Chiral superfluid in a one-dimensional bipartite optical lattice

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We study the formation of a chiral mode in a one-dimensional bipartite optical potential with a double-well dispersion. The geometry of the first excited state is tuned such that two degenerate energy minima are formed at the quasimomentum $q = \pm\hbar k_0/2$ of the first Brillouin zone. Based on the coupled-mode theory, an unconventional superfluid order $p_q \pm ip_{-q}$ is formed between the interacting condensate components within the double-well dispersion band under well-defined phase-locked steady states.

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I. INTRODUCTION

Optical lattices with a higher band occupation allow investigating physics beyond the conventional s-band Hubbard physics and mimic exotic quantum states such as unconventional Bose-Einstein condensates (UBECs) and topological superfluids [1–11]. They provide an ideal platform to manipulate and control the orbital degrees of freedom which are crucial for understanding complex condensed-matter systems such as transition metal oxides and high-T_c superconductivity [12,13]. In these configurations, an exotic chiral superfluid state with unconventional order has been proposed in the high orbital bands of a bipartite optical lattice potential [1]. The bipartite square optical potential is a checkerboard lattice, comprising two sublattices with relatively different lattice depths. This potential can be realized by crossing two standing waves with a relative phase θ. By tweaking the relative phase, the relative depth of the sublattices will change.

In a bipartite lattice, the higher bands can be populated via a swapping technique [14]. Initially the ground state is prepared with an s-atomic orbital residing at the shallow sites. By means of a population swapping technique, a large fraction of atoms can be transferred to the deeper sites where the p wave is hosted. In this system orbital hybridization is driven by an interaction between the local p orbital which plays a significant role in emulating the spin-orbit coupling and an artificial gauge field. Therefore, topologically nontrivial many-body states and superconducting states [3,4] can be implemented.

For weak interactions, the dynamics of the condensate can be described by the Gross-Pitaevskii equation (GPE) derived from mean-field theory under the assumption of a macroscopically populated coherent state,

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + \sigma|\psi|^2\right)\psi, \qquad (1)$$

where $\psi(x,t)$ is the condensate wave function, $V(x)$ is the trapping potential, σ is the interparticle interaction strength, and m is the mass of the bosonic particle. In the noninteracting limit ($\sigma = 0$), Eq. (1) becomes linear in $\psi(x)$. Considering $V(x)$ as a periodic confinement potential, Eq. (1) exhibits a linear spectrum with allowed energy bands separated by a forbidden gap structure. These bands are associated with linear propagating modes called Bloch states. Here, we consider the solution of the GPE for a one-dimensional (1D) double-period optical potential. This potential is created by an interference of two laser beams along the x direction with a relative phase θ. The relative phase θ is tuned to engineer the first excited state with doubly degenerate minima where a cold atomic cloud condenses. An atomic cloud of interacting bosons loaded into dispersion with multiply degenerate minima will be driven by the mean-field interaction to condense at a single minimum. This leads to spontaneous time-reversal symmetry (TRS) breaking and the creation of a chiral atomic superfluid [15–19]. In this state the atomic orbitals are spontaneously arranged into a vortex array with a global orbital angular momentum across the entire lattice.

Here, we report using the coupled-mode theory [20] to prove the existence of a chiral atomic superfluid. Such a state is induced by an umklapp interaction which breaks the spontaneous time-reversal symmetry in the second Bloch band of a double-period optical lattice. Our study reveals that the chiral superposition of the two Bloch modes within the highly populated second energy band of a 1D double-period optical potential represents the steady state of the system. The phase space portrait shows a fixed-point solution which indicates that the system is driven into a chiral mode $p_q \pm ip_{-q}$ under certain conditions. This system provides a simple model to create a nontrivial orbital angular momentum (OAM) order in one dimension. The prediction can be tested using the time-of-flight (TOF) technique. The same results has been realized in a 2D bipartite lattice [1].

In this paper, we demonstrate the formation of a chiral state in the dual minima first excited state of a one-dimensional double-period optically induced potential. We start by solving the standard eigenvalue problem in the Fourier basis. The

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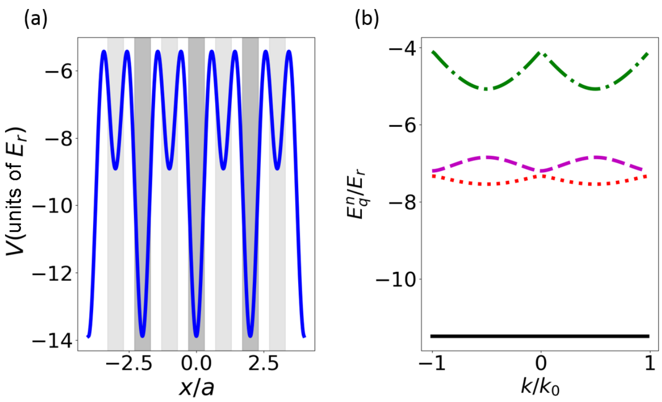

FIG. 1. (a) Bipartite 1D optical lattice potential in Eq. (2) with $V_0 = 5.7E_r$, $\theta = 0.43\pi$. The deep and the shallow sites are shown with dark and light gray stripes, respectively. (b) The linear energy spectrum of the lowest four energy bands.

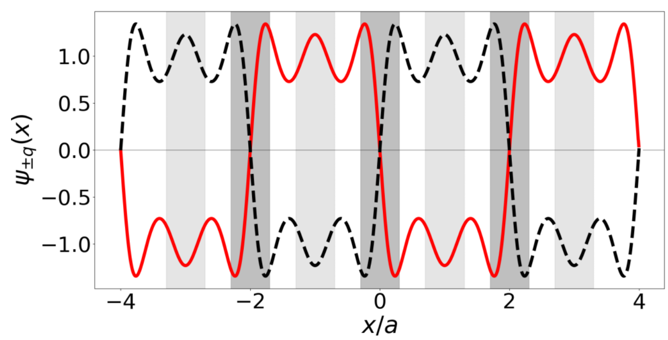

FIG. 2. The real-space Bloch function of the second energy band at points $q = \hbar k_0/2$ (solid red line), $q = -\hbar k_0/2$ (dashed black line) with $V_0 = 5.7E_r$, $\theta = 0.43\pi$.

linear dispersion and the second band Bloch state of the optical trap are analyzed. The Bloch state in the second energy band shows an s-p hybridization nature. The dynamics of the system is then investigated by solving the time-dependent GPE numerically within the framework of the coupled-mode theory. We conclude that a vortexlike state is the energetically favored state which results from the superposition of two degenerate condensate components within the second energy band.

II. THE MODEL

A. Linear spectrum and Bloch modes

We consider cold atoms loaded into the first excited state of the 1D optical lattice potential $V(x)$, described by

$$V(x) = -V_0[\cos^2(k_0x) + \cos(\theta)\cos(k_0x)], \tag{2}$$

where V_0 is the lattice depth, and $k_0 = 2\pi/a$ is the lattice wave number with a as the lattice constant. The angle θ is the relative phase between the two traveling waves along the x and $-x$ direction. The relative phase θ can tuned such that single- and double-period potentials are realized. An alternative method to create a double-period optical potential in one dimension is by interference of two optical-lattice potentials with different wavelengths (bichromatic) and different amplitudes [21–24].

For the noninteracting case ($\sigma = 0$), the eigenvalue equation $H\phi_q^n = E_q^n\phi_q^n$ can be solved and the band structure of the periodic potential (2) is derived based on the Bloch theorem, where the quasimomentum q is restricted to the first Brillouin zone: $-\hbar k_0 < q < \hbar k_0$. Using the coupling matrix H with elements $H_{ll} = E_r(k/k_0 + q)$, $H_{l;l\pm1} = -V_0\cos(\theta)$, $H_{l;l\pm2} = -V_0/4$, the eigenenergies E_q^n of the Bloch states $\phi_q^n = \sum_l c_l^{n,q} e^{i2lk_0x}$ are extracted, where the index n is the band index and l is an integer, and $E_r = \hbar^2 k_0/2m$ denotes the recoil energy. The first excited state [dotted red line in Fig. 1(b)] is characterized by two energetically degenerate minima at the $q = (\hbar k_0/2, -\hbar k_0/2)$ points where a cold bosonic system will condense. For a certain quasimomentum q, the eigenvalues E_q^n for the nth energy band and the corresponding wave vectors $c^{n,q}$ of the coupling Hamiltonian matrix are calculated. The spatial lattice potential and the corresponding band structure for $V_0 = 5.7E_r$ and $\theta = 0.43\pi$ are shown in Fig. 1. The real-space distributions of the Bloch states at the quasimomenta $q = \pm\hbar k_0/2$ of the second energy band [dotted red line in Fig. 1(b)] $\psi_{\pm q}$ manifests the s-p hybridized nature. Their wave functions are the superposition of the local s orbitals at the shallow lattice sites and the p orbitals at the deeper wells, i.e., $\psi_{\pm q} = \sqrt{n_s}\phi_s + \sqrt{n_q}\phi_{\pm q}$ as shown in Fig. 2. This hybridization occurs due to tuning the energy level of the s orbital of the shallow site with that of the p orbital of the deeper site.

B. Nonlinear dynamics

In what follows, we show the dynamics of an interacting BEC loaded into the first double-valley energy band of the potential (2) using the phase-locked superposition of two degenerate Bloch states. Based on the coupled-mode theory, we assume that the macroscopic wave function of the BEC in the first excited state can be represented as a superposition of the two degenerate Bloch modes $\psi_{\pm q}(x)$ at $q = \pm\hbar k_0/2$,

$$\psi(x,t) = C_q(t)\psi_q(x) + C_{-q}(t)\psi_{-q}(x), \tag{3}$$

with complex time-varying amplitudes $C_{\pm q}(t)$ corresponding to the Bloch states $\psi_{\pm q}(x)$, respectively. For convenience the complex amplitudes $C_{\pm q}$ are separated into real moduli $|C_{\pm q}|$ and phase $\theta_{\pm q}$.

Substituting ansatz (3) into the GP equation (1), we obtain the system of coupled-mode equations for the time-varying amplitudes $C_{\pm q}$.

$$\begin{aligned} i\frac{\partial C_{\pm q}}{\partial t} &= \sigma\gamma_{\pm q\pm q}|C_{\pm}q|^2 C_{\pm q} + \sigma\gamma_{-qq}(|C_{\mp q}|^2 C_{\pm q} \\ &\quad + C_{\mp q}^2 C_{\pm q}^* e^{2i(\theta_{-q}-\theta_q)}), \end{aligned} \tag{4}$$

where $\gamma_{ij} = \int \psi_i^2(x)\psi_j^2(x)$ is the set of the overlap integrals. The dynamics of the system can be studied by solving the rate equations for the amplitudes $C_{\pm q}$. This approach has been used to investigate the dynamics of the double-well potential [25–27], where the ground- and higher-order state collective modes are considered. To understand the system dynamics, new physical parameters are introduced: the relative populations of the two modes $Z = |C_q|^2 - |C_{-q}|^2$ and the relative

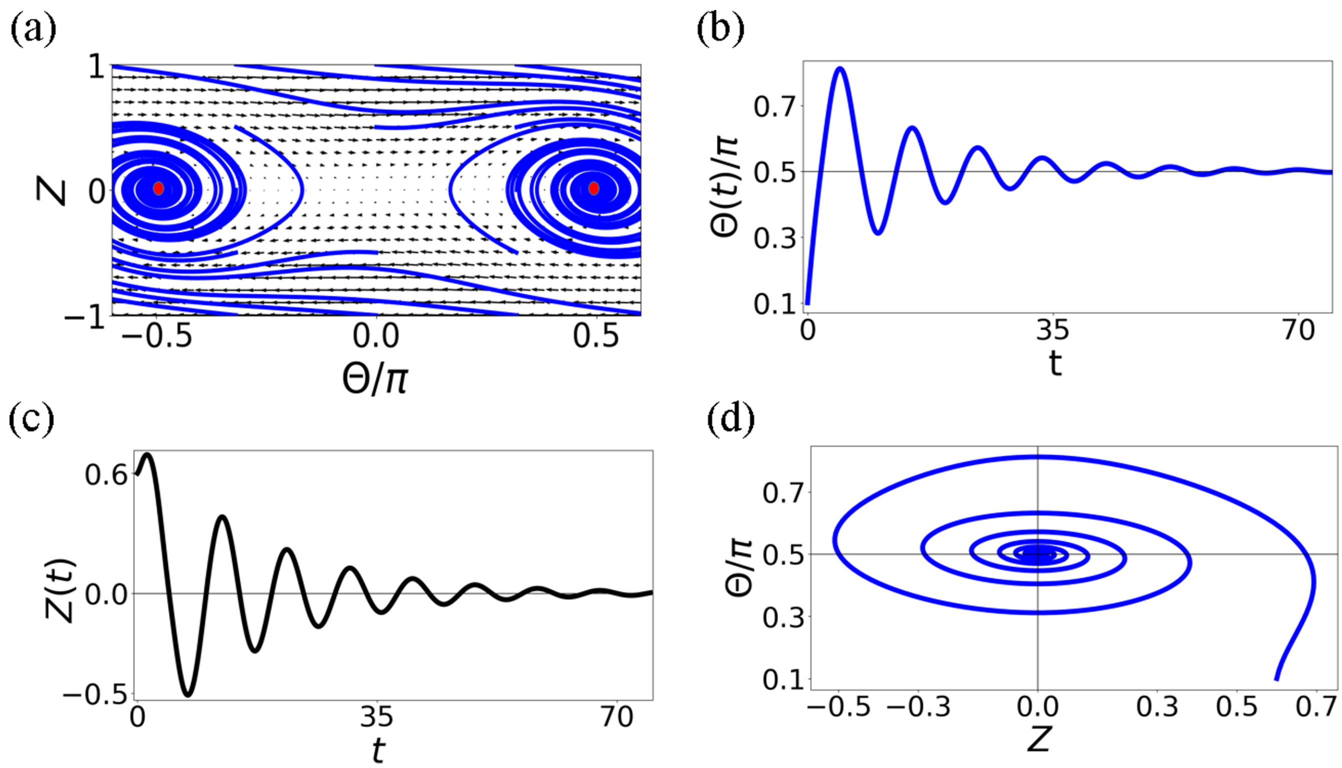

FIG. 3. The numerical solutions for the system of the coupled-mode equations (5) and (6). (a) Phase-space portrait (Z, Θ) with red spots represent the fixed-point solutions at $\pm\pi/2$. The evolution of phase difference Θ and population imbalance Z are shown in (c) and (d). In (d) the single fixed point corresponding to $(Z, \Theta) = (0, \pi/2)$ is shown.

phase $\Theta = \theta_q - \theta_{-q}$. In terms of $Z(t)$ and Θ the coupled-mode equations (4) become

$$\frac{\partial Z(t)}{\partial t} = \sigma\gamma_{-qq}(1 - Z^2)\sin(2\Theta) - \eta Z, \tag{5}$$

$$\frac{\partial \Theta(t)}{\partial t} = \sigma\{(\gamma_{qq} - \gamma_{-q-q})Z - 2Z\gamma_{-qq}[2 + \cos(2\Theta)]\}. \tag{6}$$

In this system of coupled equations, a linear damping term (ηZ) is introduced which represents the finite lifetime due to two-body collisions [28]. The time derivative of the population imbalance and the relative phase between the two points $q, -q$ are proportional to $\sin(2\Theta)$ and $\cos(2\Theta)$, respectively. Such terms are consequences of *umklapp scattering* which result in spontaneous time-reversal symmetry breaking. Due to the degeneracy between the two modes $\psi_{\pm q}$, one can safely assume that the population imbalance is relatively small, $|Z| \ll 1$. The steady states of Eqs. (5) and (6) are found under conditions of equal mode populations $Z = 0$ (i.e., $|C_{-q}| = |C_q|$) with a locked relative phase $\Theta = \pm\pi/2$. Figure 3 shows the time evolution of the relative phase Θ and the population imbalance Z. The population imbalance $Z(t)$ and the relative phase $\Theta(t)$ reach to their steady values as shown in Figs. 3(b) and 3(c). The phase-space portrait [Fig. 3(a)] (Z, Θ) reveals the existence of a two fixed-point solution at $(\pm\pi/2)$ (red spots). Knowing the steady state solution coordinates (Z, Θ) the stable solution for our system can be constructed. The superposition of the two highly populated Bloch modes with equal amplitudes and a fixed phase of $\pi/2$ results in a sequence of vortices such as $\psi_s = (\psi_q + e^{i\pi/2}\psi_{-q})$. The density distribution and the phase structure are shown in Fig. 4. It shows a dip formed in the central site with phase changes from $\pi/2$ to $-\pi/2$. This indicates that an array of vortices is formed at the center of the deep lattice sites with an alternating phase. The system is driven to this vortexlike state due to the nonzero interaction between the two BEC components residing at the two minima $q = \pm\hbar k_0/2$. Based on our formalism the chiral superposition of the two Bloch modes is energetically favorable, since it represents a steady state of the system. In the same context, it is argued that the chiral superposition provides the largest possible mode volume which in turn minimizes the local repulsive interaction [10]. Our prediction can

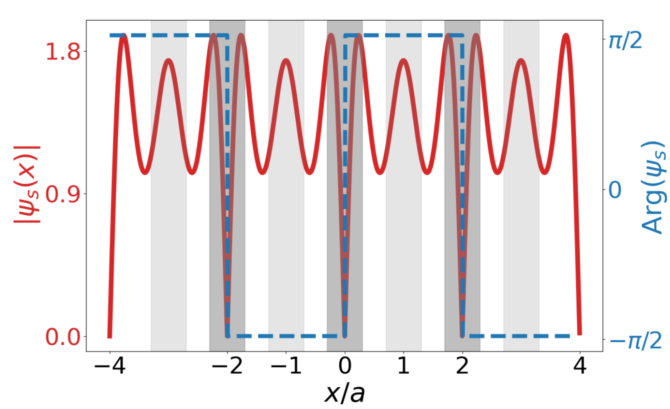

FIG. 4. The density distribution (solid red line) and the phase structure (dashed blue line) of the steady state formed in the first excited band.

be tested experimentally using interference techniques. One possible strategy is mixing the momentum wave function $\psi_{\pm q}$ by a two-photon coupling using a Bragg spectroscopy pulse [29,30] that can resolve the narrow momentum distribution with a high precision. A signature for two condensation points with a fixed relative phase would be observed in the time evolution.

III. CONCLUSION

In this paper, the standard eigenvalue problem is solved for the 1D double-period optical potential. By tuning the potential parameters a double-valley first excited state is formed with two degenerate energy minima. Cold atoms loaded into this energy band will fragment into two degenerate components at $q = \hbar k_0/2$. The Bloch state of the second energy band is characterized by *s*-*p* hybridization. Based on the coupled-mode formalism, the steady state of weakly interacting bosons in a one-dimensional bipartite lattice with a double-valley band is established. A chiral superfluid phase is energetically favorable to minimize the local repulsive interaction. This phase breaks the time-reversal symmetry. This system can be implemented experimentally to ensure creating and manipulating vortex chains in a 1D optical lattice. Controlling the spatial degree of freedom is one of the main features of the quantum information field. For instance, it may be used to generate blocks of nontrivial angular momenta carried by the chiral array to investigate particle entanglement [31]. This model may also can be used to investigate other interesting physics. The interaction strengths can be tuned by changing the lattice depth which may result in a rich phase diagram. The band diagram in Fig. 1 shows that the fourth band (dashed-dotted green line) exhibits doubly degenerate minima which suggests the formation of an F-orbital superfluid.

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