Optical theorem of an infinite circular cylinder in weakly absorbing media

Shangyu Zhang, Wenjie Zhang, Jian Dong, and Linhua Liu*
School of Energy and Power Engineering, Shandong University, Jinan 250061, China
and Optics & Thermal Radiation Research Center, Shandong University, Qingdao 266237, China

(Received 9 July 2021; revised 23 October 2021; accepted 8 February 2022; published 22 February 2022)

The conventional optical theorem *cannot* consider losses of host media. As a fundamental problem of light scattering, the generalized optical theorem of an infinite cylinder embedded in a weakly absorbing host is studied in this paper. Using the analytical way, we derive the generalized far-field and distance-independent extinction efficiencies per unit length of the cylinder under normally incident *p*- and *s*-polarized waves, which reduce to the conventional formulas in the transparent host media. For large cylinders, the increasing absorption of the host medium leads to an increasing amplitude of interference oscillation and the emergence of negative extinction, which is similar to that for spheres. Furthermore, the absorption of the host medium is proved to destroy the morphology-dependent resonance structure in extinction and suppress the electric field resonance inside the cylinder. For small cylinders in a weakly absorbing host medium, we present the conditions for negative extinction and quantitatively analyze the differences in extinction between the absorbing host medium and the nonabsorbing counterpart. It is found that the ratio of the extinction from the generalized theory to conventional formulas depends only on the refraction indices of the cylinder and the host medium. The results for small cylinders are rather sensitive to the state of polarization of the light. By illustrating the significant differences between the generalized optical theorem and the conventional theory with a specific case of a Ge cylinder in polyethylene, we show the non-negligible impacts of the absorption of the host media on the optical extinction of the cylinder for the two polarizations.

DOI: 10.1103/PhysRevA.105.023516

I. INTRODUCTION

The *optical theorem*, also termed the forward scattering theorem, is a fundamental law in light scattering theory of particles [1]. It relates the scattering amplitude in the forward direction to the extinction cross section of the particle. Extinction of light by particles is not only a ubiquitous phenomenon in nature as in the lingering light of the setting sun [2], but also a useful tool in diverse sciences and engineering disciplines, including, but not limited to, astronomy [3], atmospheric radiation and climate science [4], bio-optics [5], radiative heat transfer [6], and metamaterials [7]. To obtain meaningful insights of the physical measurement and practical observations in these disciplines, a comprehensively theoretical knowledge on the optical theorem for various circumstances is required. The optical theorem has been generalized to Gaussian beams [8], radially polarized beams [9], surface waves [10,11], inversion symmetry particles [12], nonlinear and lossy particles with time-varying optical properties [13], and source-induced electromagnetic fields [14]. However, in the above generalizations, the host medium surrounding particles is usually assumed to be *transparent*.

In certain circumstances, however, it is also necessary to consider the electromagnetic losses of the host media. In the atmosphere, ozone, carbon dioxide, and water vapor have featured absorptive bands at infrared spectral regions [15]. The commonly used polymeric matrices in nanophotonics, such as polymethyl methacrylate (PMMA) and polydimethylsiloxane (PDMS), show optical losses in the infrared band [16]. Moreover, another concrete instance is the radio-frequency heating of the metallic nanoparticles dispersed in biological tissues for photothermal therapy, where the tissues should be considered as lossy media [17,18].

Electromagnetic scattering of an infinite cylinder in the absorbing host medium has both fundamental and practical significance. The famous Mie theory is developed for spheres and infinite cylinders [19,20], which gives our insights into electromagnetic scattering. In addition, a recent work demonstrates that, under the radio-frequency electromagnetic waves, a nanocylinder or nanowire in human tissue has a higher photothermal conversion efficiency than a nanosphere [21], in which the tissue is considered as an absorbing medium.

In a transparent host medium, the optical theorem can be derived in two conventional ways. One is the analytical way to define the extinction by integrating the total Poynting vector on the conceptual spherical/cylindrical surface (CSS) in a far-field region [19]. The extinction integral over the CSS surface surrounding the particle depends only on the forward direction interference, while the contributions from the other directions on the integral will be canceled mutually [22]. The other is the operational way originating from the seminal work by van de Hulst [23], who gives the extinction by theoretically modeling the difference between the readings of a well-collimated detector (WCD) in the presence and absence of the particle [20,24]. The operational way is a supplement

*Corresponding author: liulinhua@sdu.edu.cn

to the analytical way since it discusses the ability of a WCD to measure the forward direction interference and the extinction properties of particles. The preliminary limitation on the WCD is the larger sensitive area than the particle projection [24]. However, further rigorous analysis reveals that a valid measurement requires a WCD to satisfy more restrictions, for example, to be large enough [22], to be far enough from the particle, and it should have a noncircular sensitive area [25].

Based on the above two ways, there are a lot of research efforts to develop the generalized optical theorem in absorbing host media. However, since Mundy *et al.* first derived the generalized Mie scattering theory of a sphere in absorbing host media in 1974 [26], the generalization of the optical theorem to the absorbing host medium has suffered long-lasting *controversies*. The controversies arise from the desire to preserve the derivation methods and definitions of the conventional theory.

In the generalization of the analytical way for absorbing host media, the extinction power absorbed inside the CSS is a quantity dependent on the size of the CSS. The far-field extinction efficiency of the particle is thus a CSS-dependent quantity, which is defined as, according to the conventional definition, the ratio of the extinction power absorbed inside the CSS to the mean incident energy flow entering the sphere [26] or the cylinder [27]. However, this runs counter to the physical intuition that the physical property of the particle should be a distance-independent quantity [28].

To avoid the distance-dependent extinction properties, the researchers adopt two different routines: (i) taking the CSS on the surface of the spheres [29–38] or the cylinders [27,39–44] and (ii) using the operational way to consider the readings of the WCD [45–55]. Routine (i) leads to an apparent contradiction since it neglects the near-field effects whereas the actual measurements would include these effects [15,45]. Routine (ii) takes advantage of the merits of the operational way which considers the actual experimental configuration [19]. Therefore, the operational way is used to derive the generalized optical theorem of an arbitrary finite-volume particle in absorbing host media by Videen and Sun [45] and Mishchenko [46], which is then integrated with the generalized radiative transfer theory [47,48]. Following routine (ii), a number of studies evaluate the impact of external losses on the extinction of a single particle [49–51] or a particle system [52–55]. However, the validity of the latter routine is limited by the facts that the rigorous conditions have not been discussed in relating the extinction to the readings of a WCD in the absorbing medium.

To conclude, we stress two important aspects of the problem. First, the two ways described above (the analytical and operational ways) should be two sequential steps of a complete derivation. However, sustaining the conventional definition of the analytical way gives the distance-dependent extinction which is not an intrinsically physical property of the particle. Then, it is necessary to adopt a generalized definition to eliminate this distance dependence.

Second, a singular property of the particle made from the passive material occurs if the host medium is absorbing, that is, the extinction of the particle may be negative [28]. A recent work demonstrates in detail that negative extinction arises from the amplifying interference in Fraunhofer diffraction by the increasing absorption of the host [50,52]. This interpretation focuses on large particles; however, it was mentioned recently that negative extinction can also occur for small particles compared with the wavelength [53,56]. The extinction properties for small particles should receive more attention.

In this work, we develop the optical theorem of an infinite cylinder in absorbing host media under the normally incident p- and s-polarized plane waves. In Sec. II, we derive the extinction of the unit length cylinder by means of the analytical way. The distance-independent extinction cross section is obtained by using the forward incident intensity. In Sec. III, extinction of large cylinders and small cylinders will be studied separately. For small cylinders, we try to give the conditions for the emergence of negative extinction and demonstrate the differences in the results between absorbing and nonabsorbing host media.

II. THEORY

A. Conventions and notations

Let ε_0 and μ_0 denote the vacuum permittivity and permeability, respectively. A time-harmonic homogeneous incident plane wave with the term of $e^{-i\omega t}$ is used in this paper, where ω is the angular frequency, t is the time, and $i=\sqrt{-1}$. Both the cylinder and the host medium are homogeneous, linear, local, isotropic, and *lossy* media; the physical quantities of the cylinder and the host medium are represented by the subscripts of 1 and 2, respectively. In this regard, the relative magnetic permeabilities of the cylinder and the host medium are μ_1 and μ_2, respectively. The complex refraction indices of the cylinder and the host medium are

$$n_1 = n_1' + in_1'' \quad \text{and} \quad n_2 = n_2' + in_2'', \tag{1}$$

respectively. The complex wave numbers of the sphere and the host are given, respectively, by

$$k_1 = k_1' + ik_1'' = k_0 n_1 \quad \text{and} \quad k_2 = k_2' + ik_2'' = k_0 n_2, \tag{2}$$

where the vacuum wave number k_0 has the relation of

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}, \tag{3}$$

with c the vacuum light speed and λ the wavelength of light in a vacuum. The relative refraction index m is defined as

$$m = \frac{n_1}{n_2} = \frac{k_1}{k_2}, \tag{4}$$

and the complex size parameter q is

$$q = k_2 a = q' + iq'', \tag{5}$$

with a the radius of the cylinder. Since q is complex in the absorbing host, a real-valued size parameter in a vacuum q_0 is defined as [50]

$$q_0 = k_0 a. \tag{6}$$

The cylindrical coordinate system (r, ϕ, z) and rectangular coordinate system (x, y, z) of the cylinder are illustrated in Fig. 1, which also shows the incident linear-polarized plane waves and the scattered waves. The physical quantities corresponding to the incident p and s polarization are represented by the superscripts p and s, respectively. The incident and

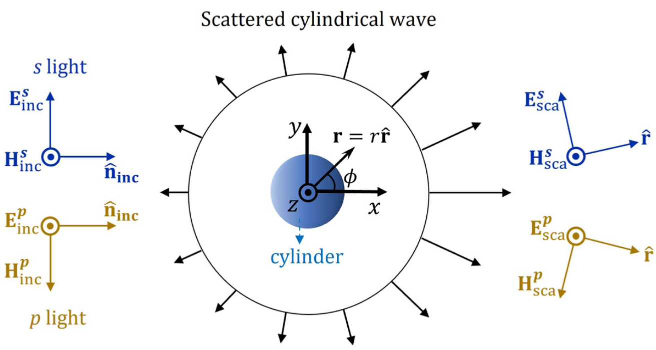

FIG. 1. Schematic diagram of light scattering by a circular cylinder under the normally incident p- and s-polarized waves. Incident and scattered electromagnetic fields are represented with the subscripts "inc" and "sca," respectively. The rectangular coordinate system (x, y, z) and cylindrical coordinate system (r, ϕ, z) are shown in the diagram.

scattered fields in this paper are represented by the subscripts "inc" and "sca," respectively.

B. Expansion of fields and asymptotic scattered field

In an absorbing medium, the refraction index n_2 is a complex parameter. In this regard, the wave number k_2 is also complex, which enters the wave equation of $\nabla^2\mathbf{E}_2 + k_2^2\mathbf{E}_2 = 0$. The separation-of-variables solution of the wave equation is invariant for the complex k_2 [57]. That is, the solution can be derived by expanding the fields in terms of the vector cylindrical harmonics, $\mathbf{M}_n$ and $\mathbf{N}_n$. In component form these vector cylindrical harmonics are [19]

$$\mathbf{M}_n = k_2\left[\frac{inH_n^{(1)}(\rho)}{\rho}\hat{\boldsymbol{e}}_r - H_n^{(1)'}(\rho)\hat{\boldsymbol{e}}_\phi\right]e^{in\phi}, \tag{7}$$

$$\mathbf{N}_n = k_2 H_n^{(1)}(\rho)\hat{\boldsymbol{e}}_z e^{in\phi}, \tag{8}$$

where $H_n^{(1)}$ is the Hankel function of the first kind and the prime in $H_n^{(1)'}(\rho)$ represents differentiation about $\rho = k_2 r$ with r the radial coordinate.

In this system, the incident electric fields can be represented by

$$\mathbf{E}^{p,s}_{\rm inc} = \mathbf{E}^{p,s}_0 e^{ik_2x}, \tag{9}$$

where the superscripts p and s represent, respectively, p and s polarizations, and $\mathbf{E}^{p,s}_0$ satisfies

$$\mathbf{E}^p_0 = E_0\hat{\boldsymbol{e}}_z \quad \text{and} \quad \mathbf{E}^s_0 = E_0\hat{\boldsymbol{e}}_y, \tag{10}$$

with E_0 the incident electric field at the origin of the coordinate system. The corresponding scattered electromagnetic waves can be expanded in terms of vector cylindrical harmonics, $\mathbf{M}_n$ and $\mathbf{N}_n$, as [19]

$$\mathbf{E}^p_{\rm sca} = -\sum_{n=-\infty}^{\infty} E_n b_n \mathbf{N}_n, \quad \mathbf{H}^p_{\rm sca} = \frac{ik_2}{\omega\mu_0\mu_2}\sum_{n=-\infty}^{\infty} E_n b_n \mathbf{M}_n, \tag{11}$$

$$\mathbf{E}^s_{\rm sca} = -\sum_{n=-\infty}^{\infty} E_n i a_n \mathbf{M}_n, \quad \mathbf{H}^s_{\rm sca} = \frac{-k_2}{\omega\mu_0\mu_2}\sum_{n=-\infty}^{\infty} E_n a_n \mathbf{N}_n, \tag{12}$$

with $E_n = E_0/(-i)^n k_2$. The expansion coefficients, a_n and b_n, are obtained from the electromagnetic boundary conditions, as [19]

$$b_n = \frac{\mu_1 J_n(mq)J_n'(q) - \mu_2 m J_n'(mq)J_n(q)}{\mu_1 J_n(mq)H_n^{(1)'}(q) - \mu_2 m J_n'(mq)H_n^{(1)}(q)}, \tag{13}$$

$$a_n = \frac{\mu_2 m J_n(mq)J_n'(q) - \mu_1 J_n'(mq)J_n(q)}{\mu_2 m J_n(mq)H_n^{(1)'}(q) - \mu_1 J_n'(mq)H_n^{(1)}(q)}, \tag{14}$$

where $J_n(z)$ is the Bessel function and the prime here represents differentiation with respect to the argument in parentheses.

In the far-field approximation, the Hankel function $H_n^{(1)}$ can be given asymptotically by [58]

$$H_n^{(1)}(\rho) \sim \sqrt{\frac{2}{\pi\rho}}e^{i(\rho - n\pi/2 - \pi/4)}. \tag{15}$$

Therefore, the asymptotic scattered fields can be written in the form of

$$\mathbf{E}^{p,s}_{\rm sca} = \frac{e^{ik_2r}}{\sqrt{r}}\mathbf{E}^{p,s}_{{\rm sca},0}(\phi), \tag{16}$$

$$\mathbf{H}^{p,s}_{\rm sca} = \frac{k_2}{\omega\mu_0\mu_2}\frac{e^{ik_2r}}{\sqrt{r}}\hat{\boldsymbol{e}}_r \times \mathbf{E}^{p,s}_{{\rm sca},0}(\phi), \tag{17}$$

where the distance-independent quantity $\mathbf{E}^{p,s}_{{\rm sca},0}(\phi)$ can be represented as

$$\mathbf{E}^{p,s}_{{\rm sca},0}(\phi) = e^{i3\pi/4}\sqrt{\frac{2}{\pi k_2}}T_{1,2}(\phi)E_0\hat{\boldsymbol{e}}_{z,y}, \tag{18}$$

with $T_{1,2}$ the amplitude scattering matrix elements given by

$$T_1 = \sum_{n=-\infty}^{+\infty} b_n e^{in\phi}, \tag{19}$$

$$T_2 = \sum_{n=-\infty}^{+\infty} a_n e^{in\phi}. \tag{20}$$

C. Far-field extinction of the cylinder

In the host medium the fields are the summation of incident and scattered waves, and then the electric and magnetic fields in the host, respectively, are

$$\mathbf{E}_2 = \mathbf{E}_{\rm inc} + \mathbf{E}_{\rm sca} \quad \text{and} \quad \mathbf{H}_2 = \mathbf{H}_{\rm inc} + \mathbf{H}_{\rm sca}. \tag{21}$$

Therefore, the time averaged Poynting vector of electromagnetic waves in the host medium can be expressed as the sum of three terms:

$$\mathbf{S}_2 = \tfrac{1}{2}{\rm Re}[\mathbf{E}_2 \times \mathbf{H}_2^*] = \mathbf{S}_{\rm inc} + \mathbf{S}_{\rm sca} + \mathbf{S}_{\rm cross}, \tag{22}$$

where the Poynting vectors contributed from the incident and the scattered field are

$$\mathbf{S}_{\rm inc} = \frac{1}{2}{\rm Re}[\mathbf{E}_{\rm inc} \times \mathbf{H}^*_{\rm inc}] = \frac{1}{2}\frac{k_2'}{\omega\mu_0\mu_2}e^{-2k_2''x}|\mathbf{E}_0|^2\hat{\boldsymbol{e}}_x, \tag{23}$$

and

$$\mathbf{S}_{\text{sca}} = \frac{1}{2}\text{Re}[\mathbf{E}_{\text{sca}} \times \mathbf{H}^{*}_{\text{sca}}] = \frac{1}{2}\frac{k_2'}{\omega\mu_0\mu_2}\frac{e^{-2k_2''r}}{r}|\mathbf{E}_{\text{sca},0}|^2\hat{\boldsymbol{e}}_r, \quad (24)$$

respectively, while the cross term,

$$\mathbf{S}_{\text{cross}} = \tfrac{1}{2}\text{Re}[\mathbf{E}_{\text{sca}} \times \mathbf{H}^{*}_{\text{inc}} + \mathbf{E}_{\text{inc}} \times \mathbf{H}^{*}_{\text{sca}}], \quad (25)$$

describes the Pointing vector component associated with the interaction (interference) between the incident and the scattered fields.

Let us now consider the electromagnetic energy budget when the wave is scattered by the cylindrical particle. We can construct an imaginary cylindrical surface with radius r around the particle. The total power per unit length cylinder absorbed within the imaginary cylinder surface is

$$W_{\text{abs}} = -r\int_0^{2\pi} \mathbf{S}_2(r)\cdot\hat{\boldsymbol{e}}_r\, d\phi, \quad (26)$$

which, according to Eq. (22), can be separated into three terms:

$$W_{\text{abs}} = W_{\text{inc}} + W_{\text{sca}} + W_{\text{cross}}. \quad (27)$$

The term W_{inc} due to the incident field can be given by

$$W_{\text{inc}} = \frac{\pi k_2' r}{\omega\mu_0\mu_2}|\mathbf{E}_0|^2 I_1(2k_2''r), \quad (28)$$

with $I_1(z)$ the first-order modified Bessel function of the first kind. The quantity W_{sca} is

$$W_{\text{sca}} = -\frac{k_2'}{2\omega\mu_0\mu_2}e^{-2k_2''r}\int_0^{2\pi}|\mathbf{E}_{\text{sca},0}|^2\, d\phi, \quad (29)$$

which describes the net power entering the imaginary surface contributed by the scattered field. Furthermore, after some algebra calculations, the cross term W_{cross} can be written as

$$W_{\text{cross}} = -r\int_0^{2\pi}\mathbf{S}_{\text{cross}}\cdot\hat{\boldsymbol{e}}_r\, d\phi$$
$$= \frac{-\sqrt{r}}{2\omega\mu_0\mu}\int_0^{2\pi} d\phi\,\text{Re}\{k_2^*[e^{ir(k_2\cos\phi - k_2^*)}\mathbf{E}_0\cdot\mathbf{E}^*_{\text{sca},0} + e^{ir(k_2 - k_2^*\cos\phi)}\cos\phi\,\mathbf{E}_0^*\cdot\mathbf{E}_{\text{sca},0} - e^{ir(k_2-k_2^*\cos\phi)}(\hat{\boldsymbol{e}}_r\cdot\mathbf{E}_0^*)(\hat{\boldsymbol{e}}_x\cdot\mathbf{E}_{\text{sca},0})]\}. \quad (30)$$

The integral in Eq. (30) can be asymptotically solved using the stationary phase method [59], since it can be simplified into the form

$$J = \int_0^{2\pi} g(\phi)e^{ik_2' r f(\phi)}\, d\phi. \quad (31)$$

In the far-field approximation of $k_2' r \gg 1$, the asymptotic behavior of the integral J comes from the critical points of phase function $f(\phi)$ where the first derivative of $f(\phi)$ vanishes. The underlying principle of stationary phase requires that the phase exponent $e^{ik_2' r f(\phi)}$ in Eq. (31) must be a rapidly oscillating function of ϕ and the amplitude function $g(\phi)$ should be a slowly varying function [60]. This restriction leads to the weakly absorbing condition of

$$\frac{k_2''}{k_2'} = \frac{n_2''}{n_2'} \ll 1. \quad (32)$$

Using the principle of stationary phase, we can calculate Eq. (31) with [59],

$$J \sim g(\phi_c)\sqrt{\frac{2\pi}{k_2' r|f''(\phi_c)|}}e^{ik_2' r f(\phi_c)}e^{i\pi\,\text{sgn}[f''(\phi_c)]/4}, \quad (33)$$

where ϕ_c is the critical point of $f(\phi)$ and $\text{sgn}[f''(\phi_c)]$ is the sign of $f''(\phi_c)$. Since $\phi = 0$ and π are the two critical points of $f(\phi) = \pm(\cos\phi - 1)$ in Eq. (31), W_{cross} can be written, according to Eq. (31) as the sum of two terms:

$$W_{\text{cross}} = W_{\text{ext}}(\phi = 0) + W_{\text{bi}}(\phi = \pi), \quad (34)$$

where

$$W_{\text{ext}}(\phi = 0) = -\frac{\sqrt{2\pi k_2'}}{\omega\mu_0\mu_2}e^{-2k_2''r}\text{Re}\{e^{\pi i/4}\mathbf{E}_0^*\cdot\mathbf{E}_{\text{sca},0}(\phi = 0)\} \quad (35)$$

represents the extinction term relating to the forward scattering interference and

$$W_{\text{bi}}(\phi = \pi) = \frac{\sqrt{2\pi}}{\omega\mu_0\mu_2}\frac{k_2''}{\sqrt{k_2'}}\text{Im}\{e^{-\pi i/4}e^{2ik_2' r}\mathbf{E}_0^*\cdot\mathbf{E}_{\text{sca},0}(\phi = \pi)\} \quad (36)$$

is the backscattering interference term. Further details about W_{bi} have been discussed in Ref. [61] for a three-dimensional scattering problem of a finite particle in the absorbing media.

The above results indicate that W_{cross} is contributed by both the forward and backward scattering interference. When the host media is nonabsorbing, W_{inc} and W_{bi} are zeros since $k_2'' = 0$, and W_{ext} is a distance-independent quantity, which allows the classical definition of extinction cross section as $C_{\text{ext}} = \frac{W_{\text{ext}}}{I^0_{\text{inc}}}$ with I^0_{inc} the distance-independent incident intensity.

However, in absorbing host media, both W_{ext} and the incident intensity I_{inc} depend upon the distance r. It is worth noting that W_{ext} relates to the distance r and the scattering amplitude in the forward direction. In other words, W_{ext} in Eq. (35) is a directly measurable quantity using the appropriate experimental configuration, which has been discussed in Refs. [6,22,25,62]. Therefore, the normalization factor should be taken as a measurable I_{inc}, which is the incident intensity situating at distance r in the forward direction when the particle is not in the path:

$$I_{\text{inc}}(\phi=0)=\frac{1}{2}\frac{k_2'}{\omega\mu_0\mu_2}|\mathbf{E}_0|^2e^{-2k_2''r}. \tag{37}$$

The distance-independent extinction cross section per unit length of the cylinder can then be defined as

$$C_{\text{ext}}=\frac{W_{\text{ext}}}{I_{\text{inc}}(\phi=0)}=\frac{-2\sqrt{2\pi}}{\sqrt{k_2'}}\frac{\text{Re}\{e^{\pi i/4}\mathbf{E}_0^*\cdot\mathbf{E}_{\text{sca},0}(\phi=0)\}}{|\mathbf{E}_0|^2}. \tag{38}$$

Substituting Eqs. (10) and (18) into Eq. (38), we can obtain the extinction cross sections *per unit length* of the cylinder for p and s polarizations,

$$C_{\text{ext}}^{p,s}=\frac{4}{\sqrt{k_2'}}\text{Re}\left\{\frac{T_{1,2}(\phi=0)}{\sqrt{k_2}}\right\}, \tag{39}$$

and the corresponding extinction efficiencies

$$Q_{\text{ext}}^{p,s}=\frac{C_{\text{ext}}^{p,s}}{2a}=\frac{2}{\sqrt{q'}}\text{Re}\left\{\frac{T_{1,2}(\phi=0)}{\sqrt{q}}\right\}. \tag{40}$$

When the host media are transparent, Eq. (40) reduces to the classical form of $Q_{\text{ext}}^{p,s}=\frac{2}{q}\text{Re}\{T_{1,2}(\phi=0)\}$, which is equivalent to Eqs. (8.36) and (8.37) in the monograph by Bohren and Huffman [19]. It is worth noting that the above extinction cross section and efficiency are intrinsic properties of the particles, which do not depend on the distance.

Finally, we should stress three aspects of the derivation. First, the rigorous mathematical analysis of real measurements of W_{ext} in absorbing media is beyond the scope of this work. In transparent media, some researchers have argued that the effective measurement requires a large enough [22] and noncircular detector [70]. In absorbing media, however, more research needs to be done. Second, using the operational way, Mishchenko *et al.* [50] have derived the extinction efficiency of a sphere in absorbing media as $Q_{\text{ext}}=\frac{4}{q'}\text{Re}\{\frac{S_{11}(0)}{q}\}$ with $S_{11}(0)$ the forward amplitude scattering matrix element. It is obvious that their result is a three-dimensional analogy to our Eq. (40), although our results are based on the analytical way. This correspondence between the analytical and operational ways requires an in-depth analysis of a real measurement configuration, which is mentioned in the first statement. Third, the far-field distance should be limited since the intensity of fields in absorbing media is damped along the optical path. Taking the incident intensity as an example, the incident intensity should be assumed to be large enough to be "measured." If the light source is located at $x=-r$, the condition of $k_2''r<\frac{1}{2}$ makes the detected intensity greater than 14% of the light source. More specific restrictions on $k_2''r$ should depend on the intensity of the light source and the lower limit of the detector capability.

III. RESULTS AND DISCUSSION

A. Large cylinders

The extinction efficiencies per unit length cylinder are calculated for several specific cases. Here, we consider three cases as cases I–III. Case I corresponds to a typical aerosol particle with refraction index $n_1=1.4$ embedded in atmospheres with $n_2=1+in_2''$. Case II is an air bubble with $n_1=1$ immersed in water with $n_2=1.33+in_2''$. Case III represents a gold particle with $n_1=0.5+1.8i$ near the surface plasmon resonance submerged in water with $n_2=1.33+in_2''$. The three cases are listed in Table I.

TABLE I. Three cases illustrated in this paper.

	Refraction index of cylinders	Refraction index of host media
Case I	$n_1=1.4$ (aerosol particles)	$n_2=1+in_2''$ (atmospheres)
Case II	$n_1=1$ (air)	$n_2=1.33+in_2''$ (water)
Case III	$n_1=0.5+1.8i$ (gold)	$n_2=1.33+in_2''$ (water)

Figure 2 shows the extinction efficiencies of case I for both p and s polarizations. The imaginary parts of the host media n_2'' are chosen to be 0, 0.002, 0.02, and 0.05. It is noted in the figures that the vertical coordinates apply to the transparent host media (i.e., $n_2''=0$) and the curves with the other n_2'' are successively translated upward with an increment of 10. The dashed curves in Fig. 2 are the extinction efficiencies of a sphere in lossy media calculated by the equation of $Q_{\text{ext}}=\frac{4}{q'}\text{Re}\{\frac{S_{11}(0)}{q}\}$ with $S_{11}(0)$ the amplitude scattering matrix in the forward direction [50], and the parameters applied to the sphere are the same as those for the cylinder.

The features of the extinction curves for the transparent host media ($n_2''=0$) have been mentioned and discussed in some classical monographs about light scattering [19,20,63]. We can observe three main features from the $n_2''=0$ curve in Fig. 2: first, a series of regular and slow oscillations which are usually referred to as the *interference structure*; second, a *damping* interference structure finally approaching value 2 at the sufficiently large size parameter, which is

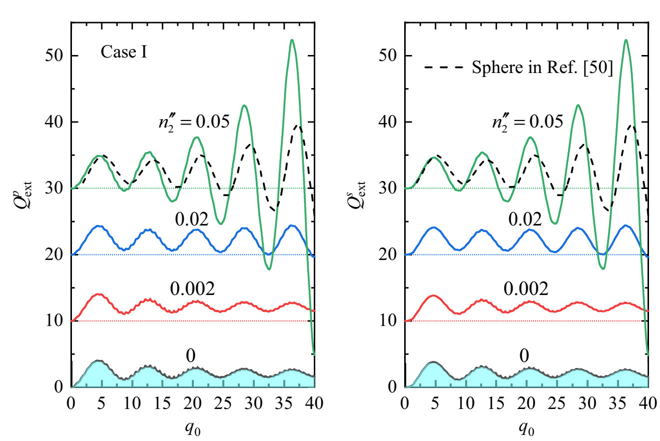

FIG. 2. Extinction efficiencies Q_{ext}^p and Q_{ext}^s versus the vacuum size parameter q_0 for $n_1=1.4$ and $n_2=1+in_2''$. The values of n_2'' are labeled beside the corresponding curves. The curves with different n_2'' are successively shifted upward by 10 in both figures. The dashed curves give the extinction efficiency of a sphere in lossy media calculated by the equation of $Q_{\text{ext}}=\frac{4}{q'}\text{Re}\{\frac{S_{11}(0)}{q}\}$ [50], where $S_{11}(0)$ is the amplitude scattering matrix of the sphere in the forward direction. The light cyan areas are results calculated by $(Q_{\text{ext}}-2)e^{-2n_2''q_0}+2$ at $n_2''=0.05$.

called the *extinction paradox*; third, sharp and irregular resonance peaks superimposed upon the interference structure, which are known as the *ripple structure* and also termed as morphology-dependent resonances (MDRs) or "whispering gallery" modes.

When the absorption of the host media is introduced, several special features emerge in Fig. 2:

(a) The period of the interference structure is independent of n_2''.

(b) The increasing of n_2'' results in the amplifying amplitude of the interference oscillation, which destroys the conventional extinction paradox since $Q_{\mathrm{ext}}^{p,s}$ cannot converge to a definite value with the increasing size parameter. A byproduct of this amplifying amplitude is the emerging *negative extinction* at some q_0 values.

(c) The ripple structure is diminished by the increasing n_2''.

As for (a), we can write a_n (and b_n) in the asymptotic form for thick cylinders as

$$a_n = \frac{F_n}{F_n + iG_n} \sim \frac{S(mq)C(q) - mC(mq)S(q)}{S(mq)C(q) - mC(mq)S(q) + i[mC(mq)C(q) + S(mq)S(q)]}, \tag{41}$$

where $S(z) = \sin[z - (n + \frac{1}{2})\frac{\pi}{2}]$ and $C(z) = \cos[z - (n + \frac{1}{2})\frac{\pi}{2}]$. By some algebra, we can get

$$F_n \sim (m+1)\sin(m-1)q + (-1)^n(m-1)\cos(m+1)q, \tag{42}$$

$$G_n = [(m+1)\cos(m-1)q + (-1)^n(m-1)\sin(m+1)q]. \tag{43}$$

Since m and q are complex, we can expand the trigonometric function as $\sin(m-1)q = \sin[q_0(n_1' - n_2')]\cosh[q_0(n_1'' - n_2'')] + i\cos[q_0(n_1' - n_2')]\sinh[q_0(n_1'' - n_2'')]$, with which the other trigonometric function terms of $\sin(m+1)q$, $\cos(m-1)q$, and $\cos(m+1)q$ have a similar form. Although it is complicated to further give a more simplified form of $\mathrm{Re}\{a_n/\sqrt{q}\}$, we can conclude that the periodicity is determined by the competitive effects among trigonometric function terms including $\sin[q_0(n_1' - n_2')]$, $\sin[q_0(n_1' + n_2')]$, $\cos[q_0(n_1' - n_2')]$, and $\cos[q_0(n_1' - n_2'')]$. The hyperbolic functions, influenced by $q_0 n_1''$ and $q_0 n_2''$, can determine which trigonometric function is the dominating factor in periodicity of the interference oscillation. It has been concluded in Ref. [64] that, for a transparent sphere in a transparent host medium, $\sin^2[q_0(n_1' - n_2')]$ dominates over the other terms within an approximate range of $0.5 \leqslant n_1'/n_2' \leqslant 2.5$. We can see in case I that the leading role of $\sin^2[q_0(n_1' - n_2')]$ is not influenced by the absorbing host media.

The increasing amplitude of the interference oscillation and negative extinction in (b) are discussed in the following. These remarkable phenomena are first found for a sphere in lossy media in Ref. [50], which can be seen from the dashed curves in Fig. 2. The authors of Ref. [50] provide a qualitative explanation on the anomalous interference oscillation as being the result of interference between the light diffracted and directly transmitted by the particle. When the host medium is transparent, both the diffracted and the transmitted fields suffer no attenuation. However, when the host medium is absorbing with $n_2'' \neq 0$ and the particle is transparent with $n_1'' = 0$, the diffracted field attenuates over the path length with a factor of $e^{-2k_0 n_2'' a}$ and the transmitted field is not subject to this factor. This differential attenuation factor of $e^{-2k_0 n_2'' a}$ leads to the exponentially growing amplitude of the interference structure with increasing $q_0 = k_0 a$ for a nonzero value of n_2''. If the differential attenuation factor was compensated on extinction results, the anomalous interference structure at large n_2'' would resemble that at $n_2'' = 0$ [50]. Indeed, replacing Q_{ext} at $n_2'' = 0.05$ by $(Q_{\mathrm{ext}} - 2)e^{-2q_0 n_2''} + 2$ will give an indistinguishable curve with the Q_{ext} at $n_2'' = 0$, which can be seen from the light cyan area in Fig. 2. We note in Fig. 2 that, at $n_2'' = 0.05$, the increasing interference amplitude with the vacuum size parameter q_0 for the cylinder is larger than that for the sphere. Since the differential attenuation factor of the cylinder equals that of the sphere, the large difference of interference oscillations at $n_2'' = 0.05$ between the cylinder and the sphere can be attributed to the moderate difference of Q_{ext} in transparent media between the two geometries.

The above qualitative explanation confirms the physical relevance of the amplifying interference oscillation with the negative extinction and the extinction paradox. According to the definition, negative extinction means that the received energy in the detector area is increased due to the presence of the scattering cylinder. Since the extinction cross sections are seen as the reduction in the detector area after interposing a particle between source and detector, it is possible for the detector to receive more radiation when the particle is less absorbing than the host medium.

Next, we discuss the diminished ripple structure with the increasing n_2'' in (c). The ripple structure in the extinction curve is well known to be formed by the resonances in the partial wave expansion coefficients, a_n and b_n [19]. Resonances from the lower-order coefficients are relatively broad with overlapping peaks. However, with the increasing mode number of n, the resonances become narrower so that each ripple peak corresponds to an individual resonance in a_n and b_n. These sharp resonances (i.e., MDR) are of fundamental importance in optical levitation experiments, fluorescence emission spectra, and Raman scattering spectra (see Ref. [65] and references therein).

The internal fields at MDR are of fundamental importance in both elastic scattering and inelastic scattering. Since the diminished MDRs are observed in Fig. 2, we plot the internal electric field $|\mathbf{E}_1^p|^2/|\mathbf{E}_0|^2$ for p polarization inside the cylinder in case I, as seen in Fig. 3. The fields outside the cylinder are not shown in the figure. The first row in Fig. 3 shows the internal electric field on the cylinder in transparent media with $n_2'' = 0$. At $q_0 = 44.35$ and $n_2'' = 0$, the parameters

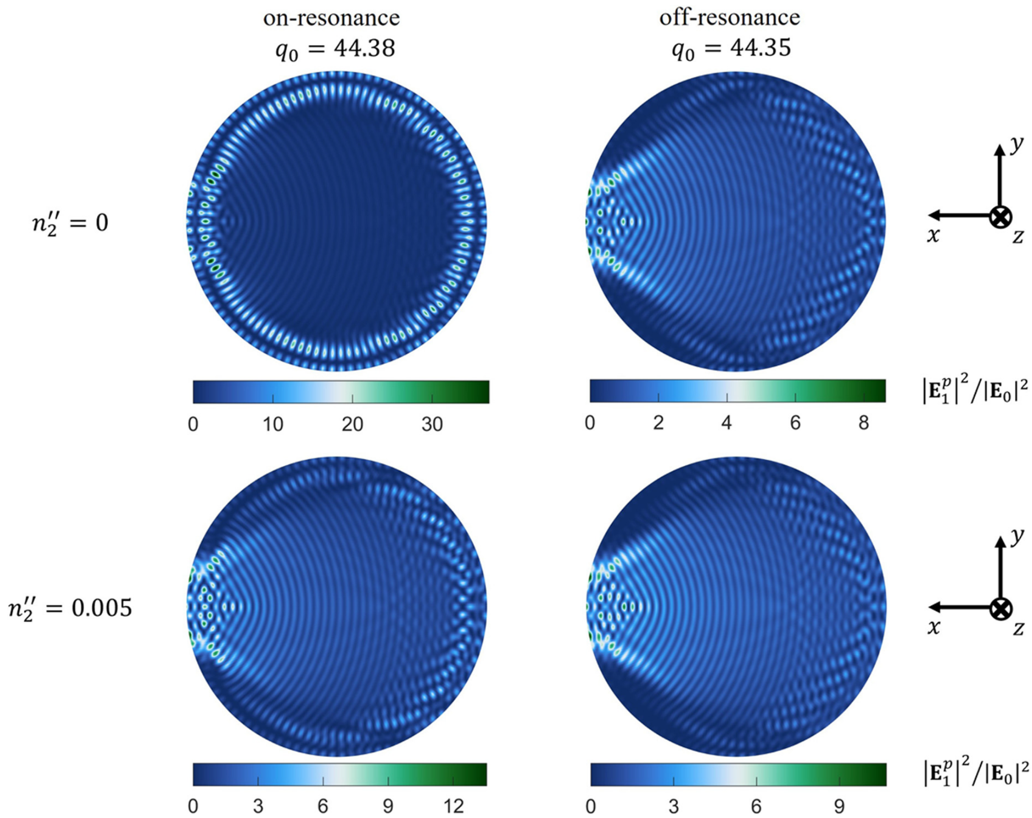

FIG. 3. Relative internal electric field to incident electric field at origin $|\mathbf{E}_1^p|^2/|\mathbf{E}_0|^2$ for p-polarized incidence in case I. The first column corresponds to the vacuum size parameters $q_0 = 44.38$, which satisfies an on-resonance condition of MDR. The second column corresponding the off-resonance condition with $q_0 = 44.35$. The first and second rows correspond to the results for transparent media with $n_2'' = 0$ and absorbing host media with $n_2'' = 0.005$, respectively.

do not satisfy the resonance condition and the maximum of the internal electric field is around 7, where the internal fields distribute mainly on the forward portion of the cylinder. However, for $q_0 = 44.38$ at $n_2'' = 0$, the MDR is "on", which largely enhances the internal field to about 37. Besides the enhancement of fields at the "on-resonance" condition, a dramatic change occurs in the distribution of the internal field. At the on-resonance condition, hundreds of peaks distribute near the circumference of the cylinder.

The second row in Fig. 3 shows the internal electric field with $n_2'' = 0.005$. We can see that the internal fields between the two different size parameters have little difference in both distribution and amplitude. This is in remarkable contrast to the internal fields at $n_2'' = 0$. Although it is at the MDR position of $q_0 = 44.38$, the internal fields in a lossy host do not present either the large enhancement or the circular distribution pattern. In addition, it is worth noting that the disappearance of MDR in internal fields at $n_2'' = 0.005$ and $q_0 = 44.38$ cannot be viewed simply as the radiation loss with the optical path, since the internal fields are normalized with the incident intensity at origin. As a result, it is concluded that the MDR features of the field inside the cylinder are diminished and even removed by the absorption of the host media.

Until now, MDRs have been weakened by the absorption of the host media, which is reflected in not only the ripple structure of far-field extinction efficiencies but also in the internal electric fields. In Fig. 2, it is revealed that the higher-order and narrower MDRs are more easily extinguished by the increasing n_2'' than the lower-order and broader MDRs. As mentioned in Ref. [50], the effect of increasing n_2'' on the ripple structures of extinction curves is similar to that of increasing n_2'' of a sphere in a nonabsorbing host (refer to Refs. [19,66] for explaining the effects of n_2'' on MDRs). It is correct that, since a_n and b_n are functions of $mq = q_0 n_1$ and $q = q_0 n_2$, the introduction of n_1'' is somewhat similar to that of n_2''. However, the similarity between n_1'' and n_2'' is not exact either in formulas of a_n or b_n or in their physical meanings, for which further detailed analysis is required.

Figure 4 shows the extinction efficiencies for case II. In this case, the amplitude of the interference oscillations is increased by n_2'' in the same way as in case I, and the periodicity of the interference structure is independent of n_2''. However, the extinction curves exhibit no MDRs in this case. According to the phenomenological interpretation of MDRs arising from totally reflected rays at the internal surface of particles, the disappearance of MDR at $n_2'' = 0$ is reasonable since total internal reflection is impossible for $n_1'/n_2' < 1$. Therefore, no MDRs exist in this case for any n_2''.

For comparison, a metallic cylinder is also studied in case III, as shown in Fig. 5. The extinction efficiencies do not exhibit any interference structure and ripple structure in this case, which has been revealed for $n_2'' = 0$ in Ref. [64]. Since the interference structure does not exist, the remarkable

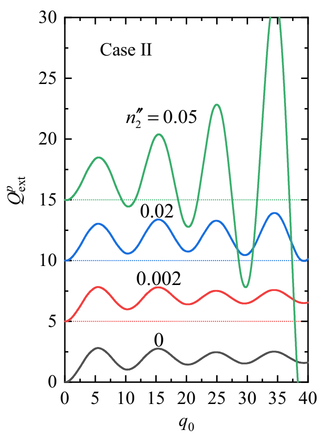

FIG. 4. Extinction efficiencies Q_{ext}^p and Q_{ext}^s versus the vacuum size parameter q_0 for $n_1 = 1$ and $n_2 = 1.33 + in_2''$. The values of n_2'' are labeled beside the corresponding curves. The curves with different n_2'' are successively shifted upward by 5 in both figures.

amplifying effects on interference oscillations by increasing n_2'' cannot exist either. Apart from this, we can observe that the increasing n_2'' has less influence on the extinction efficiencies than case I and case II. The largest increments of Q_{ext}^p and Q_{ext}^s from $n_2'' = 0$ to 0.05 are, respectively, about 0.0025 at $q_0 = 5$ and about -0.2 at $q_0 = 1$, which can be seen in the insets of Fig. 5. The effects of n_2'' on the extinction efficiencies of metallic cylinders are less remarkable than the two previous cases.

B. Small cylinders

Taking a closer look at the above results, amplifying interference oscillation is not the only way to obtain negative extinction. For small cylinders at $q_0 < 1$, negative extinction emerges in a hidden way.

Figure 6 illustrates Q_{ext}^p and Q_{ext}^s in the range of $0 < q_0 < 1$ for cases I and II. Unlike the similarity of extinction between p and s polarizations for large cylinders, extinction of small cylinders presents apparent differences between the two

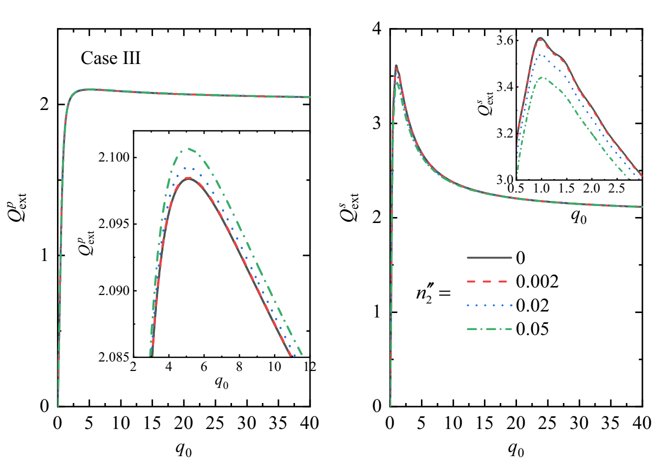

FIG. 5. Extinction efficiencies Q_{ext}^p and Q_{ext}^s versus the vacuum size parameter q_0 for $n_1 = 0.5 + 1.8i$ and $n_2 = 1.33 + in_2''$.

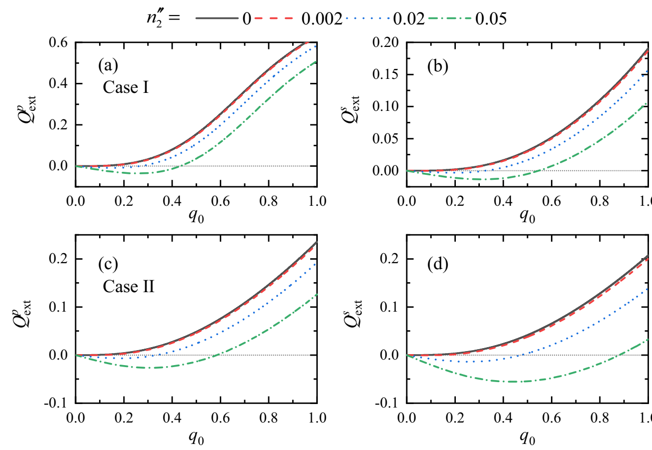

FIG. 6. Extinction efficiencies Q_{ext}^p and Q_{ext}^s versus the small vacuum size parameter $0 \leqslant q_0 \leqslant 1$. Data in (a,b) and (c,d) correspond to case I in Fig. 2 and case II in Fig. 4, respectively. Different values of n_2'' are represented by different line styles, which are shown in the legend above the figure.

different polarizations. For example, in cases I and II, both the magnitude of extinction and the position where negative extinction occurs are different for the two polarizations.

Here, two questions arise from extinction for the small cylinders: When will negative extinction emerge? Is absorption of the host media negligible for extinction of the embedded cylinders? To answer these two questions, some analysis is needed. For weakly absorbing host media with $n_2'' \ll n_2'$, we approximate the relative refraction index m as

$$m = \frac{n_1}{n_2} \sim p_1 + p_2\tau + i(p_2 - p_1\tau), \tag{44}$$

$$p_1 \triangleq \frac{n_1'}{n_2'}, \quad p_2 \triangleq \frac{n_1''}{n_2'}, \quad \tau \triangleq \frac{n_2''}{n_2'} \ll 1. \tag{45}$$

For small particles in the approximation of $|q| \ll 1$, the expansion coefficients b_0 and a_1 dominate over the other coefficients [19]. In this case, extinction efficiencies can be derived as

$$Q_{\text{ext}}^p \approx \frac{\pi q_0}{2\sqrt{n_2'}} \text{Im}\{(m^2 - 1)n_2^{3/2}\}, \tag{46}$$

$$Q_{\text{ext}}^s \approx \frac{\pi q_0}{\sqrt{n_2'}} \text{Im}\left\{\frac{m^2 - 1}{m^2 + 1} n_2^{3/2}\right\}. \tag{47}$$

If the absorbing properties of the host media are neglected, then the relative refraction index m_0 is

$$m_0 = \frac{n_1}{n_2'} = p_1 + ip_2. \tag{48}$$

When the absorbing of the host media is not considered in calculations, m and n_2 in Eqs. (46) and (47) should be replaced by m_0 and n_2', respectively. Therefore, extinction efficiencies neglecting absorption of the host media, $Q_{\text{ext},0}^{p,s}$, are

$$Q_{\text{ext},0}^p = \frac{\pi q_0 n_2'}{2} \text{Im}\{m_0^2 - 1\}, \tag{49}$$

$$Q_{\text{ext},0}^s = \pi q_0 n_2' \text{Im}\left\{\frac{m_0^2 - 1}{m_0^2 + 1}\right\}. \tag{50}$$

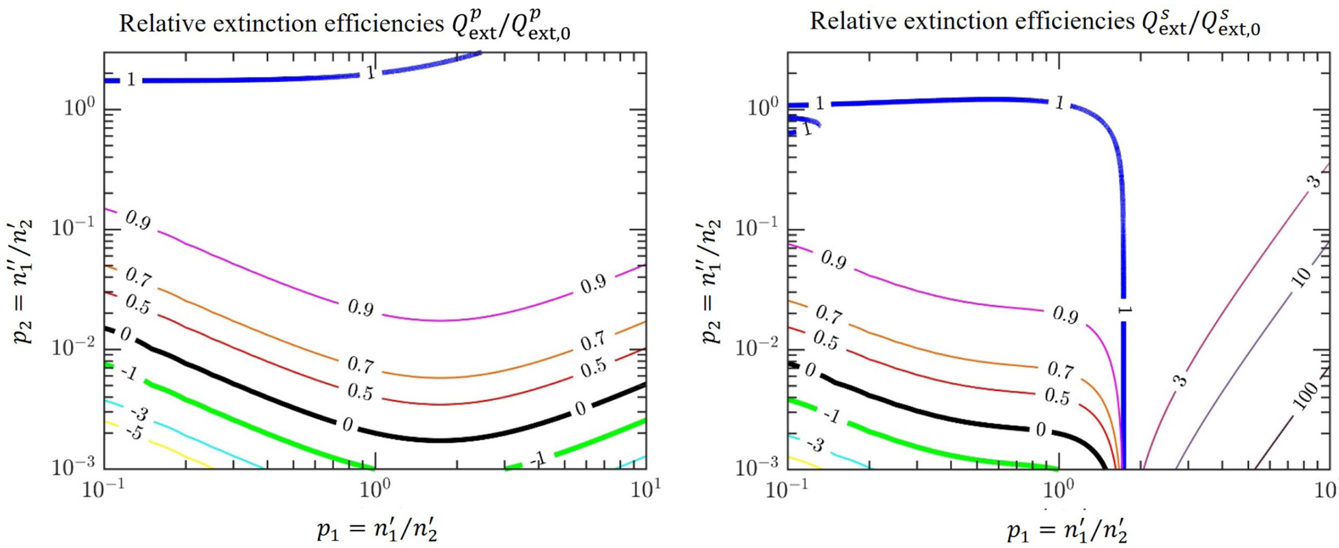

FIG. 7. Relative extinction efficiencies $Q^{p,s}_{\text{ext}}$ in absorbing host to $Q^{p,s}_{\text{ext},0}$ in transparent counterpart. $\tau = n''_2/n'_2$ is taken to be 0.002.

Then ratios of Q_{ext} to $Q_{\text{ext},0}$ are

$$\frac{Q^p_{\text{ext}}}{Q^p_{\text{ext},0}} = \frac{\text{Im}\{m^2-1\} + \frac{3}{2}\tau(\text{Re}\{m^2-1\})}{\text{Im}\{m_0^2-1\}}, \tag{51}$$

$$\frac{Q^s_{\text{ext}}}{Q^s_{\text{ext},0}} = \frac{\text{Im}\left\{\frac{m^2-1}{m^2+1}\right\} + \frac{3}{2}\tau\left(\text{Re}\left\{\frac{m^2-1}{m^2+1}\right\}\right)}{\text{Im}\left\{\frac{m_0^2-1}{m_0^2+1}\right\}}. \tag{52}$$

According to the above equations, we can conclude that these ratios $Q^{p,s}_{\text{ext}}/Q^{p,s}_{\text{ext},0}$ are functions of the relative quantities τ, p_1, and p_2, and are independent of the size parameter. We first study in detail the case of p polarization. When we solve $Q^p_{\text{ext}}/Q^p_{\text{ext},0} = c$ with c a constant, the solution will give $p_2 = \frac{(3+p_1^2)\tau}{4(1-c)p_1}$ for $c < 1$, $p_2 = \sqrt{3+p_1^2}$ for $c = 1$, and $p_2 = \frac{(3+p_1^2)\tau}{4(c-1)p_1} + \frac{4(c-1)p_1}{\tau}$ for $c > 1$. Therefore, the condition of $Q^p_{\text{ext}} = 0$ can be given as

$$p_2 = \frac{(3+p_1^2)\tau}{4p_1}. \tag{53}$$

When $p_2 < \frac{(3+p_1^2)\tau}{4p_1}$, negative extinction occurs. Furthermore, according to these equations, it is appropriate to take a specific value of τ to calculate $Q^p_{\text{ext}}/Q^p_{\text{ext},0}$ with varying p_1 and p_2, since the isoline has a simple relation with τ. The dependence of the relative extinction efficiencies of $Q^p_{\text{ext}}/Q^p_{\text{ext},0}$ for $\tau = 0.002$ on p_1 and p_2 is shown in Fig. 7(a). Since $Q^p_{\text{ext},0} \geqslant 0$, we can observe the zero points of extinction and the conditions for negative extinction.

For s polarization, the equation is more complicated. However, we can give the condition of $Q^s_{\text{ext}} = 0$ as

$$\tau\left[3p_1^4 + 3p_2^4 - 8p_1^2 + 8p_2^2 + 6p_1^2p_2^2 - 3\right] + 8p_1p_2 = 0. \tag{54}$$

It is verified that the solution of this equation coincides well with the numerical results. For $\tau = 0.002$, the dependence of $Q^s_{\text{ext}}/Q^s_{\text{ext},0}$ on p_1 and p_2 is shown in Fig. 7(b). The contour lines are drawn in the figure, where zero value is indicated by the thick black line.

Here, for small particles, an intuitive explanation of negative extinction may be provided. The extinction of small particles will be dominated by the absorption in the particles when the host media are transparent. And if it is assumed to be the same for the absorbing host media, the negative extinction will occur when n''_2 is larger than n''_1. The reason is that the difference in the absorption powers between the particles and the occupied volume of the host media is negative. We will show that this qualitative interpretation is valid for p polarization and can be supported by quantitative formulas. According to Eq. (53), the occurring condition of negative extinction is $n''_2 > \frac{4p_1}{3+p_1^2}n'_1$, which means that the absorption of the host media must be larger than the product of a factor and the absorption of the cylinder. For s polarization, although the quantitative formulas are hard to obtain, we could find from Fig. 7(b) that negative extinction occurs for $p_2 = n''_1/n'_2$ smaller than a threshold value when $p_1 = n'_1/n'_2$ is smaller than about 2. The threshold is related to the absorption of host media through $\tau = n''_2/n'_2$ as seen from Eq. (54). However, when $p_1 = n'_1/n'_2$ is larger than 2, the above qualitative explanation of negative extinction is invalid, which can be seen from Fig. 7(b).

It is easily observed from Fig. 7 that neglecting the weak absorption of the host media would lead to a large deviation from the real extinction. As an illustrating example, we calculate extinction efficiencies of a germanium (Ge) cylinder with the radius of 0.05 μm in polyethylene (PE), where the infrared optical constants of Ge and PE can be found in Refs. [67,68], respectively. In the far-infrared range, PE is a highly transparent material for applications of lens and windows. Considering the Ge cylinder embedded in PE, we should examine numerically the errors associated with ignoring the weak absorption of PE. In Fig. 8 the results corresponding to the generalized theory of this paper are compared with those from the conventional theory which neglects the absorption of PE. It can be immediately found that huge

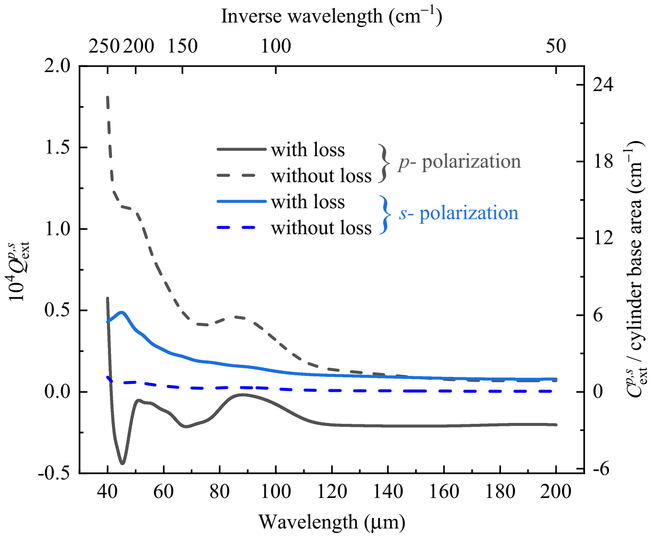

FIG. 8. $Q_{ext}^{p,s}$ of a Ge cylinder of radius 0.05 μm in polyethylene (PE) considering and neglecting the absorption of PE.

differences exist between the two theories. For p polarization, ignoring the absorption of PE will largely overestimate the extinction. When the absorption is considered, Q_{ext}^{p} crosses zero at about 40 μm and becomes negative in the range of 40–200 μm. Contrary to p polarization, ignoring the loss of PE will underestimate the extinction efficiencies of s polarization.

IV. CONCLUSION

A generalized optical theorem of an infinite cylinder embedded in a weakly absorbing host medium is developed in this paper. The far-field optical extinction cross sections and extinction efficiencies are derived in the generalized analytical way, that is, dividing the energy budget associated with the forward-scattering interference by the forward incident intensity. We elaborate on the fact that the absorption of the host medium will exert great impact on the extinction of both thick and thin cylinders.

For large cylinders, the amplitude of interference structure will be enlarged by the absorption of the host, which leads to negative extinction. Furthermore, the narrow ripple resonance structure superposed on the wide interference structure, also known as morphology-dependent resonance (MDR), is damped by the optical loss of the host media. The electric field inside the cylinder verifies the damped MDR in the absorbing host, which implies that the lossy host media may weaken or even extinguish the inelastic scattering phenomena like fluorescence scattering resonances and Raman scattering resonances.

It is found that negative extinction also exists for very small cylinders. We give the analytic expressions for the conditions of zero extinction, from which the conditions for the emergence of negative extinction are also drawn. Furthermore, the results calculated from the generalized theory in this paper are compared with those of the conventional theory that ignores the absorption of the host media. The ratios between the extinction efficiencies of the two different theories show no dependence on the size parameter but instead depend only on the refraction indices of the cylinder and the host media. Finally, as a practical example, the electromagnetic scattering by a Ge cylinder in polyethylene is considered to illustrate the differences between the two theories.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grants No. 52076123 and No. 52006127) and China Postdoctoral Science Foundation (Grant No. 2021M691906).

[1] R. G. Newton, Optical theorem and beyond, Am. J. Phys. **44**, 639 (1976).
[2] C. F. Bohren and A. B. Fraser, Colors of the sky, Phys. Teach. **23**, 267 (1985).
[3] B. D. Savage and J. S. Mathis, Observed properties of interstellar dust, Annu. Rev. Astron. Astrophys. **17**, 73 (1979).
[4] R. Ceolato and M. J. Berg, Aerosol light extinction and backscattering: A review with a lidar perspective, J. Quant. Spectrosc. Radiat. Transfer **262**, 107492 (2021).
[5] N. D. Thorat, S. A. M. Tofail, B. von Rechenberg, H. Townley, G. Brennan, C. Silien, H. M. Yadav, T. Steffen, and J. Bauer, Physically stimulated nanotheranostics for next generation cancer therapy: Focus on magnetic and light stimulations, Appl. Phys. Rev. **6**, 041306 (2019).
[6] M. I. Mishchenko, *Electromagnetic Scattering by Particles and Particle Groups: An Introduction* (Cambridge University Press, Cambridge, 2014).
[7] Q. Zhao, J. Zhou, F. Zhang, and D. Lippens, Mie resonance-based dielectric metamaterials, Mater. Today **12** (12), 60 (2009).
[8] J. A. Lock, J. T. Hodges, and G. Gouesbet, Failure of the optical theorem for Gaussian-beam scattering by a spherical particle, J. Opt. Soc. Am. A **12**, 2708 (1995).
[9] A. V. Krasavin, P. Segovia, R. Dubrovka, N. Olivier, G. A. Wurtz, P. Ginzburg, and A. V. Zayats, Generalization of the optical theorem: Experimental proof for radially polarized beams, Light Sci. Appl. **7**, 36 (2018).
[10] D. Halliday and A. Curtis, Generalized optical theorem for surface waves and layered media, Phys. Rev. E **79**, 056603 (2009).
[11] P. S. Carney, The optical cross-section theorem with incident fields containing evanescent components, J. Mod. Opt. **46**, 891 (1999).
[12] P. L. Marston, Generalized optical theorem for scatterers having inversion symmetry: Applications to acoustic backscattering, J. Acoust. Soc. Am. **109**, 1291 (2001).
[13] E. A. Marengo and J. Tu, Generalized optical theorem in the time domain, Prog. Electromagn. Res. B **65**, 1 (2016).

[14] A. E. Moskalensky and M. A. Yurkin, Energy budget and optical theorem for scattering of source-induced fields, Phys. Rev. A **99**, 053824 (2019).
[15] P. Yang, B. C. Gao, W. J. Wiscombe, M. I. Mishchenko, S. E. Platnick, H. L. Huang, B. A. Baum, Y. X. Hu, D. M. Winker, S. C. Tsay, and S. K. Park, Inherent and apparent scattering properties of coated or uncoated spheres embedded in an absorbing host medium, Appl. Opt. **41**, 2740 (2002).
[16] X. Zhang, J. Qiu, J. Zhao, X. Li, and L. Liu, Complex refractive indices measurements of polymers in infrared bands, J. Quant. Spectrosc. Radiat. Transfer **252**, 107063 (2020).
[17] H. K. Kim, G. W. Hanson, and D. A. Geller, Are gold clusters in rf fields hot or not?, Science **340**, 441 (2013).
[18] F. A. Duck, *Physical Properties of Tissues: A Comprehensive Reference Book* (Academic Press, New York, 2013).
[19] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley-VCH, New York, 1983).
[20] H. C. van de Hulst, *Light Scattering by Small Particles* (Dover Publications, New York, 1981).
[21] N. J. Rommelfanger, Z. Ou, C. H. C. Keck, and G. Hong, Differential Heating of Metal Nanostructures at Radio Frequencies, Phys. Rev. Appl. **15**, 054007 (2021).
[22] M. J. Berg, C. M. Sorensen, and A. Chakrabarti, Extinction and the optical theorem. Part I. Single particles, J. Opt. Soc. Am. A **25**, 1504 (2008).
[23] H. C. van de Hulst, On the attenuation of plane waves by obstacles of arbitrary size and form, Physica (Amsterdam) **15**, 740 (1949).
[24] M. I. Mishchenko, A. A. Lacis, and L. D. Travis, *Scattering, Absorption, and Emission of Light by Small Particles* (Cambridge University Press, Cambridge, 2002).
[25] M. I. Mishchenko, M. J. Berg, C. M. Sorensen, and C. V. M. van der Mee, On definition and measurement of extinction cross section, J. Quant. Spectrosc. Radiat. Transfer **110**, 323 (2009).
[26] W. C. Mundy, J. A. Roux, and A. M. Smith, Mie scattering by spheres in an absorbing medium, J. Opt. Soc. Am. **64**, 1593 (1974).
[27] S.-C. Lee, Light scattering by a coated infinite cylinder in an absorbing medium, J. Opt. Soc. Am. A **28**, 1067 (2011).
[28] C. F. Bohren and D. P. Gilra, Extinction by a spherical particle in an absorbing medium, J. Colloid Interface Sci. **72**, 215 (1979).
[29] R. Dynich, A. Ponyavina, and V. Filippov, Local field enhancement near spherical nanoparticles in absorbing media, J. Appl. Spectrosc. **76**, 705 (2009).
[30] M. Quinten, Local fields close to the surface of nanoparticles and aggregates of nanoparticles, Appl. Phys. B **73**, 245 (2001).
[31] M. Quinten and J. Rostalski, Lorenz-Mie theory for spheres immersed in an absorbing host medium, Part. Part. Syst. Charact. **13**, 89 (1996).
[32] I. W. Sudiarta and P. Chýlek, Mie scattering by a spherical particle in an absorbing medium, Appl. Opt. **41**, 3545 (2002).
[33] I. W. Sudiarta and P. Chýlek, Mie scattering efficiency of a large spherical particle embedded in an absorbing medium, J. Quant. Spectrosc. Radiat. Transfer **70**, 709 (2001).
[34] Q. Fu and W. Sun, Mie theory for light scattering by a spherical particle in an absorbing medium, Appl. Opt. **40**, 1354 (2001).
[35] I. W. Sudiarta and P. Chýlek, Mie-scattering formalism for spherical particles embedded in an absorbing medium, J. Opt. Soc. Am. A **18**, 1275 (2001).
[36] A. Ashkin and J. M. Dziedzic, Observation of Resonances in the Radiation Pressure on Dielectric Spheres, Phys. Rev. Lett. **38**, 1351 (1977).
[37] A. N. Lebedev, M. Gartz, U. Kreibig, and O. Stenzel, Optical extinction by spherical particles in an absorbing medium: Application to composite absorbing films, Eur. Phys. J. D **6**, 365 (1999).
[38] R. Harrison and A. Ben-Yakar, Point-by-point near-field optical energy deposition around plasmonic nanospheres in absorbing media, J. Opt. Soc. Am. A **32**, 1523 (2015).
[39] R. Ruppin, Extinction by a circular cylinder in an absorbing medium, Opt. Commun. **211**, 335 (2002).
[40] W. Sun, N. G. Loeb, S. Tanev, and G. Videen, Finite-difference time-domain solution of light scattering by an infinite dielectric column immersed in an absorbing medium, Appl. Opt. **44**, 1977 (2005).
[41] W. Sun, N. G. Loeb, and B. Lin, Light scattering by an infinite circular cylinder immersed in an absorbing medium, Appl. Opt. **44**, 2338 (2005).
[42] S.-C. Lee, Near field scattering by closely spaced infinite cylinders in an absorbing medium, J. Quant. Spectrosc. Radiat. Transfer **113**, 2379 (2012).
[43] S.-C. Lee, Near field scattering by multiple infinite cylinders in an absorbing medium at oblique incidence, J. Quant. Spectrosc. Radiat. Transfer **114**, 65 (2013).
[44] H. Jiang and S. Hu, Study on light scattering by a multilayered infinite cylinder immersed in an absorbing medium, Opt. Commun. **285**, 2496 (2012).
[45] G. Videen and W. Sun, Yet another look at light scattering from particles in absorbing media, Appl. Opt. **42**, 6724 (2003).
[46] M. I. Mishchenko, Electromagnetic scattering by a fixed finite object embedded in an absorbing medium, Opt. Express **15**, 13188 (2007).
[47] M. I. Mishchenko, Multiple scattering by particles embedded in an absorbing medium. 1. Foldy–Lax equations, order-of-scattering expansion, and coherent field, Opt. Express **16**, 2288 (2008).
[48] M. I. Mishchenko, Multiple scattering by particles embedded in an absorbing medium. 2. Radiative transfer equation, J. Quant. Spectrosc. Radiat. Transfer **109**, 2386 (2008).
[49] M. I. Mishchenko, M. A. Yurkin, and B. Cairns, Scattering of a damped inhomogeneous plane wave by a particle in a weakly absorbing medium, OSA Continuum **2**, 2362 (2019).
[50] M. I. Mishchenko, G. Videen, and P. Yang, Extinction by a homogeneous spherical particle in an absorbing medium, Opt. Lett. **42**, 4873 (2017).
[51] J. Dong, W. Zhang, and L. Liu, Discrete dipole approximation method for electromagnetic scattering by particles in an absorbing host medium, Opt. Express **29**, 7690 (2021).
[52] M. I. Mishchenko and J. M. Dlugach, Scattering and extinction by spherical particles immersed in an absorbing host medium, J. Quant. Spectrosc. Radiat. Transfer **211**, 179 (2018).
[53] J. Zhai, L. Ma, W. Xu, and L. Liu, Effect of host medium absorption on the radiative properties of dispersed media consisting of optically soft particles, J. Quant. Spectrosc. Radiat. Transfer **254**, 107206 (2020).
[54] M. I. Mishchenko and J. M. Dlugach, Multiple scattering of polarized light by particles in an absorbing medium, Appl. Opt. **58**, 4871 (2019).

[55] L. Ma, B. Xie, C. Wang, and L. Liu, Radiative transfer in dispersed media: Considering the effect of host medium absorption on particle scattering, J. Quant. Spectrosc. Radiat. Transfer **230**, 24 (2019).
[56] S. Zhang, W. Zhang, and L. Liu, Scattering by a charged sphere embedded in an absorbing medium, J. Quant. Spectrosc. Radiat. Transfer **246**, 106908 (2020).
[57] J. A. Stratton, *Electromagnetic Theory* (John Wiley & Sons, New York, 2007).
[58] D. Babusci, G. Dattoli, S. Licciardi, and E. Sabia, *Mathematical Methods for Physicists* (World Scientific, Singapore, 2019).
[59] R. Wong, *Asymptotic Approximations of Integrals* (Society for Industrial and Applied Mathematics, Philadelphia, 2001).
[60] A. V. Osipov and S. A. Tretyakov, *Modern Electromagnetic Scattering Theory with Applications* (John Wiley & Sons, New York, 2017).
[61] M. I. Mishchenko and M. A. Yurkin, Co- and counter-propagating wave effects in an absorbing medium, J. Quant. Spectrosc. Radiat. Transfer **242**, 106688 (2020).
[62] M. Yurkin, *On the Feasibility of Measuring Extinction of Single Particles*, in *The 19th Electromagnetic and Light Scattering Conference, Online*, 2021.
[63] M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Academic Press, New York, 1969).
[64] P. Chýlek and J. Zhan, Interference structure of the Mie extinction cross section, J. Opt. Soc. Am. A **6**, 1846 (1989).
[65] J. F. Owen, R. K. Chang, and P. W. Barber, Morphology-dependent resonances in Raman scattering, fluorescence emission, and elastic scattering from microparticles, Aerosol Sci. Technol. **1**, 293 (1982).
[66] G. Videen, J. Li, and P. Chýlek, Resonances and poles of weakly absorbing spheres, J. Opt. Soc. Am. A **12**, 916 (1995).
[67] E. V. Loewenstein, D. R. Smith, and R. L. Morgan, Optical constants of far infrared materials 2: Crystalline solids, Appl. Opt. **12**, 398 (1973).
[68] D. R. Smith and E. V. Loewenstein, Optical constants of far infrared materials. 3: Plastics, Appl. Opt. **14**, 1335 (1975).