Characterization of two-particle interference by complementarity

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Bohr's complementarity principle is quantitatively formulated in terms of the distinguishability of various paths a quanton can take and the measure of the interference it produces. This phenomenon results from the interference of single-quanton amplitudes for various paths. The distinguishability of paths puts a bound on the sharpness of the interference the quanton can produce. However, there exist other kinds of quantum phenomena where interference of *two-particle* amplitudes results in a two-particle interference, if the particles are indistinguishable. The Hong-Ou-Mandel (HOM) effect and the Hanbury-Brown-Twiss (HBT) effect are two well-known examples. However, two-particle interference is not as easy to define as its single-particle counterpart, and the realization that it involves interference of two-particle amplitudes came much later. In this work, a duality relation, between the particle distinguishability and the visibility of two-particle interference, is derived. The distinguishability of the two particles, arising from some internal degree of freedom, puts a bound on the sharpness of the two-particle interference they can produce, in a HOM or HBT kind of experiment. It is argued that the existence of this kind of complementarity can be used to characterize two-particle interference, which in turn leads one to the conclusion that the HOM and the HBT effects are equivalent in essence and may be treated as a single two-particle-interference phenonmenon.

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I. INTRODUCTION

It is well known that a single particle, better referred to as a quanton, passing through multiple paths, can interfere with itself, producing an interference pattern. Bohr's complementarity principle [1] can then be *quantitatively* formulated as the duality relation [2] $\mathcal{D}_Q + \mathcal{C} \leqslant 1$, where $\mathcal{D}_Q$ is the path distinguishability and $\mathcal{C}$ is the quantum coherence [3] of the quanton. The path distinguishability is defined in terms of unambiguous quantum state discrimination (UQSD) [4,5]. Coherence of the quanton can be measured in a multipath interference experiment in various ways [6–8]. If the path distinguishability is defined in a different way, one gets a different form of the duality relation [9], $\mathcal{D}^2 + \mathcal{C}^2 \leqslant 1$, where $\mathcal{D}^2 = \mathcal{D}_Q(2 - \mathcal{D}_Q)$. However, the two duality relations are the same in essence. For the special case of only two paths, this relation reduces to the well-known duality relation [10] $\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1$, where $\mathcal{V}$ is the conventional visibility of interference, defined as $\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$, with $I_{\max}$ and $I_{\min}$ being the maximum and the minimum intensity of interference in a particular region of the interference pattern, respectively.

Apart from this phenomenon which results from interference of single-particle amplitudes, later experiments showed that interference of two-particle amplitudes is also possible. Such effects have been generically called two-particle interference. However, it is not always easy to characterize two-particle interference, and the understanding that it came from interference of two-particle amplitudes came much later. Two well-known effects that capture the phenomenon well are the Hong-Ou-Mandel (HOM) effect [11–13] and the Hanbury-Brown-Twiss (HBT) effect [14–18]. Two-particle interference has been much studied as well as much debated. Although for identical particles, the two-particle symmetric and antisymmetric states are entangled, and it is not straightforward to identify two-particle interference with this entanglement [19]. Different from other two-particle quantum effects, it does not seem to originate from the entanglement between the two particles and rather appears to have its roots in the fundamental indistinguishability of identical quantum particles. The HBT effect has also been demonstrated with massive particles, both of a bosonic nature [20–22] and a fermionic nature [23]. In an interesting development, it was demonstrated that, in a HOM experiment, if the two particles coming from two different paths are made partially distinguishable, it results in the loss of visibility of the HOM interference dip [24]. If the two particles are fully distinguishable, the HOM dip completely disappears. Not only that, one can also set up a "quantum eraser" in a HOM experiment and recover the HOM dip with maximum visibility [24,25]. In another interesting experiment, a delayed-choice quantum eraser was demonstrated using thermal light [26]. In this experiment too, the interference was a two-photon interference. Although such an effect has not been explored in the HBT experiment, to our knowledge, we show that it should exist in the HBT experiment too. These experiments point towards a complementarity involving particle distinguishability and the visibility of two-particle interference. We show here that this complementarity can be *quantified* in the same way as Bohr's complementarity was quantified by the wave-particle duality relations. This complementarity

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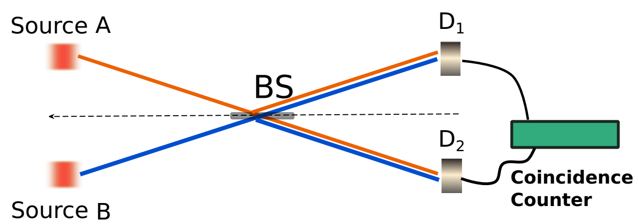

FIG. 1. A schematic diagram for the Hong-Ou-Mandel experiment. Independent particles from sources A and B meet at the beamsplitter (BS) and then arrive at the detectors D_1 and D_2.

will then characterize the two effects as a unified two-particle interference phenomenon. Quantifying complementarity in two-particle interference, and using it to characterize the two-particle interference, is the subject of this investigation.

II. THE HONG-OU-MANDEL EFFECT

We briefly introduce the HOM experiment. Two identical particles emerge from two spatially separated sources A and B, in the states $|\psi_A\rangle$ and $|\psi_B\rangle$ (see Fig. 1). Assuming that the particles are bosons, the two-particle state is the following symmetrized state:

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle_1|\psi_A\rangle_2 + |\psi_A\rangle_2|\psi_A\rangle_1), \tag{1}$$

where the labels 1 and 2 are particle labels. The two particles are split by the 50-50 beamsplitter (BS) and reach the *fixed* detectors D_1 and D_2. Now let us assume that there exists another degree of freedom by which the particles from the two sources can be distinguished. This degree of freedom belongs to the particles, e.g., polarization in the case of photons, and spin in the case of neutrons. The combined state of the two particles, and the additional degree of freedom, can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle_1|\psi_B\rangle_2|d_A\rangle_1|d_B\rangle_2 + |\psi_A\rangle_2|\psi_B\rangle_1|d_A\rangle_2|d_B\rangle_1), \tag{2}$$

where $|d_A\rangle$ and $|d_B\rangle$ are two states of the additional degree of freedom, which are assumed to be normalized, but not necessarily orthogonal. If one could distinguish between the states $|d_A\rangle$ and $|d_B\rangle$, finding a state, say, $|d_A\rangle$, would tell one that the particular particle came from source A. Since there are two identical particles involved, the additional degree of freedom has to be attached to the particle itself. It should be emphasized here that one cannot have an external path-marking device, like that used in single-particle which-way experiments, because when the particles move away from the path marker, there is no way to tell which of the two identical particles is correlated to the path marker. That is the reason why, for two-particle interference, it does not make sense to talk of path distinguishability. One can only distinguish between the two particles based on some degree of freedom of the particle. The connection between particle distinguishability and coherence of the two-particle system has been explored before [27].

The effects of the beamsplitter on the two states are as follows: $U|\psi_A\rangle = \frac{1}{\sqrt{2}}(|D_1\rangle - |D_2\rangle)$ and $U|\psi_B\rangle = \frac{1}{\sqrt{2}}(|D_1\rangle + |D_2\rangle)$, where $|D_1\rangle$ and $|D_2\rangle$ are the states of a particle at the detectors D_1 and D_2, respectively. Using this, the two-particle state, after the particles pass through the beamsplitter, can be written as

$$\begin{aligned} U|\Psi\rangle = &\frac{1}{2\sqrt{2}}(|D_1\rangle_1 - |D_2\rangle_1)(|D_1\rangle_2 + |D_2\rangle_2)|d_A\rangle_1|d_B\rangle_2 \\ &+\frac{1}{2\sqrt{2}}(|D_1\rangle_2 - |D_2\rangle_2)(|D_1\rangle_1 + |D_2\rangle_1)|d_A\rangle_2|d_B\rangle_1. \end{aligned} \tag{3}$$

The probability of a coincident count is given by

$$\begin{aligned} P_C &= |{}_1\langle D_1|{}_2\langle D_2|U|\Psi\rangle|^2 + |{}_2\langle D_1|{}_1\langle D_2|U|\Psi\rangle|^2 \\ &= \tfrac{1}{4}|(|d_A\rangle_1|d_B\rangle_2 - |d_A\rangle_2|d_B\rangle_1|^2 \\ &= \tfrac{1}{2}(1 - |\langle d_A|d_B\rangle_1||\langle d_A|d_B\rangle_2|) \\ &= \tfrac{1}{2}(1 - |\langle d_A|d_B\rangle|^2). \end{aligned} \tag{4}$$

This is the minimum intensity of the two-particle interference, as we have already assumed that the two particles arrive at the beamsplitter at the same time. If the two particles do not arrive at the beamsplitter together, each particle acts independently and is equally likely to land at D_1 or D_2. Consequently, the probability of the coincident count is $1/2$. That is the maximum intensity of the coincident count, and one can say $C_{\max} = 1/2$. The minimum coincident count intensity is given by $C_{\min} = \frac{1}{2}(1 - |\langle d_A|d_B\rangle|^2)$. The conventional definition of HOM interference visibility [12] yields

$$\mathcal{V} = \frac{C_{\max} - C_{\min}}{C_{\max}} = |\langle d_A|d_B\rangle|^2. \tag{5}$$

At any of the two detectors, suppose one wants to find out whether a particle came from source A or source B, the way to do it is to look at the other degree of freedom of the particle. If one can tell if the state of the particle is $|d_A\rangle$, it means the particle came from source A, if the state turns out to be $|d_B\rangle$, it means the particle came from source B. So the problem of distinguishing the two particles boils down to distinguishing between the quantum states $|d_A\rangle$ and $|d_B\rangle$. Since the two states may not be orthogonal, one can use UQSD to unambiguously distinguish between them. The optimal probability of successfully distinguishing between them is given by $\mathcal{D}_Q = 1 - |\langle d_A|d_B\rangle|$ [2]. Here it is more convenient to define the particle distinguishability as $\mathcal{D} = \mathcal{D}_Q(2 - \mathcal{D}_Q)$, which leads to

$$\mathcal{D} = 1 - |\langle d_A|d_B\rangle|^2, \tag{6}$$

which also happens to be the square of the distinguishability coming from minimum error discrimination of the two states [10]. Combining Eqs. (5) and (6), one arrives at

$$\mathcal{D} + \mathcal{V} = 1. \tag{7}$$

This is a duality relation involving the particle distinguishability $\mathcal{D}$ and the visibility $\mathcal{V}$ of HOM interference. If the states $|d_A\rangle$ and $|d_B\rangle$ are orthogonal, the two particles, coming from different sources, become distinguishable. Consequently they should not show any HOM effect. Indeed, in such a case the HOM dip disappears and there is no HOM interference. If $|d_A\rangle$ and $|d_B\rangle$ have partial overlap, the two particles are

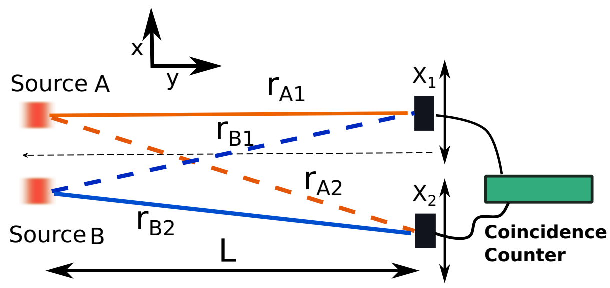

FIG. 2. A schematic diagram for the Hanbury-Brown-Twiss experiment. Independent particles from sources A and B travel and arrive at the two detectors at x_1 and x_2.

partially distinguishable. In that case the HOM is present, but partially suppressed. Thus the relation Eq. (7) quantifies the complementarity between particle distinguishability and two-particle interference.

It is straightforward to see that when $|d_A\rangle$ and $|d_B\rangle$ are orthogonal, a quantum eraser can be set up by selecting both particles in a state of the internal degree of freedom which has equal overlap with both $|d_A\rangle$ and $|d_B\rangle$.

III. THE HANBURY BROWN-TWISS EFFECT

The HBT effect was discovered much before the HOM effect, in classical radio waves. Later it was demonstrated in classical light [14]. Its applicability and meaning in the quantum domain was widely debated and misunderstood. The physical understanding of the HBT effect in the quantum domain was provided by Fano [15]. In fact, an early two-photon experiment [28] was believed to demonstrate nonlocal quantum correlations, but was later shown to be just the HBT effect [18]. In the HBT experiment two particles emerge from two spatially separated sources A and B and travel to separate, movable detectors at positions x_1 and x_2 (see Fig. 2). In our setup, the particles travel as wave packets along the y axis and spread in both the x and y directions. For the purpose of the HBT effect, their dynamics only along the x axis are relevant. In the following we do not consider the motion of the particles along the y axis explicitly. We just assume that the wave packets travel with a uniform velocity along the y axis, and after a fixed time they land up at the detectors.

Let us also assume that the particles carry another degree of freedom, which may be spin for massive particles and polarization for photons. This degree of freedom can potentially make the two particles distinguishable. In the following we assume that the particles, when they emerge from the two sources, are Gaussian wave packets centered at x_0 and $-x_0$, traveling along the y axis. The widths of the wave packets are assumed to be small, denoted by ϵ. The additional degree of freedom of the particles is assumed to have a two-dimensional Hilbert space, and the particles emerging from source A (B) have a state $|d_A\rangle$ ($|d_B\rangle$).

As in the case of the HOM experiment, the full wave function of the two particles, with the additional degree of freedom, when they just emerge from the sources, can be written as

$$\psi(x_1,x_2,0) = \frac{1}{\sqrt{\pi}\epsilon}\Big(e^{\frac{-(x_1-x_0)^2}{\epsilon^2}} e^{\frac{-(x_2+x_0)^2}{\epsilon^2}} |d_A\rangle_1|d_B\rangle_2 + \eta e^{\frac{-(x_1+x_0)^2}{\epsilon^2}} e^{\frac{-(x_2-x_0)^2}{\epsilon^2}} |d_A\rangle_2|d_B\rangle_1\Big), \tag{8}$$

where x_1 and x_2 denote the positions of the particles, and $\eta = \pm 1$. For bosonic particles, the wave function should be symmetric, and η should be 1. For fermions, the two-particle wave function should be antisymmetric, requiring η to be -1. The particles travel along the y axis to the two detectors and also spread in the x direction governed by the free-particle Hamiltonian $H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$, where m is the mass of the particles.

After a time t the particles reach the detectors. The amplitude of finding the particles at the detectors at x_1 and x_2 then works out to be

$$\psi(x_1,x_2,t) = \alpha\Big(e^{\frac{-(x_1-x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2+x_0)^2}{\epsilon^2+i\Delta}} |d_A\rangle_1|d_B\rangle_2 + \eta e^{\frac{-(x_1+x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2-x_0)^2}{\epsilon^2+i\Delta}} |d_A\rangle_2|d_B\rangle_1\Big), \tag{9}$$

where $\Delta \equiv 2\hbar t/m$, and $\alpha = \frac{1}{\sqrt{\pi(\epsilon+i\Delta/\epsilon)}}$. The joint probability density of finding the particles at x_1 and x_2 is given by

$$|\psi(x_1,x_2,t)|^2 = \frac{2}{\pi\sigma^2} e^{\frac{-2(x_1^2+x_2^2+2x_0^2)}{\sigma^2}} \cosh\left(\frac{4(x_1-x_2)x_0}{\sigma^2}\right) \left(1 + \eta|\langle d_A|d_B\rangle|^2 \frac{\cos\left(\frac{4\Delta(x_1-x_2)x_0}{\epsilon^4+\Delta^2}\right)}{\cosh\left(\frac{4(x_1-x_2)x_0}{\sigma^2}\right)}\right), \tag{10}$$

where $\sigma^2 = \epsilon^2 + \Delta^2/\epsilon^2$. The above expression represents a two-particle interference pattern exhibited in coincident counting of the two detectors, as a function of the detector separation. If the interference is observed sufficiently far from the sources, the situation is equivalent to the Fraunhofer limit, and $\Delta \gg \epsilon^2$ can be assumed to hold. In this limit, the above expression simplifies and yields a distinct interference pattern (see Fig. 3). It represents the HBT effect. The reduced visibility seen in Fig. 3 is due to $|\langle d_A|d_B\rangle|$ being less than 1. It should be stressed here that the use of Gaussian wave packets here is just for the sake of calculational convenience. A different profile of the wave packets would lead to the same effect.

Normally one has to be careful in deciding how to define visibility in a two-particle interference [29,30]. However, in the HBT setup considered here, it is quite straightforward, and one can just use the Michelson fringe contrast for the coincident counts $\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$. When the wave packets arrive at the detectors at x_1 and x_2, they are expected to be very broad and strongly overlapping. The maxima of the intensity will be for the separations of the two detectors for which the cosine term in Eq. (10) is equal to $+1$, whereas the minima will be for separations for which the cosine term is equal to -1. In the limit $\Delta \gg \epsilon^2$ the cosh term will be approximately 1. The (ideal) visibility of interference is then given by

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = |\langle d_A|d_B\rangle|^2. \tag{11}$$

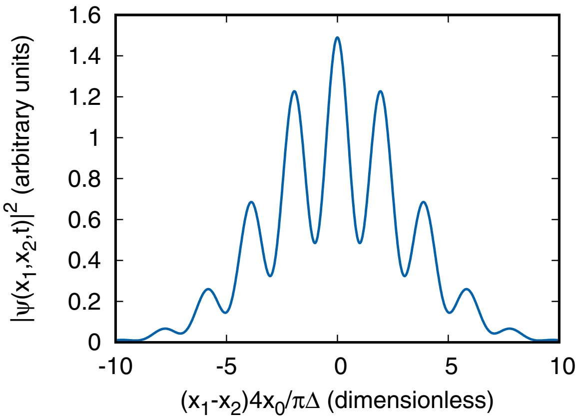

FIG. 3. The probability density of the coincident count Eq. (10), plotted against detector separation, for $|\langle d_A|d_B\rangle| = 0.7$, in the limit $\Delta \gg \epsilon^2$. The detector separation has been scaled with the fringe width, making other parameters unimportant for the interference pattern. The interference visibility is noticeably reduced.

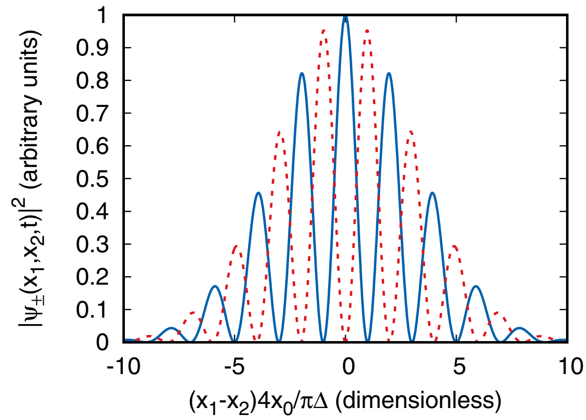

FIG. 4. The probability density of the coincident counts Eq. (10), in the experimental situation where there are spin filters in front of the two detectors, depicted by Eq. (13) (solid blue line) and Eq. (14) (dotted red line), plotted against detector separation, for $|\langle d_A|d_B\rangle| = 0$, in the limit $\Delta \gg \epsilon^2$. The two complementary interference patterns have maximum visibility and represent quantum erasure.

Under nonideal conditions the visibility is $\mathcal{V} \leqslant |\langle d_A|d_B\rangle|^2$. In certain experimental situations there may be only a few fringes within a rather narrow envelope of the wave function. This will make evaluating the visibility from the interference pattern more difficult.

As in the HOM experiment, one can distinguish between the two particles by analyzing the state of its additional degree of freedom, i.e., the spin or the polarization. One can then define the particle distinguishability by Eq. (6). Using Eqs. (6) and (11), one can write the duality relation

$$\mathcal{D} + \mathcal{V} = 1, \tag{12}$$

which is identical to that derived for the HOM effect Eq. (7). Thus, it can be thought of as a universal duality relation between particle distinguishability and two-particle interference. It should be emphasized here that in the two-particle interference discussed here, no coherence in the sources is required. Any random fluctuation in phases at the source would not affect the interference or its visibility. These experiments can also be performed with thermal light [26].

From Eq. (10) it is obvious that, if $|d_A\rangle$ and $|d_B\rangle$ are orthogonal, no interference will be seen. One can put filters in front of the two detectors which allow only the particles which have state $|d_\pm\rangle = (|d_A\rangle \pm |d_B\rangle)/\sqrt{2}$ to pass through. Identical filters need to be put in front of both detectors. In such a situation, the two-particle state at the detectors will be

$$\psi_+(x_1, x_2, t) = \frac{\alpha}{2}\Big(e^{\frac{-(x_1-x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2+x_0)^2}{\epsilon^2+i\Delta}} + \eta e^{\frac{-(x_1+x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2-x_0)^2}{\epsilon^2+i\Delta}}\Big), \tag{13}$$

if both the filters allow $|d_+\rangle$. The same state results in the situation where both the filters allow the $|d_-\rangle$ state. This state leads to an interference pattern with maximum visibility and corresponds to quantum erasure. Alternately, one can put a filter which allows only particles with state $|d_-\rangle = (|d_A\rangle - |d_B\rangle)/\sqrt{2}$ in front of the detector at x_1, and a filter which allows only particles with state $|d_+\rangle = (|d_A\rangle + |d_B\rangle)/\sqrt{2}$ in front of the detector at x_2. In this situation, the two-particle state at the detectors will be

$$\psi_-(x_1, x_2, t) = \frac{\alpha}{2}\Big(e^{\frac{-(x_1-x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2+x_0)^2}{\epsilon^2+i\Delta}} - \eta e^{\frac{-(x_1+x_0)^2}{\epsilon^2+i\Delta}} e^{\frac{-(x_2-x_0)^2}{\epsilon^2+i\Delta}}\Big). \tag{14}$$

This state also leads to an interference pattern with maximum visibility, but one which is shifted such that maxima are located at the positions of the minima of the previous interference pattern (see Fig. 4). This analysis shows that a quantum eraser is very much possible in a HBT experiment. Two-particle interference with partially distinguishable particles is a potentially useful phenomenon. A quantum-enhanced microscope was demonstrated by using two-photon interference and employing the photon polarization states [31].

IV. EQUIVALENCE OF HOM AND HBT EFFECTS

The HBT effect and the HOM effect have been treated as two distinct effects. While the HBT effect has also been seen in classical waves, the HOM effect is believed to be a purely quantum effect. The preceding analysis of the two effects, and the same kind of complementarity observed in the two, points to a closer connection between the two. We wish to emphasize that in our view the two effects are the same. This is elaborated upon in the following discussion. If one considers the single-particle two-slit interference and the single-particle Mach-Zehnder interference experiment, one may naively think of them as very different experiments, but a deeper look at the two reveals that they are in essence completely equivalent. In the two-slit experiment, the particle passes through two spatially separated slits and then emerges as two rapidly expanding wave packets which overlap with each other. In different parts of the overlapping wave packets, there is constructive or destructive interference. In a Mach-Zehnder interferometer, a particle is split into two distinct wave packets traveling two different paths, which is like the particle passing through a double-slit. Then the two parts are combined at a beamsplitter, and both are split into two parts, so that each beam has parts from both wave packets. The essential difference is that the phase difference between the two paths has to be tuned in such a way that in one output beam the two packets interfere destructively, and in the other they interfere constructively. It is as if all the dark fringes of the double-slit interference are combined into one dark output beam.

Now, the difference between the HBT experiment and the HOM experiment is very similar to that between the two-slit experiment and the Mach-Zehnder experiment. In the HBT setup, two particles emerge from two spatially separated sources and travel in the same direction as the expanding wave packets. After some time they overlap, and the joint detection of the two at different spatial locations shows a constructive or destructive interference. For certain separations of the two detectors, there is no coincident detection. These are the dark fringes. In the HOM setup, like a Mach-Zehnder setup, two particles emerge from two different sources and travel two separated well-defined paths, and they do not overlap. They are brought together at a beamsplitter and split into two parts each. Each of the two beams has parts coming from each particle. Just as in the Mach-Zehnder setup the phase difference between the two paths has to be fine-tuned to get null output at one of the two detectors, the time delay between the two photons in the HOM experiment has to be fine-tuned so that the coincident count at the output beams becomes zero. This is equivalent to the coincident count becoming zero for certain separations of the two detectors in the HBT experiment. Thus the HBT and HOM effects are quite analogous to each other and can be looked upon as a single two-particle-interference phenomenon.

V. CONCLUSION

In conclusion we have analyzed the HOM and HBT effects and shown that there exists a quantitative complementarity between the particle distinguishability and the visibility of the two-particle interference. The more distinguishable the two particles are, due to some internal degree of freedom, the more degraded is the two-particle interference. This complementarity should be universal in nature and should apply to any two-particle interference. Such a complementarity points to the fact that there is a single phenomenon underlying the HBT and HOM effects. The quantitative complementarity which is demonstrated here can be used to characterize the two-particle interference in any variant of these two experiments.

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