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Letter

# Eightfold way to dark states in SU(3) cold gases with two-body losses

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We study the quantum dynamics of a one-dimensional SU(3)-symmetric system of cold atoms in the presence of two-body losses. We exploit the representation theory of SU(3), the so-called eightfold way, as a scheme to organize the dark states of the dissipative dynamics in terms of generalized Dicke states and show how they are dynamically approached, both in the weakly and strongly interacting and dissipative regimes. Our results are relevant for a wide class of alkaline-earth(-like) gas experiments, paving the way to the dissipative preparation and exploitation of generalized Dicke states.

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## I. INTRODUCTION

Ultracold atomic gases represent a clean and flexible playground to study quantum many-body physics, at equilibrium or in dynamical settings [1–3]. Cold-atom experiments usually feature a high degree of control over system parameters and allow for an almost perfect decoupling from the surrounding environment. However, despite the tremendous experimental progresses, a perfect isolation has never been reached, for instance because of particle losses, causing energy relaxation and decoherence phenomena [4]. On one hand, this fact introduces a typical timescale determining for how long a system can be regarded as *closed*. On the other hand, on a longer timescale, the interplay between the coherent unitary evolution and the coupling to the environment can lead to a nontrivial dynamics and to stationary states featuring strong quantum correlations [5–8] and critical behaviors [9–12].

In general, this latter situation can be achieved through an active control of the environment and of its coupling to the system, via the so-called reservoir engineering [13]; however, in some situations, the dissipative processes that naturally occur in the system can also be responsible for entangled stationary states: this is the situation that we want to study in this Letter [14,15]. Since in these systems decoherence is mainly due to particle losses, developing a theoretical framework to describe this open-system dynamics and the emergence of eventual correlated quantum states represents a huge theoretical challenge that attracted increasing attention in recent years [16–25]. In particular, two-body losses induced by inelastic atomic collisions in correlated quantum gases have been observed experimentally and investigated theoretically in bosonic [5,6,26–32] and fermionic gases [15,33–37].

In this work we consider the paradigmatic case of alkaline-earth-like gases in optical lattices, experimentally realized with ytterbium [14,38–41], which are subject to two-body losses due to inelastic two-body collisions in the metastable state $^3P_0$. The (almost) perfect decoupling between the nuclear spin $I$ and the electronic angular momentum $J$ (ensured by the fact that $J = 0$ for the atomic states involved in the dynamics) implies that the relevant scattering processes are independent of $I$. As a result, this class of systems has an emergent SU($N$) spin symmetry (with $N = 2I + 1$) whose dynamics is governed by a SU($N$)-symmetric Fermi-Hubbard model describing alkaline-earth-like atoms in an optical lattice [42,43]. In the two-spin case ($N = 2$) the dissipative dynamics conserves the total spin and the system exhibits stationary states that are a mixture of highly entangled wave functions with a Dicke-like spin component [15,35], which could be exploited for various quantum-technology purposes. The impact of two-body losses for $N > 2$ has not been theoretically addressed at present, despite the availability of experimental data obtained in this regime [14].

In this Letter we study the quantum dynamics of an interacting SU(3)-symmetric one-dimensional fermionic gas in the presence of two-body losses. We show that the dark states of the dynamics can be organized via the representation theory of this group, the so-called *eightfold way* [44]. This elegant classification allows us to characterize a family of stationary states using the notion of generalized Dicke states [45] describing the spin degrees of freedom of the gas. Next, we discuss the system dynamics highlighting how the generalized Dicke-like states represent the unique attractor of the dynamics both in the weakly dissipative and weakly interacting limit as well as in the strongly dissipative and strongly interacting quantum Zeno regime. Finally, we draw our conclusions and discuss future perspectives.

## II. THE MODEL

Introducing the fermionic operators $\hat{c}^{(\dagger)}_{j,\mu}$ (with $j$ and $\mu$ labeling the lattice site and the spin, respectively), which satisfy canonical anticommutation relations, the SU($N$)-symmetric

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Fermi-Hubbard Hamiltonian reads

$$\hat{H} = -J\sum_{j,\mu}(\hat{c}^{\dagger}_{j,\mu}\hat{c}_{j+1,\mu} + \text{H.c.}) + U\sum_{j,\mu<\mu'}\hat{n}_{j,\mu}\hat{n}_{j,\mu'}. \quad (1)$$

Here, $J$ is the hopping amplitude, $U$ is the spin-independent interaction strength, and $\hat{n}_{j,\mu} = \hat{c}^{\dagger}_{j,\mu}\hat{c}_{j,\mu}$ is the spin-resolved on-site lattice-density operator. The spin index can assume $N$ values that in the following will be labeled with capital letters in progressive order ($\mu = A, B, C, \dots$). The Hamiltonian (1) is invariant under global SU($N$) rotations in spin space. As a consequence, the unitary dynamics conserves the expectation value of the $N(N-1)/2$ SU(2) pseudospin algebra generators defined in each subspace (here labeled by $\mu\mu'$ with $\mu < \mu'$) as

$$\hat{\Lambda}^{\alpha}_{\mu\mu'} = \frac{1}{2}\sum_{j}(\hat{c}^{\dagger}_{j,\mu},\ \hat{c}^{\dagger}_{j,\mu'})\sigma^{\alpha}\begin{pmatrix}\hat{c}_{j,\mu}\\ \hat{c}_{j,\mu'}\end{pmatrix}, \quad \alpha = 0, x, y, z, \quad (2)$$

where $\{\sigma^{\alpha}|\alpha = x, y, z\}$ are the Pauli matrices and $\sigma^0 = \mathbb{I}_2$.

The presence of local two-body losses is accounted for by the jump operators

$$\hat{L}_{j,\mu\mu'} = \sqrt{\gamma}\,\hat{c}_{j,\mu}\hat{c}_{j,\mu'}, \quad (3)$$

with $j = 1, \dots, L$ and $\mu < \mu'$ and $\gamma$ being the dissipation rate. The dynamics of the full density matrix $\rho(t)$ is described by a Lindblad master equation:

$$\dot{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)] + \sum_{j,\mu<\mu'}\mathsf{D}_{j,\mu\mu'}[\rho(t)], \quad (4)$$

with $\mathsf{D}_{j,\mu\mu'}[\rho(t)] = \hat{L}_{j,\mu\mu'}\rho(t)\hat{L}^{\dagger}_{j,\mu\mu'} - \frac{1}{2}\{\hat{L}^{\dagger}_{j,\mu\mu'}\hat{L}_{j,\mu\mu'}, \rho(t)\}$.

The main difference with respect to the $N = 2$ case is that the spin components defined in Eq. (2) are not conserved quantities of the full dissipative dynamics: the breaking of these conservation laws is due to the presence of several spin sectors involved in the dynamics. Thus, in terms of symmetries, the study of the $N = 3$ case can be considered representative for all the $N > 2$ models, which therefore will not be explicitly considered.

### III. EQUATIONS OF MOTION AND DARK STATES

Let us now focus on the population dynamics and define the total number of atoms $\hat{N} = \sum_{\mu}\hat{N}_{\mu}$, where $\hat{N}_{\mu} = \sum_j \hat{n}_{j,\mu}$ is the spin-resolved population. In what follows we will use the notation $O(t) \doteq \langle\hat{O}\rangle_t \doteq \text{tr}[\rho(t)\hat{O}]$. The spin-resolved populations obey the following equation [46]:

$$\dot{N}_{\mu}(t) = -\gamma\sum_{j}\sum_{\mu'\neq\mu}\left\langle\hat{n}_{j,\mu}\hat{n}_{j,\mu'}\right\rangle_t. \quad (5)$$

First, we will present a construction allowing us to map out all the possible dark states of the dissipative dynamics factorizing spin and charge degrees of freedom. Such states are not affected by the dissipative dynamics and any statistical mixture of them is stationary with respect to the master equation (4). Next, we will study the system dynamics showing how the system evolves, because of dissipation, toward such a dark subspace. We consider the class of states where orbital and spin degrees of freedom factorize, $|\Psi_{\text{dark}}\rangle = |\Psi_{\text{orb}}\rangle \otimes |\Psi_{\text{spin}}\rangle$. If $|\Psi_{\text{orb}}\rangle$ is constructed as a Slater determinant of a set of appropriate orbital modes, i.e., the eigenstates

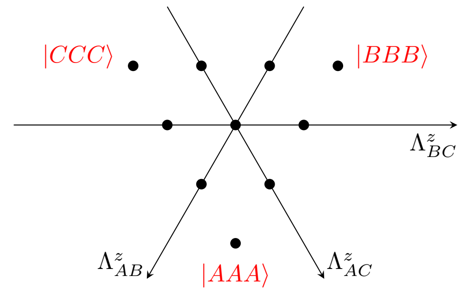


FIG. 1. The eightfold way in a dark state. Triangular irreducible representation with labels (3,0), which is composed of ten states. The three arrows allow one to identify each state through the quantum numbers $\Lambda^z_{\mu\mu'}$, where $\mu, \mu'$ take values in the three components of the gas, $A$, $B$, and $C$. Note that only two of them are linearly independent.

of the hopping Hamiltonian in Eq. (1), the state is assured to commute with the Hamiltonian and never to have a double spatial occupation, so that no particle can leak out of it. Since the full many-body wave function $|\Psi_{\text{dark}}\rangle$ of the system must be fully antisymmetric, and one such $|\Psi_{\text{orb}}\rangle$ is fully antisymmetric, the spin wave function $|\Psi_{\text{spin}}\rangle$ must be fully symmetric. In order to understand the properties of these states, we make use of group theory.

The irreducible representations of SU(3) are labeled by two integers $(p, q)$ [44]; according to group theory, the fully symmetric SU(3) states correspond to the representations with labels $(p, 0)$ and the states belonging to it can be arranged in the shape of a triangle turned upside-down with edge length $p + 1$ (see Fig. 1 for an example with $p = 3$). The number of particles accommodated in the representation is $N = p$. The dimension of a representation $(p, 0)$ is $(p + 1)(p + 2)/2$, and each state is uniquely determined by the values of $\Lambda^z_{\mu\mu'}$. In the case of the figure we have the ten fully symmetric states of $N = 3$ particles. At the three vertices of the triangle we always find the fully polarized states; in this case $|AAA\rangle$, $|BBB\rangle$, and $|CCC\rangle$. The other states are obtained by repeated application of the spin-ladder operators $\hat{\Lambda}^{\pm}_{\mu\mu'}$. If we want, for instance, to construct all the states that stand on the top edge of the triangle, from left to right we need to apply the operator $\hat{\Lambda}^{+}_{BC}$, that raises the value of $\Lambda^z_{BC}$ by one, starting from $|CCC\rangle$.

These states are *generalized Dicke states* [45] since they are fully symmetric with respect to the exchange of two particles generalizing the symmetry properties of the stationary states of the SU(2) lossy dynamics identified in Ref. [15]. Given two spin sectors $\mu, \mu'$, such states satisfy the relation [46]

$$\frac{\langle\hat{S}^2_{\mu\mu'}\rangle}{\hbar^2} = \left\langle\frac{\hat{N}_{\mu\mu'}}{2}\left(\frac{\hat{N}_{\mu\mu'}}{2} + 1\right)\right\rangle, \quad (6)$$

where $\hat{S}^{\alpha}_{\mu\mu'} = \hbar\hat{\Lambda}^{\alpha}_{\mu\mu'}$ for $\alpha = x, y, z$ and $\hat{N}_{\mu\mu'} = \hat{N}_{\mu} + \hat{N}_{\mu'} = 2\hat{\Lambda}^0_{\mu\mu'}$. Conversely, Eq. (6) can be satisfied only by the generalized Dicke states. This can be explicitly seen by considering the irreducible representations of the SU(3) group with $q \neq 0$. These representations of the group are not fully symmetric and, together with the $q = 0$ case, cover all the possible spin states that can be constructed within SU(3). By explicit construction of such states it is easy to see that for any $q \neq 0$ we get $\langle \hat{S}^2_{\mu\mu'}\rangle/\hbar^2 < \langle \hat{N}_{\mu\mu'}/2(\hat{N}_{\mu\mu'}/2+1)\rangle$. While via the eightfold way we constructed explicitly the dark states for $N = 3$, our reasoning is general and generalized Dicke states are dark states of the master Eq. (4) for any $N$ and regardless of the specific values of the system parameters.

## IV. DYNAMICS

While it is true that such states surely are stationary states of the dynamics it is not trivial to show that they are unique. Indeed, our analysis focused on states where the spin and orbital part of the wave functions factorize while we cannot exclude *a priori* that nonfactorizable dark states exist.

To corroborate this scenario, we will make use of Eq. (6) certifying that the system has flown to a mixture of generalized Dicke states. In what follows we will consider two paradigmatic regimes: (i) the weakly dissipative and weakly interacting regime and (ii) the strongly dissipative and strongly interacting limit.

*Weak dissipation and weak interactions*. We start by studying the regime of weak dissipation and weak interactions $\hbar\gamma, U \ll J$. In this limit we can write the evolution of the spin-resolved densities as [46]

$$\dot{n}_\mu(t) = \gamma \sum_{\mu'\neq\mu} \vec{s}^{\,\mathsf{T}}_{\mu\mu'} \mathsf{G} \vec{s}_{\mu\mu'}, \quad (7)$$

where we defined the four-component vector $\vec{s}_{\mu\mu'} = (s^0_{\mu\mu'}, s^x_{\mu\mu'}/\hbar, s^y_{\mu\mu'}/\hbar, s^z_{\mu\mu'}/\hbar)$ with $s^\alpha_{\mu\mu'}(t) = \langle \hat{S}^\alpha_{\mu\mu'}\rangle_t/L$, $s^0_{\mu\mu'} = \langle \hat{\Lambda}^0_{\mu\mu'}\rangle_t/L$, $n_\mu = \langle \hat{N}_\mu\rangle/L$, and $\mathsf{G} = \mathrm{diag}(-1, 1, 1, 1)$ being the relativistic Minkowsky tensor.

The fact that the time derivative of spin-resolved populations is related to the Minkowski scalar product of a four-component vector suggests some suggestive analogies with the theory of special relativity. The structure of Eq. (7) highlights indeed some of the symmetries of the problem as the internal rotations of the SU(2) pseudospins (indicating that the physics does not have a preferred direction in the internal space) and the analogs of the Lorentz boosts (which allow for the exchange between populations and coherences). Furthermore, the analogy with the Minkowski tensor suggests an effective representation of the dynamics in a population-spin diagram, where the dynamics is constrained within an effective light cone, which we dubbed *Dicke cone*.

Let us start by briefly reviewing the $N = 2$ case. In this case we just have two spin sectors labeled as $\mu = A, B$. Therefore, to determine the fixed points, we ask $\dot{n}_A = \dot{n}_B = 0$. From Eq. (7) we get the following stationarity condition:

$$\vec{s}^{\,\mathsf{T}}_{AB} \mathsf{G} \vec{s}_{AB} = 0 \;\Rightarrow\; s_{AB} = \frac{\hbar}{2} n_{AB}, \quad (8)$$

where $s_{AB} = \sqrt{(s^x_{AB})^2 + (s^y_{AB})^2 + (s^z_{AB})^2}$. The condition (8) holds both for Dicke states ($N = 2$) and generalized Dicke states ($N > 2$) [47] and defines the boundary of the *Dicke cone* within which the dynamics must take place because of the physical requirement $s_{AB} \leqslant \hbar\, n_{AB}/2$. As a result, the system dynamics can be effectively visualized in a two-dimensional parameter space spanned by the variables $s_{AB}$ and $n_{AB}$ constrained to the Dicke cone. Finally, since the $s_{AB}$ is a constant of motion for the $N = 2$ case $s_{AB}(t) = s_{AB}(0)$, the dynamics must take place on the line defined by the initial value of the spin. In the $t \to \infty$ limit, the boundary of the light cone is touched (i.e., $n_{AB} = 2s_{AB}/\hbar$) and the system reaches a stable stationary state. The $N = 2$ case has been discussed extensively in Ref. [15]. The conservation of the total spin, even in the presence of dissipative events, plays a crucial role in constraining the system dynamics. Indeed, given the initial conditions, it allows one to be predictive about the final density of the system. Starting from the $N = 2$ case we want now to explore the $N > 2$ case where the dynamics does not conserve the spin.

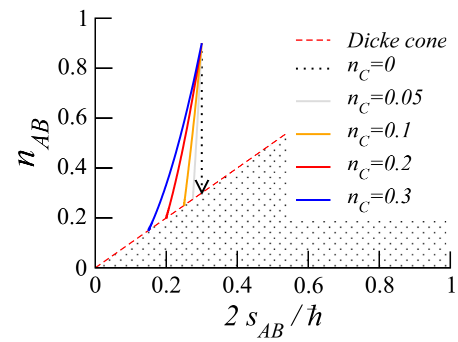


FIG. 2. SU(3) dynamics in the $n_{AB}$-$s_{AB}$ plane. In the $n_C = 0$ case the evolution must follow vertical lines defined by the initial value of $s_{AB}$. When $n_C > 0$ the spin conservation does not hold. The dynamics escapes the vertical line defined by $s_{AB}(0)$ and deviates progressively toward $s_{AB} = 0$ getting steady when $n_{AB} = 2s_{AB}$. Here we set $n_A = 0.5$, $n_B = 0.4$, $(s^x_{AB}(0), s^y_{AB}(0), s^z_{AB}(0)) = (0.1, 0.1, 0.05)$ so that $2s_{AB}(0) = 0.3$.

Let us consider the $N = 3$ case where the internal states are labeled as $\mu = A, B, C$. In this case the spin components are no longer conserved and in general $s_{\mu\mu'}(t) \neq s_{\mu\mu'}(0)$. In Fig. 2 we show the dynamics of spin and number of particles in the $AB$ subspace for a generic initial condition. When $n_C = 0$ the dynamics is spin conserving $s_{AB}(t) = s_{AB}(0)$ and the system dynamics follows vertical lines. Even if an additional internal state is now available, there are no physical processes that populate it. As a result, in this limit the system behaves effectively as in the $N = 2$ case. For $n_C > 0$ the spin in the $AB$ subspace is no longer conserved but shrinks. The trajectory in the $n_{AB}$-$s_{AB}$ plane deviates on the left of the $s_{AB}(0)$ line and evolves until the boundary of the Dicke cone is approached.

We now propose a perturbative solution of the SU(3) dynamics for different initial conditions considering the experimentally relevant situation where $s^{x,y}_{\mu\mu'} = 0$, $\forall \mu < \mu'$. We also stress that this approach is well suitable for translationally invariant states where intensive variables are unambiguously representative of the global state of the system. The equations of motion for the populations read as

$$\dot{n}_\mu = -\gamma n_\mu \sum_{\mu'\neq\mu} n_{\mu'}. \quad (9)$$

The dynamics cannot be analytically solved for a generic initial condition but only in a few cases that we will now discuss. When the system is initially prepared with a large and equal fraction of the total population in the $A$ and $B$ sectors and only a small amount of particles in the

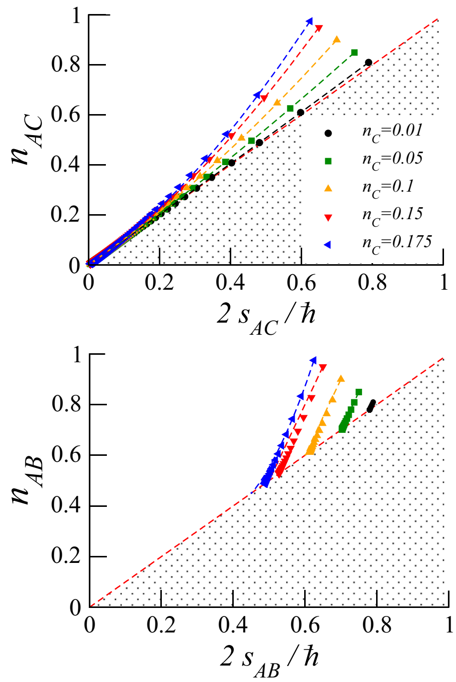


FIG. 3. SU(3) dynamics in the weakly dissipative limit. Top panel: we set $n_A(0) = n_B(0) = 0.8$ and $n_C(0)$ is varied. Bottom panel: we set $n_A(0) = 0.8$ and $n_B(0) = n_C(0)$ is varied. In both cases the numerics (filled dots) show a good agreement with the predictions of Eq. (11) (for the top panel) and Eq. (11) for (the bottom panel), even beyond the limit $n_C(0) \ll 1$. In all the panels $s^{x,y}_{\mu\mu'} = 0$, $\forall \mu < \mu'$.

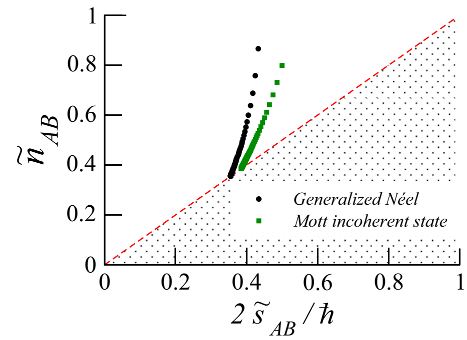


FIG. 4. SU(3) dynamics in the $\tilde{s}_{AB}$-$\tilde{n}_{AB}$ plane. Orange circles: dynamics from the generalized Néel state. Green squares: dyanmics from the Mott incoherent state. The red dashed line represents the Dicke cone satisfying Eq. (6).

$C$ subspace, $n_C(0) \ll n_A(0) = n_B(0) = \mathcal{O}(1)$, the exact solution at first order in $n_C(0)$ reads [46]

$$n_{A,B}(t) = \frac{n_{A,B}(0)}{1+\gamma t n_{A,B}(0)} - \frac{n_C(0)\ln[1+\gamma t n_{A,B}(0)]}{[1+\gamma t n_{A,B}(0)]^2},$$
$$n_C(t) = \frac{n_C(0)}{[1+\gamma t n_{A,B}(0)]^2}. \tag{10}$$

We found that the system gets empty in the long-time limit, i.e., $\lim_{t\to\infty} n_{A,B,C} = 0$. This is expected in the $A, B$ sector since the initial condition $s_{AB}(0) = 0$ implies $s_{AB}(t) = 0$, $\forall t > 0$ and the system must evolve toward the origin of the Dicke cone $s_{AB} = n_{AB} = 0$. In the $A, C$ (or equivalently $B, C$) sectors the situation is quite different since we start from a large value of the spin $s_{AC} = s^z_{AC} = \hbar(n_A - n_C)/2$ and again we flow toward the vacuum. This dynamics is shown in Fig. 3 (top panel) and the numerics show good agreement with the perturbative prediction (11). We also note that the presence of a nonvanishing population in $C$ modifies the $1/t$ mean-field-like decay of $n_{A,B}$ and determines a $1/t^2$ decay for $n_C$.

We now consider the situation where the system is initially prepared with a large fraction of the total population in the $A$ sector and a small (and equal) fraction of particles in the $B, C$ sectors, i.e., $n_B(0) = n_C(0) \ll n_A(0)$. At first order in $n_C(0)$ we find [46]

$$n_A(t) = n_A(0) - 2n_B(0)(1 - e^{-\gamma n_A(0)t}),$$
$$n_{B,C}(t) = n_{B,C}(0)e^{-\gamma n_A(0)t}. \tag{11}$$

In this case we get a steady state with a nonvanishing particle density in the $A$ sector, i.e., $\lim_{t\to\infty} n_A(t) = n_A(0) - 2n_B(0)$, while the $B, C$ sectors become empty $\lim_{t\to\infty} n_{B,C}(t) = 0$. This determines a nontrivial dynamics in the $AB$ subspace as shown in Fig. 3 (bottom panel), which is well captured by Eq. (11) for small values of $n_C$.

We conclude this part considering the case of equally populated spin sectors. This state is of particular interest since it can be easily realized in experiments [14] and corresponds to a product state in which we have one particle per lattice site with maximally mixed spin degrees of freedom. We dubbed this state *Mott incoherent state*. This state has a total spin that vanishes in the thermodynamic limit as $s^2_{\mu\mu'} \sim 1/L$ $\forall \mu \neq \mu'$. In this case Eq. (9) leads to $\dot{n}(t) = -\gamma(N-1)n^2(t)$, which is solved for $n(t)/n(0) = [1 + t\gamma n(0)(N-1)]^{-1}$. Here, the populations decay as $1/t$ with a typical rate given by $\gamma n(0)(N-1)$.

*Strongly interacting and strongly dissipative limit.* Let us now consider the strongly interacting and dissipative limit in which $\hbar\gamma, U \gg J$. In this limit states with at most one particle per lattice site are quasistationary while states with more than one excitation per lattice site are energetically disfavored and will be quickly dissipated on a timescale proportional to $1/(\hbar\gamma)$. Consequently, the dynamics at long times will mainly take place in the hard-core fermion subspace with a new relevant timescale, namely, $\Gamma_{\rm eff} \sim 1/\gamma$, which is inversely proportional to the original dissipation rate, a typical signature of the Quantum Zeno effect [15,29].

Following a method first proposed in Ref. [6], we derive an effective Lindblad master equation that governs the dynamics in this regime [46]. The effective Hamiltonian $\hat{H}' = -J\sum_{j,\mu}(\hat{f}^\dagger_{j,\mu}\hat{f}_{j+1,\mu} + \text{H.c.})$ corresponds to a hopping Hamiltonian of hard-core fermions annihilated by the operators $\hat{f}_{j,\mu}$. The effective jump operator takes into account nearest-neighbor losses $\hat{L}'_{j,\mu\mu'} = \sqrt{\Gamma_{\rm eff}}[\hat{f}_{j\mu}(\hat{f}_{j-1,\mu'} + \hat{f}_{j+1,\mu'}) - \hat{f}_{j\mu'}(\hat{f}_{j-1,\mu} + \hat{f}_{j+1,\mu})]$, with $\mu < \mu'$ and $\Gamma_{\rm eff} = \frac{4}{1+(\frac{2U}{\hbar\gamma})^2}\frac{J^2}{\hbar^2\gamma}$. This effective master equation is a generalization to the SU(3) case of the one presented in Ref. [33]. We want now to show that also in this regime the steady state is a mixture of generalized Dicke states. As a smoking gun we will study whether the condition (6) holds at long times for a generic $\mu, \mu'$ subspace. In order to verify this relation we solve numerically the master equation for open boundary conditions by means of quantum trajectories [48,49]. In particular, we consider the dynamics starting from a generalized Néel state of the form $|\psi_{\text{g-Neel}}\rangle = |A \ B \ C \ldots A \ B \ C\rangle$ and the Mott incoherent state. In Fig. 4 we plot the system evolution in the $AB$ subspace in the $2\tilde{s}_{AB}/\hbar - \tilde{n}_{AB}$ plane where we defined

$$\tilde{s}_{AB} = \frac{\sqrt{\langle \hat{S}^2_{AB}\rangle}}{L}, \quad \tilde{n}_{AB} = \frac{2\sqrt{\left\langle \frac{\hat{N}_{AB}}{2}\left(\frac{\hat{N}_{AB}}{2}+1\right)\right\rangle}}{L}. \tag{12}$$

Again, in the long-time limit the curves asymptotically collapse on the Dicke cone where Eq. (6) holds. The latter statement is true for any of the subspaces; for what concerns the Mott incoherent state, given its particular structure and symmetry, we have that the dynamics is the same in each of the subspaces [46].

## V. CONCLUSIONS

In this Letter we studied the dynamics and steady-state properties of a SU(3)-symmetric cold-atom system in the presence of two-body losses. While we explicitly considered the $N = 3$ case, our results are qualitatively valid for any $N > 2$, including $N = 6$ for which experiments have been performed [14]. This work also paves the way to future intriguing research directions. Among them we mention the study of inhomogeneous situations where the tensor $\mathsf{G}(x)$ acquires a spatial dependence allowing the exploration of analogies with general relativity and the implementation of experimentally friendly protocols for the certification and exploitation of generalized Dicke states.

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[47] Taking the thermodynamic limit of Eq. (6) we get

$$\lim_{L\to\infty} \frac{\langle \hat{S}^2_{AB} \rangle}{L^2} = \frac{\hbar^2}{4} \frac{\langle \hat{N}^2_{AB} \rangle}{L^2},$$

which gives the relation (8).

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