Impact of the lattice period on the stability dynamics of defect solitons in periodic lattices

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The dynamics of fundamental solitons is examined for periodic and defective lattices when the lattice period is varied. The existence domain and stability intervals of solitons are determined and it is shown that the solitons can exist and stay stable for a wide range of parameters. It is observed that the domain of existence is extended by increased lattice period for the periodic lattice and the square lattice with an edge dislocation. It is also demonstrated that stability of solitons around a vacancy defect and near edge dislocation can be improved by decreased lattice period, whereas a higher lattice period supports the stability of periodic lattice solitons. Further, it is shown that there are a lower limit for the period of a square lattice and an upper limit for the period of a lattice with a vacancy defect for no collapse of the solitons in their entire existence domains. Thus modification of the lattice period provides great controllability of the soliton dynamics. It is also observed that the deeper (or strong) vacancy defect in the lattice extends the stability domain of solitons for larger lattice periods.

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I. INTRODUCTION

Optical solitons are localized waves that form when the medium's nonlinear and dispersive effects are balanced. This balance results in an optical field that maintains its shape during transmission. Since their experimental observation in [1], multidimensional solitons in nonlinear optical systems with optically induced lattices have attracted researchers' interest due to their stabilizing effects on soliton stability [2–5]. Recently, real and complex (parity-time symmetric [6,7]) lattice solitons in the media with cubic (Kerr) [6,8,9], quadratic [10–12], saturable [13,14], and competing nonlinearities [15,16] and higher-order dispersion effects [17,18] have been investigated. Solitons have also been shown to exist in aperiodic or quasicrystal lattice structures [3,19,20], as well as lattices with defects [4,21–24].

In the mentioned studies, the existence and stability properties of solitons in real and complex lattices have been studied and it has shown that optical lattices can be utilized to arrest wave collapse in optical systems [3,4,9,11,23–28]. These studies focused on the lattice structure (periodic, quasiperiodic, or parity-time symmetric) and irregularities (point or line defects) in the lattices. Moreover, it is known that soliton dynamics can be manipulated by lattice depth and lattice frequency (or period) [29]. In [30] it was shown that quadratic modulation of lattice frequency serves an effective mechanism to control the shape and diffraction of lattice solitons. In [31] it was demonstrated that the dynamics of matter solitons can be managed by uniform modulation of optical lattices. In [32] the possibility of controlling the bend rate and output position of solitons was presented by a change of the depth and frequency of periodic optical lattices in the transverse direction. Furthermore, the instability of bidirectional solitons in photonic lattices has been investigated numerically and these solitons have been stabilized experimentally by increasing the lattice period for a certain range of lattice amplitude [33,34]. This stabilization phenomenon in two-dimensional square lattices was explained as follows: An increase in the lattice period can significantly reduce the mobility of beams by trapping each of them in a single lattice site.

The soliton dynamics in a homogeneous medium can be governed by the nonlinear Schrödinger equation (NLSE) [35,36]

$$iu_z(z, \mathbf{x}) = -\Delta u - |u|^{p-1}u, \tag{1}$$

where z is the longitudinal coordinate, $\mathbf{x} = (x_1, \ldots, x_d)$ are spatial coordinates, and $\Delta = \partial_{x_1x_1} + \cdots + \partial_{x_dx_d}$ is the Laplacian operator. The nonlinearity is denoted by p and it is focusing when $p > 1$. In nonlinear optics, the variables z and x_j are normalized by the diffraction length and the input beam radius, respectively. Different cases of the NLSE can be described as follows [4,35]:

$$\begin{aligned} 0 < p-1 < \frac{4}{d} \quad &\text{(the subcritical case)},\\ p-1 = \frac{4}{d} \quad &\text{(the critical case)},\\ p-1 > \frac{4}{d} \quad &\text{(the supercritical case)}. \end{aligned} \tag{2}$$

It is known that adding an external lattice to Eq. (1) supports the stability of the solitons for the critical case (e.g., a two-dimensional NLSE with cubic nonlinearity) [3,5,23], whereas the lattice does not affect the stability and instability of the solitons in the subcritical and supercritical cases, respectively [35,37]. In [35,36,38] comprehensive investigations of the (in)stability (collapse) dynamics were done for the nonlinear lattice solitons. These studies showed that the soliton profile and stability dynamics are significantly altered by the

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width of the soliton and the lattice period. The soliton can be wider, of the same order, or narrower than the lattice period. It was shown that the same nonlinear lattice can stabilize beams of specific widths and destabilize beams of other widths [36]. In [39] it was revealed that the solitons located at the lattice minimum are stable in the subcritical case regardless of their widths, but in the critical case the solitons can stay stable when they are narrower than a few lattice periods [2,40]. For the supercritical case, it was demonstrated that solitons located at the lattice minimum can be stabilized when they are sufficiently wide, but the external lattice cannot stabilize narrow solitons [35,37]. Also, in [35] it was shown that when the narrow solitons focus on a lattice maximum, the (in)stability dynamics is extremely susceptible to the lattice parameters. It is noteworthy that in nonlinear optics, the input beam size is larger than the lattice period for most cases [1,41,42], but if the beam collapses, its width shrinks to zero and thus the width of beam becomes significantly smaller than the lattice period [4,35].

In [4] the relation between the soliton width and (in)stability dynamics was investigated in a crystal (periodic), a quasicrystal, and a square lattice with a vacancy defect with fixed lattice periods. In this study, the existence and stability properties of fundamental solitons are explored in a real periodic (square) lattice, in a square lattice with a vacancy defect, and in a square lattice with an edge dislocation by variation of the lattice period. Linear stability analysis and nonlinear evolution of the solitons show that modification of the lattice period provides great controllability of the soliton dynamics for the considered lattices. Further, it is shown that there are a lower limit for the period of the square lattice and an upper limit for the period of the lattice with a vacancy defect for no collapse of the solitons in their entire existence domains. This study is also focused on the effect of a deeper (or strong) vacancy defect in the lattice and it is observed that there is a relation between the depth of the vacancy defect and the lattice period for the stability of solitons.

It is noteworthy that a vacancy defect is a point defect that is produced when an atom is missing from a normal site and all crystalline materials contain vacancies naturally; in fact, it is impossible to create a perfect crystal that is completely free of them [43]. Another type of irregularity in optical materials is the edge dislocation. It is a line defect in which a line of atoms moves from its original position [44]. It is known that these defects can be created in materials by plastic deformation and high-energy particle irradiation [45–47]. Recently, there has been significant progress in designing and fabricating irregular lattice structures with point and line defects [46,48–50].

The dynamics of lattice solitons in a cubic nonlinear medium can be characterized by a (2 + 1)-dimensional NLSE [the critical case of Eq. (1)] with an external lattice [2,25,26], which can be written as

$$iu_z + \tfrac{1}{2}\Delta u + |u|^2 u - V(x, y)u = 0, \tag{3}$$

where $u(x, y)$ is the slowly varying amplitude of the normalized static electric field, z is the propagation distance, $\Delta u \equiv u_{xx} + u_{yy}$ is the diffraction of the medium, and $V(x, y)$ is an external lattice. In this study, a two-dimensional (2D) periodic (square) lattice and two irregular lattices with defects are considered. The first lattice is a 2D square lattice with a vacancy defect that is defined by [3]

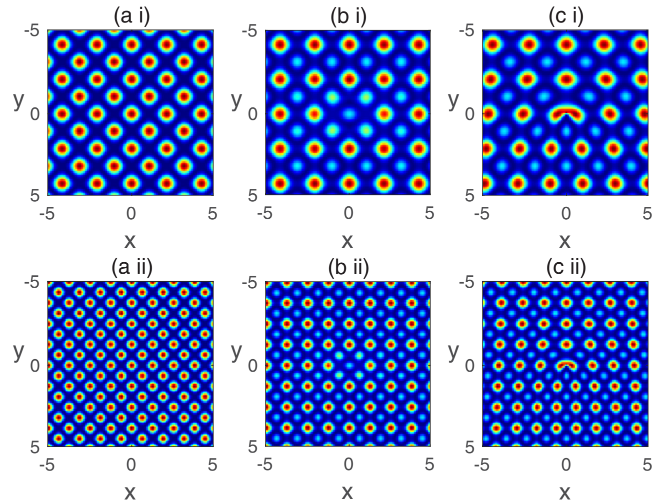

FIG. 1. Contour images of (a) periodic lattices, (b) square lattices with a vacancy defect, and (c) square lattices with an edge dislocation. The lattices are obtained when (ai)–(ci) $K = 3$ and (aii)–(cii) $K = 5$ with $V_0 = 12.5$.

$$V(x, y) = \frac{V_0}{25}|2\cos(k_x x) + 2\cos(k_y y) + \varepsilon e^{i\theta(x,y)}|^2, \tag{4}$$

where $V_0 > 0$ is the depth of the lattice, ε is the depth (or strength) of the vacancy defect, and θ is a phase distortion function that is given as

$$\theta(x, y) = \tan^{-1}\left(\frac{y - y_0}{x}\right) - \tan^{-1}\left(\frac{y + y_0}{x}\right). \tag{5}$$

Here (k_x, k_y) is a wave vector and θ produces a vacancy defect at the origin (0,0) of the square lattice. This point defect is a shallow maximum and it corresponds to two first-order phase dislocations displaced in the y direction by a distance $2y_0$ [3,4]. A vacancy defect can be created by taking $y_0 = \pi/K$, where $K = k_x = k_y$. Furthermore, far away from the origin (center), the lattice is locally similar to a square lattice with period $2\pi/K$. Thus, K can be considered as a frequency (or period) control parameter for the optical lattice. Note that, unless specified otherwise, ε is set equal to 1 for the lattice with a vacancy defect [see Figs. 1(bi) and 1(bii)] and if $\varepsilon = 0$, a perfectly periodic lattice is obtained [see Figs. 1(ai) and 1(aii)].

The second irregular potential we study is a 2D square lattice with an edge dislocation that is given by [3]

$$V(x, y) = \frac{V_0}{25}\{2\cos[k_x x + \theta(x, y)] + 2\cos(k_y y) + 1\}^2, \tag{6}$$

where the phase dislocation function $\theta(x, y)$ is defined as

$$\theta(x, y) = \frac{3\pi}{2}\tan^{-1}\left(\frac{y}{x}\right). \tag{7}$$

As can be seen from Figs. 1(ci) and 1(cii), the density of lattice sites varies vertically throughout the lattice with an edge dislocation.

In Fig. 1 the contour images of a square lattice [Fig. 1(a)], a square lattice with a vacancy defect [Fig. 1(b)], and a square lattice with an edge dislocation [Fig. 1(c)] are displayed for varied lattice periods ($K = 3$ and 5). The center ($x = y = 0$) of the lattice with a vacancy defect is a local minimum. By comparing lattices in the top and bottom rows it can be seen that the structure of the lattices does not change qualitatively by modification of the lattice period (see Fig. 1). Note that both the defects in optical lattices and modification of the lattice period can be engineered [46,47,51]. Thus, it is important to investigate the impact of the lattice period on the dynamics of defect solitons in periodic lattices.

II. NUMERICAL SOLUTION FOR FUNDAMENTAL SOLITONS

The steady-state solutions (fundamental soliton) of the model (3) are calculated by the squared operator method (SOM) [52,53]. The SOM is outlined as follows.

Inserting the ansatz $u = U(x, y)\exp(i\mu z)$ into Eq. (3), the suboperators

$$F_0 = -\mu + U^2 - V(x, y),$$
$$F_1 = -\mu + 3U^2 - V(x, y) \tag{8}$$

are obtained, where $U(x, y)$ is a real-value function and μ is the propagation constant (eigenvalue). Using F_0 and F_1, the operators $\mathbf{L}_0$ and $\mathbf{L}_1$ and the acceleration operators M_0 and M_1 are calculated as

$$\mathbf{L}_0\mathbf{U} = \frac{1}{2}\Delta U + F_0 U, \quad M_0 = \mathcal{F}^{-1}\left(\frac{\mathcal{F}(\mathbf{L}_0 U)}{K^2 + c}\right),$$
$$\mathbf{L}_1\mathbf{U} = \frac{1}{2}\Delta M_0 + F_1 M_0, \quad M_1 = \mathcal{F}^{-1}\left(\frac{\mathcal{F}(\mathbf{L}_1 U)}{K^2 + c}\right), \tag{9}$$

where $\mathcal{F}$ denotes the Fourier transformation, $K = (k_x, k_y)$ are wave numbers, $K^2 = k_x^2 + k_y^2$, and c is a positive real number. The $\mathbf{L}_0 U = 0$ is the general form of the model (3) and $\mathbf{L}_1$ is the linearized operator of $\mathbf{L}_0 U$ around the solution U [53]. Once the M_1 operator has been obtained, U is calculated by the iteration

$$U_{n+1} = U_n - M_1 \Delta t, \tag{10}$$

where Δt is an auxiliary time-step parameter. The iteration of U_n starts from a Gaussian initial condition and proceeds until the error $E = \sqrt{\|U_{n+1} - U_n\|^2} < 10^{-8}$. It has been shown that if convenient c and Δt parameters are selected heuristically, the SOM algorithm converges to a steady-state solution (soliton) for a wide range of nonlinear evolution equations [52].

Unless specified otherwise, in this study, the parameters are fixed to be

$$(V_0, c, \Delta t) = (12.5, 3, 0.2). \tag{11}$$

The potential depth V_0 is chosen as 12.5 to compare the results of this study with previous research [3,24]. In Fig. 2 fundamental lattice solitons are obtained for these fixed values on a local minimum of a periodic lattice [Fig. 2(a)], near the vacancy defect [Fig. 2(b)], and near the edge dislocation [Fig. 2(c)] when $K = 3$ (the lattice period is $2\pi/3$). It can be seen from Fig. 2 that the solitons are generated on local minima of lattices. Note that in Figs. 2(ci) and 2(cii) the initial condition of the SOM algorithm is located around the center (0,0) of the lattice with an edge dislocation and the solution

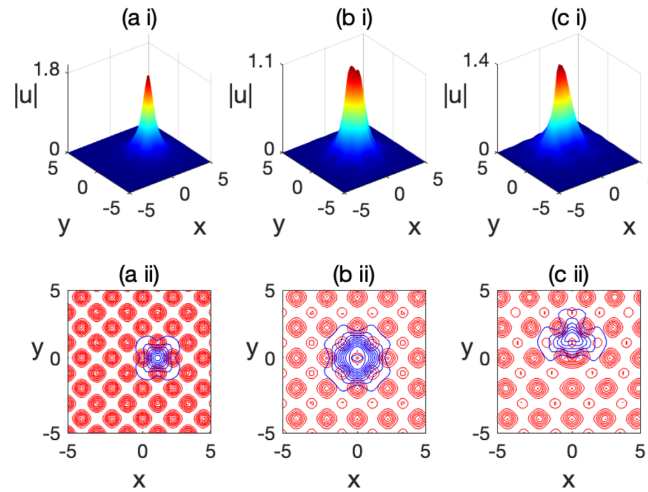

FIG. 2. (ai)–(ci) The 3D view and contour plot of the fundamental solitons superimposed (a) on the local minimum ($\pi/3, 0$) of the periodic lattice, (b) near the vacancy defect (0,0), and (c) near the edge dislocation (0, 1.41), for (a) $\mu = -1$, (b) $\mu = -1.3$, and (c) $\mu = -1.4$. In all cases $K = 3$ and the other parameters are given in (11). These solitons are calculated in the $[-15, 15]^2$ domain and a small part of this domain ($[-5, 5]^2$) is displayed for visibility.

moves to the closest local minimum during the iteration. From the previous studies in [5,20,23], it is known that although solitons can exist on local maxima of square lattices, they cannot stay stable for the NLSE with a defocusing external potential. Therefore, in this study, solitons located around the minima of the lattices will be examined.

III. POWER AND STABILITY ANALYSIS

After obtaining the fundamental solitons, their stability is examined by the linear eigenvalue spectra and nonlinear evolution of the solitons.

The linear spectrum is calculated by linearization of the model near the fundamental soliton. To do that the fundamental soliton u_0 is perturbed by

$$U = e^{i\mu z}[u_0(x, y) + R(x, y)e^{\lambda z} + I^*(x, y)e^{\lambda^* z}], \tag{12}$$

where $R, I \ll 1$ are infinitesimal perturbations. Substituting U into Eq. (3) and neglecting small terms of second order, the linearized system is obtained,

$$\mathcal{L}\mathbf{V} = \lambda\mathbf{V}, \tag{13}$$

where

$$\mathcal{L} = i\begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} R \\ I \end{pmatrix}$$

and the components of $\mathcal{L}$ are

$$\mathcal{L}_{11} = \mathcal{L}_{22} = 0,$$
$$\mathcal{L}_{12} = \tfrac{1}{2}\Delta - \mu + u^2 - V,$$
$$\mathcal{L}_{21} = \tfrac{1}{2}\Delta - \mu + 3u^2 - V. \tag{14}$$

The Fourier collocation method is utilized to compute the eigenvalues of $\mathcal{L}$ numerically [52]. If there is any eigenvalue

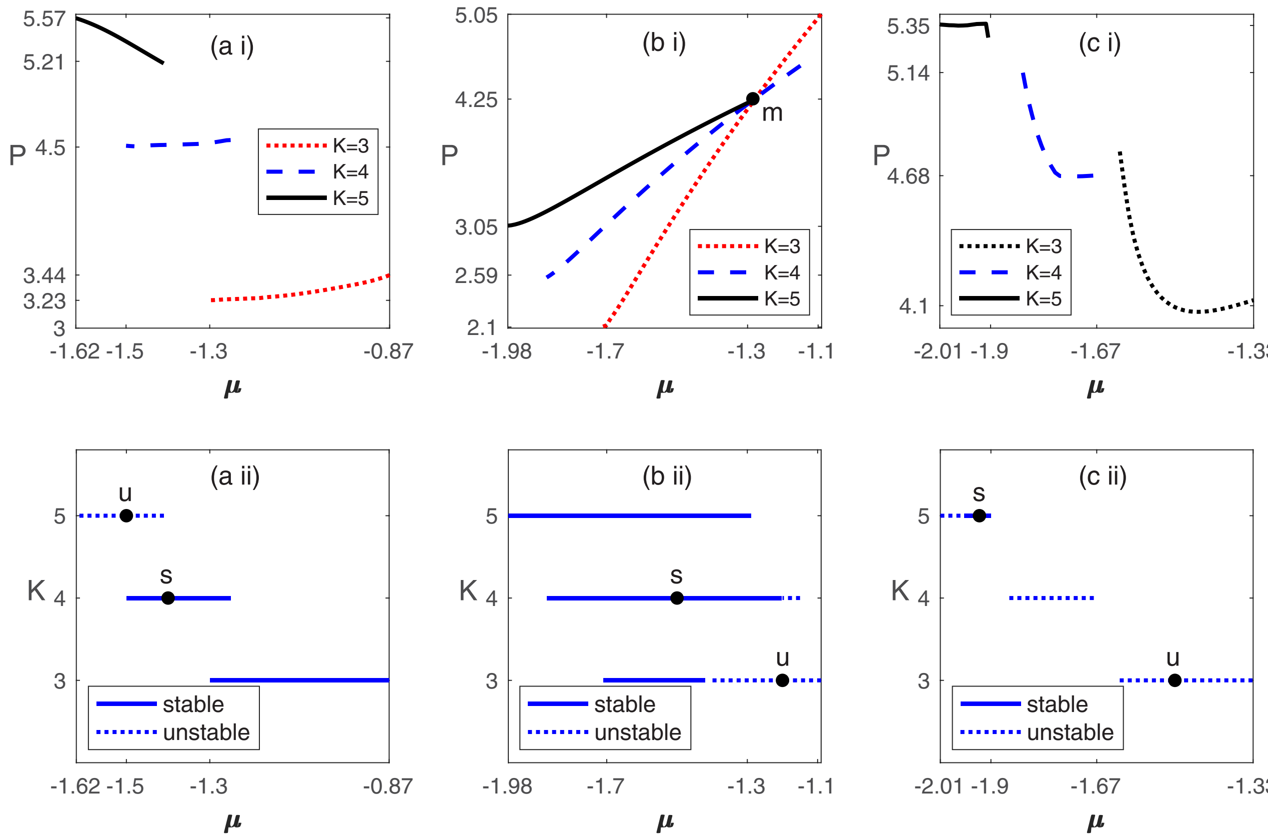

FIG. 3. (ai)–(ci) Power-eigenvalue (P-μ) diagrams and (aii)–(cii) linear stability domains of μ for fundamental solitons on (a) periodic lattices, (b) square lattices with a vacancy defect, and (c) square lattices with an edge dislocation when the lattice frequency K is varied. In (aii)–(cii) solid lines and dotted lines show linearly stable and unstable domains, respectively.

with a positive real part, the soliton will be considered as linearly unstable.

The power ($P = \iint_{-\infty}^{\infty} |u|^2 dx\, dy$) and stability dynamics of solitons have a strong relation. In [54] Vakhitov and Kolokolov demonstrated that the linear (spectral) stability of solitons is possible if their power grows with increasing propagation constant μ, i.e.,

$$dP/d\mu > 0. \tag{15}$$

Furthermore, in [55] it was shown that a necessary condition for the collapse in the 2D NLSE is that the power exceeds a critical value ($P_c \approx 11.7/2 = 5.85$).

In this regard, the power-eigenvalue diagrams of gap solitons are displayed in Figs. 3(ai)–3(ci) for various values of the lattice period. Note that this analysis shows the existence (gap) domain of solitons for μ when fundamental solitons are obtained on the minimum of the periodic lattice [Fig. 3(a)], near the vacancy defect [Fig. 3(b)], and near the edge dislocation [Fig. 3(c)]. It can be seen that the power of fundamental solitons is less than the critical power P_c for each case of the lattices. The power of periodic lattice solitons decreases and the domains of existence are extended with increased lattice period (from $2\pi/5$ to $2\pi/3$). The slope condition (15) is satisfied by periodic lattice solitons when $K = 3$ and 4 [see Fig. 3(ai)]. The power of vacancy defect solitons decreases while the lattice period increases and the power increases with increased propagation constant [see Fig. 3(aii)]. Thus, the Vakhitov-Kolokolov (VK) stability criterion is satisfied everywhere in the domain of existence for the vacancy defect solitons. For the fundamental solitons near the edge dislocation, the power is reduced and the domain of existence is extended with increased lattice period (from $2\pi/5$ to $2\pi/3$) and the slope condition (15) is satisfied only when $K = 3$ and $\mu \in [-1.46, -1.33]$ and when $K = 5$ and $\mu \in [-1.96, -1.91]$ [see Fig. 3(a3)]. It should be noted that these results are consistent with the previous studies [3,24], in which K is fixed to be 2π. Further, linear stability spectra of gap solitons are examined for each point on power curves, and linear (in)stability intervals of μ are shown for varied lattice periods in Figs. 3(aii)–3(cii). Solid and dotted lines show linearly stable and unstable domains, respectively. The results in Fig. 3(aii) show that as the lattice period is increased (from $2\pi/5$ to $2\pi/3$) both the domain of existence and linear stability interval of solitons are extended. Conversely, the linear stability interval of vacancy defect solitons is extended by decreasing the lattice period [see Fig. 3(bii)]. The domain of existence for the solitons near the edge dislocation is extended with higher lattice period, whereas linear stability of solitons is provided only if $\mu \in [-1.96, -1.91]$ with the smallest lattice period $2\pi/5$ [see Fig. 3(cii)]. Thus linear stability of solitons around the vacancy defect and edge dislocation can be improved with a reduced lattice period. In contrast, a higher lattice period supports the stability of periodic lattice solitons.

It is noteworthy that fundamental solitons on the periodic lattice are linearly stable for all their existence domain if

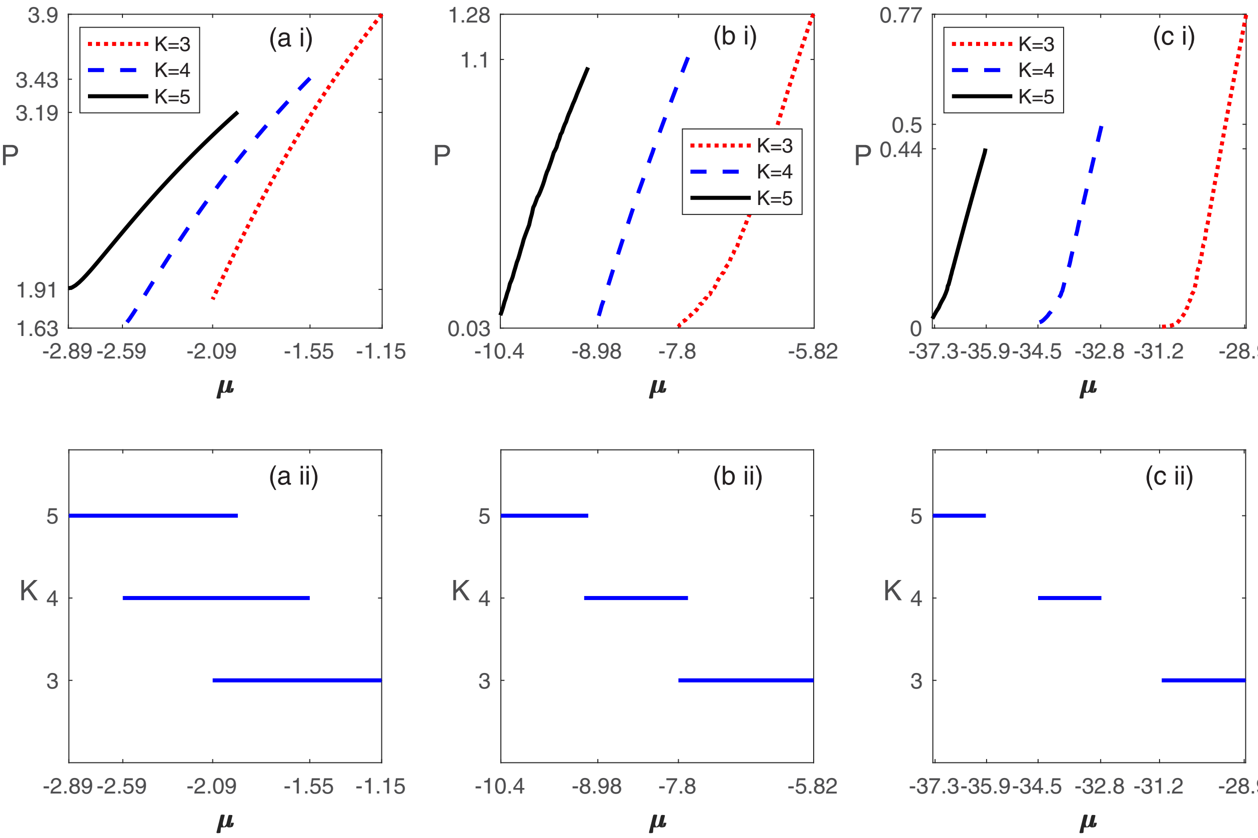

FIG. 4. (ai)–(ci) Power-eigenvalue (P-μ) diagrams and (aii)–(cii) linear stability domains of μ for vacancy defect solitons when the depth of defect (a) $\varepsilon = 2$, (b) $\varepsilon = 5$, and (c) $\varepsilon = 10$ with varied lattice frequency K.

$K < 4.27$ and vacancy defect solitons are linearly stable on their entire existence domain if $K > 4.14$ for the considered parameter regime. The solitons near the edge dislocation are linearly unstable over their entire existence domain if $K < 4.65$. It is also observed that although the power-eigenvalue (P-μ) curves of vacancy defect solitons intersect at point m (where $P = 4.253$ and $\mu = -1.285$) for three families of solutions [see Fig. 3(bi)], the solitons obtained are not identical. The peak amplitudes of the solitons are 1.123 for $K = 3$, 1.397 for $K = 4$, and 1.642 for $K = 5$.

Furthermore, to see the impact of the depth of the vacancy defect [which is denoted by ε in Eq. (4)] on soliton dynamics, the power-eigenvalue diagrams [Figs. 4(ai)–4(ci)] and stability domains [Figs. 4(aii)–4(cii)] of gap solitons are examined in Fig. 4 for large values of ε (2, 5, and 10) with varied K. In Figs. 4(ai)–4(ci) it can be seen that the power of solitons decreases as ε increases (from 2 to 10) and in Figs. 4(aii)–4(cii) it is shown that the domain of existence is extended with larger ε and the solitons are stable on all their existence domain for varied lattice periods ($K = 3, 4, 5$) when $\varepsilon = 2$ [see Fig. 4(aii)], $\varepsilon = 5$ [see Fig. 4(bii)], and $\varepsilon = 10$ [see Fig. 4(cii)]. It can also be observed that if the depth of the vacancy defect is large enough and the lattice period ($2\pi/K$) is less than an upper limit, the solitons can be stable on all their existence domains. In particular, the lattice solitons are stable on their entire existence domain for $K \geqslant 5$ when $\varepsilon \geqslant 0.4$, for $K \geqslant 4$ when $\varepsilon \geqslant 1.13$, and for $K \geqslant 3$ when $\varepsilon \geqslant 1.84$. These results reveal that the strong (deeper) vacancy defect in a square lattice improves both the existence and stability intervals of fundamental solitons around the defect for larger lattice periods.

To test the nonlinear stability, the nonlinear evolution of the solitons is examined through direct simulation of the governing equation (3). The finite-difference method is applied for the spatial domain and the solution is advanced in the z direction with a fourth-order Runge-Kutta method. The starting point of the evolution is chosen to be a steady-state solution that is calculated by the SOM and it is perturbed with 1% random noise in amplitude.

The stability of fundamental solitons, which correspond to the points s in Figs. 3(aii)–3(cii), are examined in Fig. 5 by linear stability spectra (first column), nonlinear evolution of peak amplitudes (second column), a 3D view of the fundamental soliton at $z = 0$ (third column), and the evolved soliton at $z = 500$ (fourth column). The fundamental solitons are obtained on the local minimum of the periodic lattice when $\mu = -1.4$ and $K = 4$ [Fig. 5(a)], around the vacancy defect when $\mu = -1.5$ and $K = 4$ [Fig. 5(b)], and near the edge dislocation when $\mu = -1.92$ and $K = 5$ [Fig. 5(c)]. Figure 5 shows that the spectrum of the solitons is purely imaginary in each case, the peak amplitude of the evolved solutions fluctuate relatively little, and the form of evolved solitons is preserved during the evolution. Thus, these solitons are considered to be stable.

To investigate the impact of the lattice period, the same stability analysis is repeated in Fig. 6 for the solitons that correspond to the points u in Figs. 3(aii)–3(cii). The fundamental solitons are obtained on the local minimum of the

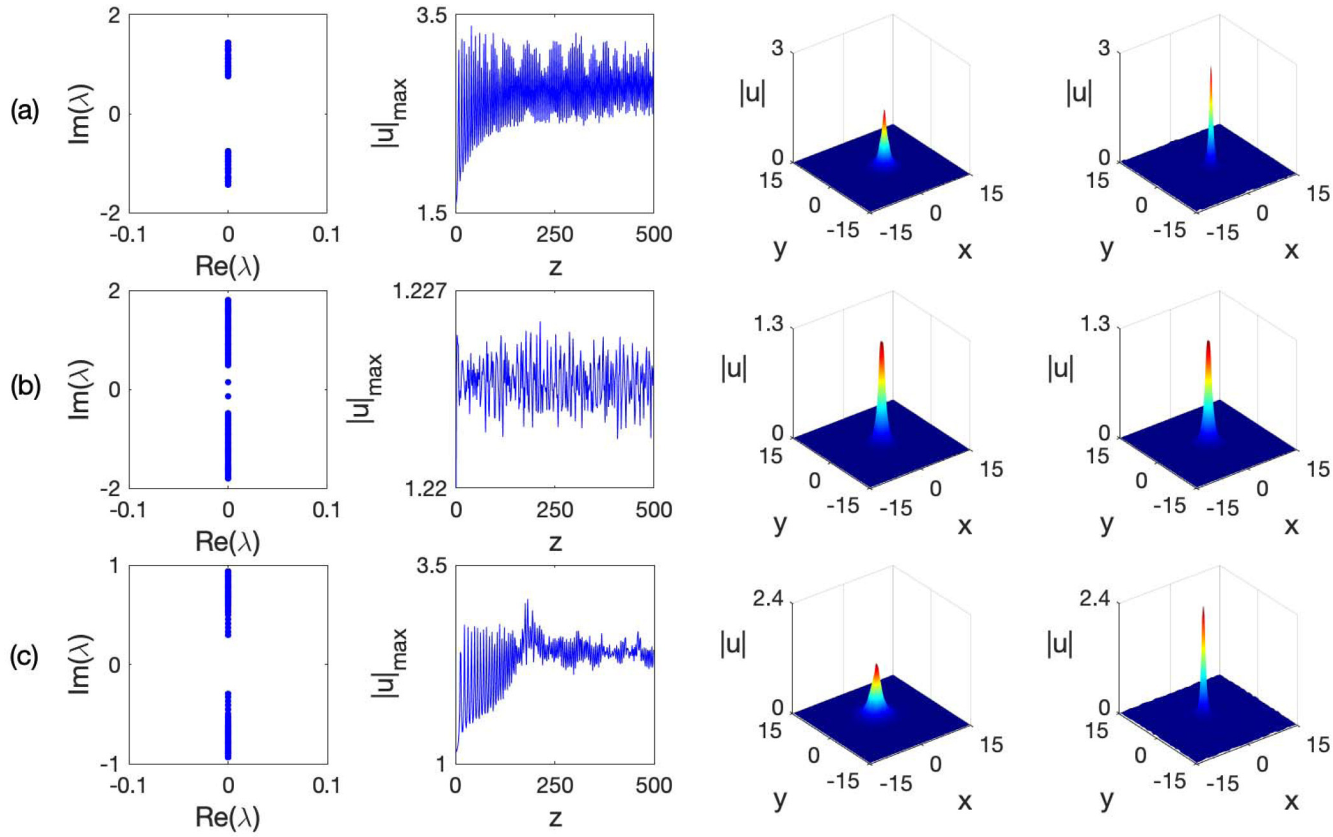

FIG. 5. Linear spectra (first column), nonlinear evolution of the peak amplitude (second column), and 3D view of the soliton before evolution (at $z = 0$) (third column) and after evolution (at $z = 500$) (fourth column) for the stable solitons that are shown by the points s in Figs. 3(aii)–3(cii). The fundamental solitons are obtained (a) on the periodic lattice when $\mu = -1.4$ and $K = 4$, (b) around the vacancy defect when $\mu = -1.5$ and $K = 4$, and (c) near the edge dislocation when $\mu = -1.92$ and $K = 5$.

periodic lattice when $\mu = -1.5$ and $K = 5$ [Fig. 6(a)], around the vacancy defect when $\mu = -1.2$ and $K = 5$ [Fig. 6(b)], and near the edge dislocation when $\mu = -1.5$ and $K = 3$ [Fig. 6(c)].

As can be seen from Fig. 6, the linear stability spectra of the solitons (first column) include eigenvalues with a positive real part and peak amplitudes of the evolved solitons (second column) blow up after a short propagation distance for the soliton on the periodic lattice [Fig. 6(a)] and around the vacancy defect [Fig. 6(b)]. The solitons near the edge dislocation decay just after $z = 40$ and the soliton profile breaks up after evolution [Fig. 6(c)]. These results indicate that the increased lattice period has an adverse effect on the stability of defect solitons.

The stability analysis performed shows that the linear spectra and nonlinear evolution of solitons are consistent for the considered lattices in each case and the (in)stability of the examined solitons obeys the VK stability criterion.

IV. CONCLUSION

Fundamental solitons were obtained numerically in a periodic lattice, around a vacancy defect, and near an edge dislocation for varied lattice periods ($2\pi/K$). The stability dynamics of these solitons has been studied by the linear spectra and nonlinear evolution of the peak amplitudes. The existence and stability domain of solitons were determined for the propagation constant μ when the lattice period is varied. It has been observed that the domain of existence is extended with increasing lattice period for the periodic lattice and the square lattice with an edge dislocation.

The stability analysis showed that the stability of solitons around the vacancy defect and near the edge dislocation can be improved with decreasing lattice period. Conversely, a higher lattice period supports the stability of periodic lattice solitons. Moreover, the numerical results revealed that there is a threshold value of the lattice period ($2\pi/4.14$) below which all vacancy defect solitons are linearly stable for the considered parameter regime. A similar lattice period threshold ($2\pi/4.27$), above which the solitons are collapse-free, was determined for the periodic lattice solitons.

Furthermore, it has been demonstrated that when the depth of the vacancy defect ε is large enough and the lattice period $2\pi/K$ is less than a threshold value, the solitons can be stable on all their existence domains. Thus, the stability of solitons around the vacancy defect can be improved by a deeper vacancy defect for larger lattice periods.

In conclusion, it has been observed that stable solitons can be obtained for a wide range of parameters on both periodic and defective lattices and it has been demonstrated that the variation of the lattice period can be applied as a collapse arrest mechanism for lattice solitons.

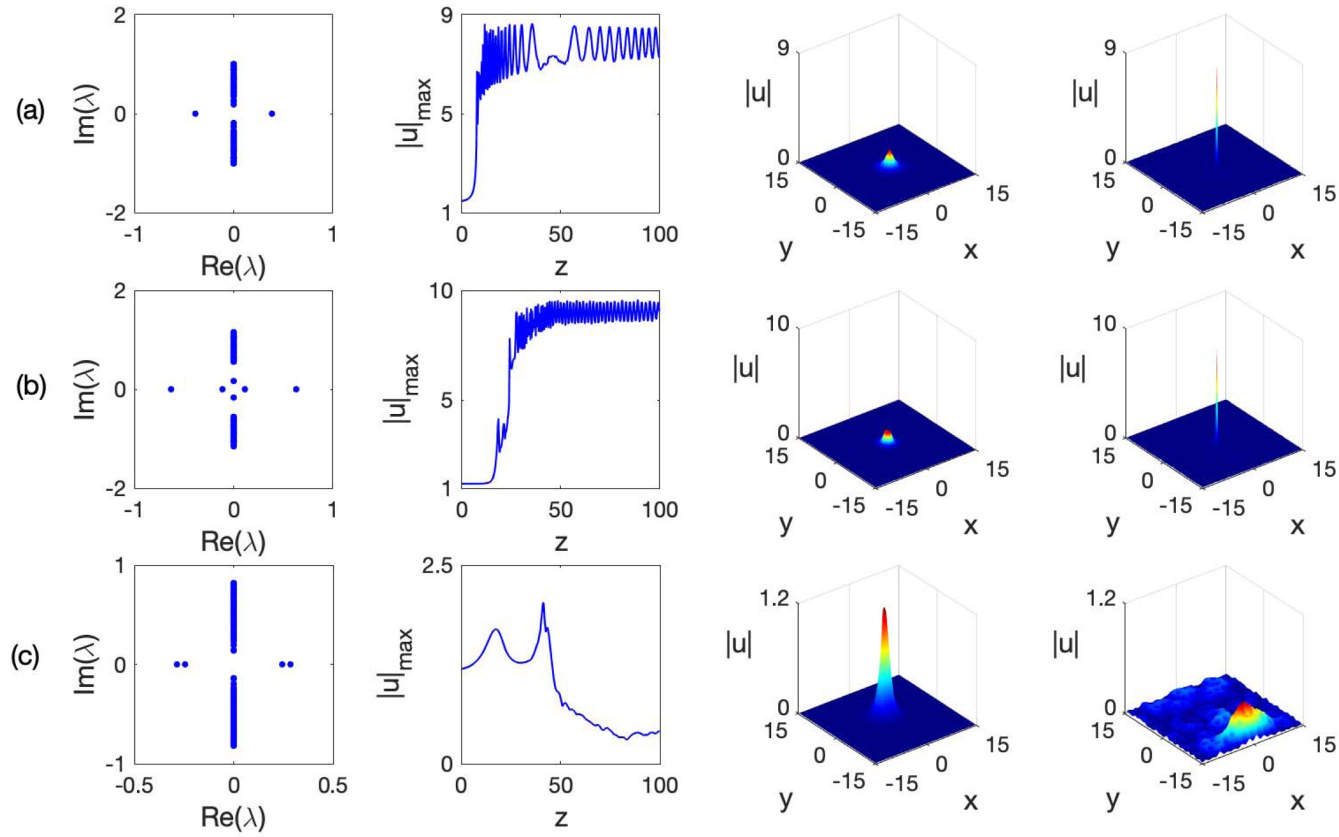

FIG. 6. Linear spectra (first column), nonlinear evolution of the peak amplitude (second column), and 3D view of the soliton before evolution (at $z = 0$) (third column) and after evolution (at $z = 100$) (fourth column) for the unstable solitons that are shown by the points u in Figs. 3(aii)–3(cii). The fundamental solitons are obtained (a) on the periodic lattice when $\mu = -1.5$ and $K = 5$, (b) around the vacancy defect when $\mu = -1.2$ and $K = 3$, and (c) near the edge dislocation when $\mu = -1.5$ and $K = 3$.

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