Editors' Suggestion

Unitary Fermi superfluid near the critical temperature: Thermodynamics and sound modes from elementary excitations

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We compare recent experimental results [Science **375**, 528 (2022)] of the superfluid unitary Fermi gas near the critical temperature with a thermodynamic model based on the elementary excitations of the system. We find good agreement between experimental data and our theory for several quantities such as first sound, second sound, and superfluid fraction. We also show that mode mixing between first and second sound occurs. Finally, we characterize the response amplitude to a density perturbation: Close to the critical temperature both first and second sound can be excited through a density perturbation, whereas at lower temperatures only the first sound mode exhibits a significant response.

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I. INTRODUCTION

The unitary Fermi gas, i.e., a gas of resonantly interacting fermions in the limit for which the scattering length diverges, constitutes a fundamental model in many-body physics [1,2], and it has been the subject of a great deal of theoretical [3–6] and experimental investigations [7–12]. It is a unifying paradigm, of remarkable importance for several different subfields of physics, from ultracold quantum gases, nuclear matter, up to high-energy physics. Indeed, the unitary Fermi gases is the nonrelativistic setup which appears to be closer to the perfect fluidity as conjectured by string-theoretical arguments [13,14], i.e., a fluid saturating the lower bound on the shear viscosity-entropy ratio [15].

The scale invariance of the system means that, as the scattering length diverges, the only energy scale in the system at $T = 0$ is the Fermi energy T_F and that all thermodynamic and transport quantities can be expressed as universal functions, depending on T/T_F only. As a consequence, the unitary Fermi gas has emerged as a standard test bed for several different many-body theoretical approaches [6]. A remarkable possibility for studying the unitary Fermi gas comes from ultracold fermions in the vicinity of a Feshbach resonance: As an external magnetic field is tuned across the resonance, the fermion-fermion interaction can assume all values from weakly to strongly attractive—in a scenario known as the BCS-BEC crossover. As a consequence, the system varies with continuity from the BCS limit where fermions form large Cooper pairs over a definite Fermi surface, to the BEC limit where fermions form tightly bound bosonic molecules. Critically, the unitary Fermi gas is to be found between these two limits, so that its superfluid transition does not simply correspond to the usual BCS or BEC paradigms, rather being due to a delicate interplay between the two [2].

Through the years, it has been shown that this interplay can be described within a thermodynamic approach [16–22] including temperature-independent single-particle and collective elementary excitations of the unitary Fermi gas. Such an approach describes with great precision a number of features, with favorable comparisons with experimental data [18–20]. Moreover, it has been demonstrated that this approach, originally proposed by Landau on phenomenological grounds [23], can be justified via beyond-mean-field treatments of a Fermi gas, such as the Nozières-Schmitt-Rink (NSR) [24] and the Gaussian pair-fluctuation approach (GPF) [25–28], in which a systematic treatment of the order parameter and its fluctuations leads to a rigorous *ab initio* theory with essentially the same physical content: BCS-like single-particle excitations and collective excitations with a Bogoliubov-like dispersion. It is also important to note that it has been recently pointed out that beyond-GPF corrections are quite small in the broken symmetry phase [29,30].

In such a complex scenario, it is fundamental to identify a diagnostic tool allowing for a comparison between theory and experiment. From this perspective, sound propagation is certainly a promising candidate for a variety of reasons. On a conceptual standpoint it can be derived on a hydrodynamic basis by connecting thermodynamic and transport quantities within the framework of Landau two-fluid theory [23,31], with no need—in principle—to refer to the particular features of the microscopic constituents. From an experimental perspective, it has been recently shown that both modes predicted by the above-mentioned Landau theory can be excited by a density-perturbing protocol driven by external laser fields [32].

Along this path, the most recent experimental breakthroughs concerning the unitary Fermi gas [11,12] allowed for the measurement of many properties at an unprecedented level of precision, providing very stringent benchmarks for the theoretical models. The present paper demonstrates that a thermodynamic theory accounting for temperature-independent elementary single-particle and collective excitation is able to reproduce with excellent precision the most recent measurements on the sound velocity. In particular, for first sound, second sound, and superfluid fraction we find very good agreement between experimental data [12] and our theory, taking into account the mode mixing between first and second sound. We also prove that around the critical temperature both the first and second sound modes may be detected with a density perturbation, but only the first sound mode has a significant density response at very low temperatures.

II. DESCRIBING THE UNITARY FERMI GAS FROM ELEMENTARY EXCITATIONS

Following an approach pioneered by Landau [23], we describe the low-temperature thermodynamics of a uniform unitary Fermi gas, consisting of N particles contained in a volume $V = L^3$, in the superfluid phase, by means of its temperature-independent single-particle BCS-like excitations and collective Bogoliubov-like excitations. Within this framework, an effective Hamiltonian describing the system can be written down [18] as

$$\hat{H} = \frac{3}{5}\xi\epsilon_F N + \sum_{\sigma=\uparrow,\downarrow}\sum_{\mathbf{p}} \epsilon_{\rm sp}(p)\hat{c}^\dagger_{\mathbf{p}\sigma}\hat{c}_{\mathbf{p}\sigma} + \sum_{\mathbf{q}} \omega_{\rm col}(q)\, \hat{b}^\dagger_{\mathbf{q}}\hat{b}_{\mathbf{q}}, \tag{1}$$

where the $\hat{c}^\dagger_{\mathbf{p}\sigma}$ ($\hat{c}_{\mathbf{p}\sigma}$) operator creates (annihilates) a single-particle excitation, respectively, with linear momentum $\mathbf{p}$, spin σ, and energy $\epsilon_{\rm sp}(p)$, whereas the $\hat{b}^\dagger_{\mathbf{p}}$ ($\hat{b}_{\mathbf{p}}$) operator creates (annihilates) a bosonic collective excitation, respectively, of linear momentum $\mathbf{q}$ and energy $\omega_{\rm col}(q)$.

The first term of Eq. (1) represents the ground-state energy of the uniform unitary Fermi gas [33,34], ξ being the celebrated Bertsch parameter $\xi \simeq 0.38$ [35] having also introduced the Fermi energy $\epsilon_F = \hbar^2(3\pi^2 n)^{2/3}/(2m)$ of a noninteracting Fermi gas of density $n = N/V$.

The second and third terms represent the contribution from off-condensate fermionic single-particle excitations and collective modes, respectively. Of course these terms do not have any use until the dispersions of the temperature-independent elementary excitations are specified. In Refs. [36,37] the dispersion relation of collective elementary excitations has been derived as

$$\omega_{\rm col}(q) = \sqrt{\frac{q^2}{2m}\left(2mc_B^2 + \frac{\lambda}{2m}q^2\right)}, \tag{2}$$

where $c_B = \sqrt{\xi/3}\, v_F$ is the Bogoliubov sound velocity with $v_F = \sqrt{2\epsilon_F/m}$ the Fermi velocity of a noninteracting Fermi gas. Here, se set $\lambda = 0.02$, by fitting the spectrum of bosonic collective modes obtained from the GPF theory [20] (see Refs. [25,28,38] for an exhaustive review on the basics of this approach).

However, the collective modes correctly describe only the low-energy density oscillations of the system; at higher energies one expects the appearance of fermionic single-particle excitations starting from the threshold above which Cooper pairs break down [16,17,33,39,40]. The dispersion of these temperature-independent single-particle elementary excitations can be written as

$$\epsilon_{\rm sp}(p) = \sqrt{\left(\frac{p^2}{2m} - \zeta\epsilon_F\right)^2 + \Delta_0^2}, \tag{3}$$

where ζ is a parameter taking into account the interaction between fermions and the reconstruction of the Fermi surface close to the critical temperature ($\zeta \simeq 0.9$ according to accurate Monte Carlo results [40]). Moreover, Δ_0 is the gap parameter, with $2\Delta_0$ the minimal energy to break a Cooper pair [33]. Notice that the gap energy Δ_0 of the unitary Fermi gas at zero temperature has been calculated with Monte Carlo simulations [40–43] and found to be $\Delta_0 = \gamma\epsilon_F$, with $\gamma \simeq 0.45$. Let us also notice that, while Δ certainly has a temperature dependence, the inclusion of a phenomenological thermal profile (as proposed, for instance, in Ref. [44]) in our framework does not produce any significant change in the sound velocities and the superfluid fraction.

III. UNIVERSAL THERMODYNAMICS AT UNITARITY

The Helmholtz free energy F of the system is given by the usual formula $F = -k_B T \ln \mathcal{Z}$, where we introduced the partition function $\mathcal{Z}$ of the system [45], defined as

$$\mathcal{Z} = {\rm Tr}[e^{-\hat{H}/k_B T}]. \tag{4}$$

Similarly to Eq. (1), the free energy of the unitary Fermi gas can be written as $F = F_0 + F_{\rm col} + F_{\rm sp}$, where F_0 is the free energy of the ground state,

$$F_{\rm sp} = -\frac{2}{\beta}\sum_{\mathbf{k}} \ln[1 + e^{-\beta E_k}] \tag{5}$$

is the free energy of fermionic single-particle excitations, and finally

$$F_{\rm col} = -\frac{1}{\beta}\sum_{\mathbf{q}} \ln[1 - e^{-\beta\omega_q}] \tag{6}$$

is the free energy of the bosonic collective excitations.

As discussed in detail in Ref. [18], the total Helmholtz free energy F of a unitary Fermi gas in the superfluid phase can be then written as

$$F = N\epsilon_F\Phi(x), \tag{7}$$

where, due to the scale invariance of the system, $\Phi(x)$ is a function of the scaled temperature $x \equiv T/T_F$ only, having defined the Fermi temperature $T_F = \epsilon_F/k_B$. Explicitly, $\Phi(x)$ takes the following form,

$$\Phi(x) = \frac{3}{5}\xi - 3x\int_0^{+\infty} \ln[1 + e^{-\tilde{\epsilon}_{\rm sp}(u)/x}]u^2 du + \frac{3}{2}x\int_0^{+\infty} \ln[1 - e^{-\tilde{\omega}_{\rm col}(u)/x}]u^2 du. \tag{8}$$

Note that the discrete summations have been replaced by integrals, and that we set $\tilde{\omega}_{\rm col}(u) = \sqrt{u^2(4\xi/3 + \lambda u^2)}$ and $\tilde{\epsilon}_{\rm sp}(u) = \sqrt{(u^2 - \zeta)^2 + \gamma^2}$.

We now aim at calculating the thermodynamics of the system in terms of the universal function $\Phi(x)$ and its derivatives. We start from the entropy S, which is readily calculated from the free energy F through the relation

$$S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}, \tag{9}$$

from which we immediately get

$$S = -Nk_B\Phi'(x), \tag{10}$$

where $\Phi'(x)$ is the first derivative of Φ with respect to x. Furthermore, the internal energy $E = F + TS$, can immediately be rewritten as

$$E = N\epsilon_F[\Phi(x) - x\,\Phi'(x)] \tag{11}$$

and, similarly, the pressure P is related to the free energy F by the simple relation

$$P = -\left(\frac{\partial F}{\partial V}\right)_{N,T}, \tag{12}$$

which we now rewrite in terms of $\Phi(x)$ and its derivatives as

$$P = \tfrac{2}{3}n\epsilon_F[\Phi(x) - x\Phi'(x)]. \tag{13}$$

As a consistency check of our simple analytical model, let us underline that, by combining Eqs. (11) and (13) one can easily recover the well-known relation $PV = (2/3)E$ for unitary fermions [7].

IV. SUPERFLUID FRACTION AND CRITICAL TEMPERATURE

According to Landau's two-fluid theory [23,31], the total number density n of a system in the superfluid phase can be written as

$$n = n_s + n_n, \tag{14}$$

where n_s is the superfluid density and n_n is the normal density [23]. Naturally, at zero temperature the whole system is in the superfluid phase, and one has $n_n = 0$ and $n = n_s$. As the temperatures increases, the normal density n_n increases as well, until at the critical temperature T_c one has $n_n = n$ and, correspondingly, $n_s = 0$. Within our scheme, the normal density of a unitary gas is given the sum of two contributions

$$n_n = n_{n,sp} + n_{n,sp}, \tag{15}$$

i.e., a contribution $n_{n,sp}$ from to the single-particle excitations and a contribution $n_{n,col}$ from collective excitations. We note that in the BCS limit of the BCS-BEC crossover one expects $n_{n,sp}$ to be the dominating contribution, whereas in the BEC limit $n_{n,col}$ should account for most of the normal density. In the present unitary case, however, we expect both single-particle and collective excitations to be relevant.

Furthermore, Landau linked the normal densities to their statistic and their energy spectrum (see, for instance, Ref. [46]), so that in the present case the single-particle contribution to the normal density reads

$$n_{n,sp} = \frac{2\beta}{3V}\sum_{\mathbf{k}}\frac{k^2}{m}\frac{e^{\beta\epsilon_{sp}(k)}}{(e^{\beta\epsilon_{sp}(k)}+1)^2}, \tag{16}$$

whereas, concerning the contribution from the collective modes,

$$n_{n,col} = \frac{\beta}{3V}\sum_{\mathbf{q}}\frac{q^2}{m}\frac{e^{\beta\omega_{col}(q)}}{(e^{\beta\omega_{col}(q)}-1)^2}. \tag{17}$$

It is then easy to derive the superfluid fraction

$$\frac{n_s}{n} = 1 - \Xi(x), \tag{18}$$

where the universal function $\Xi(x)$ is again a function of the scaled temperature $x \equiv T/T_F$ only, explicitly given by

$$\Xi(x) = \frac{2}{x}\int_0^{+\infty}\frac{e^{\tilde{\epsilon}_{sp}(\eta)/x}}{(e^{\tilde{\epsilon}_{sp}(\eta)/x}+1)^2}\eta^4 d\eta + \frac{1}{x}\int_0^{+\infty}\frac{e^{\tilde{\omega}_{col}(\eta)/x}}{(e^{\tilde{\omega}_{col}(\eta)/x}-1)^2}\eta^4 d\eta, \tag{19}$$

where we have converted sums to integrals. Finally, we stress that in the present model, the superfluid density defines the critical temperature T_c via the condition $n_s = 0$, and with our choice of parameters for the temperature-independent elementary excitation dispersions we find $T_c \approx 0.23T_F$. It must be pointed out that, while this estimation of the critical temperature agrees with more refined approaches, such as the functional GPF theory [25,28] or the NSR scheme [24], it actually differs from the most recent experimental results, placing it at $T_c/T_F \simeq 0.17$ [12]. This shortcoming, shared among a range of different formalisms, is due to the fact the induced interaction is not taken into account [47] according to the so-called Gorkov-Melik-Barkhudarov theory [48], which has been shown to provide the dominant contribution on the BCS side and a relevant correction at unitarity. The slight overestimation of our theoretical critical temperature with respect to the experimental one of Ref. [12] does not appear plotting the physical quantities vs T/T_c.

In the left panel of Fig. 1, we report the theoretically derived superfluid fraction n_s/n as a function of the dimensionless temperature T/T_c (red dashed line), compared with experimental data [12] for the unitary Fermi gas (blue dots), showing remarkable agreement; as a reference, we also plot the critical-exponent behavior observed in superfluid He (black dashed line).

V. FIRST SOUND, SECOND SOUND, AND SOUND MIXING

According to Landau [31,49] a local perturbation excites two wavelike modes—the first and the second sound—which propagate with velocities u_1 and u_2. These velocities are determined by the positive solutions of the algebraic biquadratic equation (see also Ref. [50])

$$u^4 - (c_{10}^2 + c_{20}^2)u^2 + c_T^2 c_{20}^2 = 0, \tag{20}$$

where

$$c_{10} = \sqrt{\frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_{\bar{S},V}} = v_F\sqrt{\frac{5}{9}\Phi(x) - \frac{5}{9}\frac{T}{T_F}\Phi'(x)} \tag{21}$$

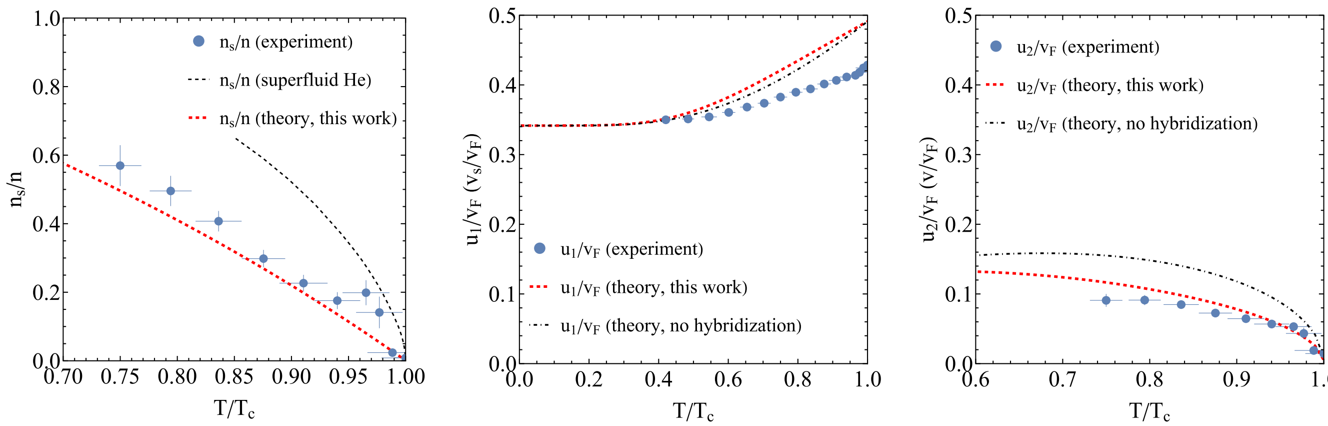

FIG. 1. Comparison between theory and experimental data in Ref. [12,51] as a function of the dimensionless temperature T/T_c. Left panel: Superfluid fraction n_s/n. Middle panel: Adimensional first velocity u_1/v_F. Right panel: Adimensional second sound velocity u_2/v_F. In the central and right panels, we report the sound velocities computed in absence of mixing, i.e. under the assumption that $c_{10} \approx c_T$. From the left panel, we infer that, contrary to the ^{4}He picture [23,55], the equality between isothermal and adiabatic compressibilities reads a much worse agreement with the experimental data, as evident from the behavior of the second sound u_2 (right panel).

is the adiabatic sound velocity with $\bar{S} = S/N$ the entropy per particle,

$$c_{20} = \sqrt{\frac{1}{m}\frac{\bar{S}^2}{\left(\frac{\partial \bar{S}}{\partial T}\right)_{N,V}}\frac{n_s}{n_n}} = v_F\sqrt{-\frac{1}{2}\frac{\Phi'(x)^2}{\Phi''(x)}\frac{1-\Xi(x)}{\Xi(x)}} \tag{22}$$

is the entropic sound velocity, and

$$c_T = \sqrt{\frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_{T,V}} = v_F\sqrt{\frac{5}{9}\left(\Phi(x) - \frac{T}{T_F}\Phi'(x)\right) + \frac{2}{9}x^2\Phi''(x)} \tag{23}$$

is the isothermal sound velocity. It is immediate to find that for $T \to 0$ one has

$$c_{10} \to c_B = v_F\sqrt{\xi/3}, \tag{24}$$

$$c_{20} \to c_B/\sqrt{3} = v_F\sqrt{\xi}/3, \tag{25}$$

$$c_T \to c_B = v_F\sqrt{\xi/3}. \tag{26}$$

The first sound u_1 is the largest of the two positive roots of Eq. (20) while the second sound u_2 is the smallest positive one. Thus

$$u_{1,2} = \sqrt{\frac{c_{10}^2 + c_{20}^2}{2} \pm \sqrt{\left(\frac{c_{10}^2 + c_{20}^2}{2}\right)^2 - c_{20}^2 c_T^2}}. \tag{27}$$

We now compare our theory with the experimental data for the sound velocities from Ref. [12,51]. In particular, in the middle panel of Fig. 1 we plot the theoretically calculated dimensionless first sound velocity u_1/v_F as a function of the dimensionless temperature T/T_c (red dashed line), comparing it with the experimental data [12,51] (blue dots) showing quite good agreement with our theory. In the same panel we also plot the first sound calculated neglecting mode mixing, i.e., under the assumption that $c_T \approx c_{10}$ (black thin dashed-dotted line). In the right panel of Fig. 1 we plot the theoretically derived dimensionless second sound velocity u_2/v_F (red dashed line), compared with experimental data [12,51] for the second sound velocity u_2/v_F (blue dots). In the same panel we also plot the dimensionless second sound u_2/v_F calculated neglecting mode mixing (black thin dashed-dotted line). As far as the second sound is concerned, our theory shows remarkable agreement with experimental data [12,51]. Importantly, this implies there is mixing between the first and second sound modes, and that for the unitary Fermi gas it is wrong to assume an approximate equality of adiabatic and isothermal compressibilities.

Concluding this section, we stress that the Einstein-like relation

$$\frac{E}{N} = \frac{10}{9}mc_{10}^2 \tag{28}$$

derived in Ref. [11] is automatically verified within our universal thermodynamic formalism, that naturally includes the scale invariance of the unitary Fermi gas.

VI. RESPONSE TO A DENSITY PERTURBATION

In general, the knowledge of the first and second sound velocities may not be sufficient to provide a reliable characterization of the experimentally observed modes. First of all, we stress that the situation is radically different from what is observed in superfluid ^{4}He, where the response in density and temperature is decoupled and first sound corresponds to a standard density waves (in-phase oscillations of the superfluid and normal components), and the second sound is understood as an entropy wave [23,52]. The technical reason has to be traced back to the isothermal and adiabatic compressibilities being approximately equal such that $c_{10} \approx c_T$ [cf. Eqs. (21) and (23)]. However, this assumption does not hold for a generic quantum fluid, so that, in principle, even a simple density-perturbing protocol may excite both modes. This is exactly the case for ultracold bosons, for which, in two spatial dimensions, second sound acts as a reliable diagnostic tool for the onset of the Berezinskii-Kosterlitz-Thouless (BKT) transition [53]. Moving to Fermi gases, the situation across the BCS-BEC crossover is significantly more involved [32]:

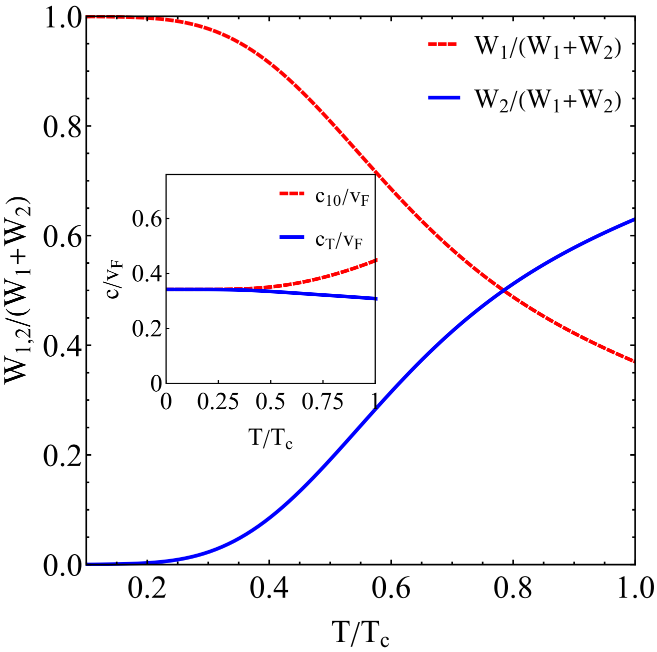

FIG. 2. Main panel: Contribution from the first (dashed red line) and second sound (solid blue line) to the amplitude of a density response, as given by Eqs. (30) and (31), as a function of the scaled temperature T/T_c. Inset: Adiabatic and entropic sound velocities c_{10} and c_T, respectively [cf. Eqs. (21) and (23)], as functions of the scaled temperature. The no-mixing condition $c_{10} \approx c_T$ (see main text) is fulfilled for $T/T_c \lesssim 0.4$, exactly where W_2 becomes vanishingly small.

While the experimental setups are certainly not comparable to helium, there have been cases where a density-perturbing protocol excited just a single mode [50,54].

Therefore, besides the values of u_1 and u_2 in Eq. (27), in order to provide a more complete characterization of the experimental picture, we also have to consider the amplitudes modes W_1 and W_2 of the response to a density perturbation [32,50,55], i.e.,

$$\delta\rho(x,t) = W_1\delta\rho_1(x \pm u_1 t) + W_2\delta\rho_2(x \pm u_2 t), \tag{29}$$

where

$$\frac{W_1}{W_1 + W_2} = \frac{\left(u_1^2 - c_{20}^2\right) u_2^2}{\left(u_1^2 - u_2^2\right) c_{20}^2} \tag{30}$$

and

$$\frac{W_2}{W_1 + W_2} = \frac{\left(c_{20}^2 - u_2^2\right) u_1^2}{\left(u_1^2 - u_2^2\right) c_{20}^2}. \tag{31}$$

In Fig. 2 we report the behavior of the relative amplitude contributions as a function of the temperature. Remarkably, we observe that in the ultralow-T regime a density probe actually excites only the first sound, since the amplitude of the u_2 mode vanishes as $T \to 0$. It is important to notice that, under the no-mixing condition $c_{10} \approx c_T$, Eqs. (30) and (31) read $W_1 = 1$ and $W_2 = 0$. Thus, this implies that mode mixing is extremely reduced deeply below the critical temperature, as confirmed by the inset in Fig. 2, showing the no-mixing condition fulfilled at $T \lesssim 0.4\,T_c$. Moving closer to the transition, our theoretical model predicts that the balance between W_1 and W_2 should tip over around $T/T_c \simeq 0.8$, where the second sound mode becomes the dominant one. This means that, while in principle a density perturbation can excite both modes, at $T \to 0$ (i.e., deeply into the superfluid regime), the amplitude corresponding to u_2 is vanishingly small and actually undetectable. The situation is overturned moving closer to the critical temperature, where the superfluid susceptibility is much higher and both modes can be simultaneously excited with comparable amplitudes.

VII. CONCLUSIONS

In this paper we have shown that a simple description in terms of temperature-independent elementary excitations is able to reproduce many properties of the unitary Fermi gas: In particular we have reproduced the recently measured superfluid fraction near the critical point [12] and, after properly accounting for mixing between sounds modes, also the first and second sound velocities. We have found that, contrary to liquid helium, near the critical temperature the first and second sound of the the unitary Fermi gas cannot be interpreted as a pure pressure-density wave and a pure entropy-temperature wave, respectively. We have also analyzed the density response to an external perturbation, our investigation showing that at very low temperatures the mixing of pressure-density and entropy-temperature oscillations is absent, whereas approaching T_c a density probe will excite both sounds. Finally, we stress that Ref. [12] reports a measurement of the sound diffusion from which they derive the viscosity-entropy ratio. Adopting the analysis developed in Refs. [19,56] our calculated viscosity-entropy ratio is about three times smaller than the one of Ref. [12] but, however, in good agreement with previous experimental determinations [57–60].

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[1] *The BCS-BEC Crossover and the Unitary Fermi Gas*, edited by W. Zwerger, Lecture Notes in Physics Vol. 836 (Springer, Berlin, 2012).
[2] M. Randeria, W. Zwerger, and M. Zwierlein, in *The BCS-BEC Crossover and the Unitary Fermi Gas* (Ref. [1]), pp. 1–32.
[3] H. Hu, P. D. Drummond, and X.-J. Liu, Nat. Phys. **3**, 469 (2007).
[4] T. Enss, R. Haussmann, and W. Zwerger, Ann. Phys. **326**, 770 (2011).
[5] F. Werner and Y. Castin, Phys. Rev. A **74**, 053604 (2006).

[6] W. Zwerger, in *Quantum Matter at Ultralow Temperatures, Proceedings of the International School of Physics "Enrico Fermi," Course 191, Varenna, 2014*, edited by M. Inguscio, W. Ketterle, S. Stringari, and G. Roati (IOS Press, Amsterdam, 2016), pp. 63–142.
[7] J. E. Thomas, J. Kinast, and A. Turlapov, Phys. Rev. Lett. **95**, 120402 (2005).
[8] S. Nascimbène, N. Navon, K. J. Jiang, F. Chevy, and C. Salomon, Nature (London) **463**, 1057 (2010).
[9] Y. Sagi, T. E. Drake, R. Paudel, and D. S. Jin, Phys. Rev. Lett. **109**, 220402 (2012).
[10] M. J. H. Ku, A. T. Sommer, L. W. Cheuk, and M. W. Zwierlein, Science **335**, 563 (2012).
[11] P. B. Patel, Z. Yan, B. Mukherjee, R. J. Fletcher, J. Struck, and M. W. Zwierlein, Science **370**, 1222 (2020).
[12] X. Li, X. Luo, S. Wang, K. Xie, X.-P. Liu, H. Hu, Y.-A. Chen, X.-C. Yao, and J.-W. Pan, Science **375**, 528 (2022).
[13] T. Schäfer and D. Teaney, Rep. Prog. Phys. **72**, 126001 (2009).
[14] T. Schäfer, Annu. Rev. Nucl. Part. Sci. **64**, 125 (2014).
[15] P. K. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
[16] A. Bulgac, J. E. Drut, and P. Magierski, Phys. Rev. Lett. **96**, 090404 (2006).
[17] A. Bulgac, J. E. Drut, and P. Magierski, Phys. Rev. Lett. **99**, 120401 (2007).
[18] L. Salasnich, Phys. Rev. A **82**, 063619 (2010).
[19] L. Salasnich and F. Toigo, J. Low Temp. Phys. **165**, 239 (2011).
[20] G. Bighin, L. Salasnich, P. A. Marchetti, and F. Toigo, Phys. Rev. A **92**, 023638 (2015).
[21] G. Bighin and L. Salasnich, Phys. Rev. B **93**, 014519 (2016).
[22] G. Bighin and L. Salasnich, Sci. Rep. **7**, 45702 (2017).
[23] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, in *Statistical Physics, Part 2*, Course of Theoretical Physics Vol. 9 (Pergamon Press, Oxford, UK, 1980).
[24] P. Nozières and S. Schmitt-Rink, J. Low. Temp. Phys. **59**, 195 (1985).
[25] H. Hu, X. J. Liu, and P. D. Drummond, Europhys. Lett. **74**, 574 (2006).
[26] S. N. Klimin, J. Tempere, and J. P. A. Devreese, J. Low Temp. Phys. **165**, 261 (2011).
[27] S. N. Klimin, J. Tempere, and J. P. A. Devreese, New J. Phys. **14**, 103044 (2012).
[28] J. Tempere and J. P. Devreese, in *Superconductors: Materials, Properties and Applications*, edited by A. Gabovich (InTech, London, 2012), p. 383.
[29] B. C. Mulkerin, X.-J. Liu, and H. Hu, Phys. Rev. A **94**, 013610 (2016).
[30] B. C. Mulkerin, X.-C. Yao, Y. Ohashi, X.-J. Liu, and H. Hu, arXiv:2201.04798.
[31] L. D. Landau, J. Phys. (USSR) **5**, 71 (1941).
[32] D. K. Hoffmann, V. P. Singh, T. Paintner, M. Jäger, W. Limmer, L. Mathey, and J. Hecker Denschlag, Nat. Commun. **12**, 7074 (2021).
[33] Q. Chen, J. Stajic, S. Tan, and K. Levin, Phys. Rep. **412**, 1 (2005).
[34] K. Levin, Q. Chen, C.-C. Chien, and Y. He, Ann. Phys. **325**, 233 (2010).
[35] G. A. Baker, Jr., Phys. Rev. C **60**, 054311 (1999).
[36] L. Salasnich and F. Toigo, Phys. Rev. A **78**, 053626 (2008).
[37] L. Salasnich, F. Ancilotto, and F. Toigo, Laser Phys. Lett. **7**, 78 (2010).
[38] M. Marini, F. Pistoiesi, and G. C. Strinati, Eur. Phys. J. B **1**, 151 (1998).
[39] A. Bulgac, J. E. Drut, and P. Magierski, Phys. Rev. A **78**, 023625 (2008).
[40] P. Magierski, G. Wlazłowski, A. Bulgac, and J. E. Drut, Phys. Rev. Lett. **103**, 210403 (2009).
[41] J. Carlson, S.-Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. **91**, 050401 (2003).
[42] S. Y. Chang, V. R. Pandharipande, J. Carlson, and K. E. Schmidt, Phys. Rev. A **70**, 043602 (2004).
[43] J. Carlson and S. Reddy, Phys. Rev. Lett. **95**, 060401 (2005).
[44] R. Prozorov and R. W. Giannetta, Supercond. Sci. Technol. **19**, R41 (2006).
[45] A. Altland and B. Simons, *Condensed Matter Field Theory* (Cambridge University Press, Cambridge, UK, 2006).
[46] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, Boston, 1971).
[47] Z.-Q. Yu, K. Huang and L. Yin, Phys. Rev. A **79**, 053636 (2009).
[48] L. P. Gorkov and T. K. Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961).
[49] I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity*, Advanced Books Classics Series (Perseus Publishing, New York, 2000).
[50] A. Tononi, A. Cappellaro, G. Bighin, and L. Salasnich, Phys. Rev. A **103**, L061303 (2021).
[51] X. Li, X. Luo, S. Wang, K. Xie, X.-P. Liu, H. Hu, Y.-A. Chen, X.-C. Yao, and J.-W. Pan, Data for "Second sound attenuation near quantum criticality", Zenodo, doi: 10.5281/zenodo.5767197 (2021).
[52] L. P. Pitaevskii and S. Stringari, arXiv:1510.01306.
[53] P. Christodoulou, M. Galka, N. Dogra, R. Lopes, J. Schmitt, and Z. Hadzibabic, Nature **594**, 191 (2021).
[54] M. Bohlen, L. Sobirey, N. Luick, H. Biss, T. Enss, T. Lompe, and H. Moritz, Phys. Rev. Lett. **124**, 240403 (2020).
[55] L. Pitaevskii and S. Stringari, Second sound in ultracold atomic gases, in *Universal Themes of Bose-Einstein Condensation*, edited by N. Proukakis, D. Snoke and P. Littlewood (Cambridge University Press, Cambridge, UK, 2017), pp. 322–347.
[56] P.-T. How and A. LeClair, J. Stat. Mech.: Theory Exp. (2010) P07001.
[57] J. Kinast, A. Turlapov, and J. E. Thomas, Phys. Rev. A **70**, 051401(R) (2004).
[58] L. Luo and J. E. Thomas, J. Low Temp. Phys. **154**, 1 (2009).
[59] C. Cao, E. Elliott, J. Joseph, H. Wu, J. Petricka, T. Schäfer, and J. E. Thomas, Science **331**, 58 (2011).
[60] T. Schäfer and C. Chafin, Scaling flows and dissipation in the dilute fermi gas at unitarity, in *The BCS-BEC Crossover and the Unitary Fermi Gas* (Ref. [1]), pp. 375–406.