Testing gravitational self-interaction via matter-wave interferometry

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The Schrödinger-Newton equation has frequently been studied as a nonlinear modification of the Schrödinger equation incorporating gravitational self-interaction. However, there is no evidence yet as to whether nature actually behaves this way. This work investigates a possible way to experimentally test gravitational self-interaction. The effect of self-gravity on the interference of massive particles is studied by numerically solving the Schrödinger-Newton equation for a particle passing through a double-slit. The results show that the presence of gravitational self-interaction has an effect on the fringe width of the interference that can be tested in matter-wave interferometry experiments. Notably, this approach can distinguish between gravitational self-interaction and environment-induced decoherence, as the latter does not affect the fringe width. This result will also provide a way to test if gravity requires to be quantized on the scale of ordinary quantum mechanics.

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I. INTRODUCTION

The emergence of classicality from quantum theory is an issue which has plagued quantum mechanics right from its inception. Quantum mechanics is linear and the Schrödinger equation allows superposition of any two distinct solutions. However, in our familiar classical world, a superposition of macroscopically distinct states, such as the state corresponding to two well-separated distinct positions of a particle, is never observed [1]. Taking into account environment-induced decoherence [2–4], one may argue that pure superposition states do not survive for long, and the interaction with the environment causes the off-diagonal elements of the *reduced* density matrix of the system to vanish. The remaining diagonal terms are then interpreted as classical probabilities. However, decoherence is based on unitary quantum evolution and if one tried to explain how a single outcome resulted for a particular measurement, one will eventually be forced to resort to some kind of many worlds interpretation [5]. Another class of approaches to address this issue invokes some kind of nonlinearity in quantum evolution, which may cause macroscopic superposition states to dynamically evolve into one macroscopic distinct state [6–9]. Different theories attribute the origin of the nonlinearity to different sources, for instance, an inherent nonlinearity in the evolution equation [10], or gravitational self-interaction [11–13]. Considerable effort has been put into finding ways to test any nonlinearity which may lead to the destruction of superpositions. For example, an experiment in space was proposed which would involve preparing a macroscopic mirror in a superposition state [14,15]. The problem with such experiments, even if they are successfully realized, is that it is difficult to distinguish between the role of decoherence and that of nonlinearity in destroying the superposition. An effect that can distinguish between these two possible causes of loss of superposition is sorely needed. This is the issue we wish to address in this work.

In 1984 Diosi [11] introduced a gravitational self-interaction term in the Schrödinger equation to constrain the spreading of the wave packet with time. The resulting integrodifferential equation, the Schrödinger-Newton (S-N) equation, compromised the linearity of quantum mechanics but provided localized stationary solutions. It was Penrose [12,16] who used the S-N equation to explore the quantum-state reduction phenomenon. He proposed that macroscopic gravity could be the reason for the collapse of the wave function as the wave packet responds to its own gravity. The effect of gravity and self-gravity on quantum systems was studied by several authors [17–22].

The coupling of classical gravity to a quantum system also addresses the question of whether gravity is fundamentally quantum or classical [23–25]. This "semiclassical" approach, where gravity is treated in the nonrelativistic (Newtonian) limit, provided simplifications to the calculations, but faced several theoretical objections [26]. However, the ultimate test would be experimental. In such a context, providing an experimental route to test the effect of S-N nonlinearity in a simple quantum mechanical context is valuable.

In the present work, we focus on the evolution of a single isolated massive quantum particle through the nonlinear Schrödinger-Newton equation. The particle is in a superposition state undergoing a double-slit interference. Any signature of nonlinearity due to gravitational self-interaction in the variation of fringe width with mass should give us an experimental

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handle on separating the effect of decoherence from gravitational state reduction.

II. TWO-SLIT EXPERIMENT WITH SELF-GRAVITY

A. Schrödinger-Newton equation

The S-N equation originated from the context of semi-classical gravity, first introduced by Möller [27] and Rosenfeld [28] independently. The fundamental interaction considered in this approach is the coupling of quantized matter with the classical gravitational field [26,29,30]. In this approach, the Einstein field equations get modified as

$$R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle, \tag{1}$$

where the term on the right-hand side is the expectation value of the energy-momentum tensor with respect to the quantum state $|\Psi\rangle$ of matter. This semi-classical modification was studied with reference to the necessity of quantizing gravity [31,32]. The prescribed modification to the Einstein field equation leads to the Schrödinger-Newton equation [26,33–35]

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - Gm^2\int\frac{|\Psi(r',t)|^2}{|r-r'|}d^3r'\right]\Psi(r,t) = i\hbar\frac{\partial\Psi(r,t)}{\partial t}. \tag{2}$$

The above equation can be seen as a *nonlinear* modification of the Schrödinger equation. The nonlinearity breaks the unitarity of the quantum dynamical evolution and opens up the possibility of a dynamical reduction of the wave function, generally referred to as collapse. It is then not surprising that such modification to linear quantum mechanics and classical gravity invites criticism [36]. Apart from this, there were several other collapse models that were investigated in the literature [6,37,38]. However, this approach has to be tested both theoretically and experimentally if one wants to rule it out. Our approach is to check whether it has any significance in the emergence of classicality at all, more so if there is an effect that can be experimentally tested. In future, if the S-N equation gets ruled out by experiments then the particular coupling considered in Eq. (1) will also get ruled out and other types of coupling between gravity and matter fields could be considered [39].

We start by making the S-N equation dimensionless, using scaling parameters $\tilde{r} = r/\sigma_r$, $\tilde{m} = m/m_r$, $\tilde{t} = t/t_r$. The factor σ_r is determined by the natural lengthscale of the problem. Once the lengthscale factor σ_r is fixed, for instance, by experimental considerations (which we discuss in the subsequent section), the other scale factors are determined in terms of σ_r and natural constants

$$t_r = \left(\frac{\sigma_r^5}{G\hbar}\right)^{\frac{1}{3}}, \quad m_r = \left(\frac{\hbar^2}{G\sigma_r}\right)^{\frac{1}{3}}. \tag{3}$$

The rescaled, dimensionless equation is

$$\left[-\frac{\tilde{\nabla}^2}{2\tilde{m}} - \tilde{m}^2\iiint\frac{|\tilde{\Psi}(\tilde{r}',\tilde{t})|^2}{|\tilde{r}-\tilde{r}'|}d^3\tilde{r}'\right]\tilde{\Psi}(\tilde{r},\tilde{t}) = i\frac{\partial\tilde{\Psi}(\tilde{r}',\tilde{t})}{\partial\tilde{t}}, \tag{4}$$

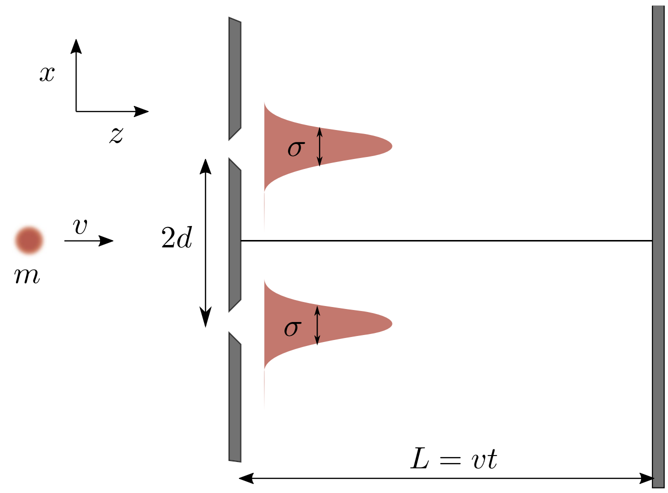

FIG. 1. Schematic diagram of two-slit interferometer for a massive particle.

where $\tilde{\Psi}(\tilde{r},\tilde{t}) = \sigma_r^{\frac{3}{2}}\,\Psi(r,t)$. The problem now has only one scaling parameter $\tilde{m}$ which is dependent on m_r.

B. Formulation of the problem

We analyze the effect of self-gravity on the interference produced by a particle of mass m passing through a two-slit interferometer (Fig. 1). The two slits are separated by a distance $2d$ along the x axis. The particle is assumed to travel along the z axis towards the screen with a constant velocity v.

As the particle emerges from the two-slit, we assume that the initial state is a superposition of two Gaussian wave packets. For the purpose of interference, the dynamics along the z axis is unimportant. It only serves to transport the particle from the slits to the screen by a distance $L = vt$ in a fixed time t. The interference results only from the spread and overlap of the wave packets in the x direction. Hence we assume the initial wave function to be spread along the x direction alone. For calculational simplicity, we assume no spread along the other two directions:

$$\tilde{\Psi}(\tilde{x},0) = A\left[e^{\frac{-(x-d)^2}{2\sigma^2}} + e^{\frac{-(x+d)^2}{2\sigma^2}}\right], \tag{5}$$

where σ is the width of each Gaussian. We completely ignore the time evolution in the y or z directions.

Since we start with a wave function restricted to the x axis, the potential due to self-gravity in Eq. (4) becomes

$$V_G = -\tilde{m}^2\iiint\frac{|\tilde{\Psi}(\tilde{x}',\tilde{t})|^2\,\delta(\tilde{y}-\tilde{y}')\,\delta(\tilde{z}-\tilde{z}')}{\sqrt{(\tilde{x}-\tilde{x}')^2 + (\tilde{y}-\tilde{y}')^2 + (\tilde{z}-\tilde{z}')^2}}d\tilde{x}'\,d\tilde{y}'\,d\tilde{z}'. \tag{6}$$

Performing the delta-function integral, Eq. (4) reduces to an effective one-dimensional (1-D) equation

$$\left[-\frac{1}{2\tilde{m}}\frac{\partial^2}{\partial\tilde{x}^2} - \tilde{m}^2\int\frac{|\tilde{\Psi}(\tilde{x}',\tilde{t})|^2}{|\tilde{x}-\tilde{x}'|}d\tilde{x}'\right]\tilde{\Psi}(\tilde{x},\tilde{t}) = i\frac{\partial\tilde{\Psi}(\tilde{x},\tilde{t})}{\partial\tilde{t}}. \tag{7}$$

The Schrödinger-Newton equation (2) is a nonlinear integrodifferential equation and is hard to solve analytically. We could use perturbative approximations, but to understand the effect of self-gravity on interference phenomena, approximation methods will not be helpful. We therefore resort to a numerical solution.

Now a massive particle is expected to lose coherence during the time evolution and it is obvious that there will also be decoherence effect due to gravitational and other kinds of interaction with the environment. This may lead to suppression of interference in a matter-wave interferometry experiment. For large mass values, one cannot confidently attribute this loss of interference to self-gravity since environment-induced decoherence also leads to exactly the same effect [40,41]. The purpose of this work is to separate out the effects of self-gravitational interaction from those of decoherence.

III. NUMERICAL RESULTS AND DISCUSSION

A. Numerics

We solve Eq. (7) numerically to obtain the solution $\tilde{\Psi}(\tilde{x},\tilde{t})$ for all rescaled time $\tilde{t}$. We used the Crank-Nicolson method [42–44], as it preserves unitarity at each time step.

We used $d = 6\,\sigma_r$ and $\sigma = 2\,\sigma_r$. The spatial extent is $[-70, 70]$, which is divided into 2000 spatial grid points and the temporal grid length is taken from 0 to 10 and is divided into 1000 time steps. Hence, $\delta\,\tilde{x} = 0.07$ and $\delta\,\tilde{t} = 0.01$. For the Crank-Nicolson method, the Courant-Friedrichs-Lewi (CFL) condition necessary for convergence, is satisfied since $\frac{\delta\,\tilde{t}}{\delta\,\tilde{x}} \sim 0.01 < 1$.

The boundary points -70, 70 actually represent numerical infinity. However, as the wave function evolves in time, the quantum mechanical spread could cause the solution to reach the numerical boundary. Once it reaches the boundary, the evolution in the next time step causes Ψ to reflect back and affects the entire solution. To avoid this undesirable effect, we made the boundary large enough such that the evolved wavepackets do not reach the boundary within the time of evolution considered.

To avoid the singularity in the 1-D form of the self-gravity potential [Eq. (6)], we use a regularized form of the potential $V_G(\tilde{x}) = -\tilde{m}^2 \int \frac{|\tilde{\Psi}(\tilde{x}',\tilde{t})|^2}{\sqrt{(\tilde{x}-\tilde{x}')^2+\epsilon^2}} d\tilde{x}'$, where ϵ is a small dimensionless parameter. In the limit $\epsilon \to 0$ one recovers the original potential. The value of ϵ is fixed at 0.01.

B. Interference

The interference patterns for different values of $\tilde{m}$ are plotted in Fig. 2. The x axis is position in units of σ_r and the y axis is the (dimensionless) probability density $|\tilde{\Psi}(\tilde{x},\tilde{t})|^2$. As one moves from mass $\tilde{m} = 0.20$ to $\tilde{m} = 0.60$, the crossover from temporal emergence of interference to complete suppression of it, due to the effect of self-gravity, is beautifully brought out. At intermediate values of mass the interference is seen with lower visibility. In contrast, in the absence of self-gravity, interference is seen even at large mass values.

In the usual two-slit interference scenario, the fringe width is equal to $\lambda L/2d$, where λ is the de Broglie wavelength of the particle, $2d$ the slit separation, and L the distance between the double-slit and the screen. For a particle of mass m traveling

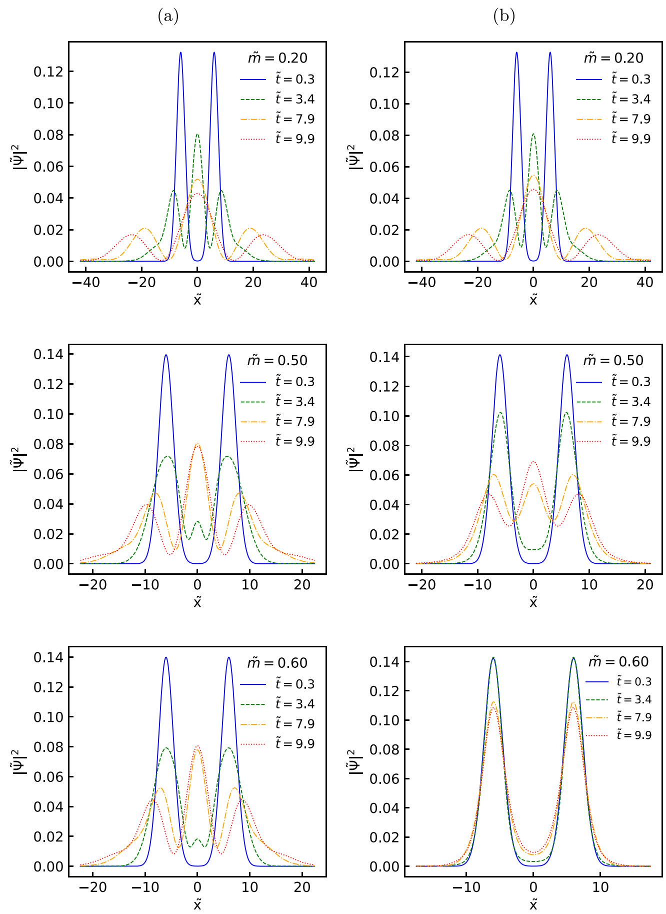

FIG. 2. Comparison of onset of quantum interference as the superposition evolves with time for different $\tilde{m}$ values for (a) free Schrödinger evoution and (b) with self-gravity.

with a velocity v, the de Broglie wavelength is $\lambda = h/mv$. Taking the distance traveled by the particle as $L = vt$, the fringe width turns out to be $w = ht/2md$. Thus, for a fixed t the fringe width varies inversely with the mass of the particle. Even if the particle experiences environment induced

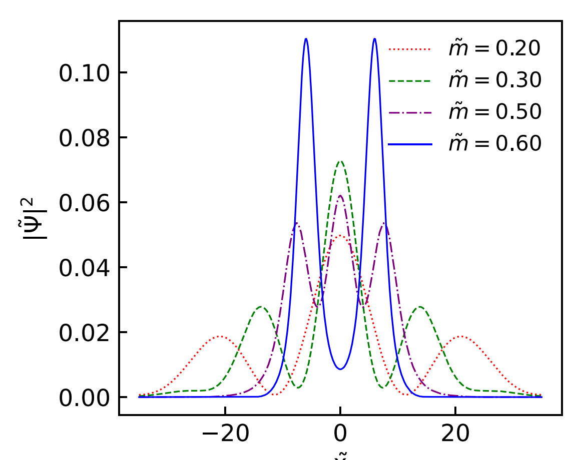

FIG. 3. Interference pattern at time $\tilde{t} = 8.9$ for different values of $\tilde{m}$. The interference gets progressively less sharp as the mass increases, until it is completely suppressed.

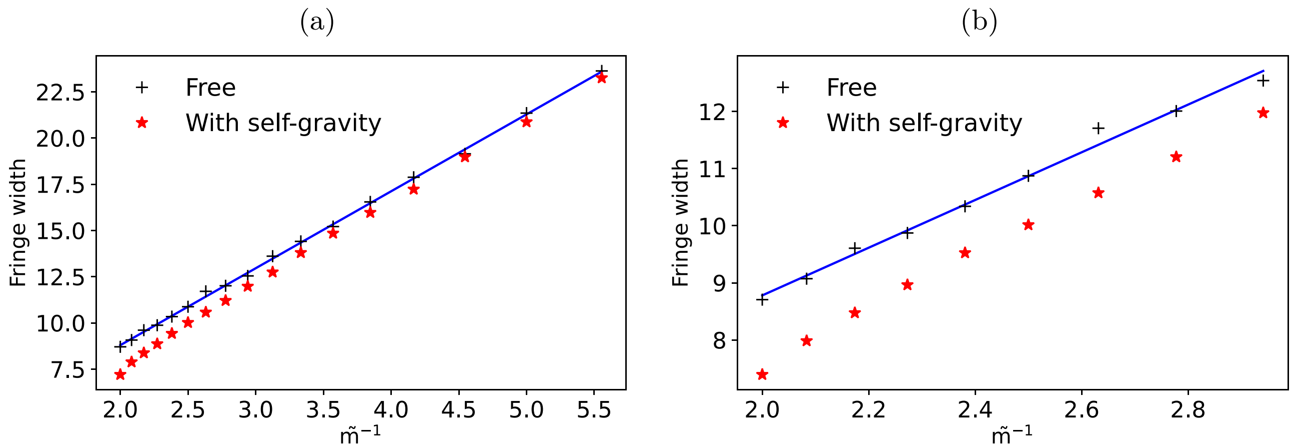

FIG. 4. Fringe width w (in units of σ_r) from simulated evolution as a function of $1/\tilde{m}$ at time $\tilde{t} = 8.9$ (a) for the full mass range considered and (b) zoomed in to high mass values. The + symbols represent w without self-gravity, the straight line through them being the trend line; the red stars represent w in the presence of self-gravity. For larger mass, in the presence of self-gravity, the deviation of w from $1/\tilde{m}$ behavior is more evident.

decoherence, although the interference visibility goes down, the fringe width remains unaffected [40]. Therefore, any deviation of the fringe width from $1/m$ dependence should be a signature of the effect of self-gravity.

The fringe width w is calculated from the simulated results as follows. It is assumed that a central peak in the probability distribution is a necessary signature of interference. We calculate w as the distance between the central peak and its nearest interference maximum. One can see from Fig. 3 that the interference peaks are well defined at $\tilde{t} = 8.9$, for various values of $\tilde{m}$. Thus, without ambiguity, we calculate the fringe width from the probability distribution for varying $\tilde{m}$, both with and without the self-gravity potential term. We plot w versus $1/\tilde{m}$ with and without self-gravity in Fig. 4. The results clearly show that in the presence of the self-gravity potential, w deviates from $1/\tilde{m}$ dependence as the mass of the particle increases. We believe this should form a clear test of gravitational self-interaction.

We also notice that, as the mass increases, the spread in the wave function is suppressed by the self-gravity effect. This is clearly seen in Fig. 3, where we plot the probability density at $\tilde{t} = 8.9$ for different mass values. For smaller masses, the wave function spreads enough so that the two wave packets overlap to result in interference. For much larger masses the gravitational self-interaction suppresses the spread of the wave packets so that they are not able to overlap and do not lead to any interference. This behavior is consistent with the original aim of introducing the S-N equation.

C. "Attraction" between peaks

It is generally expected that if the wave-function has two lobes, the self-gravitational interaction will lead to an "attraction" between the two, in the sense that dynamical evolution will bring them closer together. In Fig. 3, there is apparently no noticeable attraction within the time range considered here.

We take a closer look at the form of self-gravity potential as time evolves, for a much longer time range (Fig. 5). The initial wave function consists of two disjointed lobes and hence the potential peaks near the centers of the two wave packets. The effect of this is seen as a narrowing of the two wave

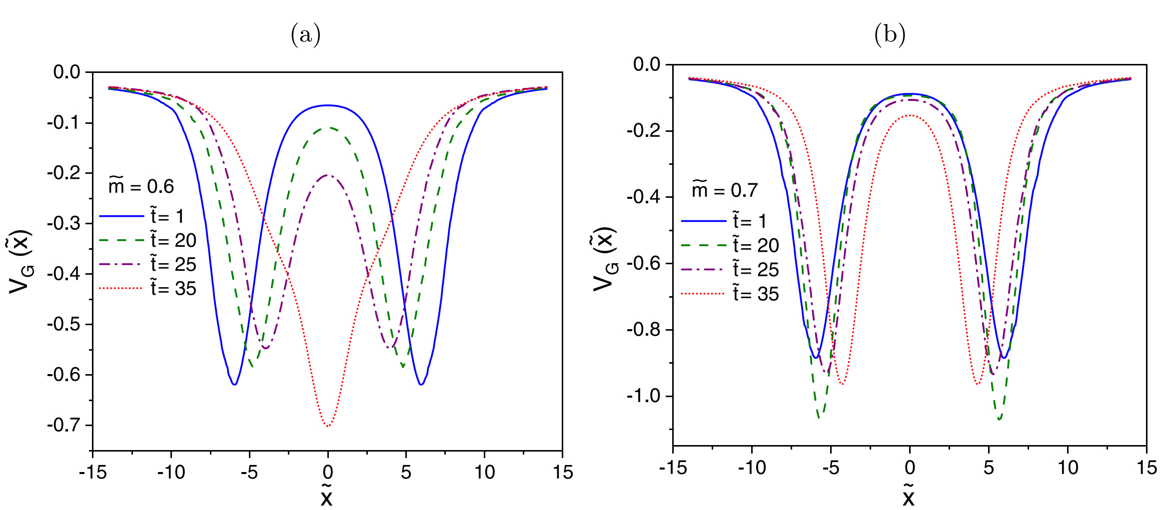

FIG. 5. The self-gravity potential $V_G(\tilde{x})$ for higher masses, plotted at various times to show its time evolution. (a) $\tilde{m} = 0.60$, (b) $\tilde{m} = 0.70$.

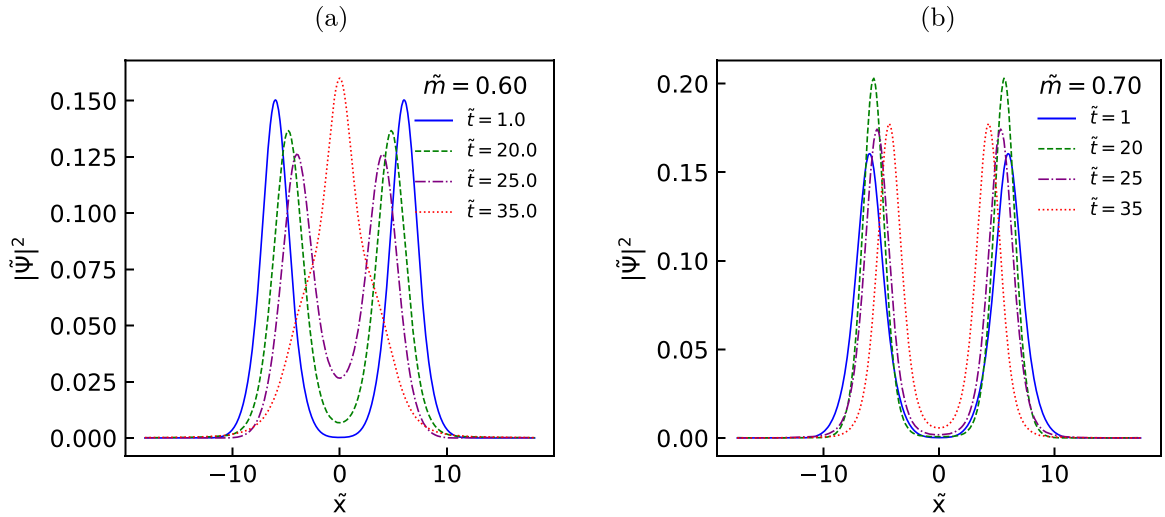

FIG. 6. Probability distribution for relatively large mass values, showing attraction due to self-gravitational effects, and finally merge into a single peak. (a) $\tilde{m} = 0.60$, (b) $\tilde{m} = 0.70$.

packets about their centers. As time evolves, there is a competition between two effects: the narrowing of each wave packet due to self-gravity, and the broadening effect of Schrödinger evolution.

For low-enough masses, the broadening effect of quantum evolution seems to dominate, the wave packets' overlap and interference is observed. For higher masses, apart from the narrowing effect due to the dominance of self-gravity, there is also overlap of the wave packets at long times. This contributes to the potential in the region between the two peaks and results in the peaks in the potential drawing closer together until eventually there is a single central peak. The effect of this is that the two wave packets appear to "attract" each other, until eventually there is a single central peak (see Fig. 6).

One may have expected that the attraction between the peaks would be stronger as the mass increased. However, for the reasons described above, the higher the mass, the slower is the attraction between the peaks.

D. Experimental feasibility

Lastly, we would like to discuss what kind of challenges our proposal poses for the experiments, if one were to try observing this effect in some experiment. As seen from Figs. 2 and 3, the effect of self-gravity on the fringe width is visible for $\tilde{m} \sim 0.5$ for $\tilde{t} \sim 8$. From Eq. (3) one can see that m_r has a $\sigma_r^{-1/3}$ dependence whereas t_r has a $\sigma_r^{5/3}$ dependence. This implies that if one chooses a large σ_r, one would see a noticeable self-gravity effect for small mass, but after a long time evolution. Thus σ_r has to be chosen such that it gives an experimentally feasible mass of the particle which can remain in a superposition of two wave packets for a time of the order of t_r. If we consider $\sigma_r = 1.112$ nm, it leads us to $m_r = 31.94 \times 10^9$ u and $t_r = 0.623$ s, which means that for particles of mass about 16×10^9 u, the self-gravity effect should be observable after about 5 seconds of time evolution. The slit separation required will be about 13 nm.

Now interferometry with large molecules has shown a steady progress, with the interference of C_{70} fullerene molecules through Talbot-Lau interferometer being a prominent example [45]. Probably the best technology at present is the optical time-domain ionizing matter-wave (OTIMA) interferometer [46]. The Vienna Kapitza-Dirac-Talbot-Lau interferometer is another one that is capable of using such high mass range, approximately 6509 u [47,48]. The latest example is using the Long-Baseline Universal Matter-Wave Interferometer (LUMI) [49], which has achieved superpositions of particles of masses as high as 25×10^3 u. It is hoped that in the future, particles of mass 10^8 u, like gold clusters, might be diffracted with the OTIMA scheme [1]. However, even this mass range is too small for observing the effect due to self-gravity. This is exemplified by the fact that if one insists on looking for self-gravity effects for particles of mass 10^8 u, one would need times of ridiculous magnitude, of the order of 10^{10} s, to see the self-gravity effects.

So, the message is that one would need to study the interference of particles of mass of the order of 10^{10} u if one hopes to see any signature of self-gravitational interaction. This looks challenging with the current state-of-the-art technology.

IV. CONCLUSION

In conclusion, we find that the analysis of the Schrödinger-Newton equation for the time evolution of a superposition of two Gaussian wave packets, as in a two-slit experiment, demonstrates the self-gravity interaction has a distinct effect on quantum interference. Interference for small mass particles is virtually indistinguishable from that governed by the pure Schrödinger evolution. For larger mass particles, quantum interference is suppressed. For intermediate mass values, interference with a reduced visibility is seen. Now in an actual experiment, the observation of interference with reduced visibility can also be attributed to environmental effects. However, the fringe width w emerges as a key element in distinguishing self-gravity effects from those of decoherence.

It yields a definite signature of the effect of self-gravity as mass increases, and should be verifiable experimentally, if matter wave interferometry experiments can be carried out at the appropriate length and mass scales.

The deviation of w versus $1/m$ from a straight line for large mass is expected, as there is a mass-dependent self-interaction potential affecting the dynamics of the particle. If this phenomenon is experimentally corroborated, then there would be reason for further analysis of the origin and effects of the S-N potential in the Schrödinger equation. We believe our work provides sufficient reason for renewed experimental work in matter-wave interferometry for larger mass particles. Apart from providing clues to the emergence of classicality from quantum mechanics, such experiments may also throw some light on the question as to whether a full quantum theory of gravity is needed, or semi-classical gravity is sufficient in several quantum mechanical contexts.

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