Letter

Typicality of nonequilibrium quasi-steady currents

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(Received 8 December 2021; accepted 28 March 2022; published 27 April 2022)

The understanding of the emergence of equilibrium statistical mechanics has progressed significantly thanks to developments from typicality, canonical and dynamical, and from the eigenstate thermalization hypothesis. Here we focus on a nonequilibrium scenario in which two nonintegrable systems prepared in different states are locally and nonextensively coupled to each other. Using both perturbative analysis and numerical exact simulations of up to 28 spin systems, we demonstrate the typical emergence of nonequilibrium quasi-steady current for weak coupling between the subsystems. We also identify that these currents originate from a prethermalization mechanism, which is the weak and local breaking of the conservation of the energy for each subsystem.

DOI: 10.1103/PhysRevA.105.L040203

Introduction. Daily experience teaches us that when two large systems prepared in different equilibrium states are put into contact, a long-lasting current, which we will refer to as quasi-steady, emerges between them. Furthermore, after this possibly long intermediate time, which depends on the size of the two objects, the overall system relaxes to an equilibrium state. To understand and characterize the emergence of the quasi-steady current, generally one considers each system to be described by an equilibrium, canonical, or microcanonical, ensemble. However, here we ask ourselves if the systems driving the quasi-steady state current can be described by single pure states. Advances in pure-state quantum statistical mechanics have shown that equilibration can emerge in isolated quantum systems [1,2]. The seminal papers by Deutsch [3] and Srednicki [4] brought to the eigenstate thermalization hypothesis (ETH), which postulates that, for a nonintegrable system, eigenstates that are close in energy have similar local properties. ETH has been tested in a variety of systems [5] and reviews can be found in Refs. [1,6–10]. Meanwhile, the attention on the foundation of statistical mechanics has led to the development of the notion of typicality [2,11–13], within canonical formalism [14,15] and for unitary dynamical processes [12,16] with more specific scenarios such as prethermalization [17] and perturbation [18,19]. For example, dynamical typicality states that pure states with the same initial expectation value for some observables will likely have similar expectation values at any later time [12,16]. This was used to show that a weak version of ETH [20–24] is both necessary and sufficient for the vast majority of states to thermalize [17].

Within the thermalization dynamics, the system may experience prethermalization [25–31]. A key signature of this phenomenon is a separation of timescales during the relaxation, for instance, a fast initial dynamics in which the system relaxes to an intermediate (often called prethermal) state, and a slower relaxation toward the true thermal state. This separation of time scales can be seen in systems with a weakly broken conserved quantity, and it has been explored in ultracold atoms experiments [32]. In Ref. [33] it was shown that prethermalization is typical in the presence of weak coupling.

Much less is known regarding the emergence of nonequilibrium quasi-steady current when coupling two systems in pure states. Here we show the emergence of such typical quasi-steady current based on the notion of dynamical typicality. We highlight that there have been remarkable studies on the emergence of typical dynamics for nonequilibrium systems [34–36], and also insightful works to extend ETH to open quantum systems [37,38]. However, in our work, we do not assume *a priori* that a steady current can be reached, and we consider a unified framework for both the emergence of quasi-steady currents and thermalization.

For the two baths we take two nonintegrable spin chains coupled at one of their edges, and we initialize the baths either in single eigenstates or random pure states taken within an energy shell. For weak coupling between the systems, we show that the resulting quasi-steady current is typical in the sense that it converges, when the system size increases, to what would be obtained from initializing the baths in microcanonical states. We also verify that the value of the current converges towards the prediction from the ETH ansatz. Furthermore, we are able to show that the dynamics that leads to the formation of a long-lasting current, and the eventual thermalization of the two coupled chains, can be understood in the framework of prethermalization. In fact the dynamics of each bath, when decoupled, conserves their own energy, while the coupling between the baths breaks this conservation law. For weak coupling between the two chains one thus expects a slow prethermalizationlike dynamics, where the prethermal state actually approaches a nonequilibrium steady state. Finally, we numerically show that the relaxation dynamics is proportional to the square of the coupling between the chains, and inversely proportional to their length, which can be derived within the prethermalization paradigm [30,31]. We thus expect, in the thermodynamic limit, that the current will exist indefinitely.

Model. Two finite-size quantum systems $H_{\rm L}$ and $H_{\rm R}$ are treated as the left and right baths, which are coupled via an interaction term V, i.e., the total Hamiltonian of the system is $H = H_{\rm L} + H_{\rm R} + V$. Each bath is chosen to be a nonintegrable spin chain with bond and site Hamiltonian ($h_n^{\rm b}$ and $h_n^{\rm s}$) given by

$$h_n^{\rm b} = J_{zz}\sigma_n^z\sigma_{n+1}^z + J_{yz}\sigma_n^y\sigma_{n+1}^z,\;\; h_n^{\rm s} = h_x\sigma_n^x + h_z\sigma_n^z, \quad (1)$$

such that $H_{\rm L} = \sum_{n=1}^{N_{\rm L}-1} h_n^{\rm b} + \sum_{n=1}^{N_{\rm L}} h_n^{\rm s}$, while for $H_{\rm R}$ the site labeling ranges from $N_{\rm L}+1$ to N. Here $N = N_{\rm L} + N_{\rm R}$ is the total number of spins, while $N_{\rm L}$ and $N_{\rm R}$ are the length of the left and right bath, respectively. The interaction term V is given by $V = \gamma B^{\rm L} \bigotimes B^{\rm R} = \gamma\sigma_{N_{\rm L}}^x\sigma_{N_{\rm L}+1}^x$ where γ is the coupling strength. We consider $J_{yz} = J_{zz}$, $h_x = -1.05J_{zz}$, $h_z = 0.5J_{zz}$ where each bath is nonintegrable with Gaussian unitary ensemble (GUE) level statistics [39]. The Hamiltonians $H_{\rm L}$ and $H_{\rm R}$, which have no conserved quantities apart from energy, have also been used to study out-of-time-ordered correlators [40]. In the following we will work in units for which $J_{zz} = \hbar = 1$.

Initial conditions. We are interested in the currents generation when the baths are prepared with different local equilibrium properties, e.g., the baths are prepared at different initial energies per spin. More specifically, for each bath, we consider an energy shell $\Xi^{\rm L\,or\,R} = [E^{\rm L\,or\,R} - \Delta^{\rm L\,or\,R}/2, E^{\rm L\,or\,R} + \Delta^{\rm L\,or\,R}/2]$ where $E^{\rm L\,or\,R}$ is the average energy and $\Delta^{\rm L\,or\,R}$ the width of the shell.

To this end we consider three different scenarios:

(i) *Eigenstate pairs*: Each bath is prepared at an arbitrary eigenstate corresponding to the shell $\Xi^{\rm L\,or\,R}$, i.e.,

$$\left|\psi^{\rm eig}\right\rangle = |i\rangle_{\rm L} \otimes |j\rangle_{\rm R}, \quad (2)$$

where $|i\rangle_{\rm L}$ ($|j\rangle_{\rm R}$) is the $i(j)$th eigenstate of $H_{\rm L}$ ($H_{\rm R}$);

(ii) *Typical-state pairs*: Each bath is a typical superposition of states within the shell, with random complex numbers $c_i^{\rm L}$ and $c_j^{\rm R}$ drawn from the Haar measure, i.e.,

$$|\psi^{\rm typ}\rangle = \sum_{i,j} c_i^{\rm L}c_j^{\rm R}\; |i\rangle_{\rm L} \otimes |j\rangle_{\rm R}; \quad (3)$$

(iii) *Microcanonical ensemble pairs*: Each bath is prepared in a microcanonical state of the energy shell $\Xi^{\rm L\,or\,R}$,

$$\rho^{\rm mic} = \rho_{\rm L}^{\rm mic} \otimes \rho_{\rm R}^{\rm mic}, \quad (4)$$

with $\rho_{\rm L\,or\,R}^{\rm mic} = 1/d_{\rm L\,or\,R}\sum_{E_i^{\rm L\,or\,R}} |i\rangle_{\rm L\,or\,R}\langle i|_{\rm L\,or\,R}$, where $E_i^{\rm L\,or\,R} \in \Xi^{\rm L\,or\,R}$ and $d_{\rm L\,or\,R}$ is the number of states in the shell $\Xi^{\rm L\,or\,R}$.

Energy currents. The energy current operator with respect to the left environment is defined as $I^{\rm L} = -{\rm d}H_{\rm L}/{\rm d}t = -{\rm i}\gamma[B^{\rm L}, H_{\rm L}] \otimes B^{\rm R} = \gamma\dot{B}^{\rm L} \otimes B^{\rm R}$ where we have defined $\dot{B}^{\rm L} \equiv -{\rm i}[B^{\rm L}, H_{\rm L}]$. The expectation value $\mathcal{I}_{\rm exact} = {\rm Tr}[I^{\rm L}\rho(t)]$ gives the exact current from evolving the initial condition via the full Hamiltonian H, and it takes three forms $\mathcal{I}_{\rm exact}^{\rm eig}$, $\mathcal{I}_{\rm exact}^{\rm typ}$ and $\mathcal{I}_{\rm exact}^{\rm mic}$ depending on whether the initial condition $\rho(0)$ is $|\psi^{\rm eig}\rangle\,\langle\psi^{\rm eig}|$, $|\psi^{\rm typ}\rangle\,\langle\psi^{\rm typ}|$ or $\rho^{\rm mic}$.[1] A true bath, by definition, should be infinitely large with a continuous spectrum. In our simulation, we would like to consider systems as large as possible to avoid finite-size effects. This poses a great numerical challenge due to the exponential growth of space and time complexity. For this purpose, various strategies have been proposed [41,42]. We developed a highly optimized time evolution algorithm that allows us to study system sizes up to $N = 28$ on a personal computer. Two main numerical techniques are used: (i) a Suzuki-Trotter-based time evolution algorithm, which only requires the storage of a single quantum state; (ii) a highly parallelized and cache-friendly implementation of the gate operations (see Ref. [43] for details).

Since such exact simulations are numerically demanding, we complement these calculations with a perturbative approach. On top of significantly reducing the time and memory demands of the calculations, the perturbative approach also allows us to gain analytical insights on the typicality of the currents as well as the importance of weak coupling between the baths. It also shares the same spirit of the weak coupling master equations formalism [44–48]. In the weak coupling limit $\gamma \to 0$, the perturbative current expression with respect to different initial conditions can be written as [43,49]

$$\mathcal{I}(t) = \gamma\dot{\mathcal{B}}^{\rm L}(t)\mathcal{B}^{\rm R}(t) - {\rm i}\gamma^2\int_0^t d\tau[\overrightarrow{\mathcal{C}}_{\rm L}(t,\tau)\mathcal{C}_{\rm R}(t,\tau) - \overleftarrow{\mathcal{C}}_{\rm L}(\tau,t)\mathcal{C}_{\rm R}(\tau,t)], \quad (5)$$

where $\dot{\mathcal{B}}^{\rm L}(t) = {\rm Tr}_{\rm L}[\rho_{\rm L}\dot{B}^{\rm L}(t)]$, $\mathcal{B}^{\rm R}(t) = {\rm Tr}_{\rm R}[\rho_{\rm R}B^{\rm R}(t)]$ and we have defined the two-time correlation functions $\overrightarrow{\mathcal{C}}_{\rm L}(t,\tau) = {\rm Tr}_{\rm L}[\rho_{\rm L}\dot{B}^{\rm L}(t)B^{\rm L}(\tau)]$, $\overleftarrow{\mathcal{C}}_{\rm L}(t,\tau) = {\rm Tr}_{\rm L}[\rho_{\rm L}B^{\rm L}(\tau)\dot{B}^{\rm L}(t)]$, and $\mathcal{C}_{\rm R}(t,\tau) = {\rm Tr}_{\rm R}[\rho_{\rm R}B^{\rm R}(t)B^{\rm R}]$. We have also defined $B^{\rm L\,or\,R}(t) = e^{{\rm i}H_{\rm L\,or\,R}t}B^{\rm L\,or\,R}e^{-{\rm i}H_{\rm L\,or\,R}t}$ and the same applies to $\dot{B}^{\rm L}(t)$. We call $\mathcal{I}_{\rm perturb}^{\rm typ}$, $\mathcal{I}_{\rm perturb}^{\rm mic}$, $\mathcal{I}_{\rm perturb}^{\rm eig}$ as typical, microcanonical, and eigenstate currents, respectively, depending on the initial conditions used. Note that when equilibrium states or single eigenstates are used, the expression for the current Eq. (5) can be significantly simplified and the first term vanishes [43].

Typical quasi-steady currents. We first consider weak coupling $\gamma = 0.01$ where we expect the perturbative results to be consistent with the exact ones. In Fig. 1 we consider baths each of size $N_{\rm L} = N_{\rm R} = 12$ and with energy windows $\Xi^{\rm L\,or\,R}$ given by [4.95,5.05] and [−5.05, −4.95], respectively. In Fig. 1(a), we depict $\mathcal{I}$ both for exact and perturbative calculations (respectively, for empty symbols and lines), and for the three different initial conditions. We observe perfect agreement between exact currents and perturbative currents for all the initial conditions considered. For the typical initial conditions, the results present larger oscillations, due to the dynamics of initial coherence for typical states. These results validate the use of perturbative methods for computing currents as all the currents computed are close to each other. In

[1]Note that when discussing the expectation value of the current we drop the superscript L because, in the quasi-steady regime we are interested in, the current from the left bath equals that to the right bath. More details in Ref. [43]

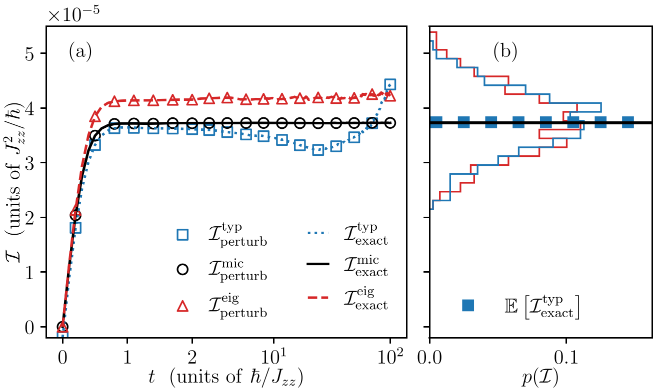

FIG. 1. (a) Currents versus time for a single eigenstate initial condition (red), microcanonical initial conditions (black), and a single typical initial condition (blue) on a symlog scale for the time axis (linear from $t = 0$ to 2 and logarithmic beyond 2). The perturbative results are denoted by markers and the exact currents are denoted by lines. (b) Histogram of the current statistics for eigenstate currents and typical currents at $t = 10$. Total number of initial states, both eigenstates and typical ones, is 399. The solid black line marks the exact microcanonical current. $N = 24$, $\gamma = 0.01$, $E_{\rm L} = -E_{\rm R} = 5$, $\Delta_{\rm L\,or\,R} = 0.1$

addition, the microcanonical current becomes constant after a time of order 1 indicating the emergence of a quasi-steady current.

Another question to be answered is whether the currents $\mathcal{I}^{\rm typ}_{\rm exact}$ or $\mathcal{I}^{\rm typ}_{\rm perturb}$ also can be considered as typical, i.e., the vast majority of the currents using different typical initial conditions show similar dynamics. This means that if we consider an ensemble of typical currents, their average $\mathbb{E}[\mathcal{I}^{\rm typ}] \approx \mathcal{I}^{\rm mic}$ with a bounded variance. Perturbatively, one can show this for the quasi-steady current in a similar fashion to the study of typicality in isolated quantum systems [11,34]. In fact, the average typical correlations functions $\mathbb{E}[\mathcal{C}^{\rm typ}]$ is equivalent to the microcanonical ones by considering, for example,

$$\mathbb{E}\big[\overrightarrow{\mathcal{C}}^{\rm typ}_{\rm L}(t,\tau)\big] = -\,\mathrm{i}\sum_{k}^{D_{\rm L}}\sum_{ij}^{d_{\rm L}}\mathbb{E}[c_i^* c_j]\mathrm{e}^{\mathrm{i}(\Delta_{ki}t+\Delta_{kj}\tau)}\Delta_{ki}B^{\rm L}_{ik}B^{\rm L}_{kj}$$
$$= \overrightarrow{\mathcal{C}}^{\rm mic}_{\rm L}(t,\tau), \tag{6}$$

where $\mathbb{E}[c_i^* c_j] = \delta_{ij}/d_{\rm L}$, $\Delta_{ki} = E_k - E_i$, $D_{\rm L}$ is the Hilbert space dimension of the left bath, and δ_{ij} is the Kronecker delta. Since the left and right correlation functions are independent, it is thus straightforward to conclude from Eq. (5) that the average typical current is equal to the microcanonical one, i.e., $\mathbb{E}[\mathcal{I}^{\rm typ}_{\rm perturb}] = \mathcal{I}^{\rm mic}_{\rm perturb}$. This is an indicator of the typicality of quasi-steady currents. However, one should also consider the variance of the typical currents. To this end, we perform numerical computations of the exact currents $\mathcal{I}^{\rm typ}_{\rm exact}$ and $\mathcal{I}^{\rm eig}_{\rm exact}$ at a time $t = 10$, and provide a histogram of these values in Fig. 1(b). Already for bath sizes $N_{\rm L\,or\,R} = 12$, we observe a clear peak near the prediction from microcanonical initial states (continuous black line), which corresponds to the average of the typical states too (solid blue square).

We then analyze the dependence of the variance with the size of the baths. Relying on the extensive character of $H_{\rm L}$ and $H_{\rm R}$, we linearly shift the energy shell for the initial conditions as the baths' size increases, i.e., $\Xi^{\rm L\,or\,R}$ are such that $E^{\rm L\,or\,R} = N_{\rm L\,or\,R}\varepsilon_{\rm L\,or\,R}$ and $\Delta^{\rm L\,or\,R} = N_{\rm L\,or\,R}\delta_{\rm L\,or\,R}$, where $\varepsilon_{\rm L\,or\,R}$ is the energy per spin in each bath and $\delta_{\rm L\,or\,R}$ is the shell width per spin chosen to be $0.1/12$. In Fig. 2(a), we show the standard deviation for different realizations of typical currents and eigenstate currents std$[\mathcal{I}]$ (rescaled over the value of the current $\mathbb{E}[\mathcal{I}]$) versus system size N. Such decaying behavior serves as numerical evidence of the typicality of nonequilibrium currents.[2] Figure 2(b), which depicts the value of $\mathcal{I}^{\rm mic}_{\rm perturb}$ (to which the typical current converges) for different energy per spins, also shows a clear convergence of the current with the system size. In fact, larger systems sizes, each denoted by different markers, result in smaller oscillations and the current versus energy curve converges towards a smooth line. To have a deeper understanding of the emergence of this smooth curve, we compare the numerical perturbative results with predictions from ETH. The ETH ansatz, which each bath follows [43], states that for a local observable O, its matrix elements in the energy basis are [4,50]

$$O_{ij} = O(E)\delta_{ij} + \mathrm{e}^{-\frac{S(E)}{2}} f(E,\omega)R_{ij}, \tag{7}$$

where $\omega = E_j - E_i$ and $E = (E_i + E_j)/2$, E_i are energy eigenvalues, $S(E)$ is the entropy given by $\mathrm{e}^{S(E)} = E\sum_i \delta(E - E_i)$, $O(E)$, and $f(E,\omega)$ are smooth functions of their arguments and R_{ij} is a matrix with normal distributed random elements. We thus estimate the smooth function $f(E,\omega)$ for each bath from a system of size $N = 24$ and evaluate the correlations stemming from Eq. (5), $\overrightarrow{\mathcal{C}}^{\rm ETH}_{\rm L}$, $\overleftarrow{\mathcal{C}}^{\rm ETH}_{\rm L}$, and $\mathcal{C}^{\rm ETH}_{\rm R}$ [7,43,51–55], from which we can determine the current expected for a large system [43]. In Fig. 2(b) we thus show with a continuous black line the value from this ETH-based approach for a system with $N = 2000$. Indeed, the microcanonical currents converge towards ETH predictions showing that the

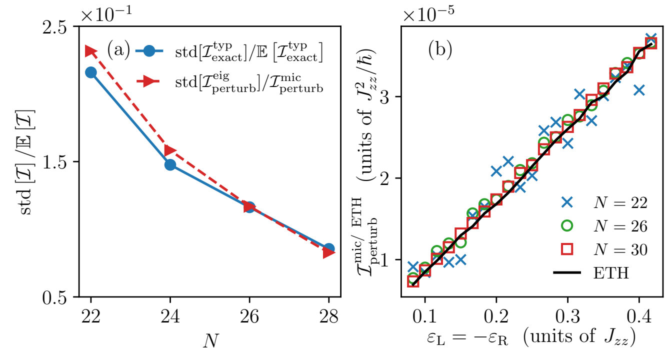

FIG. 2. (a) Normalized standard deviation versus the total system size N for exact typical currents (blue) and perturbative eigenstate currents (orange) at time $t = 10$. The number of typical initial conditions is 500. (b) Perturbative microcanonical currents versus energy per spin $\varepsilon_{\rm L} = -\varepsilon_{\rm R}$ and ETH predicted current (solid black line) for $N = 2000$. $\gamma = 0.01$.

[2]The evolution of the currents from the corresponding different typical initial conditions are shown in Ref. [43] for time up to $t = 50$

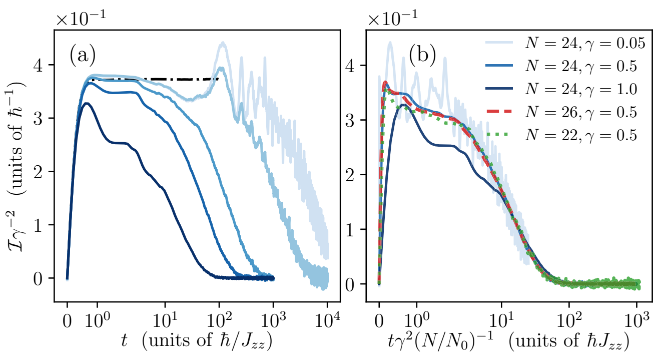

FIG. 3. (a) Typical exact currents $\mathcal{I}^{\rm typ}_{\rm exact}$ versus time t for different coupling strengths $\gamma = 0.05$, 0.1, 0.3, 0.5, and $\gamma = 1.0$ denoted by the increasingly darker blue solid lines. The typical initial states are chosen to be the same with the system size $N = 24$. The reference dynamics for perturbative microcanonical current is represented by the black dashed-dotted line. (b) Typical exact currents $\mathcal{I}^{\rm typ}_{\rm exact}$ versus scaled time $t\gamma^2/(N/N_0)^{-1}$ for different system size and coupling strength with $N_0 = 24$. The time axes are on the same symlog scale as in Fig. 1.

emergence of a smooth dependence of the current versus energy can be deduced from ETH.

Prethermalization and thermalization. To address how the system goes from the presence of a quasi-steady current to the absence of current, we consider larger magnitudes of the coupling strength γ and we perform exact simulations. We stress here that perturbative calculations do not allow one to observe thermalization because, implicitly, they assume that one takes the thermodynamic limit first, and then the infinite-time limit. In Fig. 3(a) we depict the exact typical current versus time, for a single random typical initial condition, divided by γ^2 (because of the dependence on γ of the perturbative current) for different values of the coupling strength with $N = 24$. In particular we have $\gamma = 0.05$, 0.1, 0.3, 0.5, and 1.0 for darker shades of blue. As a reference, we add the microcanonical perturbative current $\mathcal{I}^{\rm mic}_{\rm perturb}$ as a black dot-dashed line. The typical current of a single random realization and for small γ follows closely the perturbative current before deviating from it. As γ increases, the typical current deviates earlier, and more markedly from the perturbative value. In Fig. 3(b) we consider different coupling strengths γ, and system sizes N. We show a convergence at long times of the current when plotted against $\gamma^2 t/N$. This is a consequence of the perturbative character of the local coupling for larger systems; in fact the energy of the baths is extensive while the two baths are only coupled at a single site.

When analyzing the current evolution over these time scales we can clearly recognize three regimes: a first transient regime, a regime in which one can observe quasi-steady currents for weak coupling (which we have shown to be typical), and a thermalization regime in which the current goes to zero. The emergence of the intermediate regime with a quasi-steady current is analogous to the phenomenon of prethermalization in isolated systems in which a thermalizing system with a weakly broken conserved law would relax slowly due to the underlying presence of conservation laws (the prethermalized regime), and then slowly relax toward the final states. In our case the conserved quantity is the energy of each bath, which is broken by the coupling between them. This dynamical correspondence with prethermalization is strengthened by the γ^2 relaxation rate [30,31].

Conclusions. We studied the emergence of long-lasting currents between differently prepared nonintegrable systems in analogy to the emergence of prethermalization in isolated systems. Each bath has its own thermalizing dynamics, which conserves its energy. This conservation law is weakly broken due to the coupling between the baths, and this results in a typical fast relaxation towards a nonequilibrium scenario with, in the weak-coupling regime, a quasi-steady current. This is then followed by a slow relaxation towards thermalization. We have shown that such dynamics is typical in the sense that it is quantitatively the same for any typical state chosen from two different energy shells in the respective baths, and specifically we have shown that this current is consistent with the prediction from ETH, and with preparing the baths in microcanonical states. The convergence towards ETH also results in a smooth dependence of current with energy difference. To obtain these results, we have used both perturbative and exact numerical methods. For the latter we used a highly parallelized algorithm, which allowed us to simulate spin chains of the size of 28 spins (14 spins per bath) and more. We have thus studied a model that makes it possible for the observation of nonequilibrium steady current as well as the long time thermalization. This connects the advances in equilibrium statistical mechanics with those in nonequilibrium open quantum systems.

We would like to stress here that the study of transport and thermalization properties of systems by preparing two subsystems in different states and coupling them has already been undertaken in the past years [56–63]. With this work we lay a stronger basis on the application of these approaches, highlighting that the evolution of a relatively small number of different pure-state initial conditions can lead to accurate estimates of the currents expected from the microcanonical initial conditions. In the future we will consider adding an intermediate system between the baths so as to study conditions for the emergence of a quasi-steady current within this intermediate system. Furthermore, it would be important to consider whether the conditions for the emergence of the quasi-steady current can be loosened, for example considering also integrable baths [57,64–69].

Acknowledgments. The authors are grateful to R. Chen, J. Gemmer, T. Monnai, J. Thingna, and H. Zhou for fruitful discussions. X.X. is grateful for the hospitality of Sichuan Normal Univeristy. D.P. acknowledges support from the Ministry of Education of Singapore AcRF MOE Tier-II (Project No. MOE2018-T2-2-142). C.G. acknowledges support from National Natural Science Foundation of China under Grants No. 11805279, No. 61833010, No. 12074117, and No. 12061131011. The computational work for this Letter were partially performed on the National Supercomputing Centre, Singapore [70].

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