Cancellation of D_1 line transitions of alkali-metal atoms by magnetic-field values

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In this work, π, σ^+, and σ^- transitions between magnetic sublevels of the D_1 line of all alkali-metal atoms are considered analytically. General block Hamiltonian matrices in the presence of a magnetic-field for the ground and excited states are built in order to describe all the transitions. Eigenvalues and eigenkets describing ground and excited levels are calculated and modified and unperturbed transfer coefficients as a function of the nuclear spin I, the magnetic quantum number m, and the magnetic-field magnitude B are defined. Transition cancellations are observed only for some π transitions of each isotope. The main result is that we obtain a single formula which expresses the magnetic-field values canceling these transitions. These values also correspond to the case when the intensities of some of the other transitions reach their maximum. In addition, we examine the derivative of π transition modified transfer coefficients in order to find the magnetic-field values which correspond to the intensities' maximum. The accuracy of the magnetic-field B values is only limited by the uncertainty of the physical quantities involved.

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I. INTRODUCTION

Alkali-metal vapors are widely used in atomic physics, e.g., in laser experiments [1], information storage [2], spectroscopy [3–5], magnetometry [6,7], and laser frequency stabilization [8], and are also the main material to study Bose-Einstein condensates [9,10]. This is due to the fact that alkali-metal atoms have a high transition intensity close to the infrared range. Continuous wave narrowband diode lasers operating in this domain have good features and are cheap, which makes the experiments easier. The above-mentioned properties make the study of alkali-metal vapor transitions very important, especially in an external magnetic field.

It is well known that in a moderate external magnetic field B, atomic energy levels split into magnetic sublevels (Zeeman splitting). The frequency difference between ground and excited sublevels deviates greatly from the linear behavior [11–13]. Significant changes also occur for atomic transition probabilities. In the case of small values of B (up to approximately 1000 G), the Zeeman split hyperfine transitions overlap because of Doppler broadening. To study the behavior of each atomic transition, one should use sub-Doppler techniques [14,15]. It was demonstrated [16] that by derivative selective reflection, strong line narrowing can be achieved.

In this paper modified and unperturbed transition coefficients for the D_1 line are analytically obtained. A general formula for the magnetic-field values canceling some transitions while leading to the maximum of others is extracted. We determine an expression describing the cancellation or maximum value of the transitions for the same magnetic quantum number m.

From the point of view of numerical simulations, all stable and long-lived isotope modified transfer coefficients within a magnetic field are examined and B-field values for which the transition intensities are maximum or zero are calculated.

II. THEORY

Fine structure is the splitting of main spectral lines of an atom. It is the result of the coupling between the orbital angular momentum $\mathbf{L}$ and spin angular momentum $\mathbf{S}$ of the single optical electron. The total electron angular momentum can be written in the form

$$\mathbf{J} = \mathbf{L} + \mathbf{S}. \tag{1}$$

For $s \rightarrow p$ transitions, for the ground state we have $L = 0$ and $S = 1/2$ and for the excited state $L = 1$ and $S = 1/2$.

The hyperfine structure is the result of the combination between the total electron angular momentum $\mathbf{J}$ and the total nuclear angular momentum $\mathbf{I}$ of the atom. The total angular momentum $\mathbf{F}$ is the sum of $\mathbf{I}$ and $\mathbf{J}$:

$$\mathbf{F} = \mathbf{I} + \mathbf{J}. \tag{2}$$

In this work only the D_1 line of alkali-metal atoms is considered. In these cases the value of the total electron angular momentum magnitude is $J = 1/2$. The total atomic angular momentum magnitude takes the values

$$I - \tfrac{1}{2} \leqslant F \leqslant I + \tfrac{1}{2}, \tag{3}$$

where I is the magnitude of the total nuclear angular momentum. For all alkali-metal atoms the total nuclear angular momentum is an integer or half-integer quantity. For the F

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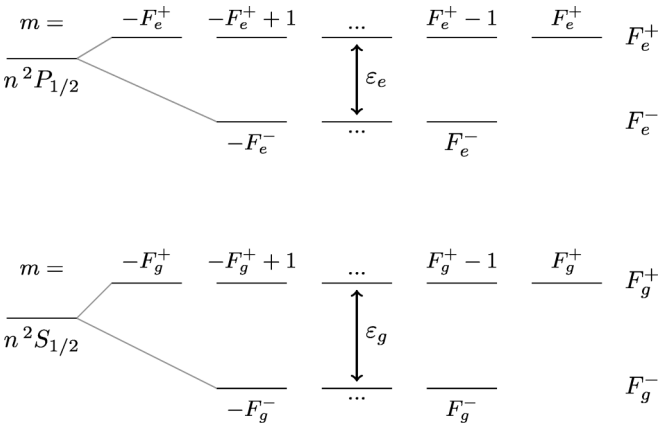

FIG. 1. Scheme of the D_1 line in a magnetic field, when I is a half-integer quantity.

number the following notation will be used:

$$F_{g,e}^{\pm} = I \pm \tfrac{1}{2}. \tag{4}$$

The indices g and e stand for the ground and excited states, respectively.

Within a magnetic field, the D_1 line energy levels split into several magnetic sublevels, which are described by magnetic quantum numbers m, which can take the values

$$-F \leqslant m \leqslant F. \tag{5}$$

Figure 1 depicts the D_1 line scheme when I is a half-integer quantity. The following denotations are used: n is the principal quantum number, which generally describes the system; $\varepsilon_g = E_0(F_g^+) - E_0(F_g^-)$ is the energy difference between ground levels; and $\varepsilon_e = E_0(F_e^+) - E_0(F_e^-)$ is the energy difference between excited levels. It should be noted that when I is an integer number, the hyperfine structure is inverted.

Within a static magnetic field $\mathbf{B}$, the Hamiltonian $\mathcal{H}$ is the sum of the unperturbed Hamiltonian and the Zeeman Hamiltonian. We choose the direction of quantization axis to be the same as the direction of magnetic field [11]. Taking into account the value of J, in the unperturbed basis $|F, m\rangle$, the diagonal elements of the Hamiltonian matrix $\mathcal{H}$ are

$$\langle F, m|\,\mathcal{H}\,|F, m\rangle = E_0(F) - \mu_B g_F(F) m B, \tag{6}$$

where $E_0(F)$ is the zero-field energy of the level with total angular momentum value F, μ_B is the Bohr magneton, $g_F(F)$ is the associated Landé factor, and B is the magnitude of the magnetic field. Nondiagonal elements can be expressed in the form

$$\begin{aligned}\langle F, m|\,\mathcal{H}\,|F-1, m\rangle &= \langle F-1, m|\,\mathcal{H}\,|F, m\rangle \\ &= -\frac{\mu_B}{2}(g_J - g_I)B\sqrt{1-\left(\frac{2m}{1+2I}\right)^2},\end{aligned} \tag{7}$$

where g_J and g_I are the total angular and nuclear Landé factors [17], respectively. For the ground and excited states $g_J^g = g_S$ and $g_J^e = \frac{4g_L - g_S}{3}$. As F quantum numbers for ground and excited states are the same, in Eq. (6) we can use the following formulas for $g_F(F)$:

$$g_F(F_{g,e}^-) = g_I + \frac{g_I - g_J^{g,e}}{1+2I}$$

and

$$g_F(F_{g,e}^+) = \frac{g_J^{g,e} + 2g_I I}{1+2I}.$$

As we consider D_1 line transitions within a magnetic field, for a complete description of the system it is enough to write general 2×2 block matrices for the ground and excited states, where each block matrix corresponds to a given value of m, using Eqs. (6) and (7). We will not write Hamiltonian elements for the $m = \pm F_{g,e}$ values because they correspond to pure states and the corresponding transitions do not depend on the magnetic-field value B. The zero-field energies E_0 have been set to zero for both ground- and excited-state lower levels. Below, the matrices $\mathcal{H}_G$ and $\mathcal{H}_E$ describe the ground and excited states and can be written as

$$\mathcal{H}_{G,E} = \begin{array}{c} \\ \langle F_{g,e}^+, m_{g,e}| \\ \langle F_{g,e}^-, m_{g,e}| \end{array} \begin{array}{c} \begin{array}{cc} |F_{g,e}^+, m_{g,e}\rangle & \qquad |F_{g,e}^-, m_{g,e}\rangle \end{array} \\ \begin{pmatrix} \varepsilon_{g,e} - \mu_B \dfrac{f_{g,e}}{1+2I} m_{g,e} B & \dfrac{\mu_B}{2} g_{g,e} B \sqrt{1-\left(\dfrac{2m_{g,e}}{1+2I}\right)^2} \\ \dfrac{\mu_B}{2} g_{g,e} B \sqrt{1-\left(\dfrac{2m_{g,e}}{1+2I}\right)^2} & -\mu_B\left(g_I + \dfrac{g_{g,e}}{1+2I}\right) m_{g,e} B \end{pmatrix} \end{array}, \tag{8}$$

where $g_g = g_I - g_S$, $g_e = \frac{3g_I - 4g_L + g_S}{3}$, $f_g = g_s + 2g_I I$, and $f_e = \frac{4g_L - g_S + 6g_I I}{3}$. After diagonalization, the eigenvalues of the matrices are given by

$$\begin{aligned}\Lambda_{G,E}^{\pm} = &\frac{\varepsilon_{g,e} - 2\mu_B g_I m_{g,e} B}{2} \\ &\pm \frac{1}{2}\left(\varepsilon_{g,e}^2 + \mu_B^2 g_{g,e}^2 B^2 + \frac{4\varepsilon_{g,e}\mu_B g_{g,e} m_{g,e} B}{1+2I}\right)^{1/2}.\end{aligned} \tag{9}$$

The eigenkets corresponding to $\Lambda_{G,E}^{\pm}$, expressed in terms of unperturbed state vectors $|\psi(F_{g,e}, m_{g,e})\rangle = \sum_{F'_{g,e}} c_{F_{g,e}F'_{g,e}} |F'_{g,e}, m_{g,e}\rangle$, are

$$\begin{aligned}|\psi(F_{g,e}^{\pm}, m_{g,e})\rangle = &\frac{1}{\sqrt{1+\kappa_{g,e\pm}^2}} |F_{g,e}^+, m_{g,e}\rangle \\ &+ \frac{\kappa_{g,e\pm}}{\sqrt{1+\kappa_{g,e\pm}^2}} |F_{g,e}^-, m_{g,e}\rangle,\end{aligned} \tag{10}$$

TABLE I. Values used to calculate transitions between D_1 line magnetic sublevels with their uncertainties. The asterisks mark the calculated values of g_L using the exact formula of Phillips [33] and values for the isotopes of Audi *et al.* [34].

Isotope	I	g_L	g_I [3]	ε_g (MHz)	ε_e (MHz)
^{23}Na	3/2	0.99997613 [19]	−0.00080461080(80)	1771.6261288(10) [3]	188.697(14) [20,21]
^{39}K	3/2	0.99997905339670(14)*	−0.00014193489(12)	461.73(14) [22]	57.696(10) [20]
^{40}K	4	0.99997974531640(14)*	0.000176490(34)	−1285.87(35) [22]	−155.31(35) [22]
^{41}K	3/2	0.99998039390246(13)*	−0.00007790600(8)	253.99(12) [3,22,23]	30.50(16) [22]
^{85}Rb	5/2	0.99999354 [24]	−0.00029364000(60)	3035.7324390(60) [3]	361.58(17) [25,26]
^{87}Rb	3/2	0.99999369 [27]	−0.0009951414(10)	6834.682610904290(90) [28]	814.50(13) [3,25,26]
^{133}Cs	7/2	0.99999587 [29]	−0.00039885395(52)	9192.631770 (exact) [29]	1167.680(30) [30,31]

where we defined

$$\kappa_{g,e\pm} = \frac{2(1+2I)(\Lambda^{\pm}_{G,E} - \varepsilon_{g,e}) + 2\mu_B f_{g,e} m_{g,e} B}{\mu_B g_{g,e} B \sqrt{(1+2I)^2 - 4m^2_{g,e}}}.$$

The relation which defines the electric dipole component D_q [11] is

$$|\langle e|\, D_q\, |g\rangle|^2 = \frac{3\varepsilon_0 \hbar \Gamma_e \lambda^3_{eg}}{8\pi^2} \times a^2[|\psi(F_e, m_e)\rangle\,; |\psi(F_g, m_g)\rangle\,; q], \quad (11)$$

where ε_0 is the vacuum electric permittivity, Γ_e is the natural decay rate, λ_{eg} is the wavelength between ground and excited states, and $q = 0, \pm 1$ corresponds to π and $\sigma^{\pm}$ transitions, respectively. The definition of the modified transfer coefficient is

$$a[|\psi(F_e, m_e)\rangle; |\psi(F_g, m_g)\rangle\,; q] = \sum_{F'_e, F'_g} c_{F_e F'_e} a(F'_e, m_e; F'_g, m_g; q) c_{F_g F'_g}, \quad (12)$$

where $a(F_e, m_e; F_g, m_g; q)$ are the unperturbed transfer coefficients

$$a(F_e, m_e; F_g, m_g; q) = (-1)^{3/2+I+F_e+F_g-m_e} \sqrt{2}\sqrt{2F_e+1} \times \sqrt{2F_g+1} \begin{pmatrix} F_e & 1 & F_g \\ -m_e & q & m_g \end{pmatrix} \begin{Bmatrix} F_e & 1 & F_g \\ \frac{1}{2} & I & \frac{1}{2} \end{Bmatrix}, \quad (13)$$

which depend on Wigner 3-j and 6-j symbols.

The σ^+ and σ^- transitions are not canceled for any value of the magnetic field. Cancellations occur only for π transitions. Let us examine the unperturbed transfer coefficients $a(F_e, m; F_g, m; 0)$. Two of them have the following expression:

$$a(F^{\pm}_e, m; F^{\pm}_g, m; 0) = \pm \frac{1}{\sqrt{3}} \frac{2m}{1+2I}. \quad (14)$$

For the next two unperturbed coefficients the expression is

$$a(F^{\pm}_e, m; F^{\mp}_g, m; 0) = \frac{1}{\sqrt{3}} \sqrt{1 - \left(\frac{2m}{1+2I}\right)^2}. \quad (15)$$

From Eqs. (12) and (13) and formulas (14) and (15), modified transfer coefficients that have a cancellation are

$$\begin{aligned} a[|\psi(F^{\pm}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0] &= \frac{\kappa_{e\pm}}{\sqrt{1+\kappa^2_{e\pm}}} a(F^-_e, m; F^-_g, m; 0) \frac{\kappa_{g\pm}}{\sqrt{1+\kappa^2_{g\pm}}} \\ &+ \frac{\kappa_{e\pm}}{\sqrt{1+\kappa^2_{e\pm}}} a(F^-_e, m; F^+_g, m; 0) \frac{1}{\sqrt{1+\kappa^2_{g\pm}}} \\ &+ \frac{1}{\sqrt{1+\kappa^2_{e\pm}}} a(F^+_e, m; F^-_g, m; 0) \frac{\kappa_{g\pm}}{\sqrt{1+\kappa^2_{g\pm}}} \\ &+ \frac{1}{\sqrt{1+\kappa^2_{e\pm}}} a(F^+_e, m; F^+_g, m; 0) \frac{1}{\sqrt{1+\kappa^2_{g\pm}}}. \end{aligned} \quad (16)$$

The solutions of $a[|\psi(F^{\pm}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0] = 0$ are

$$B^{\pm}_{\pm} = -\frac{2m}{\mu_B(1+2I)} \frac{2\varepsilon_e \varepsilon_g}{g_g \varepsilon_e + g_e \varepsilon_g}. \quad (17)$$

The condition on the considered solutions of the modified transfer coefficients which defines permissible values of the magnetic quantum number m depends on nuclear spin:

$$0 \leqslant (-1)^{2I} m \leqslant I - \tfrac{1}{2}. \quad (18)$$

From the formula (17) one notes that for isotopes having a half-integer nuclear spin, transition cancellations exist for π transitions between levels for which the magnetic quantum number is zero ($m = 0$); however, as the atomic states are degenerated, it is not possible to observe cancellation of these transitions.

Modified transfer coefficients $a[|\psi(F^{\mp}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0]$ cannot be equal to zero, but these quantities have a very interesting behavior. While for certain values of B transition intensities corresponding to $a[|\psi(F^{\pm}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0]$ are zero (transition cancellation), the transition intensities corresponding to $a[|\psi(F^{\mp}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0]$ reach their maximum value, which corresponds to the intensity of a π transition occurring between pure states (so-called "guiding" atomic transitions [18]). This is ensured by the calculation of the derivative of modified transfer coefficients squared with respect to the magnetic field:

$$\frac{da^2[|\psi(F^{\mp}_e, m)\rangle\,, |\psi(F^{\pm}_g, m)\rangle\,, 0]}{dB} = 0. \quad (19)$$

TABLE II. Values of the B field canceling transitions of ^{85}Rb with their uncertainties.

Isotope	No.	F	m	B (G)
^{85}Rb	1	2	−2	380.73(13)
^{85}Rb	2	3	−2	380.73(13)
^{85}Rb	3	2	−1	190.368(66)
^{85}Rb	4	3	−1	190.368(66)

The solution of Eq. (19) is exactly the formula (17) with the condition mentioned in the formula (18). We call quantities $a[|\psi(F_e^{\pm}, m)\rangle, |\psi(F_g^{\pm}, m)\rangle, 0]$ and $a[|\psi(F_e^{\mp}, m)\rangle, |\psi(F_g^{\pm}, m)\rangle, 0]$ pair-modified transfer coefficients and transitions corresponding to them pair-transitions. As one can notice, cancellations occur only for transitions obeying $\Delta F = F_e - F_g = 0$ and maximum values take place when $\Delta F = F_e - F_g = \pm 1$.

III. ANALYSIS OF STABLE AND LONG-LIVED ISOTOPES

In this section we fully analyze D_1 line transition cancellations and maxima of ^{23}Na, ^{39}K, ^{40}K, ^{41}K, ^{85}Rb, ^{87}Rb, and ^{133}Cs atoms. All the mentioned isotopes except ^{40}K and ^{87}Rb are stable. The half-life of ^{40}K is $1.248(3) \times 10^9$ and that of ^{87}Rb is $49.23(22) \times 10^9$ years. It should be noted that we do not study all the possible isotopes of all alkali-metal atoms mainly due to the lack of data on these isotopes and also because their half-life is too short to envisage an experiment in the near future. However, the present theory is still valid to study them.

In Table I all considered isotope data are given with uncertainties. As one can see, the most imprecise values in general are ε_e; however, for ^{39}K, ^{40}K, and ^{41}K frequency differences between ground-state levels are not precisely known. These quantities have the most influence on the size of the uncertainties of the calculated B values. It should be noted that when the value of I is an integer (only for ^{40}K in this work), the values of ε_g and ε_e should have a minus sign to be in agreement with our notation. For further calculations, for the Bohr magneton and g_S spin Landé factor we used the values $\mu_B/h = -1.399\,624\,504\,2(86)$ MHz/G and $g_S = 2.002\,319\,304\,362\,2(15)$ [32], respectively. We noticed that in the paper of Phillips [33] $1/m$ is missing in the second term of the exact formula for g_L.

In order that the cancellation and maximization of transitions could be well understood by the reader, as an example we will show figures only for the ^{85}Rb isotope. The total atomic angular momentum magnitude is $F = 2$ for the lower levels of ground and excited states and $F = 3$ for the upper levels. Some transitions cancel for $m = -2$ and -1. For this isotope we will also analyze those transitions which have a maximum value, as mentioned before. The π transitions corresponding to the above-mentioned magnetic quantum numbers which do not cancel reach their maximum values for the magnetic-field values canceling the other transitions. Figure 2(a) depicts the modified transfer coefficients (i.e., $a[|\psi(F_e, m)\rangle, |\psi(F_g, m)\rangle, 0]$ quantities) for $m = -2$ and -1. Lines numbered 5–8 have no cancellation and are nothing more than transition coefficients between ground and excited states, where $F_g \neq F_e$. As an illustration, the dashed line in Fig. 2(a) indicates that the intensity of transitions 1 and 2 is equal to zero for the same value of B. Figure 2(b) depicts the modified transfer coefficients squared in order to compare them with each other and with the guiding atomic transition coefficient for which $m = -3$. To extend the previous illustration with the dashed line in Fig. 2(a), we draw it in the case of the modified transfer coefficients squared to show explicitly that the minimum of transitions 1 and 2 (point A) coincides with the maximum of transitions 5 and 6 (point B).

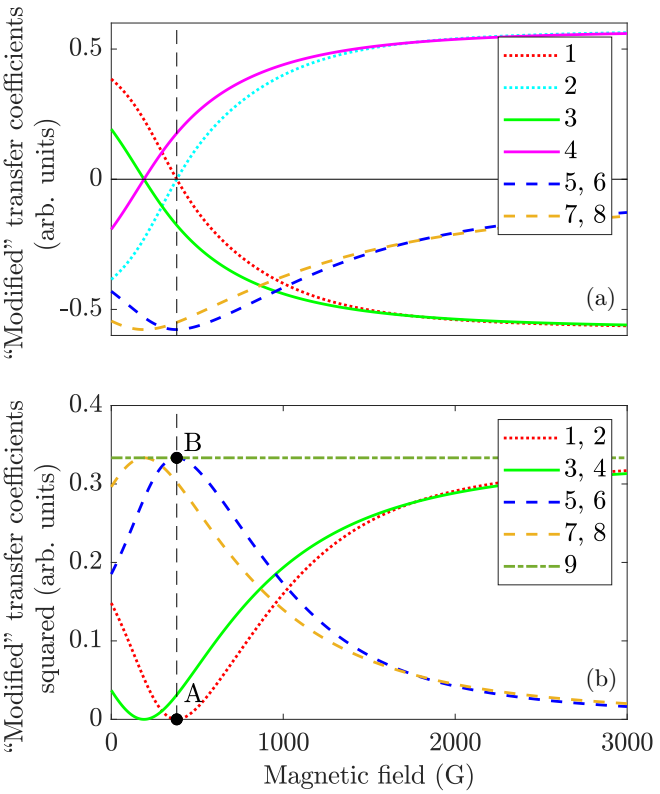

FIG. 2. (a) ^{85}Rb D_1 line modified transfer coefficients for $m = -2$ and -1 π transitions. For this isotope four cancellations exist: 1, 2, 3, and 4. (b) Modified transfer coefficients squared for $m = -3$, -2, and -1. The vertical dashed line indicates the value $B = 380.73$ G, which corresponds to the cancellation of the transitions 1 and 2 (point A) and coincides with the maximum of transitions 5 and 6 (point B). Numbers 1–8 given in the inset are defined in Tables II and III; the guiding transition coefficient squared 9 corresponds to $m = -3$.

In Table II all B-field values which cancel transitions of the ^{85}Rb D_1 line are listed. One also can see that for these

TABLE III. Values of the B field maximizing transitions of ^{85}Rb with their uncertainties.

Isotope	No.	F	m	B (G)
^{85}Rb	5	−1	−2	380.73(13)
^{85}Rb	6	1	−2	380.73(13)
^{85}Rb	7	−1	−1	190.368(66)
^{85}Rb	8	1	−1	190.368(66)

TABLE IV. B-field values canceling transitions of ^{23}Na, ^{39}K, ^{40}K, ^{41}K, ^{87}Rb, and ^{133}Cs with their uncertainties.

Isotope	F	m	B (G)
^{23}Na	1	−1	153.2007(86)
^{23}Na	2	−1	153.2007(86)
^{39}K	1	−1	44.991(10)
^{39}K	2	−1	44.991(10)
^{40}K	9/2	7/2	190.20(33)
^{40}K	7/2	7/2	190.20(33)
^{40}K	9/2	5/2	135.85(24)
^{40}K	7/2	5/2	135.85(24)
^{40}K	9/2	3/2	81.51(15)
^{40}K	7/2	3/2	81.51(15)
^{40}K	9/2	1/2	27.171(48)
^{40}K	7/2	1/2	27.171(48)
^{41}K	1	−1	24.046(95)
^{41}K	2	−1	24.046(95)
^{87}Rb	1	−1	642.590(76)
^{87}Rb	2	−1	642.590(76)
^{133}Cs	3	−3	1359.237(26)
^{133}Cs	4	−3	1359.237(26)
^{133}Cs	3	−2	906.158(17)
^{133}Cs	4	−2	906.158(17)
^{133}Cs	3	−1	453.0790(84)
^{133}Cs	4	−1	453.0790(84)

values of magnetic-field transitions 5–8 (Table III) reach their maximum value equal to the transfer coefficient 9 squared.

In Table IV all B-field values which cancel D_1 line transitions of ^{23}Na, ^{39}K, ^{40}K, ^{41}K, ^{87}Rb, and ^{133}Cs are listed. The second column shows the values of the total angular momentum magnitude for both ground and excited states. The third column indicates between which magnetic sublevels the transition occurs and the fourth column displays calculated values of B field with their uncertainties, which are a consequence of the uncertainties of the physical quantities involved in the calculations.

IV. CONCLUSION AND OUTLOOK

In this work an analytical model to calculate all optical transitions within a magnetic field for all types of polarized light and for the D_1 line of all alkali-metal atoms was used. We determined a unique formula expressing magnetic-field values for which some π transition intensities become zero and some other π transition intensities become maximum simultaneously. No σ^+ or σ^- transitions have a cancellation. Several reasons lead us to find these values as precisely as possible. The first one is that for very sensitive magnetometer calibration some standards should exist and these values are good standards [35,36] for atomic systems: They do not depend on any external condition or parameter. A second reason is that relation (17) is an exact theoretical relation expressing the values of B that cancel the transitions as a function of fundamental quantities like energy differences, Landé factors, Bohr magneton, nuclear spin value, and quantum numbers. Thus, determining these B values by any experimental means as precisely as possible can lead to an improvement of the values of these fundamental quantities. However, in the experiments it is always more complicated to precisely measure small signals than big ones; thus the cancellation of transitions cannot be measured directly because of the existence of noise in any experiment. In other words, as for small peaks the signal-to-noise ratio is smaller than for higher peaks, it is profitable to measure peaks with bigger intensity. Thus, in the experiment described in one of our latest papers [35], we are going to measure those transitions (e.g., find maxima) which have a maximum value. If we are able to find a magnetic-field magnitude for which the transition intensity is maximum, it will mean that we found a pair-transition cancellation value. Obviously, the graph of the derivative of the intensity with respect to B should be calculated in the neighborhood of the maximum value, despite the fact that the change of transition intensity can be very smooth, as the change of sign of the slope of the derivative will give precisely the value for which it crosses the B axis; thus will give the B value for which the pair-transitions reach their minimum.

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[1] A. Sargsyan, D. Sarkisyan, Y. Pashayan-Leroy, C. Leroy, S. Cartaleva, A. D. Wilson-Gordon, and M. Auzinsh, J. Exp. Theor. Phys. **121**, 966 (2015).

[2] R. Legaie, C. J. Picken, and J. D. Pritchard, J. Opt. Soc. Am. B **35**, 892 (2018).

[3] E. Arimondo, M. Inguscio, and P. Violino, Rev. Mod. Phys. **49**, 31 (1977).

[4] M. Auzinsh, D. Budker, and S. M. Rochester, *Optically Polarized Atoms: Understanding Light-Atom Interactions* (Oxford University Press, Oxford, 2010).

[5] A. Aleksanyan, S. Shmavonyan, E. Gazazyan, A. Khanbekyan, H. Azizbekyan, M. Movsisyan, and A. Papoyan, J. Opt. Soc. Am. B **37**, 203 (2020).

[6] Y. Yin, B. Zhou, K. Yin, Y. Wang, J. Tang, M. Ye, X. Ning, and B. Han, J. Phys. D **54**, 485001 (2021).

[7] Q. Guo, X. Li, N. Zhang, J. Lu, D. Ma, Z. Li, and S. Jiang, IEEE Sensors Journal **22**, 6476 (2022).

[8] E. Talker, R. Zektzer, Y. Barash, N. Mazurski, and U. Levy, J. Vac. Sci. Technol. B **38**, 050601 (2020).

[9] B. Evrard, A. Qu, J. Dalibard, and F. Gerbier, Science **373**, 1340 (2021).

[10] M. Hans, F. Schmutte, C. Viermann, N. Liebster, M. Sparn, M. K. Oberthaler, and H. Strobel, Rev. Sci. Instrum. **92**, 023203 (2021).

[11] P. Tremblay, A. Michaud, M. Levesque, S. Thériault, M. Breton, J. Beaubien, and N. Cyr, Phys. Rev. A **42**, 2766 (1990).

[12] N. Papageorgiou, A. Weis, V. A. Sautenkov, D. Bloch, and M. Ducloy, Appl. Phys. B **59**, 123 (1994).
[13] A. Sargsyan, G. Hakhumyan, A. Papoyan, D. Sarkisyan, A. Atvars, and M. Auzinsh, Appl. Phys. Lett. **93**, 021119 (2008).
[14] K. A. Khanbekyan, E. Mariotti, A. A. Khanbekyan, L. Moi, and A. M. Khanbekyan, Opt. Spectrosc. **120**, 701 (2016).
[15] A. Sargsyan, E. Klinger, C. Leroy, I. G. Hughes, D. Sarkisyan, and C. S. Adams, J. Phys. B **52**, 195001 (2019).
[16] A. Sargsyan, E. Klinger, G. Hakhumyan, A. Tonoyan, A. Papoyan, C. Leroy, and D. Sarkisyan, J. Opt. Soc. Am. B **34**, 776 (2017).
[17] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).
[18] A. Sargsyan, G. Hakhumyan, A. Papoyan, and D. Sarkisyan, JETP Lett. **101**, 303 (2015).
[19] D. Steck, Sodium D line data, https://steck.us/alkalidata/ (2019).
[20] D. Das and V. Natarajan, J. Phys. B **41**, 035001 (2008).
[21] W. A. van Wijngaarden and J. Li, Z. Phys. D **32**, 67 (1994).
[22] S. Falke, E. Tiemann, C. Lisdat, H. Schnatz, and G. Grosche, Phys. Rev. A **74**, 032503 (2006).
[23] T. Tiecke, Properties of potassium, https://tobiastiecke.nl/ (2019).
[24] D. Steck, Rubidium 85 D line data, https://steck.us/alkalidata/ (2019).
[25] A. Banerjee, D. Das, and V. Natarajan, Europhys. Lett. **65**, 172 (2004).
[26] G. P. Barwood, P. Gill, and W. R. C. Rowley, Appl. Phys. B **53**, 142 (1991).
[27] D. Steck, Rubidium 87 D line data, https://steck.us/alkalidata/ (2019).
[28] S. Bize, Y. Sortais, M. S. Santos, C. Mandache, A. Clairon, and C. Salomon, Europhys. Lett. **45**, 558 (1999).
[29] D. Steck, Cesium D line data, https://steck.us/alkalidata/ (2019).
[30] T. Udem, J. Reichert, R. Holzwarth, and T. W. Hänsch, Phys. Rev. Lett. **82**, 3568 (1999).
[31] R. J. Rafac and C. E. Tanner, Phys. Rev. A **56**, 1027 (1997).
[32] P. J. Mohr, D. B. Newell, and B. N. Taylor, J. Phys. Chem. Ref. Data **45**, 043102 (2016).
[33] M. Phillips, Phys. Rev. **76**, 1803 (1949).
[34] G. Audi, A. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[35] A. Aleksanyan, R. Momier, E. Gazazyan, A. Papoyan, and C. Leroy, J. Opt. Soc. Am. B **37**, 3504 (2020).
[36] R. Momier, A. Aleksanyan, E. Gazazyan, A. Papoyan, and C. Leroy, J. Quant. Spectrosc. Radiat. Transfer **257**, 107371 (2020).