, acts as an indicator of the equivalence principle violation. The scattering times of wave packets are distinctive enough to distinguish between the Bohmian and Copenhagen interpretations. The scattering time of monoenergetic stationary states, formulated here as a modification of the Bohmian time by probability undercurrents, turns out to be a sensitive probe of the equivalence principle violation. We derive the quantum scattering times and analyze equivalence principle violating terms systematically. We discuss the experimental setup needed for measuring the violation, and describe implications of a possible measurement for time in quantum theory, including the tunneling time.

DOI: 10.1103/PhysRevA.106.022215

I. INTRODUCTION

Equivalence of the inertial mass m_I and the gravitational charge m_G of each and every particle [1,2] renders the Newtonian dynamics purely geometrical. This equivalence has been tested for centuries and has now reached some 10^{-15} relative accuracy [3,4]. It implies that all small bodies (classical particles) fall at the same time if released from the same height with the same velocity. This universality is a fundamental aspect of Newtonian dynamics under gravity, and provides a universal timescale testable with distinct bodies [3].

The universality above is expected to be invalid for quantum particles since their masses continue to appear in the Schrödinger equation even when $m_I \equiv m_G$ [5–10]. The thing is that $\hbar$ comes out of the woodwork as a new constant with the dimension of mass-distance velocity and, in consequence, quantum dynamics remain nongeometrical with or without the equivalence $m_I \equiv m_G$. The free-fall time of quantum particles have been analyzed by Davies [11] by utilizing the Peres clock [12–14], by Viola and Onofrio [15] by using the semiclassical dynamics, by Ali and others [16] by considering wave packets, by Flores and Galapon [17] by making use of a time operator [18], and by Seveso and others by utilizing the information-theoretic methods [19,20]. These studies use different methods but agree on the existence of a finite deviation from the universal free-fall time of the classical dynamics. As will be analyzed in detail in the sequel, deviation from universal free-fall time or, equivalently, violation of the equivalence principle is characterized by the ratio $\hbar/m$. This quantity disappears and gives then way to universality only in the classical limit, namely, only when action of the particle is significantly sizable than $\hbar$.

The crux of the problem is that in quantum theory time is not an observable represented by a Hermitian operator, and there is, thus, no baseline methodology to calculate temporal intervals [21–24]. The resolution, according to various proposals [25,26], is that the march of time in quantum systems should be defined in terms of the changes in certain representative observables (position, momentum, spin orientation, and the like [23,25]). The values of these observables act as duration markers [24] but the choice of the observables depends on how the particle is modeled or perceived (such as, for example, wave packets or energy eigenstates). In general, however, an unambiguous definition and elucidation of time is necessary for both the foundations and applications of the quantum theory [12,24,27].

In search for a proper reification of time, the Copenhagen (standard) and Bohmian [28] interpretations provide two alternative routes [24]. In the Copenhagen interpretation, quantum particles do not possess well-defined trajectories. In the de Broglie–Bohm interpretation, on the other hand, quantum particles possess well-defined trajectories controlled by their probability flows [28–30]. The two interpretations are empirically equivalent in that they have different views on reality and yet they give identical results on main physical questions [29]. This equivalence of theirs gets, however, disrupted once trajectories and probability flows of particles are concerned. In this regard, one phenomenon on which the two interpretations disagree is the probability backflow (having momentum and probability current in opposite directions) which plagues the standard interpretation [31] but does not occur at all in the Bohmian interpretation. In this sense, future experiments on probability backflow [32–34] may differentiate between the two interpretations.

Another occasion in which the two interpretations differ concerns the notion of the quantum travel time. In the standard interpretation (where particles evolve as distributions until they collapse indeterministically under some measurement process) the march of time has been defined variously (see the reviews [25,26,35]) by using different observables as markers. In the Bohmian interpretation (where particles follow a well-defined trajectory under the control of the Schrödinger equation) the march of time can be expressed uniquely in terms of their positions [28–30]. (Comprehensive explorations in Refs. [24,36,37] shed light on different aspects of the two interpretations with regard to the problem of time in quantum theory.)

In this paper, we will study time it takes for a quantum particle to scatter from its own gravitational potential and use this scattering time to determine or measure the equivalence principle violation. In the setup we consider, quantum particles are shot upwards and their return times are recorded such that deviations of the recorded times from the classical universal flight time will be an indicator of the equivalence principle violation. We will study this problem within the de Broglie–Bohm interpretation of the quantum theory [24,28–30]. The scattering times we will compute will be average times in that the Bohmian time formula involve integrations over probability and probability current densities.

In Sec. II below, we study scattering times of wave packets (having classical analogs), and show that dispersion of the wave packet is the main source equivalence principle violation. We compare our finding with the Copenhagen result (with time operator method [17]) and conclude that future experiments may be able to probe what interpretation of quantum theory is realized in nature.

In Sec. III, we study flight times of monoenergetic stationary-state particles (having no classical analogs). We extend the Bohmian time formula of Sec. II to probability undercurrents to obtain a Bohmian-inspired time formula. We show that the Bohmian-inspired time formula is tailor made for such states. Therein, we determine scattering times of quantum particles in terms of the corresponding classical scattering times. We apply the Bohmian-inspired time formula in both of the classically allowed and classically forbidden regions, study its short- and high-flight limits, and reveal the sources of equivalence principle violation. We find that quantum particles spend time behind the classical turning point during their penetration into and withdrawal from the gravitational potential barrier.

In Sec. IV, we discuss the state-of-the-art experimental situation and discuss how the quantum scattering times (for both the wave packets and the stationary states) can be tested in cold atom experiments. We also discuss their implications for applications and foundations of quantum mechanics.

In Sec. V we conclude.

II. QUANTUM SCATTERING TIME: WAVE PACKETS

In vacuum (negligible friction), small bodies (macroscopic particles with negligible tidal forces) obey Newton's motion equation,

$$\frac{d^2 z(t)}{dt^2} = -g \tag{1}$$

for a uniform gravitational field g pointing in the negative z direction. This equation is universal (same for all particles) thanks to the equality $m_I = m_G \equiv m$ between the inertial mass m_I and the gravitational charge m_G. As a result, all classical particles, tossed upwards from a vertical position $z = z_i$ with initial velocity v_i, follow one and the same trajectory,

$$z_c(t) = z_i + v_i t - \tfrac{1}{2} g t^2, \tag{2}$$

as a solution of (1). This means that rise of the particle comes to a halt at the moment $t_\cap = v_i/g$ corresponding to a height of $z_c(t_\cap) \equiv z_\cap = z_i + v_i^2/2g$. This height $z_\cap$ is the turning point.

Universality of the classical motion above is not expected to hold for quantum particles. The reason is that, unlike the Newtonian motion equation (1), the Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(t,z)}{\partial z^2} + V(z)\Psi(t,z) = i\hbar \frac{\partial \Psi(t,z)}{\partial t} \tag{3}$$

depends explicitly on the particle masses irrespective of if $m_I \equiv m_G$ or $m_I \not\equiv m_G$. This nonuniversality has the meaning that the equivalence principle is violated for quantum particles. In fact, with the gravitational potential energy,

$$V(z) = mgz, \tag{4}$$

the Schrödinger equation (3) is seen to invariably involve the dimensionful parameter $\hbar/m$. It admits different solutions, one of which being the wave-packet solution [6–8],

$$\Psi(t,z) = \left(\frac{d}{\sqrt{\pi} D^2}\right)^{1/2} \exp\left\{-\frac{(z-z_c)^2}{2D^2} + \frac{m}{i\hbar} z_i v_i\right\} \times \exp\left\{-\frac{m}{i\hbar}\left(z - \frac{v_i t}{2}\right)(v_i - gt) + \frac{m g^2}{i\hbar} t^3\right\} \tag{5}$$

characterized by the time-varying width $D^2 = d^2 + \frac{i\hbar t}{m}$. It is obviously a nonstationary state as its phase is not linear in time t. It is an approximation to the notion of particle and possesses classical analog in that its center $z = z_c$ follows the Newtonian motion equation (1).

The foremost feature of the Bohmian mechanics is that it ascribes trajectories $z(t)$ to quantum particles such that

$$\frac{dz}{dt} = \frac{J(t,z)}{R(t,z)}, \tag{6}$$

under the control of the Schrödinger equation (3). In this regard, $R(t,z) = \Psi^*(t,z)\Psi(t,z)$ is the probability density, and

$$J(t,z) = \frac{\hbar}{2mi}\left(\Psi^*(t,z)\frac{d}{dz}\Psi(t,z) - \Psi(t,z)\frac{d}{dz}\Psi^*(t,z)\right) \tag{7}$$

is the probability current density. They satisfy the continuity equation,

$$\frac{\partial R(t,z)}{\partial t} + \frac{\partial J(t,z)}{\partial z} = 0, \tag{8}$$

which ensures the conservation of probability. Their ratio,

$$\frac{J(t,z)}{R(t,z)} = \frac{t[z - z_c(t)]}{\frac{m^2 d^4}{\hbar^2} + t^2} + v_i - gt \tag{9}$$

reveals the nonclassical features of wave-packet (5). In fact, with this current-to-probability ratio the Bohmian equation (6) takes the compact form

$$\frac{d}{dt}\left[\frac{z - z_c(t)}{\left(1 + \frac{\hbar^2 t^2}{m^2 d^4}\right)^{1/2}}\right] = 0, \tag{10}$$

TABLE I. The QST/CST ratio. QST starts deviating from the CST at the order $\hbar$ ($\hbar^2$) for the Bohmian (Copenhagen [17]) interpretation. Future experiments may be able to probe what interpretation of quantum behavior is allowed in nature.

	Bohmian interpretation	Copenhagen interpretation
$\frac{(\Delta t)_q^{(wp)}}{(\Delta t)_c}$	$1+\frac{\hbar}{m\sqrt{2gd^3}}+O(\hbar^2)$	$1+\frac{\hbar^2}{4m^2d^2v_i^2}+O(\hbar^4)$

showing explicitly that deviation from the classical solution $z = z_c(t)$ occurs due solely to the wave-packet dispersion ($\hbar t/md^2$). In fact, this equation acquires the solution,

$$z(t) = z_c(t) + \ell\left(1+\frac{\hbar^2 t^2}{m^2 d^4}\right)^{1/2}, \tag{11}$$

with some length parameter ℓ. It is clear that the width function D^2 in the wave-packet (5) vanishes if both $\hbar \to 0$ and $d \to 0$, and it is expected that in these limits the classical solution $z = z_c(t)$ is going to be attained. This comes to mean that the parameter ℓ should be proportional to the width d, and one can set this way $\ell = d$ in (11).

The wave packet in (5), after shot upwards at $z = z_i$, propagates up to the classical turning point $z = z_\cap$ and scatters back therein to fall down to $z = z_i$ in a total duration of $(\Delta t)_q^{(wp)}$. This duration is the quantum scattering time (QST) of the wave packet. The time-formula (11) leads to the Bohmian QST,

$$(\Delta t)_q^{(wp)} = (\Delta t)_c\left(1+\frac{\hbar}{m\sqrt{2gd^3}}+O(\hbar^2)\right) \tag{12}$$

for $\ell = d \ll z_\cap - z_i = v_i^2/2g$. In here, $(\Delta t)_c = 2v_i/g$ is the classical scattering time (CST) defined beneath Eq. (2). This is the average quantum scattering time. [It is average in the sense that the Bohmian equation (6) involves probability and probability current densities and integrations over them effectively give an average duration. This becomes more evident with the Bohmian time (19) describing the stationary-state particles.]

The quantum time formula (12) is a proof that the equivalence principle is violated at the $\hbar$ order where duration of penetration (tunneling) into the semi-infinite classically forbidden region ($z > z_\cap$) is expected to be subleading since the wave-packet (5) approximates a classical particle moving on the classical trajectory $z_c(t)$. As a matter of fact, equivalence principle violation occurs due mainly to the dispersion of the wave packet ($\hbar/m$ in the width function D^2) as was concluded also by previous studies [9,15,16]. It is clear from the wave-packet QST in (12) that the more QST/CST deviates from unity the stronger the violation of the equivalence principle [1,2].

The deviation of the wave-packet QST from the CST turns out to be a sensitive probe of the formulation of the quantum behavior. Table I gives an example of this. Indeed, as shown by the table, for the Bohmian interpretation the deviation is an $O(\hbar)$ effect. For the Copenhagen interpretation with the operator method [17] (similarly with the current density method [16,38]), however, the deviation is an $O(\hbar^2)$ effect. (The formula in Table I is obtained by taking $z_i \ll z_\cap$, which is not inconsistent with the Bohmian formula.) These two distinct $\hbar$ sensitivities along with the other parametric differences show that the future experiments may be able to probe what interpretation of quantum behavior is realized in nature. To this end, experiments with cold atoms and neutrons [39] may prove useful.

III. QUANTUM SCATTERING TIME: STATIONARY-STATE PARTICLES

In this section, we will study average scattering time of a beam of monoenergetic quantum particles from their gravitational potential and show explicitly how this scattering duration signifies violation of the equivalence principle. As a matter of fact, we will study a setup in which quantum particles of mass m and energy E are shot upwards (such as a fountain) in their gravitational potential and their return time (total flight time) is recorded. As was with the wave packet of the last section, difference between the stationary-state QST and the CST will be an indicator of the equivalence principle violation.

It proves useful to start with the calculation of the CST. The difference from the CST in Sec. II is that this time the object of concern is a classical particle of fixed energy E in the framework of the Newtonian dynamics in (1). This particle, thrown upwards from $z = z_i$, rises up, turns backwards at the turning point $z = z_\cap$, and falls at $z = z_i$. This whole motion takes the total time (the CST) [1,2],

$$(\Delta t)_c = \int_{z_i}^{z_\cap}\frac{dz}{\sqrt{2g(z_\cap - z)}} + \int_{z_\cap}^{z_i}\frac{dz}{-\sqrt{2g(z_\cap - z)}} = 2\left(\frac{2(z_\cap - z_i)}{g}\right)^{1/2}, \tag{13}$$

which is a universal duration that depends only on the gravitational acceleration g and the logged height $z_\cap - z_i$. This means that all monoenergetic classical particles approach and scatter back from their gravitational potentials in the same duration $(\Delta t)_c$ irrespective of their masses and other features. In phase space, it can be put into the form

$$(\Delta t)_c = \int_{z_i}^{z_\cap}\frac{m\,dz}{p_z} + \int_{z_\cap}^{z_i}\frac{m\,dz}{-p_z}, \tag{14}$$

in which $p_z = \sqrt{2m[E - V(z)]}$ is momentum of the particle, $E = mgz_\cap$ is its total energy, and $V(z)$ is its potential energy in (4). This expression for $(\Delta t)_c$ expresses the march of time in terms of the coordinate of the particle (as if a clock attached on it) [21–24].

In general, the classical dynamics underlying the CST in (14) corresponds to stationary-state quantum dynamics. Quantum particles obeying such dynamics possess the wave function,

$$\Psi(z,t) = \psi(z)e^{-(i/\hbar)Et}, \tag{15}$$

whose replacement in the time-dependent Schrödinger equation in (3) leads to

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(z)}{dz^2} + V(z)\psi(z) = E\psi(z), \tag{16}$$

as the time-independent Schrödinger equation governing $\psi(z)$. With the stationary-state wave-function (15), the

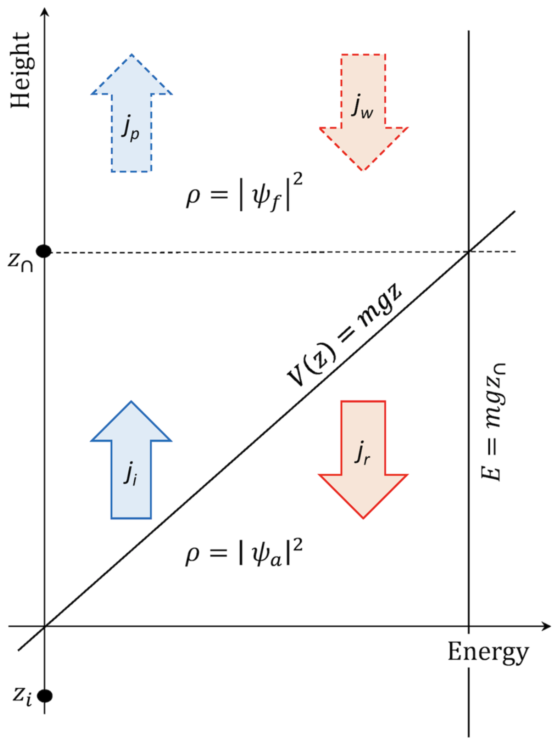

FIG. 1. Scattering of monoenergetic stationary-state quantum particles from their gravitational potential energy $V(z) = mgz$. In the classically allowed region ($z < z_\cap$), the particles are distributed with the probability density $\rho = |\psi_a|^2$ and probability undercurrents $j_i = -j_r$. Similarly, in the classically forbidden region ($z > z_\cap$), the particles are distributed with the probability density $\rho = |\psi_f|^2$ and the probability undercurrents $j_p = -j_w$. These probability currents are obtained by judiciously splitting the wave functions into two complex pieces as $\psi_a(z) = \psi_i(z) + \psi_r(z)$ in the allowed region and as $\psi_f(z) = \psi_p(z) + \psi_w(z)$ in the forbidden region. In Bohmian mechanics, quantum travel time marches with the ascribed particle position so that the total quantum scattering time is composed of the rising (up-blue full arrow), penetrating (up-blue dashed arrow), withdrawing (down-red dashed arrow), and falling (down-red full arrow) transitions.

probability current density $J(t, z)$ in (7) takes the form

$$j(z) = \frac{\hbar}{2mi}\left(\psi^*(z)\frac{d}{dz}\psi(z) - \psi(z)\frac{d}{dz}\psi^*(z)\right), \qquad (17)$$

and the probability density $R(t, z) = \Psi^*(t, z)\Psi(t, z)$ reduces to $\rho(z) = \psi^*(z)\psi(z)$. Obviously, $j(z)$ must be strictly constant [although $\rho(z)$ can depend on z] according to the continuity of the probability flow in (8).

The stationary-state wave functions, such as (15) are tailor made for stationary scattering events as they represent the steady flux of particles shot upwards (such as, a fountain) and scattered back downwards (such as, rain) [11]. The problem is to define scattering time for such states in the setup depicted in Fig. 1. To this end, as already discussed in the previous section, Bohmian mechanics [28–30] provides a viable framework. The reason is that Bohmian mechanics assigns trajectories to quantum particles—even to spatially spread-out stationary-state particles described by (15) [28,29]. For such states, the Bohmian relation in (6) turns to

$$\frac{dz}{dt} = \frac{j}{\rho}, \qquad (18)$$

in which j/ρ is a function only of the coordinate z. From this one can readily construct that the travel time formula,

$$(\Delta t)_q = \int_a^b dz \frac{\rho}{j}, \qquad (19)$$

in which j is assumed to flow from a to b. This is the average QST corresponding to the CST in (14). It expresses the march of the time in terms of the probability density ρ and the probability current density j in the region extending from $z = a$ to $z = b$. It is the Bohmian QST for stationary-state particles, and gives the average quantum scattering time because it is effectively the average value of the inverse current density $(1/j)$. In applying (19) one keeps in mind that ρ can vary with z but j remains strictly constant.

The Schrödinger equation (16) possesses the piecewise solution [40,41],

$$\psi(z) = \begin{cases} \psi_a(z) & \text{for } z \leqslant z_\cap, \\ \psi_f(z) & \text{for } z \geqslant z_\cap, \end{cases} \qquad (20)$$

in which

$$\psi_a(z) = N\zeta^{1/3}[J_{1/3}(\zeta) + J_{-(1/3)}(\zeta)] \qquad (21)$$

is the wave function in the classically allowed region ($z < z_\cap$), and

$$\psi_f(z) = N\zeta^{1/3}[I_{-(1/3)}(\zeta) - I_{1/3}(\zeta)] \qquad (22)$$

is the wave function in the classically forbidden region ($z < z_\cap$). In these solutions, N is a normalization constant, and $J_{\pm 1/3}$ and $I_{\pm 1/3}$ are the Bessel functions of order $\pm 1/3$ with the argument,

$$\zeta = \frac{2}{3}\left(\frac{|z - z_\cap|}{L_q}\right)^{3/2}, \qquad (23)$$

in which

$$L_q = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3} \qquad (24)$$

is the natural length scale for a quantum particle under gravity. It breaks universality with $(\hbar/m)^{2/3}$ power law.

A. Quantum flight time in allowed region

The wave-function $\psi_a(z)$ in (21), describing the state of the particle in the allowed region ($z < z_\cap$), can have at most a global phase. In fact, it can be taken purely real without loss of generality. It gives then the zero probability current as follows from (17). This actually means that there are two equal and opposite undercurrents constituting the stationary-state probability distribution. This implies that it must be possible to split the wave-function $\psi_a(z)$ into two complex functions of equal and opposite currents. One can, therefore, write (see Ref. [11] for a similar decomposition)

$$\psi_a(z) = \psi_i(z) + \psi_r(z), \qquad (25)$$

in which

$$\psi_i(z) = N\zeta^{1/3}\{e^{-(i\pi/3)}J_{1/3}(\zeta) + e^{i\pi/3}J_{-(1/3)}(\zeta)\} \tag{26}$$

has the positive (upward) probability current,

$$j_i = \frac{\hbar}{\pi m L_q}\left(\frac{3}{2}\right)^{4/3}|N|^2 \tag{27}$$

as follows from (17), and

$$\psi_r(z) = N\zeta^{1/3}\big\{(1 - e^{-(i\pi)/3})J_{1/3}(\zeta) + (1 - e^{(i\pi)/3})J_{-(1/3)}(\zeta)\big\} \tag{28}$$

has the negative (downward) probability current,

$$j_r = -\frac{\hbar}{\pi m L_q}\left(\frac{3}{2}\right)^{4/3}|N|^2, \tag{29}$$

as follows again from (17). The two currents are indeed equal in size and opposite in sign. They ensure that the wave-function $\psi_a(z)$ is composed of an incident wave (ψ_i) inducing an upward probability flow and a reflected wave (ψ_r) creating a downward probability flow.

The decomposition of the wave function into two complex wave functions of equal-size and opposite-sign probability undercurrents has proven useful for revealing the probability underflows in the stationary-state scattering problem at hand. It worked for the allowed-region wave function in (21), and it will be seen to work for the forbidden-region wave-function (22) in Sec. III B. It worked because the wave-functions (21) and (22) involve the Bessel functions, and Wronskians of Bessel functions lead to the required probability currents [11,40,41]. In general, decomposition becomes a necessity if probability underflows in the stationary system are needed. The structure of the two undercurrents (equal in size and opposite in sign) determines how the decomposition should be performed, but this does not guarantee uniqueness of the decomposition since there can exist different decompositions leading to the same undercurrents. Moreover, it not clear if the decomposition of a general wave function does uniquely lead to proper probability undercurrents. (This point seems to require a separate investigation. The generality and uniqueness of the decomposition is an open problem.)

Having obtained the probability currents (27) and (29), it is now time to compute the associated quantum flight times. It might be tempting to use the Bohmian time formula in (19) directly. This, however, is not so easy. The reason is that in Bohmian mechanics quantum particles are guided not by the undercurrents (j_i and $j_r = -j_i$) but by the total probability current ($j_i + j_r$ which equals zero). In view of this difficulty, we introduce a Bohmian-inspired new time definition by replacing the total current in the Bohmian time (19) with the j_i and j_r undercurrents. With this replacement, it becomes possible follow propagation of particles in the directions of the undercurrents. In this regard, quantum particles rise from $z = z_i$ to the turning point $z = z_\cap$ within the average Bohmian-inspired time [42],

$$(\Delta t)_q^{(\mathrm{rise})} = \int_{z_i}^{z_\cap}\frac{|\psi_a(z)|^2}{2j_i}dz = -\frac{2\pi T_q}{\left[3^{1/3}\Gamma\left(\frac{1}{3}\right)\right]^2} + 2\pi T_q\{\beta_q[\mathrm{Ai}(-\beta_q)]^2 + [\mathrm{Ai}'(-\beta_q)]^2\}, \tag{30}$$

in which β_q is quadratic in $(\Delta t)_c$,

$$\beta_q = \left(\frac{(\Delta t)_c}{4T_q}\right)^2, \tag{31}$$

and involves

$$T_q = \left(\frac{\hbar}{4mg^2}\right)^{1/3}, \tag{32}$$

as the natural timescale for a quantum particle under gravity. In the rise time (30), at the right-hand side, the function $\mathrm{Ai}(\cdots)$ is the Airy function of the first kind, and $\mathrm{Ai}'(\cdots)$ is its derivative [40,41].

In parallel with the rise time above, quantum particles are found to fall from the turning point $z_\cap$ to z_i within the average Bohmian-inspired time [42],

$$(\Delta t)_q^{(\mathrm{fall})} = \int_{z_\cap}^{z_i}\frac{|\psi_a(z)|^2}{2j_r}dz = (\Delta t)_q^{(\mathrm{rise})}, \tag{33}$$

where the $1/2$ factor in the integrands of $(\Delta t)_q^{(\mathrm{rise})}$ and $(\Delta t)_q^{(\mathrm{fall})}$ is there to avoid double counting whereas keeping the interference terms between ψ_i and ψ_r. These two times give the average flow durations in the classically allowed region in Fig. 1.

B. Quantum flight time in the forbidden region

The wave-function $\psi_f(z)$ in (22), describing the state of the particle in the classically forbidden region ($z > z_\cap$), can have at most a global phase. It can, in fact, be taken real (such as, $\psi_a(z)$ in the classically allowed region) without loss of generality. It possesses zero probability current as follows from (17). As in Sec. III A, this zero current can be structured as being composed of two equal and opposite undercurrents by an appropriate splitting of the stationary-state wave function into two complex wave functions. One can write, therefore,

$$\psi_f(z) = \psi_p(z) + \psi_w(z), \tag{34}$$

in parallel with (25) such that

$$\psi_p(z) = Ni\zeta^{1/3}\{e^{(i\pi)/6}I_{1/3}(\zeta) + e^{-(i\pi)/6}I_{-(1/3)}(\zeta)\} \tag{35}$$

has the positive (upward) probability current,

$$j_p = \frac{\hbar}{\pi m L_q}\left(\frac{3}{2}\right)^{4/3}|N|^2, \tag{36}$$

as follows from (17), and

$$\psi_w(z) = -N\zeta^{1/3}\{(1 - e^{-(i\pi)/3})I_{1/3}(\zeta) - (1 - e^{(i\pi)/3})I_{-(1/3)}(\zeta)\} \tag{37}$$

has the negative (downward) probability current,

$$j_w = -\frac{\hbar}{\pi m L_q}\left(\frac{3}{2}\right)^{4/3}|N|^2, \tag{38}$$

as follows again from (17). The two undercurrents are indeed equal in size and opposite in sign. They ensure, thus, that the wave-function $\psi_f(z)$ is composed of a penetrating evanescent wave (ψ_p, decaying towards $z = \infty$) inducing an upward probability flow and a withdrawing evanescent wave (ψ_w, decaying towards $z = z_\cap$) creating a downward probability flow.

Having derived the probability currents (36) and (38), average quantum flight times in the classically forbidden region ($z > z_\cap$) can now be computed by using the Bohmian-inspired time formula in Sec. III A. Indeed, it turns out that an evanescencing quantum particle penetrates into the forbidden region for an average Bohmian-inspired time [42],

$$(\Delta t)_q^{(\text{penetrate})} = \int_{z_\cap}^{\infty} \frac{|\psi_f(z)|^2}{2j_p} dz = \frac{2\pi T_q}{\left[3^{1/3}\Gamma\left(\frac{1}{3}\right)\right]^2}, \quad (39)$$

whose right-hand side is set by $\mathrm{Ai}'(0)$ [40,41].

In parallel with the penetration time above, quantum particles withdraw back to the turning point $z_\cap$ in the average Bohmian-inspired time [42],

$$(\Delta t)_q^{(\text{withdraw})} = \int_{\infty}^{z_\cap} \frac{|\psi_f(z)|^2}{2j_w} dz = \frac{2\pi T_q}{\left[3^{1/3}\Gamma\left(\frac{1}{3}\right)\right]^2}, \quad (40)$$

where the factor $1/2$ in the integrands of $(\Delta t)_q^{(\text{penetrate})}$ and $(\Delta t)_q^{(\text{withdraw})}$ is placed to prevent double counting whereas keeping the cross terms between ψ_p and ψ_w. These two times sum up to the total time spent in the classically forbidden region (as depicted in Fig. 1).

Before going any further, it proves instructive to discuss time spent in the classically forbidden region ($z > z_\cap$) also in the dwell time formulation [43]. In this formulation, quantum particles of incidence current j_{inc} spend a time [43,44],

$$(\Delta t)^{(\text{dwell})} = \frac{1}{j_{\text{inc}}} \int_a^b dz\, \rho_f, \quad (41)$$

in a classically forbidden region extending from $z = a$ to $z = b$ with the probability density ρ_f. This time formula differs from the Bohmian time (19) by the fact that the current j_{inc} is the incident current, not the current in the forbidden region extending from a to b. Despite this, explicit calculation shows that the dwell time satisfies the relation [42],

$$(\Delta t)_q^{(\text{dwell})} = (\Delta t)_q^{(\text{penetrate})} + (\Delta t)_q^{(\text{withdraw})}, \quad (42)$$

after letting $\rho^f \to |\psi_f(z)|^2$ and $j_{\text{inc}} \to j_i$ in (41), where $\psi_f(z)$ and j_i are defined in (34) and (27), respectively. The relation (42) gives an independent confirmation of the Bohmian-inspired travel time formula [splitting of the wave function in two complex pieces as in (34) and use of the respective currents (36) and (38)].

C. Quantum scattering time

On physical grounds, QST is fundamentally different than CST in (14). Indeed, whereas quantum particles perform a "rise-penetrate-withdraw-fall" motion the classical particles perform a simple "rise-turn-fall" motion. The reason is that there is essentially no turning point for a quantum particle as it is always able to penetrate into the $z > z_\cap$ domain [as in (35)] and withdraw back [as in (37)] as a semi-infinite tunneling transition induced by evanescent waves. (This effect is expected to be subleading for a wave packet as discussed in Sec. II.) All this implies that the QST is composed of four segments,

$$(\Delta t)_q = (\Delta t)_q^{(\text{rise})} + (\Delta t)_q^{(\text{penetrate})} + (\Delta t)_q^{(\text{withdraw})} + (\Delta t)_q^{(\text{fall})}, \quad (43)$$

as an ordered set of transitions depicted in Fig. 1. Now, collecting the individual time intervals from (30), (39), (40), and (33), this $(\Delta t)_q$ formula leads to the QST/CST ratio,

$$\frac{(\Delta t)_q}{(\Delta t)_c} = \pi\sqrt{\beta_q}[\mathrm{Ai}(-\beta_q)]^2 + \frac{\pi}{\sqrt{\beta_q}}[\mathrm{Ai}'(-\beta_q)]^2, \quad (44)$$

as because $(\Delta t)_q^{(\text{penetrate})} + (\Delta t)_q^{(\text{withdraw})}$ cancels out the constant part in $(\Delta t)_q^{(\text{rise})} + (\Delta t)_q^{(\text{fall})}$. This exact result shows how QST differs from CST as a function of the universality-breaking parameter $\hbar/m$. This dependence on $\hbar/m$ ensures that QST/CST is an unambiguous vestige of equivalence principle violation. More specifically, the more QST/CST deviates from unity the stronger the violation of the equivalence principle [1,2].

TABLE II. The quantum characteristic timescale T_q in (32) and quantum-mechanical collision time in (45) for the electron and neutron. In general, $T_q(\text{atom}) \approx T_q(\text{neutron})A^{-1/3}$ for an atom with mass number A.

Particle	Mass (kg)	T_q (s)	$(\Delta t)_q[z_i = z_\cap]$ (s)
Electron	9.109×10^{-31}	1.496×10^{-8}	1.259×10^{-8}
Neutron	1.674×10^{-27}	1.221×10^{-9}	1.028×10^{-9}

One physically important regime of QST in (44) is the short-flight regime, namely, $z_i \to z_\cap$ limit. In this regime, the particle starts already at the turning point $z = z_\cap$, penetrates into the semi-infinite barrier for a duration $(\Delta t)_q^{(\text{penetrate})}$ and reappears at the turning point after a time lapse of $(\Delta t)_q^{(\text{withdraw})}$. In this short-flight limit, the CST vanishes identically as follows from (13) $[(\Delta t)_c[z_i - z_\cap] = 0]$ but the QST takes the nonzero value,

$$(\Delta t)_q[z_i = z_\cap] = (\Delta t)_q^{(\text{penetrate})} + (\Delta t)_q^{(\text{withdraw})} = \frac{4\pi T_q}{\left[3^{1/3}\Gamma\left(\frac{1}{3}\right)\right]^2}, \quad (45)$$

which shows that the quantum particle wanders in the classically forbidden region for a finite duration. This wandering is due to particle's penetration into and withdrawal from the $z > z_\cap$ domain. It turns out that the tunneling into the semi-infinite potential barrier $V(z) > E$ makes quantum particle to acquire a finite collision duration at the turning point. Indeed, as depicted in Fig. 2 for particles of masses m (dot-dashed black), $10m$ (full red), and $m/10$ (dashed blue), QST remains nonzero even when the CST vanishes (zero-flight limit). This means that each particle spends a finite time at the turning point corresponding to collision duration with the gravitational potential barrier. By definition, $(\Delta t)_q[z_\cap = z_i]$ is an $O[(\hbar/m)^{1/3}]$ quantum effect and varies from particle to particle as exemplified in Table II for the electron and the neutron.

Another physically important regime of QST in (44) is the high-flight regime, namely, the $z_\cap - z_i \gg L_q$ regime. In this

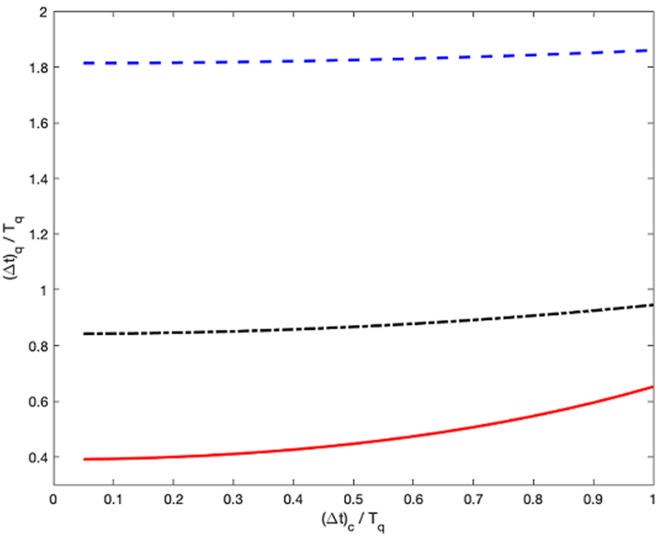

FIG. 2. Variation of the QST $[(\Delta t)_q/T_q]$ with the CST $[(\Delta t)_c/T_q]$ for quantum particles having masses m (dot-dashed black), $10m$ (full red), and $m/10$ (dashed blue). It is clear that, in each case, QST remains nonzero even when the CST vanishes exactly. These QST values at $(\Delta t)_c = 0$ (namely, $z_i = z_\cap$) are proof that the quantum particles possess a finite collision time at the turning point, which serves as an indicator of the equivalence principle violation.

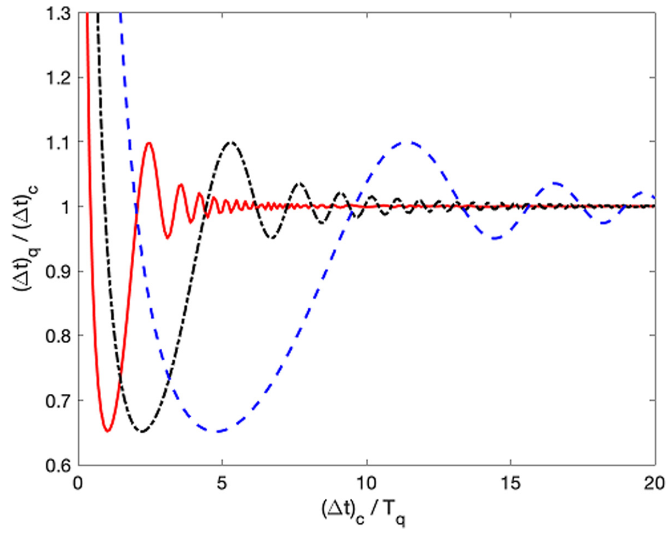

FIG. 3. Variation of the QST/CST $[(\Delta t)_q/(\Delta t)_c]$ with the CST $[(\Delta t)_c/T_q]$ for quantum particles having masses m (dot-dashed black), $10m$ (full red), and $m/10$ (blue dashed). It is clear that each QST relaxes to the CST in an oscillatory fashion such that lighter (heavier) the particle slower (faster) the relaxation. Equivalence principle violation is pronounced at low values of $(\Delta t)_c/T_q$.

limit, on physical grounds, one expects QST to approach CST. Indeed, for $z_\cap - z_i \gg L_q$ the exact QST/CST in (44) takes the form

$$\frac{(\Delta t)_q[z_\cap - z_i \gg L_q]}{(\Delta t)_c} = 1 - \frac{\cos \alpha_q}{3\alpha_q} + O[(\hbar/m)^2], \quad (46)$$

where $\alpha_q = \frac{4}{3}(\beta_q)^{3/2}$ parametrizes the universality-breaking quantum contributions, which vary from particle to particle via $\alpha_q \propto m/\hbar$. The parameter α_q gives information about equivalence principle violation by a measurement of QST/CST for long flights. In general, heavier the particle smaller the quantum contribution as revealed by the T_q values in Table II. Direct calculation reveals that the high-flight QST in (46) holds for distances grater than 0.274 fm (0.183 fm) for electrons (neutrons).

Depicted in Fig. 3 is QST as a function of the CST for particles of masses m (dot-dashed black), $10m$ (full red), and $m/10$ (dashed blue). The plot extends from the short-flight to the high-flight regime as $(\Delta t)_c/T_q$ increases. It is clear that the QST exhibits strong swings at low $(\Delta t)_c/T_q$, which can be detected experimentally by using beams of different energies. It is also clear that the QST relaxes to the CST at large $(\Delta t)_c/T_q$ in an oscillatory fashion such that the lighter (heavier) the particle, the slower (faster) the relaxation. Evidently, the equivalence principle violation becomes stronger at low $(\Delta t)_c/T_q$. This reduction of QST to e CST at large $(\Delta t)_c/T_q$ is also what is emphasized in Ref. [11] by Davies. The operator approach in Ref. [17] is valid only if the particle does not reach $z_\cap$ and is, thus, not possible to contrast with the results here. Nevertheless, both Refs. [11,17] find $O(\hbar)$ and higher-order (positive or negative) corrections to CST.

IV. EXPERIMENTAL DETERMINATION

Universality of free fall has been under experimental exploration for decades [45–47]. In the past decade experiments have diversified and reached higher precision levels [48–53]. The experiments with cold atoms are, particularly, promising. It is likely that experiments as such, including cold neutrons [39], can start measuring flight times of quantum particles in the near future. Such scattering experiments can be conducted reliably under ultra-high-vacuum conditions corresponding to pressures about 10^{-10} Pa and mean free paths about 10^5 m.

The present paper reports actually two classes of new results. The first concerns scattering time of wave packets [48–53]. It was analyzed in Sec. II with the main result that a proper measurement of the scattering time can distinguish between the Bohmian and the Copenhagen interpretations of the quantum behavior. Indeed, equivalence principle violation is of size $\hbar$ ($\hbar^2$) in the Bohmian (Copenhagen) approach, wave-packet QST becomes a new distinguishing quantity after the quantum backflow [31–33]. Future experiments might probe what interpretation is realized in nature.

The second class of new results concern scattering times of the monoenergetic beam of stationary-state particles. One can shoot such particles upwards, make them scatter off from their gravitational potential, and measure their average return times. The determinations of $(\Delta t)_q[z_\cap = z_i]$ and $(\Delta t)_q[z_\cap - z_i \gg L_q]$ are, particularly, important for various reasons. Indeed, experimental verification of $(\Delta t)_q[z_\cap = z_i]$ in (45) would ensure that

(1) quantum free-fall is not universal,
(2) quantum tunneling takes finite time, and
(3) quantum travel time could be Bohmian.

Experimental confirmation of $(\Delta t)_q[z_\cap - z_i \gg L_q]$, on the other hand, would ensure that

(1) quantum free-fall is not universal,

(2) quantum particles can fall much faster or slower than the classical particles, and

(3) universal classical free-fall times are attained for long flights.

In general, QST is around nanoseconds for cold neutrons and significantly shorter for cold atoms (cesium, potassium, rubidium, and the like). Table II, Figs. 3 and 2 provide the necessary information. These scattering times should give an idea about the precision goal in future experiments.

V. CONCLUSION

In this paper, we have performed a systematic study of the scattering times of quantum particles from their gravitational potentials. We have utilized the opportune Bohmian mechanics as it ascribes trajectories to quantum particles. We have first analyzed scattering times of wave packets in the Bohmian formalism in a way involving the equivalence principle violating ratio $\hbar/m$. We have found that scattering times can distinguish between the Bohmian and thye Copenhagen interpretations.

We have next analyzed monoenergetic stationary-state particles corresponding to the steady flux of quantum particles and shown that their quantum and classical scattering times differ from each other in a way involving the equivalence principle violating ratio $\hbar/m$. We have analyzed the quantum scattering time in short- and high-flight regimes and low- and high-mass limits and found explicit expressions testable by appropriate scattering experiments. It turns out that experiments with different particle energies and different particle masses seem to have good potential to test the quantum violation of the equivalence principle. The formula found can prove useful for both theoretical and experimental tests of the equivalence principle in quantum systems.

Experimental determination of the quantum scattering time of wave packets can determine what interpretation of the quantum behavior is realized in nature. The scattering times of stationary-state particles, on the other hand, can put an end to the quest for the correct formula for traversal and tunneling times in quantum theory. And analyses of the tunnel ionization of atoms can provide a cross-check for experimental data [54–57]. Fundamentally, quantum scattering time, if measured accurately, can innovate our conception of time in quantum theory with widespread implications for tunneling-enabled processes.

ACKNOWLEDGMENT

This work was supported by the IPS Project No. B.A.CF-20-02239 at Sabancı University. I am grateful to conscientious reviewers for their useful criticisms, questions, and suggestions.

[1] A. Spallicci, *Mass and Motion in General Relativity*, Fundamental Theories of Physics, Vol. 162 (Springer, Berlin, 2011), p. 561.
[2] E. Di Casola, S. Liberati, and S. Sonego, Am. J. Phys. **83**, 39 (2015).
[3] A. M. Nobili and A. Anselmi, Phys. Lett. A **382**, 2205 (2018).
[4] P. Touboul, G. Métris, M. Rodrigues, Y. André, Q. Baghi, J. Bergé, D. Boulanger, S. Bremer, P. Carle, R. Chhun, B. Christophe, V. Cipolla, T. Damour, P. Danto, H. Dittus, P. Fayet, B. Foulon, C. Gageant, P.-Y. Guidotti, D. Hagedorn *et al.*, Phys. Rev. Lett. **119**, 231101 (2017).
[5] P. Candelas and D. W. Sciama, Phys. Rev. D **27**, 1715 (1983).
[6] M. Wadati, J. Phys. Soc. Jpn. **68**, 2543 (1999).
[7] G. Vandegrift, Am. J. Phys. **68**, 576 (2000).
[8] O. Vallee, Am. J. Phys. **68**, 672 (2000).
[9] V. A. Emelyanov, Eur. Phys. J. C **82**, 318 (2022).
[10] C. Anastopoulos and B. L. Hu, Class. Quantum Grav. **35**, 035011 (2018).
[11] P. C. W. Davies, Class. Quantum Grav. **21**, 2761 (2004).
[12] H. Salecker and E. P. Wigner, Phys. Rev. **109**, 571 (1958).
[13] A. Peres, Am. J. Phys. **48**, 552 (1980).
[14] D. Alonso, R. Sala Mayato, and J. G. Muga, Phys. Rev. A **67**, 032105 (2003).
[15] L. Viola and R. Onofrio, Phys. Rev. D **55**, 455 (1997).
[16] M. M. Ali, A. S. Majumdar, D. Home, and A. K. Pan, Class. Quantum Grav. **23**, 6493 (2006).
[17] P. C. M. Flores and E. A. Galapon, Phys. Rev. A **99**, 042113 (2019).
[18] Y. Aharonov and D. Bohm, Phys. Rev. **122**, 1649 (1961).
[19] L. Seveso, V. Peri, and M. Paris, J. Phys.: Conf. Ser. **880**, 012067 (2017).
[20] L. Seveso and M. Paris, Ann. Phys. **380**, 213 (2017).
[21] A. Challinor, A. Lasenby, S. Somaroo, C. Doran, and S. Gull, Phys. Lett. A **227**, 143 (1997).
[22] M. Abolhasani and M. Golshani, Phys. Rev. A **62**, 012106 (2000).
[23] P. Busch, The time–energy uncertainty relation, in *Time in Quantum Mechanics* (Springer, Berlin, Heidelberg, 2008), pp. 73–105.
[24] G. Field, On the status of quantum tunneling times, http://philsci-archive.pitt.edu/id/eprint/18446 (2020).
[25] J. Hilgevoord, Stud. Hist. Philos. Sci. B: Stud. Hist. Philos. Mod. Phys. **36**, 29 (2005).
[26] *Time in Quantum Mechanics*, edited by J. G. Muga, R. Sala Mayato, and I. L. Egusquiza (Springer, Berlin, 2008).
[27] M. Calçada, J. T. Lunardi, and L. A. Manzoni, Phys. Rev. A **79**, 012110 (2009).
[28] D. Bohm, Phys. Rev. **85**, 166 (1952).
[29] X. Oriols and J. Mompart, *Overview of Bohmian Mechanics* (Jenny Stanford, Dubai, UAE, 2019).
[30] S. Sonego, Phys. Lett. A **208**, 1 (1995).
[31] G. R. Allcock, Ann. Phys. (NY) **53**, 253 (1969).
[32] J. M. Yearsley, J. J. Halliwell, R. Hartshorn, and A. Whitby, Phys. Rev. A **86**, 042116 (2012).
[33] S. Das and D. Dürr, Sci. Rep. **9**, 2242 (2019).
[34] S. Das, M. Nöth, and D. Dürr, Phys. Rev. A **99**, 052124 (2019).
[35] E. H. Hauge and J. A. Stovneng, Rev. Mod. Phys. **61**, 917 (1989)
[36] J. T. Cushing and G. E. Bowman, Bohmian mechanics and chaos, in *From Physics to Philosophy*, edited by J. Butterfield and C. Pagonis (Cambridge University Press, Cambridge, UK, 1999).

[37] J. T. Cushing, Philos. Sci. **67**, S430 (2000).
[38] J. G. Muga, S. Brouard, and D. Macias, Ann. Phys. (NY) **240**, 351 (1995).
[39] H. Kaneko, A. Tohsaki, and A. Hosaka, Prog. Theor. Phys. **128**, 533 (2012).
[40] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (NBS, Washington, 1964).
[41] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1964).
[42] J. R. Albright, J. Phys. A **10**, 485 (1977).
[43] F. T. Smith, Phys. Rev. **118**, 349 (1960).
[44] M. Büttiker, Phys. Rev. B **27**, 6178 (1983).
[45] R. Colella, A. W. Overhauser, and S. Werner, Phys. Rev. Lett. **34**, 1472 (1975).
[46] B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988).
[47] C. G. Aminoff, A. M. Steane, P. Bouyer, P. Desbiolles, J. Dalibard, and C. Cohen-Tannoudji, Phys. Rev. Lett. **71**, 3083 (1993).
[48] D. Aguilera, H. Ahlers, B. Battelier, A. Bawamia, A. Bertoldi, R. Bondarescu *et al.*, Class. Quantum Grav. **31**, 115010 (2014).
[49] D. Schlippert, J. Hartwig, H. Albers, L. L. Richardson, C. Schubert, A. Roura, W. P. Schleich, W. Ertmer, and E. M. Rasel, Phys. Rev. Lett. **112**, 203002 (2014).
[50] X.-C. Duan, X.-B. Deng, M.-K. Zhou, K. Zhang, W.-J. Xu, F. Xiong, Y.-Y. Xu, C.-G. Shao, J. Luo, and Z.-K. Hu, Phys. Rev. Lett. **117**, 023001 (2016).
[51] G. Rosi, G. D'Amico, L. Cacciapuoti *et al.*, Nat. Commun. **8**, 15529 (2017).
[52] H. Albers, A. Herbst, L. L. Richardson, H. Heine, D. Nath, J. Hartwig, C. Schubert, C. Vogt, M. Woltmann, C. Lmmerzahl, S. Herrmann, W. Ertmer, E. M. Rasel, and D. Schlippert, Eur. Phys. J. D **74**, 145 (2020).
[53] C. Deppner, W. Herr, M. Cornelius, P. Stromberger, T. Sternke, C. Grzeschik, A. Grote, J. Rudolph, S. Herrmann and M. Krutzik, A. Wenzlawski, R. Corgier, E. Charron, D. Guéry-Odelin, N. Gaaloul, C. Lämmerzahl, A. Peters, P. Windpassinger, and E. M. Rasel, Phys. Rev. Lett. **127**, 100401 (2021).
[54] A. S. Landsman, M. Weger, J. Maurer, R. Boge, A. Ludwig, S. Heuser, C. Cirelli, L. Gallmann, and U. Keller, Optica **1**, 343 (2014).
[55] D. Demir and T. Güner, Ann. Phys. (NY) **386**, 291 (2017).
[56] N. Camus, E. Yakaboylu, L. Fechner, M. Klaiber, M. Laux, Y. Mi, K. Z. Hatsagortsyan, T. Pfeifer, C. H. Keitel, and R. Moshammer, Phys. Rev. Lett. **119**, 023201 (2017).
[57] D. Demir and S. Pacal, Quantum travel time and tunnel ionization times of atoms, arXiv:2001.06071.