Evolution of two-mode quantum states under a dissipative environment: Comparison of the robustness of squeezing and entanglement resources

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We explore the relative robustness of single-mode squeezing and entanglement (which are quantum resources interconvertible via passive optics) for two-mode Gaussian states under different dissipative environments. When the individual modes interact with identical local baths, entanglement and squeezing decay at the same rate. However, when only one of the modes interacts with a local bath, the comparative robustness of entanglement and squeezing depends on the initial squeezing of the state. Similarly, when the system interacts with a global bath, the robustness of entanglement and squeezing depends on the initial squeezing. Thus depending on the nature of dissipative environments and the initial squeezing of the state, one can select the more robust form of resource out of squeezing and entanglement to store quantumness. This can be used to effectively enhance the performance of various quantum information processing protocols based on continuous variable Gaussian states.

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I. INTRODUCTION

Continuous variable (CV) quantum information processing (QIP) based on quantum optical sources has been gaining attention over time [1,2]. Developments in continuous variable quantum key distribution (CV-QKD) protocols [3] and photonic computing [4] are particularly noteworthy. Nonclassical Gaussian states play a key role in this context, as they can be easily produced, manipulated, and measured in the laboratory [5,6]. Nonclassicality or genuine quantumness in quantum states is a resource that needs to be defined, identified, and preserved for use in QIP protocols [7–10].

In the quantum optical sense, if the Glauber-Sudarshan P function [11,12] behaves like a classical probability distribution, the corresponding state can be simulated by ensembles of solutions of Maxwell equations, and the state cannot exhibit any nonclassical features [13]. On the other hand, nonpositivity of the Glauber-Sudarshan P function indicates nonclassicality [14,15]. Squeezing, when quadrature noise drops below the shot noise limit, is one specific form of quantumness based on P representation [16]. Gaussian squeezed states are thus an important class of nonclassical states that are extremely useful for QIP. A different notion of quantumness arises from an information theory viewpoint, where correlations in composite quantum systems can go beyond classically allowed values leading to nonclassical situations. Quantum entanglement is one such resource that can lead to situations that violate local realism [17]. While single-mode squeezing and intermode entanglement are very different notions of nonclassicality, for Gaussian states they can be interconverted into each other via passive optical elements such as beam splitters, phase shifters, wave-plates, and mirrors [18–25]. However, this may not be true for general CV system states [26].

Environmental interactions can cause disturbances that invariably lead to the diminishing of quantum resources, which may be present in the form of squeezing and entanglement. The environmental interactions are detrimental to the performance of various QIP tasks. Therefore, it is of significant importance to analyze the evolution of quantum systems under different dissipative environments and find ways to protect resources against environmental effects. A lot of work has already been done in this regard [9,27–33]. Entanglement dynamics has been studied for local as well as global dissipative environments under both Markovian and non-Markovian assumptions. Many interesting phenomena have been observed, such as sudden death of entanglement in local as well as global dissipative environments [9,32]. Further, researchers have also shown that entanglement can be produced in two-mode separable squeezed states and can even be enhanced in two-mode squeezed vacuum (TMSV) states evolving under the presence of a global thermal bath [30,34]. The effect of decoherence on nonlocality, steering, entanglement of formation, and discord has also been studied [35–39]. Moreover, much research has been conducted to study decoherence in non-Gaussian states, particularly photon subtracted states [40,41].

In this article, we strive to find which of the two resources, squeezing or entanglement, is more robust to environmental noise [42–44]. We consider two different cases that provide

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an insight into the relative sensitivity of squeezing and entanglement resources to dissipative environments under the Markovian assumption. The first case considers squeezing of the individual modes followed by an evolution under a dissipative environment, and finally the two modes are entangled using passive optics (beam splitter). The second case considers squeezing of the individual modes, which then are entangled using passive optics, and finally we let them evolve in a dissipative environment. We have considered our system to be interacting with local and global thermal baths, which are Gaussian channels, i.e., Gaussian states remain Gaussian under such interactions. We provide the time-dependent covariance matrix, which is used for entanglement analysis. Although a full characterization of entanglement for CV systems is not possible, for Gaussian states, necessary and sufficient criteria for the detection of entanglement exist [7,8]. Further, we can quantify entanglement using logarithmic negativity in two-mode Gaussian states, which we use in our work [45–47].

It is natural to expect that entanglement in the TMSV state, where intermodal correlations are present, will be more fragile compared to squeezing in a two-mode separable squeezed state. However, our analysis reveals that the relative robustness of entanglement and squeezing depends on the dissipative environment with which the system is interacting. When the two modes interact with identical baths, the squeezing and entanglement decay in exactly the same way. On the other hand, when only one of the modes interacts with a local bath, the results depend on the initial squeezing of the state. There exists a threshold of the initial squeezing of the state, below which entanglement is more robust than squeezing. Otherwise, squeezing is more robust than entanglement. Similarly, such a threshold also exists when the system interacts with a global bath, below which squeezing is more robust than entanglement. Otherwise, entanglement is more robust than squeezing. We provide analytical expressions for these thresholds of the squeezing parameter, which will enable experimentalists to identify more robust resources against a dissipative environment.

The paper is organized as follows. In Sec. II, we review the formalism for the CV systems and describe the notions of squeezing and entanglement for two-mode Gaussian states. In Sec. III A, we set out to study the relative robustness of squeezing and entanglement against a disturbance caused by a noisy dissipative environment. We consider the system evolution under local thermal baths in Sec. III B, while Sec. III C deals with the evolution of the system under a global bath. Finally, in Sec. IV, we provide some concluding remarks and future directions.

II. TWO-MODE SYSTEMS: ENTANGLEMENT AND SQUEEZING

We describe the formalism of two-mode CV systems and discuss the two nonclassical notions, namely squeezing and entanglement, which we intend to study under different noisy dissipative environments.

A. Two-mode CV systems and symplectic transformations

We consider a two-mode CV quantum system described by the quadrature operators $\hat{q}_1$, $\hat{p}_1$, $\hat{q}_2$, and $\hat{p}_2$ [1,2,48,48,49]. To handle the analysis of the two-mode system compactly, we introduce the column vector

$$\hat{\boldsymbol{\xi}} = (\hat{\xi}_j) = (\hat{q}_1,\ \hat{p}_1,\ \hat{q}_2,\ \hat{p}_2)^T. \tag{1}$$

The canonical commutation relations can be written as

$$\underset{\substack{j,k=1,2,3,4\\ \hbar=1}}{[\hat{\xi}_j, \hat{\xi}_k] = i(\omega^{\oplus 2})_{jk}} \quad \text{with} \quad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{2}$$

The annihilation and creation operators $\hat{a}_j$ and $\hat{a}_j^\dagger$ $(j = 1, 2)$ can be expressed in terms of quadrature operators as follows:

$$\hat{a}_j = \frac{1}{\sqrt{2}}(\hat{q}_j + i\hat{p}_j), \quad \hat{a}_j^\dagger = \frac{1}{\sqrt{2}}(\hat{q}_j - i\hat{p}_j). \tag{3}$$

The Hilbert space of the two-mode system has an orthogonal basis in the Fock representation $|n_1, n_2\rangle$ with $\{n_1,\ n_2 = 0, 1, \ldots, \infty\}$, which are simultaneous eigenvectors of the number operators $\hat{a}_1^\dagger \hat{a}_1$ and $\hat{a}_2^\dagger \hat{a}_2$.

The linear homogeneous transformations S specified by real 4×4 matrices acting on the quadrature operators (1) and preserving the canonical commutation relation (2) form the symplectic group $\mathrm{Sp}(4,\ \mathcal{R})$. The quadrature operators transform as $\hat{\xi}_i \to \hat{\xi}_i' = S_{ij}\hat{\xi}_j$. Further, the symplectic condition for matrix S is

$$S\omega^{\oplus 2}S^T = \omega^{\oplus 2} \Rightarrow S \in \mathrm{Sp}(4,\ \mathcal{R}). \tag{4}$$

The symplectic group can be decomposed as $S = PK(X, Y)$, where P is the noncompact part and $K(X, Y)$ is the maximally compact subgroup of $\mathrm{Sp}(4, \mathcal{R})$. The elements of the set P act on the states through their infinite-dimensional unitary representation (metaplectic representation), change the total number of photons, and are called active operations. Active operations can transform a classical state into a nonclassical state and vice versa. On the other hand, the elements of $K(X, Y)$ which form the maximally compact subgroup while acting on the states via their metaplectic representation conserve the total number of photons and are called passive operations. Passive operations cannot create or destroy the nonclassicality of a state and can be implemented using passive optical elements such as beam splitters, phase shifters, waveplates, and mirrors. We discuss two basic symplectic operations that are important for our work [1,2,48,49].

Single-mode squeezing operation: The transformation matrix corresponding to the single-mode squeezing operator, which acts on the quadrature operators $\hat{q}_i$ and $\hat{p}_i$, is given by

$$S_i(r) = \begin{pmatrix} e^{-r} & 0 \\ 0 & e^{r} \end{pmatrix}. \tag{5}$$

The corresponding $\mathrm{Sp}(4, \mathcal{R})$ element will be $S_1(r) \oplus \mathbb{1}_2$ or $\mathbb{1}_2 \oplus S_2(r)$ depending upon which of the two modes the squeezing operator acts. Here $\mathbb{1}_2$ represents a 2×2 identity matrix. Single-mode squeezing operators are active operations that can transform a classical state into a nonclassical state.

Beam splitter operation: The transformation matrix corresponding to a beam splitter acting on the two-mode quadrature operators $\hat{\xi} = (\hat{q}_1,\ \hat{p}_1,\ \hat{q}_2,\ \hat{p}_2)^T$ is given by

$$B_{12}(\theta) = \begin{pmatrix} \cos\theta\, \mathbb{1}_2 & \sin\theta\, \mathbb{1}_2 \\ -\sin\theta\, \mathbb{1}_2 & \cos\theta\, \mathbb{1}_2 \end{pmatrix}, \tag{6}$$

where θ is related to the transmittance of the beam splitter according to $\tau = \cos^2\theta$. Angle $\theta = \pi/4$ corresponds to a balanced (50 : 50) beam splitter. Beam splitter transformation is a passive operation belonging to the $K(X, Y)$ subgroup of $Sp(4, \mathcal{R})$, which cannot transform the classical or nonclassical status of a state.

B. Squeezing and entanglement in Gaussian states

Gaussian states can be described by Gaussian-Wigner functions in phase space. They can be completely specified by the mean values and covariances of the quadrature operators. However, without any loss of generality, we can consider a zero-centered state as any general Gaussian state can be made zero-centered by application of the displacement operator without affecting the entanglement content of the state. The Wigner function for zero-centered Gaussian states can be written as [1]

$$W(\boldsymbol{\xi}) = \frac{\exp[-(1/2)\boldsymbol{\xi}^T V^{-1}\boldsymbol{\xi}]}{(2\pi)^2\sqrt{\det V}}, \tag{7}$$

where V is the covariance matrix whose elements are given by

$$V = (V_{ij}) = \tfrac{1}{2}\langle\{\Delta\hat{\xi}_i, \Delta\hat{\xi}_j\}\rangle, \tag{8}$$

where $\Delta\hat{\xi}_i = \hat{\xi}_i - \langle\hat{\xi}_i\rangle$, and $\{,\}$ denotes the anticommutator. The uncertainty principle can be expressed in terms of the covariance matrix as

$$V + \frac{i}{2}\Omega \geqslant 0. \tag{9}$$

The covariance matrix for the zero-centered two-mode Gaussian state is given by

$$V = \begin{pmatrix} \langle q_1^2\rangle & \frac{1}{2}\langle\{q_1, p_1\}\rangle & \langle q_1 q_2\rangle & \langle q_1 p_2\rangle \\ \frac{1}{2}\langle\{q_1, p_1\}\rangle & \langle p_1^2\rangle & \langle q_2 p_1\rangle & \langle p_1 p_2\rangle \\ \langle q_1 q_2\rangle & \langle q_2 p_1\rangle & \langle q_2^2\rangle & \frac{1}{2}\langle\{q_2, p_2\}\rangle \\ \langle q_1 p_2\rangle & \langle p_1 p_2\rangle & \frac{1}{2}\langle\{q_2, p_2\}\rangle & \langle p_2^2\rangle \end{pmatrix}. \tag{10}$$

A quantum state is said to be squeezed in quadrature $\hat{\xi}_i$ if the fluctuations in the corresponding quadrature reduce below the coherent state value, i.e., $(\Delta\hat{\xi}_i)^2 < 1/2$. The single-mode squeezing operator $S(r)$ (5) acting on a state through its metaplectic representation alters fluctuations in the quadratures, and hence can transform a nonsqueezed state into a squeezed state. Specifically, as an example, if $S(r)$ acts on the first mode, the fluctuations in the $\hat{\xi}_1$ quadrature transform as $(\Delta\hat{\xi}_1)^2 \rightarrow e^{-2r}(\Delta\hat{\xi}_1)^2$. Since one is allowed to redefine quadratures by mixing them via passive operations, a state is squeezed even if the noise in one of the transformed quadratures falls below the coherent state value. We will consider squeezing caused by single-mode squeezing transformations $S(r)$ as described above.

While the detection of entanglement in general states of CV systems remains an open problem, for Gaussian states the Simon criterion provides necessary and sufficient conditions for the detection of entanglement [7,8]. By Simon's criterion,

$$\det A \det B + \left(\tfrac{1}{4} - |\det C|\right)^2 - \mathrm{Tr}[A\omega C\omega B\omega C^T\omega] \\ -\tfrac{1}{4}(\det A + \det B) \geqslant 0 \tag{11}$$

is necessary and sufficient for a state to be separable. Here A, B, and C are 2×2 matrices and are related to the covariance matrix of a two-mode system as

$$V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}. \tag{12}$$

Furthermore for Gaussian states, the logarithmic negativity can be used as a measure of entanglement between the two modes. The logarithmic negativity for a two-mode Gaussian state is defined as

$$E_N = \max\{0, -\log_2(2n_-)\}, \tag{13}$$

where n_- is the smallest symplectic eigenvalue of the partially transposed covariance matrix, which can be compactly written using the block matrices form of the covariance matrix (12) as follows:

$$n_-^2 = \tfrac{1}{2}[\Sigma - \sqrt{\Sigma^2 - 4\det V}], \tag{14}$$

where $\Sigma = \det A + \det B - 2\det C$. We employ this measure to quantify entanglement throughout this paper.

III. EFFECT OF ENVIRONMENT ON QUANTUM RESOURCES OF SQUEEZING AND ENTANGLEMENT

In this section, we set out to explore the effect of environmental coupling on diminishing the quantum resources of squeezing and entanglement. We couple the two-mode system with thermal baths in several different ways to study the relative performance of squeezing and entanglement in terms of their ability to withstand the environmental effects.

A. Environment coupled to the two-mode optical system

We consider a two-mode system interacting with a thermal bath. The bath is considered to be comprised of a large set of harmonic oscillators. The corresponding total Hamiltonian can be written as

$$\hat{H} = \hat{H}_{\rm S} + \hat{H}_{\rm B} + \hat{H}_{\rm SB}, \tag{15}$$

where $\hat{H}_{\rm S}$ and $\hat{H}_{\rm B}$ are the system and the bath Hamiltonians, respectively, and $\hat{H}_{\rm SB}$ is the interaction Hamiltonian. We consider two distinct cases:

Case 1: In this case, a two-mode separable squeezed state interacting with a thermal bath is considered. We start with a two-mode system initialized to the vacuum state $|\mathbf{0}\rangle$, which can be represented by the following covariance matrix:

$$V_{|\mathbf{0}\rangle} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

To generate a two-mode separable squeezed state, the two-mode vacuum state is squeezed by equal and opposite amounts by the squeezing operation $S_1(r) \oplus S_2(-r)$. Thereafter, the covariance matrix of the two-mode separable squeezed state can be written as

$$V_1(t=0) = \frac{1}{2}\begin{pmatrix} e^{-2r} & 0 & 0 & 0 \\ 0 & e^{2r} & 0 & 0 \\ 0 & 0 & e^{2r} & 0 \\ 0 & 0 & 0 & e^{-2r} \end{pmatrix}. \tag{17}$$

The system is then allowed to interact with a thermal bath. We set the time to $t = 0$, when the interaction with the bath is switched on. Thus, the covariance matrix of the state at time $t = 0$ is given by Eq. (17). The interaction with the thermal bath is then switched on, which can lead to decay of squeezing. After a time $t = \tau$, the interaction is switched off, and the modes are mixed using a 50 : 50 beam splitter with a view to convert the remaining nonclassicality into entanglement.

Case 2: In this case, we consider a two-mode squeezed vacuum state (TMSV), an entangled state, interacting with a thermal bath. The two-mode entangled state is generated by first squeezing the two-mode system in the vacuum state by an equal and opposite amount by the squeezing operation $S_1(r) \oplus S_2(-r)$. The modes are then mixed via a 50 : 50 beam splitter in order to convert the squeezing into entanglement, and subsequently the thermal bath is switched on. The covariance matrix at time $t = 0$ for this case is given by

$$V_2(t=0) = \frac{1}{2}\begin{pmatrix} \cosh(2r)\,\mathbb{1}_2 & \sinh(2r)\,\mathbb{Z} \\ \sinh(2r)\,\mathbb{Z} & \cosh(2r)\,\mathbb{1}_2 \end{pmatrix}, \tag{18}$$

where $\mathbb{Z}$ is diag(1, −1). It should be noted here that for the two-mode separable squeezed state, the quantum resource is in the form of squeezing, while for the TMSV state, squeezing has been converted into entanglement [50,51]. Thus, for a two-mode separable squeezed state, squeezing decays due to environmental interactions. On the other hand, for the TMSV state, both squeezing and entanglement decay due to environmental interactions. The two cases differ from each other only in the fact that quantumness has been completely stored in the form of squeezing in the former case, while in the latter case, quantumness has been converted into the form of entanglement. The settings above have been constructed to address our main question: Which form of quantumness, squeezing or entanglement, is more resilient to a dissipative environment? To this end, we consider the evolution of the two aforementioned cases in the following two dissipative environments:

(i) The two modes of the system interact with two local thermal baths.

(ii) The two modes of the system interact with a global thermal bath.

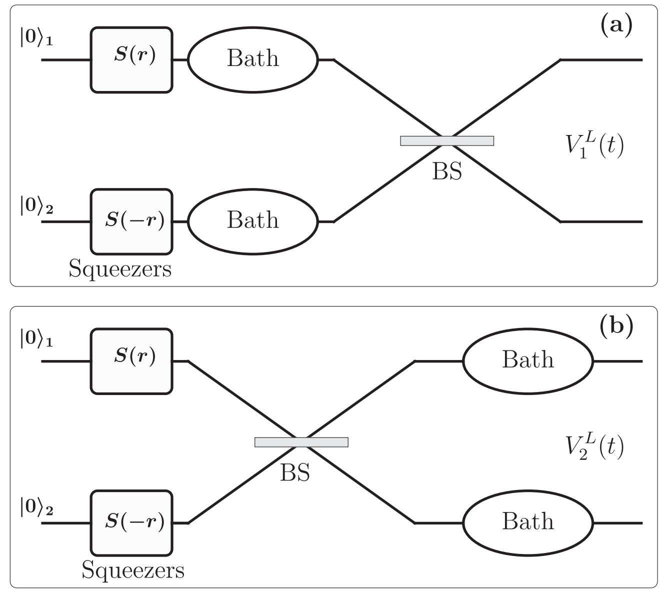

FIG. 1. Schematic representation of the dissipation under local thermal baths. (a) The two-mode separable squeezed state is allowed to evolve under local thermal baths, and after a time t, the two modes are mixed using a 50 : 50 beam splitter. (b) The two-mode separable squeezed state is first mixed using a beam splitter and then it is allowed to evolve under local thermal baths.

These studies will enable us to answer whether we should store the quantum resource as squeezing or as entanglement in a given situation.

B. Evolution under local thermal baths

In this subsection, we consider that our two-mode system interacts with two local thermal baths. The interaction Hamiltonian is given by

$$\hat{H}_{\rm SB} = g_1 \sum_{k=1}^{\infty} (\hat{a}_1 \hat{b}_k^\dagger + \hat{a}_1^\dagger \hat{b}_k) + g_2 \sum_{l=1}^{\infty} (\hat{a}_2 \hat{c}_l^\dagger + \hat{a}_2^\dagger \hat{c}_l), \tag{19}$$

where g_1 and g_2 are the coupling constants, and $\hat{b}_k$ and $\hat{b}_k^\dagger$ are annihilation and creation operators of the kth mode of the reservoir interacting with the first mode of the system. Similarly, $\hat{c}_l$ and $\hat{c}_l^\dagger$ are annihilation and creation operators of the lth mode of the reservoir interacting with the second mode of the system.

Under Markovian assumption, we can write the master equation for the evolution of the system density operator ρ as

$$\frac{\partial}{\partial t}\rho = \Bigg\{ \sum_{i=1,2} \frac{\gamma_i}{2}(N_i+1)(2\hat{a}_i\rho\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\rho - \rho\hat{a}_i^\dagger\hat{a}_i) + \frac{\gamma_i}{2}N_i(2\hat{a}_i^\dagger\rho\hat{a}_i - \hat{a}_i\hat{a}_i^\dagger\rho - \rho\hat{a}_i\hat{a}_i^\dagger)\Bigg\}, \tag{20}$$

where γ_i's are the decay constants, and N_i's represent the mean photon number of the individual baths. Equation (20) can be used to find the time evolution of variances of quadrature operators, and hence the evolution of the covariance matrix.

Case 1: We consider the case in which each mode of the two-mode separable squeezed state interacts with distinct local thermal baths. The schematic diagram is shown in Fig. 1(a). The covariance matrix of the initial state at time $t = 0$ is given by $V_1(0)$ (17). Using the master equation (20), we obtain the covariance matrix after an interaction with the bath for time t as [52]

$$V_1(t) = X(t)V_1(0)X(t)^T + \tfrac{1}{2}Y(t), \tag{21}$$

where $X(t)$ and $Y(t)$ are 4×4 diagonal matrices given by

$$X(t) = \begin{pmatrix} (1-\tau_1)^{1/4}\,\mathbb{1}_2 & 0 \\ 0 & (1-\tau_2)^{1/4}\,\mathbb{1}_2 \end{pmatrix},$$
$$Y(t) = \begin{pmatrix} (1+\frac{N_1}{2})(1-\sqrt{1-\tau_1})\,\mathbb{1}_2 & 0 \\ 0 & (1+\frac{N_2}{2})(1-\sqrt{1-\tau_2})\,\mathbb{1}_2 \end{pmatrix}, \tag{22}$$

where $\tau_1 = 1 - e^{-2\gamma_1 t}$ and $\tau_2 = 1 - e^{-2\gamma_2 t}$ are dimensionless time parameters. We note that while t goes from 0 to ∞, τ_i goes from 0 to 1. The final covariance matrix after passing both the modes through a 50 : 50 beam splitter (6) is given by

$$V_1^L(t) = B_{12}\left(\frac{\pi}{4}\right)\left[X(t)V_1(0)X(t)^T + \frac{1}{2}Y(t)\right]B_{12}\left(\frac{-\pi}{4}\right), \tag{23}$$

where the superscript L over $V_1(t)$ stands for the local bath, and t represents the time duration of the system-bath interaction.

Case 2: We consider the case in which each mode of the TMSV state interacts with a distinct local thermal bath. The schematic is depicted in Fig. 1(b). The initial covariance matrix of the TMSV state is given by Eq. (18). Using the master equation (20), the covariance matrix at time t is evaluated as

$$V_2^L(t) = X(t)V_2(0)X(t)^T + \tfrac{1}{2}Y(t). \tag{24}$$

From Eqs. (23) and (24), various conclusions can be drawn. We consider the following two special cases of symmetric and asymmetric interaction of the local baths with the system:

Symmetric interaction: Consider the case in which $g_1 = g_2 = g$ and both baths are at the same temperature so that $\gamma_1 = \gamma_2 = \gamma$ and $N_1 = N_2 = N$. Thus the two local baths are identical. This leads to the same final covariance matrix for the two-mode separable squeezed state (23) and the TMSV state (24). Therefore both of the resources, squeezing and entanglement, are equally sensitive to decoherence when two identical local thermal baths act on each mode of the system.

Extreme asymmetric interaction: Consider the case in which $g_2 = 0$, which implies $\gamma_2 = 0$. Further, we take $g_1 = g$, $\gamma_1 = \gamma$, and $\tau_1 = \tau$. Thus, only the first mode of the system interacts with the thermal bath. The evolved covariance matrix for the two-mode separable squeezed state (23) and the TMSV state (24) are not the same in this situation, and hence we expect different rates of decay for logarithmic negativity of the final state.

Using Eq. (11), the condition on the initial squeezing parameter r, such that the two-mode separable squeezed state never becomes disentangled, is given by

$$|r| > r_c^L = \frac{1}{2}\left[\ln\left(1 + \frac{N_1}{2}\right)\right]. \tag{25}$$

However, for values of $|r| \leqslant r_c^L$, entanglement dies out for interaction times longer than

$$\tau_a = \frac{8\,e^{4|r|}[2 + N_1 - 2\cosh(2|r|)]\sinh(2|r|)}{[(2+N_1)e^{2|r|} - 2]^2}. \tag{26}$$

On the other hand, the time of disentanglement for the TMSV state evaluates to [38]

$$\tau_b = \frac{8(2+N_1)}{(4+N_1)^2}, \tag{27}$$

which is independent of the initial squeezing parameter r. Further, $\lim_{N_1\to 0} \tau_b = 1$, which means the entanglement of the TMSV state survives indefinitely for a zero-temperature bath. In general, Eqs. (26) and (27) imply the existence of finite disentanglement time under the aforementioned conditions for two-mode separable as well as TMSV states. This corresponds to the phenomenon of entanglement sudden death.

Further, if the initial squeezing parameter r is such that $|r|$ is less than a certain value r_t^L, then it is a better strategy to store the resource in the form entanglement, otherwise it is better to store resources in the form of squeezing. The expression for r_t^L can be evaluated by equating τ_a and τ_b, and it is given by

$$r_t^L = \frac{1}{2}\left[\ln\left(\frac{2 + N_1 + \sqrt{2(2 + 4N_1 + N_1^2)}}{4 + N_1}\right)\right]. \tag{28}$$

We plot the logarithmic negativity for different values of the initial squeezing parameter in Fig. 2. For $N_1 = 4$, the numerical values of r_c^L and r_t^L turn out to be $r_c^L = 0.55$ and $r_t^L = 0.29$. Therefore, for $r = 0.20 < r_t^L < r_c^L$, we observe that entanglement is more robust against dissipation as compared to squeezing. For $r_t^L < r = 0.40 < r_c^L$, we see that squeezing is more robust against dissipation as compared to entanglement. Finally, for $r = 0.60 > r_c^L > r_t^L$, squeezing is more robust against dissipation as compared to entanglement, and the two-mode separable case always stays entangled.

C. Evolution under a global bath

In this section, we consider the scenario in which the two modes are coupled to a common thermal reservoir, which we refer to as the global bath. The interaction Hamiltonian $\hat{H}_{\text{SB}}$ is given by

$$\hat{H}_{\text{SB}} = g\sum_{i=1,2}\left(\hat{a}_i\sum_{k=1}^{\infty}\hat{b}_k^\dagger + \hat{a}_i^\dagger\sum_{k=1}^{\infty}\hat{b}_k\right), \tag{29}$$

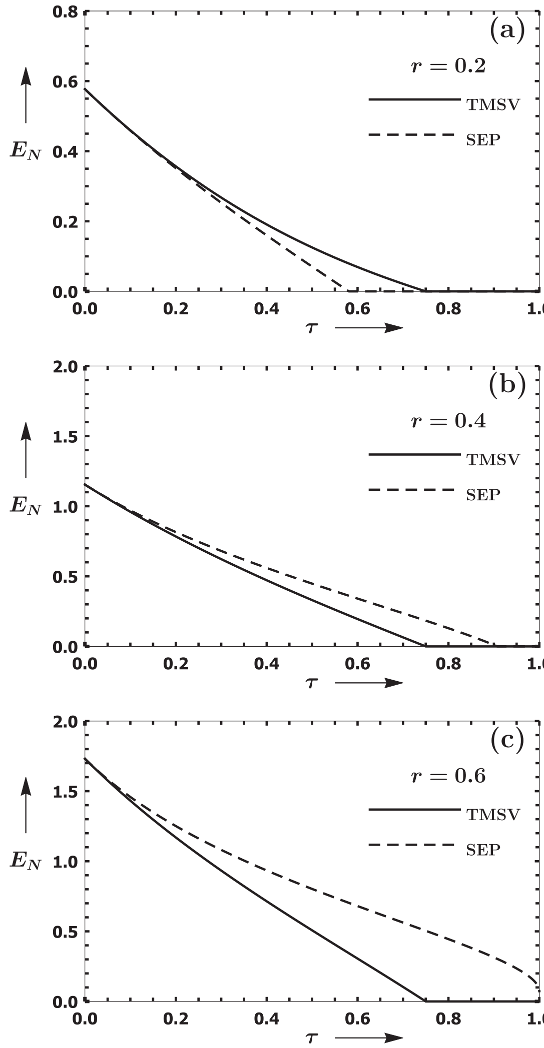

FIG. 2. Logarithmic negativity E_N of the two-mode Gaussian state as a function of dimensionless time $\tau(= 1 - e^{-2\gamma t})$. Only one of the two modes is allowed to evolve under the presence of a local bath. The mean number of photons in the bath is taken to be $N = 4$. (a) For values of the initial squeezing parameter such that $|r| < r_t^L < r_c^L$ [(25) and (28)], entanglement is more robust than squeezing. (b) For $r_t^L < |r| < r_c^L$, squeezing is more robust than entanglement. (c) For $|r| > r_c^L > r_t^L$, squeezing is more robust than entanglement and the two-mode separable case always remains entangled. The vertical axis is in ebits, while the horizontal axis is dimensionless.

where $\hat{b}_k$ and $\hat{b}_k^\dagger$ are annihilation and creation operators of the kth mode of the reservoir, and g is the coupling constant between the system and the environment. Under Markovian assumption, we can write the master equation for the evolution of system density operator ρ as

$$\frac{\partial}{\partial t}\rho = \frac{\gamma}{2}\Bigg\{\sum_{i,j=1,2}(N+1)(2\hat{a}_i\rho\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_i\rho - \rho\hat{a}_j^\dagger\hat{a}_i) + N(2\hat{a}_j^\dagger\rho\hat{a}_i - \hat{a}_i\hat{a}_j^\dagger\rho - \rho\hat{a}_i\hat{a}_j^\dagger)\Bigg\}, \tag{30}$$

where γ is the decay constant, and N represents the mean photon number of the bath. Equation (30) can be used to find

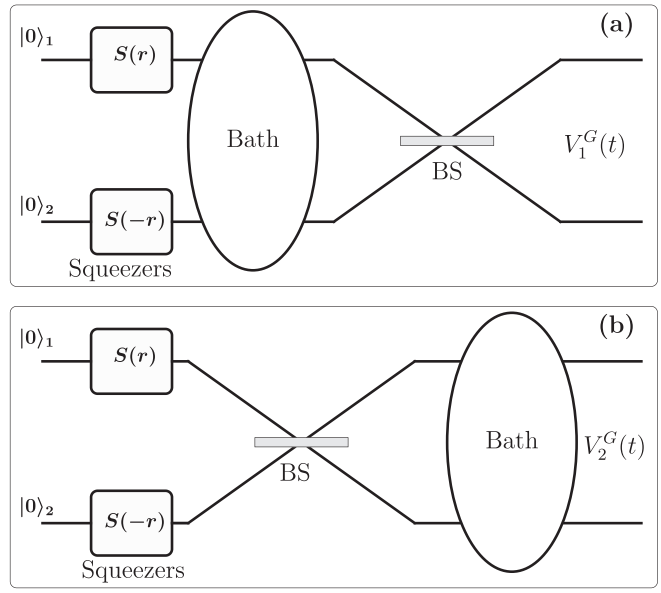

FIG. 3. Schematic representation of the dissipation under a global bath. (a) The two-mode separable squeezed state is allowed to evolve under the global thermal bath, and after a time t, the two modes are mixed using a 50 : 50 beam splitter. (b) The two-mode separable squeezed state is first mixed using a 50 : 50 beam splitter, and then it is allowed to evolve under the global thermal bath.

the time evolution of the variances of quadrature operators, and hence the time evolution of the covariance matrix.

Case 1: We consider the case in which the two-mode separable squeezed state interacts with a global bath. The schematic diagram is shown in Fig. 3(a). The initial covariance matrix of the two-mode separable squeezed state is given by Eq. (17). Using Eq. (30), the covariance matrix after an interaction for time t with the bath is evaluated to be

$$\frac{1}{4}\begin{pmatrix} \sigma_1(t) & 0 & \sigma_3(t) & 0 \\ 0 & \sigma_2(t) & 0 & \sigma_3(t) \\ \sigma_3(t) & 0 & \sigma_2(t) & 0 \\ 0 & \sigma_3(t) & 0 & \sigma_1(t) \end{pmatrix}, \tag{31}$$

where

$$\sigma_{1,2}(t) = (2N+1)\tau - \cosh(2r)(\tau - 2) \mp 2\sinh(2r)\sqrt{1-\tau},$$
$$\sigma_3(t) = [2N + 1 - \cosh(2r)]\tau, \tag{32}$$

where $\tau = 1 - e^{-2\gamma t}$. We now pass both modes through a 50 : 50 beam splitter, and the resultant covariance matrix is given by

$$V_1^G(t) = \frac{1}{2}\begin{pmatrix} \sigma_1'(t) & 0 & \sigma_3'(t) & 0 \\ 0 & \sigma_1'(t) & 0 & -\sigma_3'(t) \\ \sigma_3'(t) & 0 & \sigma_2'(t) & 0 \\ 0 & -\sigma_3'(t) & 0 & \sigma_2'(t) \end{pmatrix}, \tag{33}$$

where

$$\sigma_1'(t) = (2N+1)\tau - \cosh(2r)(\tau - 1),$$
$$\sigma_2'(t) = \cosh(2r),$$
$$\sigma_3'(t) = \sinh(2r)\sqrt{1-\tau}. \tag{34}$$

Case 2: We consider the case in which the TMSV state interacts with a global bath. The schematic is presented in Fig. 3(b). The initial covariance matrix of the TMSV state is given by Eq. (18). The covariance matrix after an interaction for time t with the bath is evaluated to be

$$V_2^G(t) = \frac{1}{2}\begin{pmatrix} \delta_1(t) & 0 & \delta_3(t) & 0 \\ 0 & \delta_2(t) & 0 & \delta_4(t) \\ \delta_3(t) & 0 & \delta_1(t) & 0 \\ 0 & \delta_4(t) & 0 & \delta_2(t) \end{pmatrix}, \tag{35}$$

where

$$\begin{aligned} \delta_1(t) &= \tfrac{1}{2}(2N+1-e^{2r})\tau + \cosh(2r), \\ \delta_2(t) &= \tfrac{1}{2}(2N+1-e^{-2r})\tau + \cosh(2r), \\ \delta_3(t) &= \tfrac{1}{2}(2N+1-e^{2r})\tau + \sinh(2r), \\ \delta_4(t) &= \tfrac{1}{2}(2N+1-e^{-2r})\tau - \sinh(2r). \end{aligned} \tag{36}$$

The evolved covariance matrix for the two-mode separable squeezed state (33) and that for the TMSV state (35) are not the same in this situation, and hence we expect different rates of decay for logarithmic negativity. For the TMSV state, there exists a critical value of initial squeezing parameter r_c^G, above which the entanglement never becomes zero [28]:

$$|r| > r_c^G = \tfrac{1}{2}[\ln(2N+1)]. \tag{37}$$

For values of $|r|$ less than r_c^G, entanglement sudden death occurs after a time

$$\tau_c = \frac{2\sinh(2|r|)}{1-e^{-2|r|}+2N}. \tag{38}$$

It is also observed that for the two-mode separable squeezed state, entanglement always becomes zero at a particular value of time irrespective of the initial squeezing of the state:

$$\tau_d = \frac{1}{1+N}. \tag{39}$$

Thus entanglement sudden death always occurs for the two-mode separable squeezed state except for the zero-temperature bath.

If the initial squeezing parameter r is such that $|r|$ is less than a certain value r_t^G, then it is a better strategy to store the quantum resource in the form of squeezing, otherwise it is better to store the quantumness in the form of entanglement. The expression for r_t^G can be evaluated by equating τ_c and τ_d, and it is given by

$$r_t^G = \frac{1}{2}\left[\ln\left(\frac{1+2N+\sqrt{1+8N+8N^2}}{2(1+N)}\right)\right]. \tag{40}$$

We have shown the plots of logarithmic negativity for different values of initial squeezing parameter in Fig. 4. For $N = 4$, the numerical values of r_c^G and r_t^G turn out to be $r_c^G = 1.10$ and $r_t^G = 0.39$. Therefore, in corroboration with analytical results, we observe that for $r = 0.20 < r_t^G < r_c^G$, squeezing is more robust against dissipation as compared to entanglement. Further, for $r_t^G < r = 0.60 < r_c^G$, entanglement is more robust against dissipation as compared to squeezing. Finally, for $r = 1.60 > r_c^G > r_t^G$, entanglement is more robust against dissipation as compared to squeezing; however, the TMSV state always remains entangled. We have summarized the results of this section in Table I.

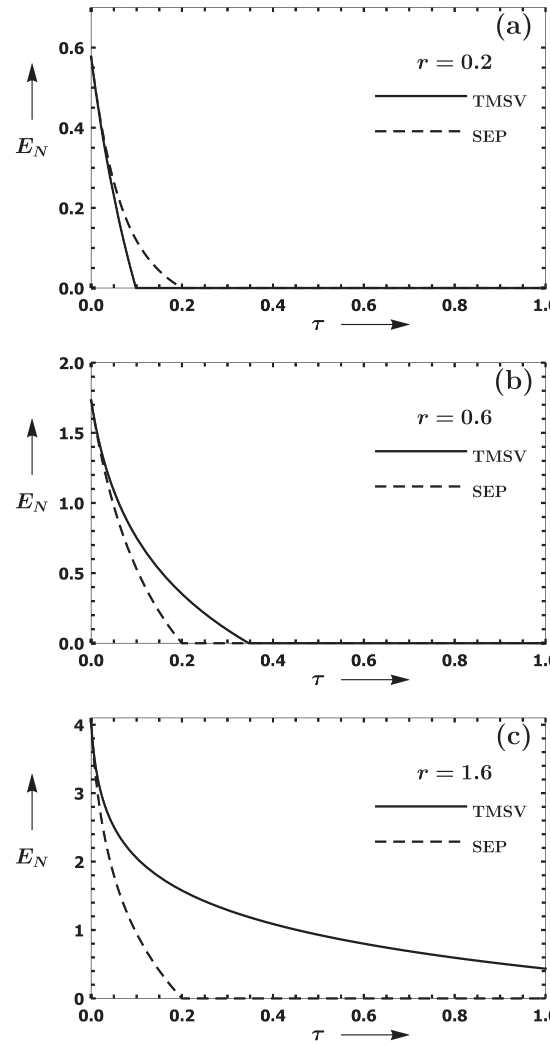

FIG. 4. Logarithmic negativity E_N as a function of dimensionless time $\tau(= 1 - e^{-2\gamma t})$. The system is allowed to evolve under the presence of a global thermal bath. The mean number of photons in the bath has been taken to be $N_1 = 4$. (a) For values of the initial squeezing parameter such that $|r| < r_t^G < r_c^G$ [(37) and (40)], squeezing is more robust than entanglement. (b) For $r_t^G < |r| < r_c^G$, entanglement is more robust than squeezing. (c) For $|r| > r_c^G > r_t^G$, entanglement is more robust than squeezing, and the TMSV state always remains entangled. The vertical axis is in ebits, while the horizontal axis is dimensionless.

IV. CONCLUSION

In this paper, we compared the robustness of squeezing and entanglement resources in two-mode Gaussian states evolving in different noisy dissipative environments. To this end, we considered two different cases: In the first case, two-mode separable squeezed states were allowed to evolve in the dissipative environment, and then were entangled using passive operations. In the second case, the squeezed modes were first entangled using passive operations and then were evolved in the dissipative environment. The results show that the robustness of squeezing and entanglement depends on the initial squeezing of the state and the nature of the dissipative environment. We also observe entanglement sudden death in

TABLE I. More robust resource against dissipation: squeezing or entanglement.

Environment nature	r Range	Result
Identical local baths		Squeezing ≡ Entanglement
Single local bath	$\|r\| < r_t^L$	Squeezing < Entanglement
	$\|r\| > r_t^L$	Squeezing > Entanglement
Global bath	$\|r\| < r_t^G$	Squeezing > Entanglement
	$\|r\| > r_t^G$	Squeezing < Entanglement

specific cases. The fact that interconversion of squeezing and entanglement can be made by using passive optical elements like beam splitters, wave plates, phase shifters, and mirrors makes it convenient to save the resources in one or another form depending on the situation. One of the directions that we are pursuing is to generalize this work by considering more general environmental models and non-Gaussian states, which will have implications for recent CV-based key distribution [53] and quantum teleportation [54,55] protocols.

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