Comment on "Stark shift and width of x-ray lines from highly charged ions in dense plasmas"

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We point out in this Comment a misleading remark made by Gu and Beiersdorfer [Phys. Rev. A **101**, 032501 (2020)] that the Debye-Hückel (DH) model "performs rather poorly" compared with the ion sphere model is due to their questionable use of the Debye length by assuming the same mobility for the plasma electrons and ions. Actually, our recent works based on a judicious application of the DH approximation, which meets the spatial and temporal criteria, have led to good agreements with all the available line shift experimental measurements on α and β emission lines of He-like ions.

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In a recent article by Gu and Beiersdorfer [1], the authors illustrate in its Fig. 2 that the application of the Debye-Hückel (DH) approximation [2] would lead to redshifts for the $K\beta$ emission of the He-like Ar ion over an order of magnitude higher than the ones from other theoretical estimations. The purpose of this Comment is to point out that this large difference is actually due to the substantial overestimation of the Debye screening in their application of the Debye length, identified as R_d in the Eq. (7) of Ref. [1] with an extra factor of $(Z_{\rm eff} + 1)$ where $Z_{\rm eff}$ is the effective charge of the plasma ions representing the screening due to the ions, which results from assuming the same mobility for the plasma electrons and ions. Due to this discrepancy they concluded that the DH model "performs rather poorly." Instead, they stated that the ion sphere (IS) model is a reasonable approximation.

For atomic processes subject to outside plasma environment due to the fact that the mobility of the plasma ion is substantially less than that of the plasma electron, it is known that the Debye length D is given by [3,4]

$$D = 1.4048\sqrt{k_bT_e/N_e}a_0. \tag{1}$$

without the $(Z_{\rm eff} + 1)$ factor, which was also pointed out earlier in Ref. [5]. The k_bT_e and N_e in Eq. (1) are in the units of eV and 10^{22} cm^{-3}, respectively. More details of the DH approximation is reviewed recently [4]. Figure 1 shows the results from the calculation with the correct Debye length, which is larger when $T_e < 1000$ eV, but, not an order of magnitude greater, than the estimation from the result of the ion sphere calculation of Ref. [1].

Following the early application of the IS approach [6], two more recent versions of the IS model have been applied to study the redshifts of the α and β emission lines of the He-like ions. The first one is the analytical ion sphere model by Li and Rosmej [7]. Whereas its calculated redshifts of the α emission of the He-like Al ion are in agreement with the picosecond time-resolved measurement (see Fig. 4 of Ref. [8]), its estimated redshifts are substantially greater than the recent measured data from a high-resolution satellite lines free measurement of the β line of the He-like Cl ions (see, Fig. 4 of Ref. [9]). Shortly after the He-like Cl ion measurement, the estimated redshifts from an average atom ion sphere model (AIS) calculation was reported to be in good agreement with the measured data (see Fig. 1 of Ref. [10]). However, the subsequent application of this AIS model to the α emission of the He-like Al ion are substantially smaller than the earlier experimentally observed data (see, Fig. 2 of Ref. [11]). In other words, both IS models could only lead to the redshifts in agreement with one of the recent experimental measurements, but, not both. As a result, one needs to question the suggestion by Ref. [1] if the IS approach is indeed the "reasonable approximation" for the line shifts subject to plasma environment.

In contrast to the implication by Ref. [1] that the DH approximation fails to work for the transition energy shifts for atomic transitions, we would like to stress in this Comment that the applications of the DH approximation to atomic processes which meet its temporal and spatial criteria [4,5,12] have, in fact, led to estimated redshifts for a number of atomic transitions in agreement with three best known quantitative experimental measurements [8,9,13]. For the Lyman-α transition of the H-like Al ion, the theoretically calculated redshifts based on the DH approximation shown by Fig. 3 of Ref. [5] are in agreement with the measurement by Saemann *et al.* [13]. For the He-like ions, the estimated redshifts of the α and β emission lines of the He-like ions with DH approximation have led to theagreement shown by Fig. 2 of Ref. [4] with the experimental measurements by Stillman *et al.* [8] and Beiersdorfer *et al.* [9], respectively. We should also point out that in the applications of the DH approximation, our theoretical results were carried out with two different multiconfiguration calculation procedures, one relativistic and the

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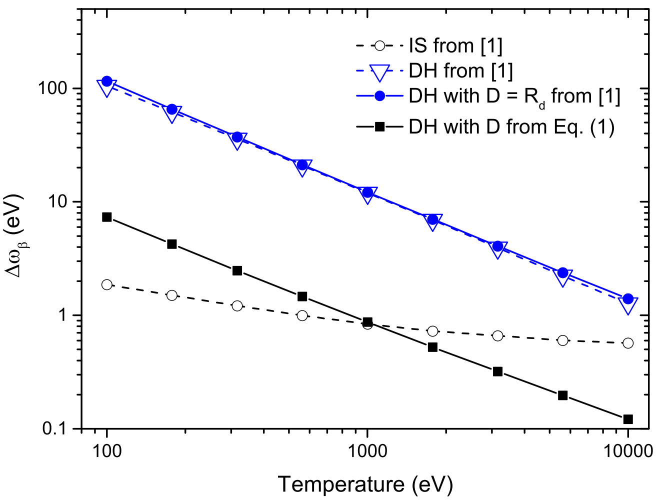

FIG. 1. Redshift of the $K\beta$ transition of He-like Ar ion at $N_e = 10^{23}$ cm^{-3} in terms of the plasma electronic temperature k_bT_e with the DH model with D from Eq. (1) (solid black), DH with $D = R_d$ from Ref. [1] (solid blue), DH from Ref. [1] (dashed blue), and the IS model from Ref. [1] (dashed black).

other nonrelativistic, which lead to the theoretical results that are in close agreement with each other, indicating that the electron-electron correlation between atomic electrons are, in fact, taken into account adequately [4,12].

For the DH approximation, the electrons and ions in the outside charged-neutral plasma are treated as charged particles with no quantum-mechanical interaction (such as those involving the spin of the individual particles in the solid system) and, thus, the Maxwell-Boltzmann statistics is applied. This has led to a nearly linear $(1/k_bT_e)$ temperature dependence at fixed density of the redshifts shown by the top plot of Fig. 3 of Ref. [4]. Although there is no definitive quantitative measurement on the temperature dependence, the two recent different experiments have covered a range of temperatures that appear to suggest a more pronounced temperature dependence than the fairly small variations in the resulting redshifts at different temperatures obtained from the IS model, which is based on the Fermi-Dirac statistics for the outside plasma (see, e.g., Fig. 4 of Ref. [9]). A statistical electron screening model was recently proposed [14] to study the atomic processes in warm-hot dense plasmas. This approach includes corrections for the Fermi-Dirac distribution by considering the nonequilibrium feature of the plasma-electron distribution around the atomic ion due to the three-body recombination process. It has the effect of broadening the phase space of the plasma electron and made its distribution close to the Boltzmann distribution under high temperatures. In fact, for the conditions relevant to the two recent experiments [8,9], this statistical model would lead to a plasma electron distribution almost identical to the DH model with the D from Eq. (1). Together with the agreement pointed out earlier between the experimental data and our theoretically estimated redshifts based on the DH approximation, it appears to support the use of the classical Maxwell-Boltzmann statistics over the IS models with the Fermi-Dirac distributions for the outside plasma. More quantitative experimental redshift measurements on the temperature variation at fixed plasma density would certainly help for a more definitive conclusion.

Following the detailed discussion on the general feature of the α and β emission lines of the He-like ions presented in Ref. [4], the ratio $R = \Delta\omega/\omega_0$ between the redshifts $\Delta\omega$ and its plasma-free transition energy ω_0 is dominated by the leading contributing term c/λ^2, where c is the theoretically fitted coefficient given by Table 1 of Ref. [4], and λ is the reduced Debye length, or $\lambda = Z_{\rm eff}D$ with $Z_{\rm eff} = Z - 1$ for the He-like ions. In other words, R is approximately proportional to $(N_e/k_bT_e)/Z^2_{\rm eff}$. Following the quasihydrogenic picture, one would expect that the plasma-free ω_0 also varies approximately as $Z^2_{\rm eff}$. Therefore, the estimated redshifts $\Delta\omega = R\omega_0$ would approximately be proportional to N_e/k_bT_e, or, determined by a pair of N_e and k_bT_e for all He-like ions with Z between 6 and 18 that meet the spatial and temporal criteria of the DH approximation. This observation based on our study makes it easier for the choice of the He-like ions for experimental measurements. Finally, we would like to point out that the anticipated larger temperature-dependent variation in redshifts at fixed plasma density based on the DH approximation could effectively facility the study of atomic emission lines as a viable plasma diagnostic possibility.

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