Effects of a three-level laser on mechanical squeezing in a doubly resonant optomechanical cavity coupled to biased noise fluctuations

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We address the behavior of squeezing in two movable mirrors coupled to a pumped correlated emission laser inside a doubly resonant optomechanical driven cavity in the presence of biased noise fluctuations. Aiming at generating controllable and robust mechanical squeezing that can be utilized in making the quantum features of radiation in the cavity accessible for application, we explore mechanical squeezing that can be induced as a result of the transfer of coherent superposition. We found that a coupled mechanical oscillator mode exhibits squeezing in the good cavity limit and adiabatic regime. It is also shown that the degree of mechanical squeezing is robust for large amplitudes of atomic pumping until maximum squeezing is achieved but weakens afterwards. The squeezing turns out to be powerful mainly for strong atom-field coupling, large atomic injection rates, and intense biased noise fluctuations. In light of the observed possibility of controlling the realizable degree of squeezing, we hope and expect that the considered system could be employed in applications such as quantum metrology.

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I. INTRODUCTION

There has been considerable interest in the study of the nonclassical behaviors of massive mechanical oscillators in cavity optomechanics (COM) where the radiation pressure force that emanates from the light carrying momentum couples the electromagnetic field and the mechanical oscillator [1–3]. In this respect, many projects of COM have been formulated and experimentally demonstrated [1,2,4–6] as in cooling the mechanical resonators to near their quantum ground states and entangling the mechanical modes [7–11]. Quantum fluctuations using optomechanical methods have also been employed in generating the squeezed states of the optical and mechanical modes [12–15]. It is a well-established fact that mechanical squeezing, which is one of the key macroscopic quantum effects, can be used for many applications such as improved precision of quantum metrology [16], measurement of weak classical force [2], biological measurements [17], and quantum-to-classical transitions [18,19].

Several schemes have been proposed to generate mechanical squeezing in resonators based on parametric processes, feedback, measurements, and quantum-reservoir engineering [16,20–29]. In an optomechanical cavity where a movable mirror in its steady state can be regarded as a low-noise Kerr nonlinear medium [28], experimental realizations of squeezing in optical fields [13,16] and mechanical modes [29] have been witnessed. Hybrid optomechanical systems that include atoms have also been proposed to induce mechanical squeezing. For instance, many works have used two-level atom(s) to induce squeezing of the mechanical resonator [30,31]. It is also found to be interesting to include a correlated emission laser (CEL), which has been studied extensively and the emitted radiation is found to exhibit strong squeezing [32–34]. In light of this, even though two-photon coherence in a three-level laser is shown to induce entanglement between the optical mode and movable mirror, and between two movable mirrors of a doubly resonant cavity [35,36], in the present paper, we seek to address its effects on the degree of mechanical squeezing. In the same spirit, impinging biased noise fluctuations, which can be introduced externally as discussed in Ref. [37] in the form of a squeezed reservoir or an optical feedback loop [38–41], on the walls of the cavity is expected to lead to correlated vibrations that have a potential to enhance atomic coherence due to the phase sensitivity of the cascade transitions.

Since a scheme that generates robust and controllable mechanical squeezing is highly sought for, we intend to explore the extent to which the manipulation of the characteristics of CEL and biased noise fluctuations helps in controlling the degree of squeezing of coupled mechanical oscillators. It is also of significant interest to make the quantum properties of the radiation in the cavity accessible to the experimenter, which can be achieved by coherently coupling the radiation in the cavity with mechanical oscillators via optical radiation pressure. With this in mind, we propose a mechanism by which the generated mechanical squeezing can be altered via the coherence of the two-mode radiation produced by CEL and shared with the two movable mirrors. Particularly, the three-level atoms in the cascade configuration that initially

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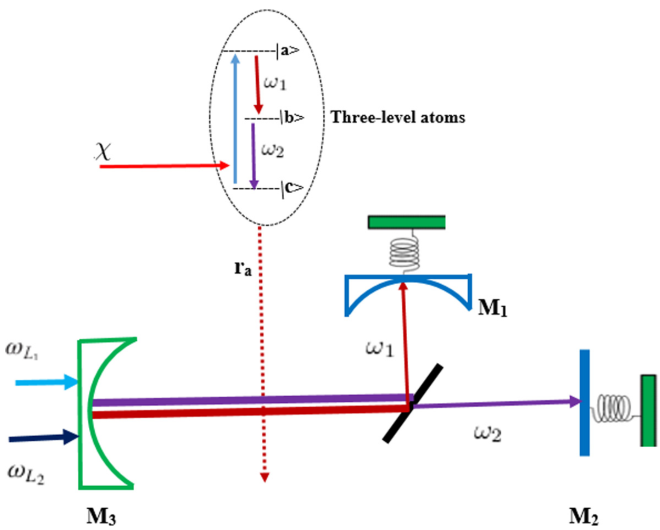

FIG. 1. Schematic representation of a pumped two-mode correlated emission laser coupled to two movable mirrors (M_1 and M_2), where the doubly resonant cavity is driven by two external lasers of frequencies ω_{L_1} and ω_{L_2} via port mirror M_3. The cavity radiation fields with frequencies ω_1 and ω_2 are filtered by a beam splitter (BS). The generated radiation modes are coupled to the separate movable mirrors by radiation pressure.

occupy the lower-energy level and pumped with external light are assumed to be injected into the cavity. The cavity is also assumed to be driven by two monochromatic lasers of different wavelengths and coupled to biased noise fluctuations. Notably, introducing biased noise fluctuations turns out to be vital for witnessing strong mechanical squeezing at low driving power since the noise in one of the quadratures of the mechanical modes can be suppressed due to the vibrations resulting from external bias.

To achieve the intended goal, we find it necessary to construct the interaction Hamiltonian that describes the system. Once the Hamiltonian is known, the corresponding master equation is derived confining to a linear analysis and good cavity limit. Afterwards, with the aid of the derived master equation for two-mode fields and mechanical oscillators, we obtain the quantum Langevin equations in the adiabatic regime from which the covariance matrix is obtained to carry out the accompanying analysis. In the process, we find that the mechanical oscillation of the movable mirrors exhibits a robust degree of squeezing that can be manipulated and controlled by changing the parameters of the correlated emission laser and the degree of bias of external noise.

II. DYNAMICAL EQUATIONS

A. Interaction Hamiltonian

The system under consideration comprises pumped cascade three-level atoms injected into a doubly resonant Fabry-Pérot type cavity, two perfectly reflecting movable mirrors M_1 and M_2, a port mirror M_3, and a beam splitter (BS) as shown in Fig. 1 [34,36]. Injected atoms, which are removed after a time longer than the spontaneous emission time, are presumed to interact nonresonantly with the two cavity modes of frequencies ω_1 and ω_2. To establish a coherent superposition in the upper and lower atomic states, the atoms are assumed to be driven by a strong coherent laser field of amplitude χ and frequency ω_p. The doubly resonant cavity on the other hand is driven by two coherent lasers of frequencies ω_{L_1} and ω_{L_2}. To include the influence of external noise on the mechanical oscillation, we assume that the resonant cavity is coupled to biased noise fluctuations [37].

In this setting, the movable mirrors are expected to undergo mechanical oscillation with a nonclassical property such as squeezing since the emerging coherent correlation can be transferred to mechanical oscillations. The corresponding quantum harmonic oscillators could thus be modeled with their respective thermal baths at equilibrium with temperatures T_1 and T_2 having the annihilation (creation) operator of each vibrational mode $\hat{b}_j$ ($\hat{b}_j^\dagger$) satisfying the relation $[\hat{b}_j, \hat{b}_j^\dagger] = 1$ with $j = 1, 2$.

For such a system, the Hamiltonian of the system in the interaction picture can be obtained under the rotating-wave approximation, dipole approximation, and applying the fact that $\hat{\sigma}_{aa} + \hat{\sigma}_{bb} + \hat{\sigma}_{cc} = 1$. In this regard, we use the transformation $e^{i\hat{H}_0 t}\hat{H}_s e^{-i\hat{H}_0 t}$ with

$$\hat{H}_0 = \hbar(\Omega_1 + \Omega_2)\hat{\sigma}_{aa} + \hbar\Omega_2\hat{\sigma}_{bb} + \hbar\Omega_1\hat{a}_1^\dagger\hat{a}_1 + \hbar\Omega_2\hat{a}_2^\dagger\hat{a}_2, \tag{1}$$

$$\begin{aligned}\hat{H}_s =& \hbar(\xi_1 + \xi_2)\hat{\sigma}_{aa} + \hbar\xi_2\hat{\sigma}_{bb} + \hbar\delta\omega_1\hat{a}_1^\dagger\hat{a}_1 + \hbar\delta\omega_2\hat{a}_2^\dagger\hat{a}_2 \\ &+ i\hbar g_1(\hat{\sigma}_{ab}\hat{a}_1 - \hat{a}_1^\dagger\hat{\sigma}_{ba}) + i\hbar g_2(\hat{\sigma}_{bc}\hat{a}_2 - \hat{a}_2^\dagger\hat{\sigma}_{cb}) \\ &+ i\hbar\frac{\chi}{2}(e^{-i\omega_p t}\hat{\sigma}_{ac} - \text{H.c.}) + i\hbar\sum_{k=1}^{2}(\epsilon_k\hat{a}_k^\dagger e^{-i\omega_{L_k}t} - \text{H.c.}) \\ &+ \hbar\sum_{k=1}^{2}[\omega_{m_k}\hat{b}_k^\dagger\hat{b}_k + G_{0_k}\hat{a}_k^\dagger\hat{a}_k(\hat{b}_k^\dagger + \hat{b}_k)]. \end{aligned}\tag{2}$$

In addition, the total interaction Hamiltonian in the atom-field and field-mirror interactions can be expressed as $\hat{H} = \hat{H}_I^{(af)} + \hat{H}_I^{(fm)}$ with [5,34,36]

$$\begin{aligned}\hat{H}_I^{af} =& \hbar(\xi_1 + \xi_2)\hat{\sigma}_{aa} + \hbar\xi_2\hat{\sigma}_{bb} + i\hbar\frac{\chi}{2}(\hat{\sigma}_{ac} - \hat{\sigma}_{ca}) \\ &+ i\hbar g_1(\hat{\sigma}_{ab}\hat{a}_1 - \hat{a}_1^\dagger\hat{\sigma}_{ba}) + i\hbar g_2(\hat{\sigma}_{bc}\hat{a}_2 - \hat{a}_2^\dagger\hat{\sigma}_{cb}),\end{aligned}\tag{3}$$

$$\begin{aligned}\hat{H}_I^{fm} =& \hbar\sum_{j=1}^{2}[\delta\omega_j\hat{a}_j^\dagger\hat{a}_j + i(\epsilon_j\hat{a}_j^\dagger e^{i\delta\omega_j t} - \epsilon_j^*\hat{a}_j e^{-i\delta\omega_j t})] \\ &+ \hbar\sum_{j=1}^{2}[\omega_{m_j}\hat{b}_j^\dagger\hat{b}_j + G_{0_j}\hat{a}_j^\dagger\hat{a}_j(\hat{b}_j^\dagger + \hat{b}_j)],\end{aligned}\tag{4}$$

where the atomic operators $\hat{\sigma}_{kk} = |k\rangle\langle k|$ for $(k = a, b, c)$ are denoted by $\hat{\sigma}_{ab} = |a\rangle\langle b|$, $\hat{\sigma}_{bc} = |b\rangle\langle c|$, and $\hat{\sigma}_{ac} = |a\rangle\langle c|$ with the frequency of the kth atomic states being ω_k. In addition, g_1 (g_2) is the coupling strength between the transitions $|a\rangle \to |b\rangle$ ($|b\rangle \to |c\rangle$) and the annihilation (creation) operator $\hat{a}_j$ ($\hat{a}_j^\dagger$) stands for the jth cavity mode. The optomechanical coupling strength between the mechanical and cavity fields is

$G_{0_j} = \frac{\omega_j}{L_j}\sqrt{\frac{\hbar}{m_j\omega_{m_j}}}$ and the amplitudes of the lasers that drive the doubly resonant cavity are $|\epsilon_j| = \sqrt{\frac{\kappa_j P_j}{\hbar\omega_{L_j}}}$ with ω_{m_j}, L_j, m_j, κ_j, P_j, and ω_{L_j} being the mechanical frequencies, the length of the cavities, the masses of movable mirrors, the damping rates of the cavities, the power of lasers driving the cavity, and the frequencies of the pump lasers, respectively. In the same way, the mechanical quality factors are defined as $Q_j = \omega_{m_j}/\gamma_{m_j}$ with $j = 1, 2$. It is also denoted that $\xi_1 = \omega_{ab} - \Omega_1$, $\xi_2 = \omega_{bc} - \Omega_2$ with $\omega_{ab} = \omega_a - \omega_b$ and $\omega_{bc} = \omega_b - \omega_c$ being the frequencies for the $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$ transitions. Notably, $\Omega_j = \omega_j - \delta\omega_j$ indicates the shifted cavity frequency and $\delta\omega_j = G_{0_j}\langle\hat{b}_j^\dagger + \hat{b}_j\rangle$ shows the frequency shift due to radiation pressure while $\delta_j = \Omega_j - \omega_{L_j}$ and the two-photon resonance is $\omega_p = \Omega_1 + \Omega_2$.

B. Master equation

To derive the master equation corresponding to the two-mode laser in the cavity, we mainly opt to apply the approach introduced to study CEL [34,36]. Here, we consider the case when all the atoms are initially made to occupy the lower-energy level [42]. On the other hand, quantum state transfer from the two-mode cavity field to the mechanical modes can be achieved in the adiabatic regime under the condition that epitomizes mirrors with a high Q factor in which the mechanical baths are considered as Markovian [43] in the regime of weak effective optomechanical coupling, that is, the cavity decay rates are very much larger than the mechanical decay rates $\kappa_j \gg \gamma_{m_j}$ [36,44]. The Markovian master equation for an open quantum harmonic mechanical oscillator in the microscale can be derived from effective environmental models of bosonic oscillators at low temperatures [45–49]. In this respect, by weakening the demand on the the equilibrium state as a well-defined positive Markovian and translationally invariant, the dynamics describing damping can be achieved [50].

Following these works, we obtain the master equation for the cavity modes coupled to a biased noise fluctuation as in Ref. [36] (the proof can be found in Appendix A), while the two oscillating mirrors are being coupled to their respective thermal environments, as

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) =& \alpha_{11}(\hat{\rho}\hat{a}_1\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{\rho}\hat{a}_1) + \alpha_{11}^*(\hat{a}_1\hat{a}_1^\dagger\hat{\rho} - \hat{a}_1^\dagger\hat{\rho}\hat{a}_1)\\ &+ \alpha_{22}(\hat{a}_2\hat{\rho}\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_2\hat{\rho}) + \alpha_{22}^*(\hat{a}_2\hat{\rho}\hat{a}_2^\dagger - \hat{\rho}\hat{a}_2^\dagger\hat{a}_2)\\ &+ \alpha_{12}(\hat{\rho}\hat{a}_2^\dagger\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{\rho}\hat{a}_2^\dagger) + \alpha_{12}^*(\hat{a}_1\hat{a}_2\hat{\rho} - \hat{a}_2\hat{\rho}\hat{a}_1)\\ &+ \alpha_{21}(\hat{a}_1^\dagger\hat{\rho}\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_1^\dagger\hat{\rho}) + \alpha_{21}^*(\hat{a}_2\hat{\rho}\hat{a}_1 - \hat{\rho}\hat{a}_1\hat{a}_2)\\ &+ \frac{1}{2}\sum_{j=1}^{2} k_j[(N+1)(2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\hat{\rho} - \hat{\rho}\hat{a}_j^\dagger\hat{a}_j)\\ &+ N(2\hat{a}_j^\dagger\hat{\rho}\hat{a}_j - \hat{a}_j\hat{a}_j^\dagger\hat{\rho} - \hat{\rho}\hat{a}_j\hat{a}_j^\dagger)]\\ &+ \sqrt{k_1k_2}M[2\hat{a}_1^\dagger\hat{\rho}\hat{a}_2^\dagger + 2\hat{a}_2^\dagger\hat{\rho}\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\rho}\\ &- \hat{a}_2^\dagger\hat{a}_1^\dagger\hat{\rho} - \hat{\rho}\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{\rho}\hat{a}_2^\dagger\hat{a}_1^\dagger]\\ &+ \frac{1}{2}\sum_{j=1}^{2}\gamma_{m_j}[(n_j+1)(2\hat{b}_j\hat{\rho}\hat{b}_j^\dagger - \hat{b}_j^\dagger\hat{b}_j\hat{\rho} - \hat{\rho}\hat{b}_j^\dagger\hat{b}_j)\\ &+ n_j(2\hat{b}_j^\dagger\hat{\rho}\hat{b}_j - \hat{b}_j\hat{b}_j^\dagger\hat{\rho} - \hat{\rho}\hat{b}_j\hat{b}_j^\dagger)]. \end{aligned} \tag{5}$$

The coefficients α_{ij} are given by

$$\alpha_{11} = -\frac{g_1^2 r_a}{\mathcal{F}}\left[(\gamma_{bc} - i\xi_2)\frac{T_{aa}}{D_2} - \frac{\chi}{2}\frac{T_{ac}^*}{D_1}\right], \tag{6}$$

$$\alpha_{12} = \frac{g_1 g_2 r_a}{\mathcal{F}}\left[(\gamma_{bc} - i\xi_2)\frac{T_{ac}}{D_1} + \frac{\chi}{2}\frac{T_{cc}}{D_2}\right], \tag{7}$$

$$\alpha_{22} = \frac{g_2^2 r_a}{\mathcal{F}}\left[(\gamma_{ab} - i\xi_1)\frac{T_{cc}}{D_2} - \frac{\chi}{2}\frac{T_{ac}^*}{D_1}\right], \tag{8}$$

$$\alpha_{21} = -\frac{g_1 g_2 r_a}{\mathcal{F}}\left[(\gamma_{ab} - i\xi_1)\frac{T_{ac}}{D_1} - \frac{\chi}{2}\frac{T_{aa}}{D_2}\right], \tag{9}$$

where $\mathcal{F} = \frac{\chi^4}{4} + (\gamma_{ab} + i\xi_1)(\gamma_{bc} - i\xi_2)$ and γ_{ij} $(i \neq j)$ belong to the dephasing rate. The expressions for T_{ij} and D_i are provided in Appendix A.

Note that κ_j and γ_{m_j} account for damping of the cavity modes coupled to biased noise fluctuations and the damping rates of the mechanical oscillators coupled to thermal baths at temperatures T_1 and T_2 and the corresponding thermal phonon numbers n_1 and n_2 [51]. In the same spirit, the mean photon number and the correlation between the two-mode biased noise fluctuations are accounted for by N and $|M|$, where $|M|^2 = N(N+1)$.

III. LINEARIZED SOLUTION

A. Linearization of quantum Langevin equations

To analyze the mechanical squeezing, it appears more suitable to apply the quantum Langevin approach in which the master equation is applied to determine the quantum Langevin equation for the atom-cavity mode and optomechanical system separately. This procedure is justified in the regime where the atom-field coupling turns out to be much stronger than the optomechanical coupling. With the aid of the master equation (5), the field-mirror interaction Hamiltonian (4), and making use of the general relation $\langle\dot{\hat{Z}}\rangle = \mathrm{Tr}(\dot{\hat{\rho}}\hat{Z})$, the nonlinear quantum Langevin equations are obtained as [52]

$$\begin{aligned}\frac{d\hat{a}_j(t)}{dt} =& -\left(\frac{k_j}{2} + \alpha_{jj} + i\delta\omega_j\right)\hat{a}_j(t) - \alpha_{jk}\hat{a}_k^\dagger(t)\\ &- iG_{0_j}\hat{a}_j(\hat{b}_j^\dagger + \hat{b}_j) + \epsilon_j e^{i\delta_j t} + \hat{F}_j,\end{aligned} \tag{10}$$

$$\frac{d\hat{b}_j(t)}{dt} = -\left(\frac{\gamma_{m_j}}{2} + i\omega_{m_j}\right)\hat{b}_j(t) - iG_{0_j}\hat{a}_j^\dagger\hat{a}_j + \sqrt{\gamma_{m_j}}\hat{f}_j. \tag{11}$$

The noise operator $\hat{F}_j$ is due to the coupling of biased noise fluctuation with the cavity modes, whereas $\hat{f}_j$ stands for the noise operator corresponding to a thermal reservoir coupled to a mechanical oscillator (see Appendix B for correlations between noise operators).

The analysis of the quantum dynamics of the whole system is not trivial since the nonlinear nature of the radiation pressure makes it intractable to get the rigorous analytical solutions of Eqs. (10) and (11). To overcome this difficulty, we adopt the linearization approach as discussed in Ref. [51].

In light of this, a reasonable optomechanical interaction can be achieved when the cavity is intensely driven by strong power lasers [28]. In addition, the optimal transfer of quantum properties from the two cavity fields to the mechanical modes can be achieved in an adiabatic regime [44].

The steady-state mean value of each bosonic operator is thus taken to be larger when compared to the corresponding quantum fluctuation: $|\langle\hat{a}_j\rangle| = |a_{j_{ss}}| \gg |\delta\hat{a}_j|$ and $|\langle\hat{b}_j\rangle| = |b_{j_{ss}}| \gg |\delta\hat{b}_j|$ for $j = 1, 2$. We thus consider the dynamics of small fluctuations around the steady state of the system by decomposing each operator into two parts, the sum of its average operator in the steady state and a small fluctuation operator with zero mean value $\langle\delta\hat{a}_j\rangle = \langle\delta\hat{b}_j\rangle = 0$ as

$$\hat{a}_j = \langle a\rangle_{j_{ss}} + \delta\hat{a}_j, \quad \hat{b}_j = \langle b\rangle_{j_{ss}} + \delta\hat{b}_j, \tag{12}$$

where the mean operator values $\langle a\rangle_{j_{ss}}$ and $\langle b\rangle_{j_{ss}}$ are complex numbers and can be evaluated by setting the time derivatives to zero and factorizing the averages in Eqs. (10) and (11).

In using a transformed frame defined by $\tilde{a}_j = \hat{a}_j e^{-i\delta_j t}$ and Eq. (12), one may see that the equations for both fluctuations and c-number steady-state values have a coupling between the two cavity modes (terms proportional to α_{12} and α_{21}) that contain highly oscillating factor $e^{-(\delta_1+\delta_2)t}$. One can still obtain solutions for $\langle\tilde{a}_j\rangle$ in the steady state that amount to dropping the highly oscillating terms completely or choosing a condition such that $\delta_2 = -\delta_1$ and retaining the coupling terms. In the regime of the rotating-wave approximation, one can verify that

$$\langle\tilde{a}_j\rangle = \frac{\epsilon_j}{\frac{\kappa_j}{2} + \alpha_{jj} - i\Delta_j}, \tag{13}$$

$$\langle\hat{b}_j^\dagger + \hat{b}_j\rangle = -\frac{2\omega_{m_j} G_{0_j}\langle\tilde{a}_j^\dagger\tilde{a}_j\rangle}{\frac{\gamma_{m_j}^2}{4} + \omega_{m_j}^2}, \tag{14}$$

where $\Delta_j = \omega_{L_j} - \omega_j - G_{0_j}\langle\hat{b}_j^\dagger + \hat{b}_j\rangle$ denote the cavity mode detunings.

Afterwards, upon introducing the slowly varying fluctuation operators $\delta\hat{a}(t) = \delta\tilde{a}(t)\, e^{i\delta_j t}$ and $\delta\hat{b}(t) = \delta\tilde{b}(t)\, e^{-i\omega_{m_j} t}$ into Eqs. (10) and (11), the dynamics of the linearized quantum Langevin equations for the fluctuation modes can be written as [51]

$$\frac{d}{dt}(\delta\hat{a}_j) = -\left(\frac{\kappa_j'}{2} - i\Delta_j\right)\delta\hat{a}_j - \alpha_{jk}\,\delta\hat{a}_k^\dagger + \hat{F}_j - iG_{0_j}\langle\tilde{a}_j\rangle(\delta\tilde{b}_j^\dagger e^{i(\delta_j+\omega_{m_j})t} + \delta\tilde{b}_j e^{i(\delta_j-\omega_{m_j})t}), \tag{15}$$

$$\frac{d}{dt}(\delta\tilde{b}_j) = -\frac{\gamma_{m_j}}{2}\delta\tilde{b}_j - iG_{0_j}\langle\tilde{a}_j^\dagger\rangle\delta\hat{a}_j\, e^{-i(\delta_j-\omega_{m_j})t} - iG_{0_j}\langle\tilde{a}_j\rangle\delta\hat{a}_j^\dagger\, e^{i(\delta_j+\omega_{m_j})t} + \sqrt{\gamma_{m_j}}\tilde{f}_j, \tag{16}$$

with $\kappa_j' = \kappa_j + 2\alpha_{jj}$, $\tilde{f}_j = \hat{f}_j\, e^{i\omega_{m_j}t}$, and $\tilde{F}_j = \hat{F}_j\, e^{-i\delta_j t}$. It might not be difficult to verify that the operators $\delta\hat{a}_j$ and $\delta\hat{b}_j$ satisfy the usual boson commutation relations.

Since no rotating-wave approximation has been made in the fluctuation equations, the coupling terms (proportional to α_{12} and α_{21}) induced by the two-photon coherence are kept. For optomechanical coupling, when $\delta_j = \omega_{m_j}$, the interaction describes parametric amplification and can be used to realize optomechanical squeezing [5], whereas when $\delta_j = -\omega_{m_j}$, the interaction is relevant in inducing quantum state transfer and cooling [5,36,44,53].

In this work, since we are interested in transferring the squeezing of the cavity fields to the mechanical modes, we take $\delta_j = -\omega_{m_j}$. We also choose $\omega_{L_j} \approx \omega_j + G_{0_j}\langle\hat{b}_j^\dagger + \hat{b}_j\rangle$ so that α_{jj} and α_{jk} are real. Upon carrying out an adiabatic approximation of Eq. (15), we obtain coupled Langevin equations for mechanical oscillators $\delta\tilde{b}_j$,

$$\delta\dot{\tilde{b}}_1 = -\frac{\Lambda_1}{2}\delta\tilde{b}_1 - G_{12}\delta\tilde{b}_2^\dagger - c_1\hat{F}_1^\dagger + c_2\hat{F}_2 + \sqrt{\gamma_{m_1}}\tilde{f}_1, \tag{17}$$

$$\delta\dot{\tilde{b}}_2 = -\frac{\Lambda_2}{2}\delta\tilde{b}_2 - G_{21}\delta\tilde{b}_1^\dagger + d_1\hat{F}_1 - d_2\hat{F}_2^\dagger + \sqrt{\gamma_{m_2}}\tilde{f}_2, \tag{18}$$

where $\Lambda_1 = \gamma_{m_1} - \Lambda_{b_1}$, $\Lambda_2 = \gamma_{m_2} - \Lambda_{b_2}$ with $\Lambda_{b_1} = \frac{4G_1G_1^*\kappa_2'}{K}$, $\Lambda_{b_2} = \frac{4G_2G_2^*\kappa_1'}{K}$ in which $K = \kappa_1'\kappa_2' - 4\alpha_{12}\alpha_{21}$ denotes the effective damping rates for the mechanical modes induced by the radiation pressures. In the same way, $G_{12} = \frac{4G_1G_2\alpha_{12}^*}{K}$ and $G_{21} = \frac{4G_1G_2\alpha_{21}^*}{K}$ are effectively coupled between the two mechanical modes induced by the laser system, whereas $c_1 = \frac{2G_1\kappa_2'}{K}$, $c_2 = \frac{4G_1\alpha_{12}^*}{K}$, $d_1 = \frac{4G_2\alpha_{21}^*}{K}$, and $d_2 = \frac{2G_2\kappa_1'}{K}$ in which many-photon coupling is denoted by $G_j = iG_{0_j}\langle\tilde{a}_j\rangle$.

B. Steady-state covariance matrix

To analyze quantum correlations between the mechanical modes, we use quadrature operators defined as $\delta\tilde{H}_j = \frac{1}{\sqrt{2}}(\delta\tilde{b}_j^\dagger + \delta\tilde{b}_j)$ and $\delta\tilde{L}_j = \frac{i}{\sqrt{2}}(\delta\tilde{b}_j^\dagger - \delta\tilde{b}_j)$. The corresponding Hermitian input noise operators are $\delta\tilde{H}_j^{\text{in}} = \frac{1}{\sqrt{2}}(\tilde{F}_{b_j}^\dagger + \tilde{F}_{b_j})$ and $\delta\tilde{L}_j^{\text{in}} = \frac{i}{\sqrt{2}}(\tilde{F}_{b_j}^\dagger - \tilde{F}_{b_j})$, where $\tilde{F}_{b_1} = -c_1\hat{F}_1^\dagger + c_2\hat{F}_2 + \sqrt{\gamma_{m_1}}\tilde{f}_1$ and $\tilde{F}_{b_2} = d_1\hat{F}_1 - d_2\hat{F}_2^\dagger + \sqrt{\gamma_{m_2}}\tilde{f}_2$.

Once the expressions for these fluctuation operators are attained, we can write a matrix equation of the form

$$\dot{\mathbf{R}}(t) = \mathcal{A}\,\mathbf{R}(t) + \mathbf{v}(t), \tag{19}$$

where $\mathbf{R}(t) = (\delta\tilde{H}_1, \delta\tilde{L}_1, \delta\tilde{H}_2, \delta\tilde{L}_2)^T$, $\mathbf{v}(t) = (\delta\hat{H}_1^{\text{in}}, \delta\tilde{L}_1^{\text{in}}, \delta\hat{H}_2^{\text{in}}, \delta\tilde{L}_2^{\text{in}})^T$,

$$\mathcal{A} = \begin{pmatrix} -\frac{\Lambda_1}{2} & 0 & -G_{12} & 0 \\ 0 & -\frac{\Lambda_1}{2} & 0 & G_{12} \\ -G_{21} & 0 & -\frac{\Lambda_2}{2} & 0 \\ 0 & G_{21} & 0 & -\frac{\Lambda_2}{2} \end{pmatrix},$$

which epitomizes the coupling between the fluctuations and vector $\mathbf{v}(t)$ that contains the noise operators of both cavity and mirrors.

We then attempt to find a stable solution for Eq. (19) so that it attains unique steady-state-independent initial conditions. Since the quantum noises $\hat{F}_j$ and $\hat{f}_j$ are taken to be zero-mean Gaussian noises and the dynamics of $\delta\tilde{H}_j$ and $\delta\tilde{L}_j$ is linearized, the steady-state fluctuation becomes a zero-mean Gaussian state that can be fully characterized by the covariance matrix (CM) of the system. One may note that the system can reach a stable steady-state condition when real parts of the eigenvalues of the drift matrix $\mathcal{A}$ are all negative. The stability condition can then be obtained by using the Routh-Hurwitz criterion [54]. In connection to this, the steady-state CM can be found by solving the Lyapunov equation [55]

$$\mathcal{A}V + V^T\mathcal{A} = -\mathcal{D}, \tag{20}$$

where the elements of CM are defined as $V_{ij} = \frac{\langle R_i(\infty)R_j(\infty)+R_j(\infty)R_i(\infty)\rangle}{2}$ and the diffusion matrix ($\mathcal{D}$) with its entries as $\mathcal{D}_{ij}\delta(t-t') = \frac{\langle \upsilon_i(t)\upsilon_j(t')+\upsilon_j(t')\upsilon_i(t)\rangle}{2}$. Using the Routh-Hurwitz criterion, the eigenvalues of matrix $\mathcal{A}$ are negative on condition that $\Lambda_1\Lambda_2 > 4G_{12}G_{21}$, that is, the CM at steady state satisfies Eq. (20), when

$$\mathcal{D} = \begin{pmatrix} B_{11} & 0 & B_{13} & B_{14} \\ 0 & B_{11} & B_{14} & -B_{13} \\ B_{13} & B_{14} & B_{33} & 0 \\ B_{14} & -B_{13} & 0 & B_{33} \end{pmatrix}, \tag{21}$$

with

$$B_{11} = c_1^2\left[\frac{\kappa_1(2N+1)-2\alpha_{11}}{2}\right] + \gamma_{m_1}\left(\frac{2n_1+1}{2}\right) - c_1c_2\left[\frac{\alpha_{12}-\alpha_{21}-2\sqrt{\kappa_1\kappa_2}(M^*+M)}{2}\right] + c_2^2\left[\frac{\kappa_2(2N+1)+2\alpha_{22}}{2}\right], \tag{22}$$

$$B_{13} = c_1d_1\left[\frac{\kappa_1(2N+1)-2\alpha_{11}}{2}\right] + \gamma_{m_1}\left(\frac{2n_1+1}{2}\right) + \frac{(c_1d_2+c_2d_1)}{2}\left[\frac{\alpha_{12}-\alpha_{21}-2\sqrt{\kappa_1\kappa_2}(M^*+M)}{2}\right] - c_2d_2\left[\frac{\kappa_2(2N+1)+2\alpha_{22}}{2}\right], \tag{23}$$

$$B_{14} = i\frac{(c_2d_1-c_1d_2)}{2}\left[\frac{\alpha_{21}-\alpha_{12}-2\sqrt{\kappa_1\kappa_2}(M^*-M)}{2}\right], \tag{24}$$

$$B_{33} = d_1^2\left[\frac{\kappa_1(2N+1)-2\alpha_{11}}{2}\right] + \gamma_{m_2}\left(\frac{2n_2+1}{2}\right) - d_1d_2\left[\frac{\alpha_{12}-\alpha_{21}-2\sqrt{\kappa_1\kappa_2}(M^*+M)}{2}\right] + d_2^2\left[\frac{\kappa_2(2N+1)+2\alpha_{22}}{2}\right]. \tag{25}$$

IV. MECHANICAL SQUEEZING

The squeezing properties of two-mode mechanical quantum fluctuations can be defined by

$$\delta C_+ = \frac{1}{\sqrt{2}}(\delta\tilde{b}_{1_+} + \delta\tilde{b}_{2_+}), \tag{26}$$

$$\delta C_- = \frac{1}{\sqrt{2}}(\delta\tilde{b}_{1_-} + \delta\tilde{b}_{2_-}), \tag{27}$$

where $\delta\tilde{b}_{1_+} = (\delta\tilde{b}_1 + \delta\tilde{b}_1^\dagger)$, $\delta\tilde{b}_{1_-} = i(\delta\tilde{b}_1^\dagger - \delta\tilde{b}_1)$, $\delta\tilde{b}_{2_+} = (\delta\tilde{b}_2 + \delta\tilde{b}_2^\dagger)$, and $\delta\tilde{b}_{2_-} = i(\delta\tilde{b}_2^\dagger - \delta\tilde{b}_2)$. The operators δC_+ and δC_- are Hermitian and satisfy the commutation relation $[\delta C_+, \delta C_-] = 2i$. It might be worth noting that a two-mode Gaussian state is said to be in a squeezed state if the variances of the quadrature fluctuation operators are $\Delta(\delta C_+)^2 < 1$ and $\Delta(\delta C_-)^2 > 1$ or $\Delta(\delta C_-)^2 < 1$ and $\Delta(\delta C_+)^2 > 1$ such that $\Delta(\delta C_+)\Delta(\delta C_-) \geqslant 1$ [52]. With this background, the quadrature variances of the operators δC_+ and δC_- can be expressed as

$$\Delta(\delta C_\pm)^2 = \langle\delta C_\pm^2\rangle - \langle\delta C_\pm\rangle^2. \tag{28}$$

With the aid of the predefined quantum fluctuation operators $\delta\tilde{H}_j$, $\delta\tilde{L}_j$, and Eqs. (26)–(28), we see that the variances of the quadrature fluctuation operators at steady state take the form

$$\Delta(\delta C_+)^2 = \langle\delta\tilde{H}_1\delta\tilde{H}_1\rangle + \langle\delta\tilde{H}_1\delta\tilde{H}_2\rangle + \langle\delta\tilde{H}_2\delta\tilde{H}_1\rangle + \langle\delta\tilde{H}_2\delta\tilde{H}_2\rangle, \tag{29}$$

$$\Delta(\delta C_-)^2 = \langle\delta\tilde{L}_1\delta\tilde{L}_1\rangle + \langle\delta\tilde{L}_1\delta\tilde{L}_2\rangle + \langle\delta\tilde{L}_2\delta\tilde{L}_1\rangle + \langle\delta\tilde{L}_2\delta\tilde{L}_2\rangle. \tag{30}$$

Applying the elements of the CM from the solution of the Lyapunov equation for the expectation values in Eqs. (29) and (30), we obtain the variances of the plus and minus quadratures of mechanical quantum fluctuation operators.

In general, it is perceived that squeezing becomes minimum when the variance of the mechanical quadrature operator is equal to one and maximum when it is zero, and anything in between is considered as a degree of squeezing in the sense that how much it is close to zero. Noting that the mechanical oscillators in our system were not directly coupled initially, the two-mode squeezed radiation emitted from cascade atoms [32,33] is found to be transferred to the quantum harmonically oscillating mirrors due to the emerging optical radiation pressures, which leads to the enhanced squeezing of the modes of mechanical oscillations. In particular, the effect of the amplitude of the atomic pumping laser (χ), the atom-field coupling strength, the rate of atomic injection, atomic decay rates, and biased noise fluctuation on the mechanical squeezing is investigated.

To demonstrate the steady-state squeezing behavior of two mirrors in a doubly resonant cavity when all atoms injected into cavity are initially at the lower-energy level $|\psi(0)\rangle = |c\rangle$, we choose the parameters of two mirrors and two cavity radiations based on recent experiments for an optomechanical system [56,57]: The atom-cavity coupling constants $g_1 = g_2 = g = 5.0\pi$ MHz, mechanical damping constants $\gamma_{m_1} = \gamma_{m_2} = 2\pi \times 60$ Hz, angular frequency of the mechanical oscillators $\omega_{m_1} = \omega_{m_2} = 6\pi$ MHz, masses of the mechanical oscillator $m_1 = m_2 = 5$ ng, the initial length of the cavities $L_1 = 1.064$ mm, $L_2 = 0.810$ mm, the powers of light driving the cavity $P_1 = P_2 = 5.5127\ \mu$W, and the thermal bath temperatures to the mechanical oscillators $T_1 = T_2 = 5.80$ mK. We also take other theoretical parameters, for instance, the cavity damping constants $\kappa_1 = 1.97$ kHz, $\kappa_2 = 2.59$ kHz, angular frequency of the atomic levels $\omega_a = 63.271$ GHz, $\omega_b = 43.271$ GHz, $\omega_c = 25.555$ GHz, the rates of injection $r_a = 0.35$ MHz, atomic emission rates (dephasing and spontaneous) $\gamma_a = \gamma_b = \gamma_c = \gamma_{ab} = \gamma_{bc} = \gamma_{ac} = \gamma = 11.50$ MHz, and the biased noise fluctuation photon number $N = 1$.

As one can see from Figs. 2–5, the quantum fluctuations of the two coupled movable mirrors are squeezed at the steady state for a wide range of χ due to the transferred quantum state from the cavity fields. It can also be seen that the degree of squeezing in the coupled oscillating mirrors increases with an increase in χ to the extent where the degree of squeezing becomes maximum. One can particularly notice in Fig. 2

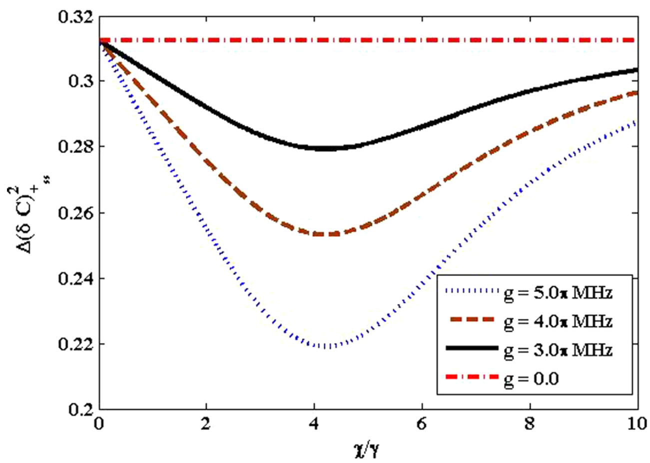

FIG. 2. Plots of the steady-state quadrature variance of the two-mode mechanical oscillators $\Delta(\delta C_+)^2$ against $\frac{\chi}{\gamma}$ at the given parameters for different atom-field coupling strengths $g_1 = g_2 = g = 5.0\pi$ MHz (blue dotted curve), $g = 4.0\pi$ MHz (brown dashed curve), $g = 3.0\pi$ MHz (black solid curve), and $g = 0$ (red dashed-dotted curve).

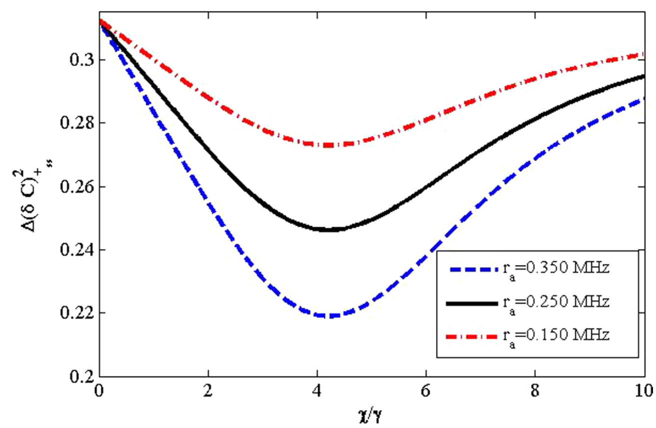

FIG. 3. Plots of the steady-state quadrature variance of the two-mode mechanical oscillators $\Delta(\delta C_+)^2$ against $\frac{\chi}{\gamma}$ at the given parameters for different rates of atomic injection $r_a = 0.35$ MHz (blue dashed curve), $r_a = 0.25$ MHz (black solid curve), and $r_a = 0.15$ MHz (red dashed-dotted curve).

that the degree of mechanical squeezing is enhanced with increased atom-field coupling strength. It is also revealed (see data in Table I) that there is a threshold coupling between the atoms and the fields above which the mechanical squeezing would not be manifested. On the other hand, the mechanical squeezing can survive even at greater atom-field coupling strength in the case where the power of the laser driving the cavity is increased. Markedly, appropriate coupling between the atoms and cavity needs to be used to witness a meaningful degree of mechanical squeezing. One can also see from Fig. 3 that the degree of mechanical squeezing increases with the rate at which the atoms are injected into the cavity. The outcome that indicates the effect of the rate at which the atoms are injected into the cavity in enhancing the degree of mechanical squeezing of coupled mirrors may provide better control over mechanical squeezing in relation to the number of atoms in the cavity during interactions.

We also explore the effect of biased noise fluctuations on the degree of mechanical squeezing. From this study, it is possible to observe that the presence of biased noise fluctuation is crucial for the mechanical squeezing to exist under certain conditions since its effect looks much better transferred to the mechanical oscillators. With this understanding, one may see from Fig. 4 that the degree of mechanical squeezing increases with the strength of biased noise fluctuations. A very weak degree of two-mode mechanical squeezing is found to exist even in the absence of biased noise fluctuations ($N = 0$) when the power of lasers driving the cavity is high (551 mW) and the atom-field coupling strength gets weak (2π MHz). This entails that the squeezing of mechanical oscillation can also be controlled via external biased fluctuation input. To see the situation more closely, we also studied the dependence of the degree of mechanical squeezing on the atomic damping rate. As shown in Fig. 5, the degree of mechanical squeezing increases almost in the same manner for different atomic damping rates up to the point where the degree of squeezing becomes maximum. This result may reveal the effect of the atomic decay rates in increasing the degree of squeezing of coupled oscillating mirrors with regards to how rapidly the atoms are removed from the lower-energy level.

TABLE I. Numerical value of the degree of mechanical squeezing from Fig. 2 for different values of atom-field coupling strength.

Atom-field coupling strength (MHz)	Squeezing (below shot noise)	Occurs at ($\frac{\chi}{\gamma}$)
$g = 5\pi$	78.10%	4.13
$g = 4\pi$	74.68%	4.13
$g = 3\pi$	72.06%	4.13
$g = 0$	68.75%	[0.01, 10]

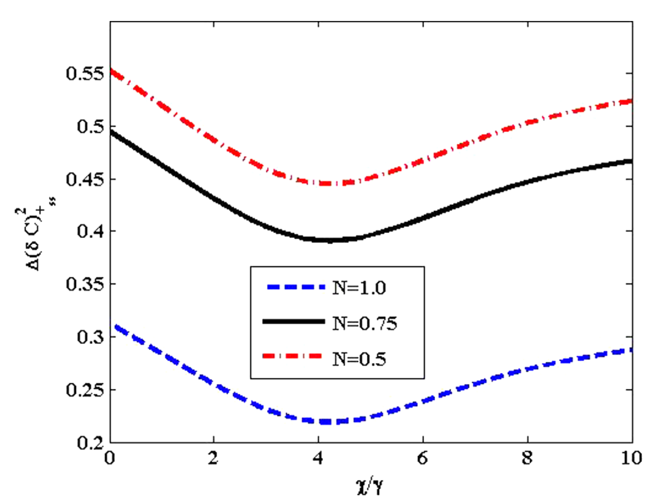

FIG. 4. Plots of the steady-state quadrature variance of the two-mode mechanical oscillators $\Delta(\delta C_+)^2$ against $\frac{\chi}{\gamma}$ at the given parameters for different biased noise photon numbers $N = 1.00$ (blue dashed curve), $N = 0.75$ (black solid curve), and $N = 0.50$ (red dashed-dotted curve).

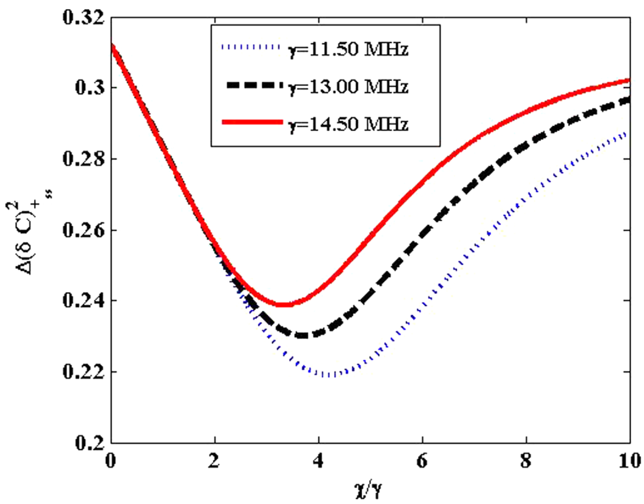

FIG. 5. Plots of the steady-state quadrature variance of the two-mode mechanical oscillators $\Delta(\delta C_+)^2$ against $\frac{\chi}{\gamma}$ at the given parameters for different atomic decay rates (assuming that the spontaneous and stimulated emission rates are the same) $\gamma = 11.50$ MHz (blue dotted curve), $\gamma = 13.00$ MHz (black dashed curve), and $\gamma = 14.50$ MHz (red solid curve).

V. CONCLUSION

We studied the degree of squeezing in mechanical oscillators coupled to two-mode CEL via optical radiation pressures in a doubly resonant optomechanical cavity where the external biased noise fluctuations are coupled to the two-mode radiation. We analyzed the squeezing of the movable mirrors by using the intermode correlation induced by the two-photon coherence in CEL under the good cavity limit and an adiabatic regime for some realistic parameters. Mechanical squeezing persists in a wide range of amplitudes of the atomic pumping laser. Particularly, an enhanced degree of mechanical squeezing is realized for strong atom-field coupling, a large atomic injection rate, and intense biased noise fluctuations. Due to externally induced correlated vibrations, it turns out that mechanical squeezing exists as long as there are biased noise fluctuations. The degree of mechanical squeezing is also found to increase almost equally for different atomic emission rates until a maximum degree of squeezing is reached.

With the observed flexibility and diversity in controlling the degree by which the squeezing in the cavity radiation can be transferred to the modes of mechanical oscillators, we hope that this scheme can be useful in making the quantum properties in the cavity available for application. So if the technique of transferring coherence superposition from CEL to the oscillation of the mirrors could be properly established, as we intend to do in forthcoming work, it could have a potential to overcome the hurdle of making the quantum properties generated in the cavity mechanism accessible. We then come to understanding that this work could lead to further scrutiny in inducing other nonclassical correlations between mechanical oscillators such as entanglement, quantum discord, and steering. It could also serve in designing a scheme that can mechanically mimic the underlying quantum features of the radiation, which might be useful in enhancing the deeper understanding of quantum phenomena.

APPENDIX A: DERIVATION OF THE MASTER EQUATION

To derive the master equation for two-mode fields, we suppose that $\rho_{AR}(t, t_j)$ is the density operator for the cavity modes at time t, whereas the atom injected at a rate r_a in an earlier time t_j and leaves the cavity after time τ, and hence $t - \tau \leqslant t_j \leqslant t$. The density operator for the atoms in the cavity plus the two-mode cavity field at time t can be written as $\rho_{AR}(t) \equiv r_a \sum_j \rho_{AR}(t, t_j)\Delta t_j$, where $r_a \Delta t_j$ represents the number of atoms injected into cavity in a small time interval Δt_j. At the continuum limit, with the aid of the Leibnitz rule along with the assumption that the atomic and cavity mode states are uncorrelated at the time the atoms are injected into the cavity and when they are removed, one can write

$$\dot{\rho}_{AR}(t) = r_a[\rho_A(0) - \rho_A(t, t-\tau)]\rho(t) - \frac{i}{\hbar}\big[\hat{H}_I^{af}, \rho_{AR}(t)\big]. \tag{A1}$$

Then, upon carrying out trace operations, we obtain

$$\frac{d}{dt}\hat{\rho}(t) = g_1(\hat{a}_1\rho_{ba} - \rho_{ba}\hat{a}_1 - \hat{a}_1^\dagger\rho_{ab} + \rho_{ab}\hat{a}_1^\dagger) + g_2(\hat{a}_2\rho_{cb} - \rho_{cb}\hat{a}_2 - \hat{a}_2^\dagger\rho_{bc} + \rho_{bc}\hat{a}_2^\dagger), \tag{A2}$$

where the density operator elements $\rho_{mn} = \langle m|\hat{\rho}_{AR}|n\rangle$ with $(m, n = a, b, c)$, and their complex conjugates for $m \neq n$ that appear in Eq. (A2) can be acquired by multiplying Eq. (A1) from the left by $\langle m|$ and the right by $|n\rangle$. In addition, assuming that the atom decays to energy levels other than lasing levels when it leaves the cavity, we have

$$\dot{\rho}_{mn} = r_a\rho_{mn}^{(0)}\rho(t) - \frac{i}{\hbar}[\langle m|[\hat{H}_I^{af}, \hat{\rho}_{AR}(t)|n\rangle] - \gamma_{mn}\hat{\rho}_{mn}, \tag{A3}$$

where the last terms account for the spontaneous emission and dephasing process. Making use of Eqs. (3) and (A3), we can then obtain

$$\dot{\rho}_{ab} = -(\gamma_{ab} + i\xi_1)\rho_{ab} - g_1(\rho_{aa}\hat{a}_1 - \hat{a}_1\rho_{bb}) + g_2\rho_{ac}\hat{a}_2^\dagger + \frac{\chi}{2}\rho_{cb}, \tag{A4}$$

$$\dot{\rho}_{bc} = -(\gamma_{bc} + i\xi_2)\rho_{bc} - g_2(\rho_{bb}\hat{a}_2 - \hat{a}_2\rho_{cc}) - g_1\hat{a}_1^\dagger\rho_{ac} - \frac{\chi}{2}\rho_{ba}, \tag{A5}$$

where γ_{ab} and γ_{bc} are dephasing rates for the corresponding transitions. One may note that other elements of the density operator can be generated in the same way.

In the present work, we seek to consider the case when all the atoms are initially made to occupy the lower-energy level and then pumped by a laser of amplitude χ [32,33]. Afterwards, confining to a linear analysis, an adiabatic regime, and a good cavity limit, one readily obtains

$$\rho_{aa} = \frac{\chi}{2\gamma_a}(\rho_{ca} + \rho_{ac}), \quad \rho_{bb} = 0,$$
$$\rho_{cc} = \frac{r_a}{\gamma_c}\rho - \frac{\chi}{2\gamma_c}(\rho_{ca} + \rho_{ac}), \tag{A6}$$
$$\rho_{ac} = \frac{\chi}{2[\gamma_{ac} + i(\xi_1 + \xi_2)]}(\rho_{cc} - \rho_{aa}),$$

where γ_j is the jth atomic level spontaneous emission decay rate and γ_{ac} is the two-photon dephasing rate. Note that the good cavity limit is a condition where the cavity damping rate is much smaller than the spontaneous emission and dephasing rates of the two photons in atomic coherence. With this aid, the cavity variables vary more slowly than the atomic ones where the atomic variables reach the steady state earlier than the cavity ones, so we can set the time derivatives of the aforementioned conditioned density operators to zero, thus being able to solve the system of equations analytically.

Upon using ρ_{ac} in Eq. (A6) along with its complex conjugate, one can then write ρ_{aa}, ρ_{cc}, and ρ_{ac} as

$$\rho_{aa} = \frac{r_a\rho}{D_2}T_{aa}, \quad \rho_{cc} = \frac{r_a\rho}{D_2}T_{cc}, \quad \rho_{ac} = \frac{r_a\rho}{D_1}T_{ac}, \tag{A7}$$

where

$$T_{cc} = \frac{1}{2}[2\gamma_a D_1 + \chi^2\gamma_{ac}], \quad D_1 = \gamma_{ac}^2 + (\xi_1+\xi_2)^2,$$

$$T_{aa} = \frac{1}{2}\chi^2\gamma_{ac}, \quad T_{ac} = \frac{\chi}{2D_2}[\gamma_{ac} - i(\xi_1+\xi_2)]\big(T_{cc} - T_{aa}\big),$$

$$D_2 = \chi^2\gamma_{ac}\frac{(\gamma_a+\gamma_c)}{2} + \gamma_a\gamma_c\big[\gamma_{ac}^2 + (\xi_1+\xi_2)^2\big].$$

Now, applying the adiabatic approximation to Eqs. (A4) and (A5), substituting the resulting complex conjugate as required, and using Eq. (A7) with its conjugates, one can verify that

$$g_1\hat{\rho}_{ab} = \alpha_{11}\hat{\rho}\hat{a}_1 + \alpha_{12}\hat{\rho}\hat{a}_2^\dagger, \tag{A8}$$

$$g_2\hat{\rho}_{bc} = \alpha_{22}\hat{a}_2\hat{\rho} + \alpha_{21}\hat{a}_1^\dagger\hat{\rho}. \tag{A9}$$

In this context, upon substituting Eqs. (A8) and (A9) along with their complex conjugates into Eq. (A2), we obtain the master equation for the cavity modes coupled to biased noise fluctuations.

APPENDIX B: NOISE CORRELATIONS

For any operators $\hat{A}$ and $\hat{B}$ and their corresponding noise operators $\hat{F}_A$ and $\hat{F}_B$, it follows from Einstein's relation that

$$2\langle D_{\hat{A}\hat{B}}\rangle = \frac{d}{dt}\langle\hat{A}\hat{B}\rangle - \langle(\dot{\hat{A}} - \hat{F}_A)\hat{B}\rangle - \langle\hat{A}(\dot{\hat{B}} - \hat{F}_B)\rangle, \tag{B1}$$

where $\langle D_{\hat{A}\hat{B}}\rangle$ is the diffusion coefficient (with $\hat{A}$ and $\hat{B} = \hat{a}_j, \hat{b}_j$) for $j = 1, 2$ [51]. Using this relation, the equations for second-order moments of the cavity mode operators $\hat{a}_j$, and

$$\langle\hat{F}_{\hat{A}}(t)\hat{F}_{\hat{B}}(t')\rangle = 2\langle D_{\hat{A}\hat{B}}\rangle\delta(t-t'), \tag{B2}$$

the nonzero correlation properties of the cavity mode noise operators are found to be

$$\langle\hat{F}_1^\dagger(t)\hat{F}_1(t')\rangle = [\kappa_1 N - 2\,\mathrm{Re}(\alpha_{11})]\delta(t-t'), \tag{B3}$$

$$\langle\hat{F}_1(t)\hat{F}_1^\dagger(t')\rangle = \kappa_1(N+1)\delta(t-t'), \tag{B4}$$

$$\big\langle\hat{F}_2^\dagger(t)\hat{F}_2(t')\big\rangle = \kappa_2 N\,\delta(t-t'), \tag{B5}$$

$$\langle\hat{F}_2(t)\hat{F}_2^\dagger(t')\rangle = [\kappa_2(N+1) + 2\,\mathrm{Re}(\alpha_{22})]\delta(t-t'), \tag{B6}$$

$$\langle\hat{F}_1(t)\hat{F}_2(t')\rangle = -\sqrt{\kappa_1\kappa_2}M\delta(t-t'), \tag{B7}$$

$$\langle\hat{F}_1^\dagger(t)\hat{F}_2^\dagger(t')\rangle = (\alpha_{12}^* - \alpha_{21}^* - \sqrt{\kappa_1\kappa_2}M^*)\delta(t-t'), \tag{B8}$$

$$\langle\hat{F}_2(t)\hat{F}_1(t')\rangle = (\alpha_{12} - \alpha_{21} - \sqrt{\kappa_1\kappa_2}M)\delta(t-t'), \tag{B9}$$

$$\langle\hat{F}_2^\dagger(t)\hat{F}_1^\dagger(t')\rangle = (\alpha_{21}^* - \alpha_{12}^* - \sqrt{\kappa_1\kappa_2}M^*)\delta(t-t'). \tag{B10}$$

In the same manner, the nonvanishing correlations between the mechanical noise operators with the aid of Eqs. (4) and (5) can be written in the form

$$\langle\hat{f}_j^\dagger(t)\hat{f}_j(t')\rangle = n_j\delta(t-t'), \tag{B11}$$

$$\langle\hat{f}_j(t)\hat{f}_j^\dagger(t')\rangle = (n_j+1)\delta(t-t'), \tag{B12}$$

where $n_j^{-1} = e^{\frac{\hbar\omega_{mj}}{k_B T_j}} - 1$ is the mean thermal occupation number, k_B is the Boltzmann constant, and T_j is the temperature of the jth reservoir of the mechanical oscillator.

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