Accelerating the approach of dissipative quantum spin systems towards stationarity through global spin rotations

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We consider open quantum systems whose dynamics is governed by a time-independent Markovian Lindblad master equation. Such systems approach their stationary state on a timescale that is determined by the spectral gap of the generator of the master equation dynamics. In a recent paper [Carollo *et al.*, Phys. Rev. Lett. **127**, 060401 (2021)] it was shown that under certain circumstances it was possible to exponentially accelerate the approach to stationarity by performing a unitary transformation of the initial state. This phenomenon can be regarded as the quantum version of the so-called Mpemba effect. The transformation of the initial state removes its overlap with the dynamical mode of the open system dynamics that possesses the slowest decay rate and, thus, determines the spectral gap. Whereas this transformation can be exactly constructed in some cases, it is, in practice, challenging to implement. Here we show that even far simpler transformations constructed by a global unitary spin rotation allow to exponentially speed up relaxation. We demonstrate this using simple dissipative quantum spin systems, which are relevant for current quantum simulation and computation platforms based on trapped atoms and ions.

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I. INTRODUCTION

The study of open quantum systems is an important subfield in modern physics. It considers ensembles of quantum particles that are weakly coupled to an external bath. This coupling gives rise to irreversible processes and a relaxation dynamics that typically leads to a stationary state. In the simplest possible setting this dynamics is governed by a Markovian Lindblad master equation [1–3], which evolves the quantum state ρ of the system in time. This approach is applicable to a wide range of phenomena, including simple processes, such as spontaneous decay, or sophisticated many-body effects, such as sub- and superradiance [4]. Moreover, this formulation allows to describe stationary state phases, transitions among them [5,6], even protocols for (quantum) computation [7], and the creation of correlated many-body states [8–10] which may be relevant in the realm of quantum technologies.

In the context of these applications one may be interested in approaching the stationary state, i.e., the end result of a computation or a desired correlated state, on a timescale which is as short as possible. In the case of Markovian (Lindblad) time evolutions, this timescale is dictated by the spectrum—in particular, by the spectral gap—of the dynamical generator. This gap is, however, difficult to alter without changing the characteristics of the stationary state itself. An alternative approach to this problem is offered by the so-called Mpemba effect which was first discussed in the context of classical nonequilibrium problems. Originally, it describes the counterintuitive phenomenon that water, which is initially at a high temperature, freezes faster than it would when starting from a lower temperature. It is named after Mpemba who has discovered this effect while preparing ice cream, at that time being a schoolboy in Tanzania [11]. The observed accelerated approach to the stationary state turns out to originate from the fact that a high-temperature thermal state has lower overlap with slowly decaying dynamical modes than a thermal state with low temperature [12–21].

This idea can be generalized to the dynamics of open quantum systems as recently discussed in Ref. [22]. Here it was shown that for any given pure initial state ρ_0 relaxation to stationarity can be accelerated by the application of a preliminary unitary operation U, see Fig. 1(a). It was shown how this unitary needs to be constructed such that the new initial state $\bar{\rho}_0 = U\rho_0 U^\dagger$ arrives at stationarity exponentially faster. The idea underlying the construction of the unitary is that it should make the initial state "orthogonal" to, i.e., not overlapping with the dynamical mode associated with the slowest decay rate, which, in fact, corresponds to the spectral gap of the dynamical generator. For certain cases this unitary can be explicitly constructed. However, this construction may be not so useful in practical terms as one needs knowledge of the precise structure of some eigenmodes of the dynamical generator. The resulting unitary is typically rather contrived, i.e., it remains doubtful that it can be readily implemented in an experimental setting.

In this paper we investigate whether it is possible to achieve an exponential acceleration in the relaxation towards stationarity through the application of substantially simpler unitaries. To be specific, we focus on driven dissipative spin

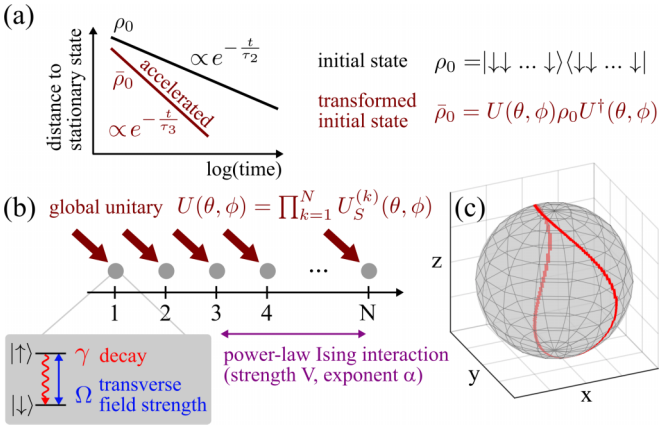

FIG. 1. Quantum Mpemba effect and open quantum Ising spin chain. (a) The application of a unitary transformation $U(\theta,\phi)$ to the initial state ρ_0 may accelerate the approach of an open quantum system to its stationary state. This is achieved when the unitary removes the overlap of the transformed initial state $\bar{\rho}_0$ with the slowest decaying mode with lifetime τ_2 (inverse of the negative real part of the spectral gap λ_2, see the main text). In that case the long-time dynamics is governed by the lifetime $\tau_3 < \tau_2$. In this paper we consider global unitary spin rotations, which are parametrized by the polar and the azimuthal angles θ and ϕ, respectively [see panel (b) and the main text]. (b) Sketch of a one-dimensional quantum Ising spin chain with transverse magnetic-field strength Ω and power-law interactions (exponent α). Spins decay from their up-state $|\uparrow\rangle$ to the down-state $|\downarrow\rangle$ at a rate γ. (c) Unit sphere depicting spin rotation angles θ and ϕ for which the unitary $U(\theta,\phi)$ leads to an accelerated approach to the stationary state. The corresponding angles are indicated in red color.

chains, shown in Fig. 1(b), which feature power-law interactions and single-spin decay. These systems model typical many-body settings realized on quantum simulators using trapped ions [23] and neutral atoms [24] and are, thus, of direct relevance to current efforts in quantum computation and simulation. We show that already a global rotation—solely dependent on the two angles θ, ϕ which parametrize the unit sphere—can be sufficient to observe a quantum Mpemba effect. In order to obtain a quantitative picture, we compute—as a function of interaction strength and range—the area on the unit sphere for which an exponentially accelerated approach to stationarity is achieved [see Fig. 1(c)]. We find that the Mpemba effect in Markovian open quantum systems is robust in the sense that it occurs with a simple unitary for a variety of angles in all studied parameter regimes. (We note that, following the discussion presented in the Supplemental Material of Ref. [22], it is possible to investigate the acceleration of the approach to the stationary state through simple unitaries also in the case of non-Markovian dynamics.) Moreover, our simplified approach also works in situations where it is not possible to analytically construct the ideal unitary with the approach put forward in Ref. [22].

Our results may be of practical relevance for applications in quantum technology, e.g., for accelerating the dissipative preparation of entangled states and the processing speed of dissipative quantum computation [7] and pattern recognition [25–28].

II. QUANTUM MPEMBA EFFECT

The open quantum systems we are considering in this paper are described by a Markovian Lindblad master equation [3] which evolves the density matrix ρ with the dynamical generator (master operator) W,

$$\dot{\rho}(t) = W[\rho(t)] = -i[H,\rho(t)] + \sum_k \left(L_k \rho(t) L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho(t)\} \right). \tag{1}$$

Here H is the quantum Hamiltonian which governs the coherent dynamics, and the L_k are the so-called jump operators which implement incoherent and dissipative processes [3].

The time evolution of any initial state ρ_0 is given by

$$\rho(t) = e^{Wt}[\rho_0] = \rho_{\rm SS} + \sum_{k=2}^{D^2} \mathrm{tr}(l_k \rho_0) r_k e^{\lambda_k t}. \tag{2}$$

Here, D is the dimension of the system Hilbert space, and l_k, r_k, and λ_k are the left eigenmatrices, the right eigenmatrices, and the eigenvalues of the dynamical generator, respectively,

$$W[r_k] = \lambda_k r_k, \quad W^\dagger[l_k] = \lambda_k l_k, \tag{3}$$

where $W^\dagger$ is the dual (also called adjoint) master operator which acts on observables rather than on states. The stationary state of the time-evolution $\rho_{\rm SS}$ is the right eigenmatrix of the dynamical generator associated with the eigenvalue zero and has been taken out of the sum. The remaining eigenvalues λ_k have a real part that is smaller or equal than zero. In the following we assume that the λ_k's are sorted in ascending order according to the modulus of their real part: $|\mathrm{Re}\{\lambda_{k+1}\}| \geqslant |\mathrm{Re}\{\lambda_k\}|$. In this case λ_2 corresponds to the spectral gap, and the negative inverse of its real part sets the longest timescale of the relaxation dynamics. This means, that for long times one has

$$\|\rho(t) - \rho_{\rm SS}\| \sim \exp\left(\mathrm{Re}\{\lambda_2\} t\right), \tag{4}$$

where $\|\cdot\|$ is a suitably chosen norm. From the expansion (2) it then follows that if there exists a unitary U which eliminates the overlap of the initial state with this slowest decaying mode, i.e.,

$$\mathrm{tr}(l_2 U \rho_0 U^\dagger) = 0, \tag{5}$$

the approach to the stationary state is exponentially accelerated. The new timescale over which the stationary state is approached is given by the inverse of the negative real part of λ_3: $\tau_3 = -1/\mathrm{Re}(\lambda_3)$ [see sketch in Fig. 1(a)],

$$\|\rho(t) - \rho_{\rm SS}\| \sim \exp(\mathrm{Re}\{\lambda_3\} t). \tag{6}$$

In Ref. [22] it was shown that the unitary U accomplishing condition (5) can be explicitly constructed for a pure initial state—which we write as $\rho_0 = |0\rangle\langle 0|$—provided that the eigenvalue λ_2, which corresponds to the slowest decaying mode, is real and nondegenerate. In this case the corresponding left-hand eigenmatrix l_2 is Hermitian and can be spectrally

decomposed as

$$l_2 = \sum_{m=1}^{D} \alpha_m |\phi_m\rangle\langle\phi_m|, \tag{7}$$

with the eigenvalues $\alpha_m \in \mathbb{R}$ and the orthonormal set of eigenstates $\{|\phi_m\rangle\}$. If one of the eigenvalues is zero—without loss of generality we assume $\alpha_1 = 0$—then condition (5) can be met by a unitary that rotates the initial state $|0\rangle$ onto the corresponding eigenstate $|\phi_1\rangle$ such that $|\langle\phi_1|U|0\rangle| = 1$.

In general, the expansion (7) does not contain a zero eigenvalue. A unitary, which satisfies condition (5) can, nevertheless, be constructed. Underlying this construction is the observation that among the eigenvalues α_m some have to be positive and some negative. This results from the fact that the overlap between l_2 and the stationary state vanishes, i.e.,

$$0 = \mathrm{tr}(l_2\rho_{\mathrm{SS}}) = \sum_{m=1}^{D} \alpha_m \langle\phi_m|\rho_{\mathrm{SS}}|\phi_m\rangle. \tag{8}$$

Given that the stationary state is a positive operator, i.e., $\langle\phi_m|\rho_{\mathrm{SS}}|\phi_m\rangle \geqslant 0$, it follows that there are both positive and negative α_m's. Without loss of generality we assume that $\alpha_1 < 0$ and $\alpha_2 > 0$. Introducing the unitary,

$$U(s) = \exp[-is(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)]R, \tag{9}$$

with R being also a unitary operator such that $R|0\rangle = |\phi_1\rangle$, one can show that [22]

$$\mathrm{tr}[l_2 U(s)\rho_0 U^\dagger(s)] = \alpha_1 \cos^2(s) + \alpha_2 \sin^2(s), \tag{10}$$

and, thus, by choosing

$$s = \arctan\left(\sqrt{-\frac{\alpha_1}{\alpha_2}}\right), \tag{11}$$

condition (5) is met.

Unfortunately, this way of constructing the unitary operator accelerating the relaxation to stationarity is not justified in other situations. For example, a scenario that is often encountered is one where the eigenvalues of the lowest excited modes of the master operator form a complex conjugate pair,

$$\rho(t) = \rho_{\mathrm{SS}} + \mathrm{tr}(\bar{l}_2\rho_0)\bar{r}_2 e^{\lambda_2 t} + \mathrm{tr}(\bar{l}_2^\dagger\rho_0)\bar{r}_2^\dagger e^{\lambda_2^* t} + \cdots. \tag{12}$$

In these cases, the state of the system approaches stationarity displaying (damped) oscillations for certain matrix elements and not a purely exponential decay. To achieve acceleration one would seek a unitary U that accomplishes

$$\mathrm{tr}(\bar{l}_2 U\rho_0 U^\dagger) = [\mathrm{tr}(\bar{l}_2^\dagger U\rho_0 U^\dagger)]^* = 0. \tag{13}$$

However, due to the fact that $\bar{l}_2$ is not Hermitian in these cases, a decomposition, such as the one in Eq. (7) is not possible and the previously presented route for constructing U cannot be pursued.

III. ACCELERATED APPROACH TO STATIONARITY VIA SIMPLE UNITARIES

Whereas the unitary transformation (9) can be in theory constructed, it will be, in practice, challenging to implement given that it requires precise knowledge of mode l_2. Moreover, when the eigenvalues of the slowest decaying modes form a complex conjugate pair the theory of the previous section is not applicable. It is, therefore, of relevance to understand whether simpler unitaries exist that also may lead to the desired acceleration towards stationarity. This appears possible given that the mere requirement for achieving acceleration is a vanishing overlap between the transformed initial state and the slowest decaying mode. For sufficiently large state spaces, it is reasonable to be expected that this should be possible to achieve with a simple unitary.

To investigate this we consider for concreteness a one-dimensional spin system, composed by N spins, whose coherent evolution is governed by the Hamiltonian [see Fig. 1(b)],

$$H = \Omega \sum_{k=1}^{N} \sigma_k^x + V \sum_{k<m}^{N} \frac{\sigma_k^z \sigma_m^z}{|k-m|^\alpha}, \tag{14}$$

where σ_k^ν ($\nu = x, y, z$)'s are the usual Pauli matrices. The parameter Ω is the transverse field strength, V is the interaction strength, and α is the exponent that controls the interaction range [29]. Dissipation is governed by the jump operators,

$$L_k = \sqrt{\gamma}\sigma_k^- = \frac{\sqrt{\gamma}}{2}\left(\sigma_k^x - i\sigma_k^y\right), \tag{15}$$

which model single-spin decay from the up-state $|\uparrow\rangle$ to the down-state $|\downarrow\rangle$ at rate γ. Related models have been theoretically studied in the past [5,30–33], and instances of these systems can be experimentally realized in systems (quantum simulators) consisting of trapped ions [23,34] and Rydberg atoms [24,35,36].

In order to obtain first insights into the model we show in Fig. 2 the negative real part of the eigenvalues λ_2 of the master operator [see spectral expansion given by Eq. (2)]. Moreover, we indicate where λ_2 possesses a nonzero imaginary part and the time-evolved state is, thus, of the form (12). As can be seen in the figure, this is indeed a relevant case, which occurs across a wide region of parameters. The exact value of the imaginary part of λ_2 is reported in the Appendix.

For the purpose of this paper we consider a pure initial state in which all spins occupy the down-state,

$$\rho_0 = |\downarrow\downarrow\cdots\downarrow\rangle\langle\downarrow\downarrow\cdots\downarrow|. \tag{16}$$

The unitary that we apply to this initial state has the form

$$U(\theta,\phi) = \prod_{k=1}^{N} U_S^{(k)}(\theta,\phi), \tag{17}$$

with

$$U_S^{(k)}(\theta,\phi) = \exp\left(\frac{i}{2}\phi\sigma_k^z\right)\exp\left(\frac{i}{2}\theta\sigma_k^y\right). \tag{18}$$

This unitary, which acts globally on all spins simultaneously, rotates them first by an angle $\theta \in [0,\pi]$ (polar angle) about the y axis and, subsequently, by an angle $\phi \in [0, 2\pi)$ (azimuthal angle) around the z axis. On quantum simulator platforms such operations can be readily realized by suitably timed laser or microwave pulses and is, therefore, significantly easier to implement than the ideal unitary (9). The goal is now to determine the angles θ and ϕ for which acceleration

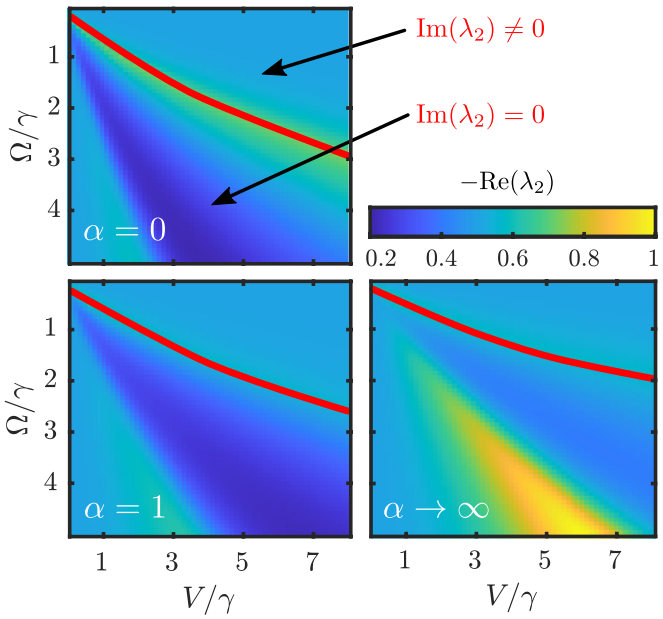

FIG. 2. First excited mode of the master operator. The density plot shows the negative real part of the first nonzero eigenvalue λ_2 of the master operator W [see Eq. (1)] on the Ω-V plane for three different values of the power-law exponent α. The red line delimits the region where λ_2 has a zero and a nonzero imaginary part. The latter case corresponds to the situation described by Eq. (12). Note, that for obtaining the data presented here we have chosen the lowest excited eigenvalue of W which lies in the same symmetry subspace as the initial state (16). The data shown are for a system size of $N = 5$ and open boundary conditions.

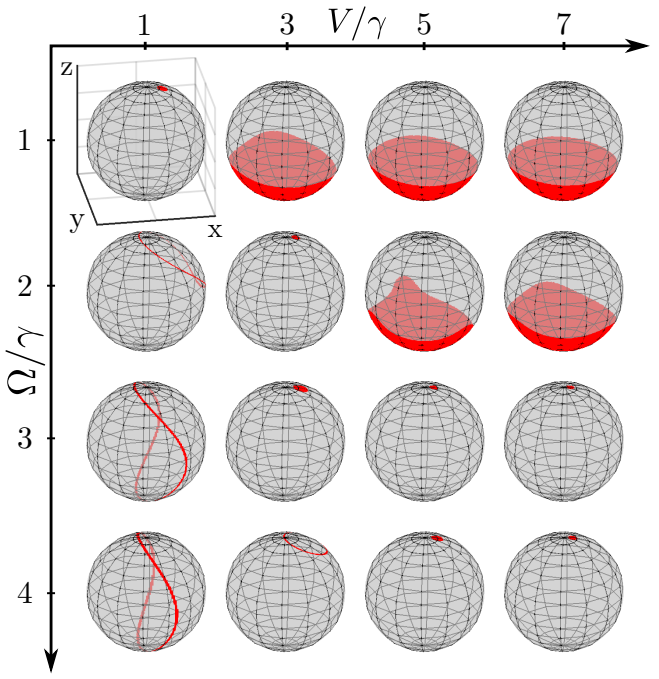

FIG. 3. Rotation angles which lead to an accelerated approach to stationarity. The application of the unitary (17) to the initial-state (16) can reduce its overlap with the slowest decaying mode of the master operator (1). Angles for which this overlap $\chi(\theta, \phi)$ is smaller than $\epsilon = 10^{-2}$—see Eq. (19)—are displayed in red. The abrupt change in the size of the red region for large V is due to a crossing of the eigenvalues of the master operator, which can also be observed in Fig. 2. The data shown are for $N = 5$ and power-law exponent $\alpha = 0$. Note, that if one were to decrease the threshold value ϵ the width of the red ribbonlike lines (see small values of V) would shrink and the extended red areas (see large values of V) would contract.

is achieved. To accomplish this we use the criterion,

$$\chi(\theta, \phi) = |\mathrm{tr}[l_2 U(\theta, \phi)\rho_0 U^\dagger(\theta, \phi)]| \leqslant \epsilon, \tag{19}$$

where ϵ is a threshold, which we set to 10^{-2}. In Fig. 3 we depict these angles for various parameter regimes of the considered spin system. We find that it is indeed generally possible to find rotation angles (θ, ϕ) which reduce the overlap of the initial state and the lowest decaying mode below the chosen threshold. In fact, there are entire areas on the unit sphere for which acceleration of the approach to stationarity is achieved. As can be seen in Fig. 3—where all the rotation angles leading to acceleration are marked in red—the size of this area on the unit sphere may depend strongly on the values of the parameters Ω and V. On one hand, when both Ω and V are simultaneously large or small only a small rotation ($\theta \ll \pi$) away from the initial state ($\theta = 0, \phi = 0$) suffices to speed up the approach to stationarity. On the other hand, one observes that for small values of Ω and large values of V the area on the unit sphere for which acceleration is achieved becomes large. This drastic quantitative change is due to the fact that the real parts of the eigenvalues corresponding to the slowest decaying mode of the master operator cross, and, therefore, the character of the slowest decaying mode is changing. In Fig. 2 this eigenvalue crossing also manifests, e.g., through the appearance of an imaginary part in the eigenvalue corresponding to the slowest decaying mode. In the following we investigate the relative area on the unit sphere spanned by the angles θ and ϕ, where acceleration is achieved, i.e., the size of the red areas in Fig. 3. This quantity is defined as

$$A = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, \sin\theta \Theta[\epsilon - \chi(\theta, \phi)], \tag{20}$$

where $\Theta[x]$ is the Heaviside step function and ϵ is the threshold parameter introduced above. The corresponding data are shown in Fig. 4(a) for a system of $N = 5$ spins [37]. Throughout the entire Ω-V plane we find A to be nonzero, which means that it is always possible to find a global unitary (17) that accomplishes a speedup of relaxation. However, the size of the area may differ considerably. In particular, for large values of V the area is significantly extended which might offer some robustness with respect to variations in the angles θ and ϕ that could be relevant for experiments. This change from a small to a large area is not gradual but appears at a critical value of V, which increases with Ω, regardless of the value of α, i.e., the exponent describing the power-law decay of the interactions. This abrupt change might be a (finite-size) signature of a dissipative phase transition in the stationary state of the dissipative spin chain, which has been studied previously in several works [29–31,38].

As a final point of investigation, we analyze the achievable acceleration. To this end we compare the timescales corresponding to the slowest and second slowest decaying modes, which are given by $\tau_2 = -1/\mathrm{Re}(\lambda_2)$ and $\tau_3 = -1/\mathrm{Re}(\lambda_3)$,

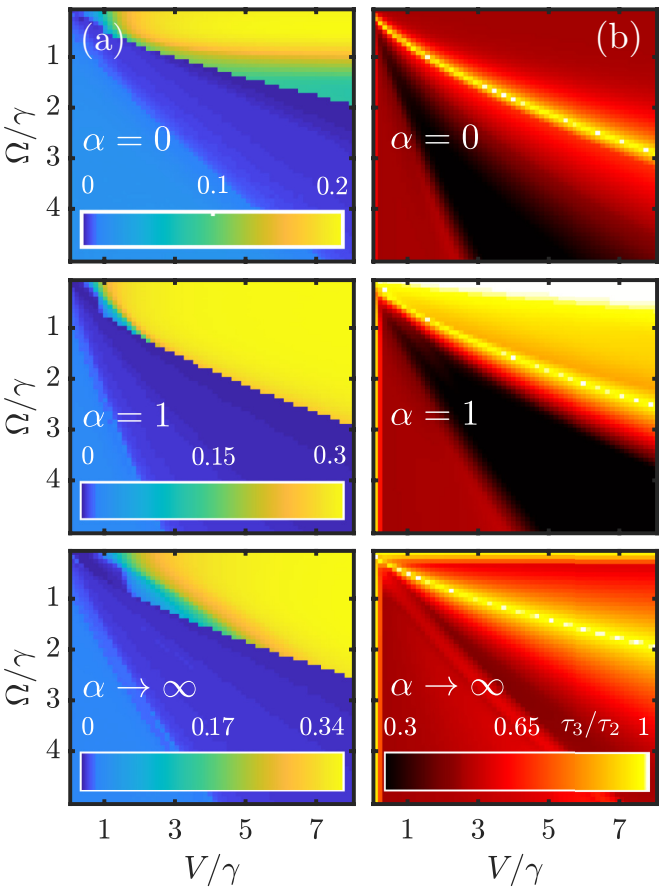

FIG. 4. Relative size of the parameter region in which acceleration is achieved and gain in relaxation time. (a) Relative area (20) on the unit sphere for which the overlap of the initial state with the slowest decaying mode is smaller than $\epsilon = 10^{-2}$. This area corresponds to the regions marked in red in Fig. 3. (b) Ratio of the timescales $\tau_3 = -1/\mathrm{Re}(\lambda_3)$ and $\tau_2 = -1/\mathrm{Re}(\lambda_2)$ which quantifies the minimal reduction in relaxation timescale which can be achieved by removing the overlap between the initial state and the slowest relaxing mode of master operator (1). Lower values mean larger acceleration. Note that for obtaining the data presented here we have chosen the modes of master operator W that lie in the same symmetry subspace as the initial state (16). The data shown in the panels are for a system size of $N = 5$ (open boundary conditions) and various values of the power-law exponent α.

respectively. In Fig. 4(b) we show the ratio of the two timescales τ_3/τ_2, i.e., the smaller this quantity, the higher the achievable acceleration. As can be seen from the data, the relaxation timescale can be reduced up to a factor of 3, depending on the specific values of Ω and V. In the region where the abrupt transition is visible in panel (a) hardly any acceleration is possible. This means that the eigenvalues of the slowest and second slowest decaying modes become degenerate, and corroborate the picture of the onset of a phase transition which is typically associated with collapsing eigenvalues of the dynamical generator.

IV. CONCLUSIONS AND OUTLOOK

We have shown—using an open transverse field Ising model as an exemplary case—that the approach to stationarity of an open quantum system can be accelerated by a unitary rotation of the initial state. The underlying idea of this so-called Mpemba effect is that such rotation renders the initial state orthogonal to the slowest decaying mode of the master operator. Given the high-dimensional state space of a many-body system, it appears plausible that it should not be too challenging to achieve this. Indeed, even for the global unitary considered in our paper it was always possible to find spin rotation angles that, in principle, yield an exponential speedup of the relaxation dynamics. This is an encouraging finding, given that this type of unitary is simple to implement on current quantum computing and simulation platforms.

There are, nevertheless, a number of open questions which give room for further studies. For example, the current investigation focuses on the case of a pure initial state and an extension of the investigation of the Mpemba effect to the case of mixed states is highly desirable. Moreover, one may ask whether and under what conditions it is possible to render the initial state orthogonal to a large set of slowly decaying modes. This may become particularly important in many-body systems in the vicinity of phase transitions where the spectrum is dense and many eigenstates of the master operator acquire a real part close to zero. On one hand these questions can be explored numerically in certain model systems, such as the spin chains considered here. On the other hand, it would be interesting to understand under what circumstance it is—at least, theoretically—possible to explicitly construct unitaries or, more generally, quantum maps that remove the overlap of a given initial state with a set of slowly decaying modes.

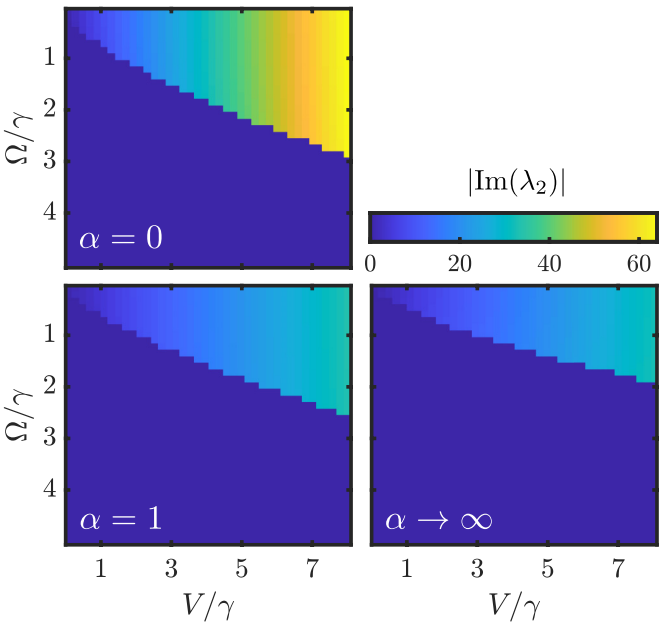

FIG. 5. Imaginary part of the first excited mode of the master operator. The density plot shows the absolute value of the imaginary part of the first nonzero eigenvalue λ_2 of the master operator W [see Eq. (1)] on the Ω-V plane for three different values of the power-law exponent α. Note, that for obtaining the data presented here we have chosen the lowest excited eigenvalue of W which lies in the same symmetry subspace as initial state (16). The data shown are for a system size of $N = 5$ and open boundary conditions.

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APPENDIX: IMAGINARY PART OF λ_2

In this Appendix, we provide additional results showing the imaginary part of the eigenvalue λ_2 considered in Fig. 2. This is shown in Fig. 5. We recall here that the imaginary part of the eigenvalue λ_2 does not determine the lifetime of the slowest decaying mode but solely introduces (damped) oscillations in the quantum state density matrix approaching stationarity.

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