


Genuine activation of nonlocality: From locally available to locally hidden informationSomshubhro Bandyopadhyay ^{*}*Department of Physics, Bose Institute, EN 80, Sector V, Bidhannagar, Kolkata 700091, India*Saronath Halder[†]*Harish-Chandra Research Institute, HBNI, Chhatnag Road, Jhansi, Allahabad 211 019, India* (Received 4 January 2021; revised 1 May 2021; accepted 25 October 2021; published 8 November 2021)

Quantum nonlocality has different manifestations that, in general, are revealed by local measurements of the parts of a composite system. In this paper, we study nonlocality arising from a set of orthogonal states that cannot be perfectly distinguished by local operations and classical communication (LOCC). Such a set is deemed nonlocal, for a joint measurement on the whole system is necessary for perfect discrimination of the states with certainty. On the other hand, a set of orthogonal states that can be perfectly distinguished by LOCC is believed to be devoid of nonlocal properties. Here, we show that there exist orthogonal sets that are locally distinguishable but without local redundancy (i.e., they become nonorthogonal on discarding one or more subsystems) whose nonlocality can be activated by local measurements. In particular, a state chosen from such a set can be locally converted, with certainty, into another state, the identity of which can now only be ascertained by global measurement and no longer by LOCC. In other words, a locally distinguishable set without local redundancy may be locally converted into a locally indistinguishable set with certainty. We also suggest an application, namely, local hiding of information, that allows us to locally hide locally available information without losing any part. Once hidden, the information in its entirety can only be retrieved using entanglement.

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Introduction. Quantum systems consisting of two or more subsystems may have nonlocal properties that, in general, are revealed by local measurements of the parts. Perhaps the most well-known manifestation of quantum nonlocality, viz, Bell nonlocality [1–3] arises from entangled states [4] through their violation of Bell-type inequalities [5–11]. The latter implies that the predictions of quantum theory cannot be explained by any local theory. Bell nonlocality is of particular importance in quantum foundations [1], quantum information [1], and applications thereof. For example, Bell nonlocality tests are routinely used to certify device-independent quantum protocols [12–16].

Bell nonlocality, however, is not the only kind of nonlocality of interest. In this paper, we focus on the nonlocality that arises in the task of discrimination of quantum states by LOCC [17–47]. Recall that LOCC protocols are where local observers perform quantum operations on their respective subsystems and communicate via classical channels but cannot exchange quantum information. Now suppose that two or more observers share the parts of a quantum system prepared in one of several known orthogonal states, the identity of which they do not know. Their goal is to determine which state the system is in without error. But because they are separated from each other, they can only perform measurements realized by LOCC. So the question here is: Can they perfectly distinguish the orthogonal states by LOCC as is

always possible by a joint measurement on the whole system? The answer, however, turns out to be no in general. While two orthogonal-pure states can be perfectly distinguished by LOCC [21], entangled orthogonal bases, such as the Bell basis, cannot be [23,26,29]. We say that a set of orthogonal states is locally distinguishable if the constituent states can be perfectly distinguished with certainty by a LOCC protocol; otherwise, locally indistinguishable.

A locally indistinguishable set of states is nonlocal in the sense that a suitable joint measurement on the whole system always yields more information about the state of the system than any sequence of LOCC [19,24,26,47–49]. This new kind of nonlocality has its own share of counterintuitive results. For example, entanglement is neither necessary nor sufficient for local indistinguishability. The former is proved by the existence of orthogonal product states that are locally indistinguishable [19,20,35,47,50–63]. They give rise to the so-called “quantum nonlocality without entanglement” [19] and the recently discovered stronger version of the same [47]. That entanglement is not sufficient follows from the result that any two mutually orthogonal pure states are locally distinguishable [21].

Besides exhibiting a different kind of nonlocality, locally indistinguishable states also imply the existence of locally hidden information: information encoded in locally indistinguishable states cannot be fully accessed by the local observers, so part of it remains hidden. For example, one can encode two classical bits in four Bell states, but only one bit can be extracted locally simply because the Bell states cannot be perfectly distinguished by LOCC. The only way

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to avail the complete information is by using additional quantum resources such as entanglement. Quantum cryptography primitives such as data hiding [64–67] and secret sharing [68] rely on this particular fact.

For the reasons outlined above, it is not surprising why locally indistinguishable states have received all the attention so far. That includes finding new classes [20,32,38,40,43,45,46], extending the formalism to density matrices [39,41], obtaining new techniques to prove local indistinguishability [23,26,29–31], understanding the presence of entanglement [31,33] or lack of it [19,20,25,49], and more recently, finding the entanglement cost of distinguishing locally indistinguishable states [46]. A set of locally distinguishable states, on the other hand, is generally understood to be neither interesting nor important. The reasons being, it is neither nonlocal because the constituent states can be perfectly distinguished by LOCC nor useful in ways a locally indistinguishable set can be. In this paper, however, we will show that this long-held understanding is, at best, incomplete, for there exist orthogonal states that are locally distinguishable but deserve consideration on par with their locally indistinguishable cousins.

In this paper, we will consider a specific class of LOCC measurements, namely, orthogonality-preserving local measurements (OPLM) [25,47]. The OPLMs are local measurements that keep the postmeasurement states mutually orthogonal, but, on the other hand, they might eliminate one or more states. Indeed, a LOCC protocol that distinguishes orthogonal states is a sequence of OPLMs [69]. We will, of course, leave out the trivial OPLMs, those that do not change the states and consider only the nontrivial ones, those where not all the measurement operators are proportional to the identity.

Let us now consider the following problem. Suppose Alice and Bob share a state chosen from a known set S of orthogonal states. They do not know the identity of the state. They now perform an OPLM \mathbb{M} . Then, for a given outcome μ of this measurement, they end up with a state that belongs to a new orthogonal set S'_μ whose cardinality $|S'_\mu| \leq |S|$. Note that the action of an OPLM does not lead to loss of information [70]. Now, if S is locally indistinguishable, then, by definition, so is S'_μ for all μ . But if S is locally distinguishable instead, can S'_μ be locally indistinguishable? We will make this question more precise in a moment but before we proceed, let us briefly discuss the issue of local redundancy [71].

We say that local redundancy exists in an orthogonal set that remains orthogonal if we discard one or more subsystems. It may be present provided at least one of the local dimensions is composite. An example would make this clear. Let $\{|\Psi_i\rangle\}_{i=1}^4$ and $\{|\Phi_i\rangle\}_{i=1}^4$ be the two-qubit Bell basis and the computational basis respectively. Consider the set

$$\{|\Psi_i\rangle_{AB} \otimes |\Phi_i\rangle_{A'B'} : i = 1, \dots, 4\},$$

where A, A' and B, B' are the qubit pairs held by Alice and Bob, respectively. First, note that the above set is locally distinguishable because one can locally measure $A'B'$ in the computational basis and correctly learn about the identity of the given state. Now observe that if we trace out, for example, AB or $A'B'$ the resulting states remain mutually orthogonal. This is the redundancy we are talking about, which, however, has consequences. In particular, discarding $A'B'$ makes the

set locally indistinguishable because Alice and Bob would then share one of the four Bell states. But, on the other hand, discarding AB keeps it locally distinguishable. This situation arises only because of the local redundancy present in the set. For our analysis, therefore, we will consider orthogonal sets that do not have this redundancy.

So now we suppose S is locally distinguishable and does not have local redundancy. As noted earlier, the action of an OPLM \mathbb{M} achieves the following set transformation: $S \rightarrow S'_\mu$ for the outcome μ , where for every μ it holds that $|S'_\mu| \leq |S|$. This brings us to the question that motivated this work: Do there exist an S and \mathbb{M} such that for any outcome μ , S'_μ is locally indistinguishable? In other words, does there exist an S such that for any given state chosen from S , Alice and Bob can convert it, with certainty, into a state, the identity of which can now only be ascertained by a global measurement and not by LOCC?

So what we require is the following: For any given input $\rho_i \in S$ and any outcome μ of an OPLM \mathbb{M} , $\rho_i \rightarrow \sigma_i(\mu)$ such that the orthogonal set $S'_\mu = \{\sigma_i(\mu)\}$ is locally indistinguishable. Note that the requirement cannot be satisfied if Alice and Bob perform an OPLM that reveals the identity of the input state (which is possible because the set is locally distinguishable), or an OPLM whose every outcome results in a definite output.

An affirmative answer to our question therefore looks improbable and more so because LOCC operations have inherent limitations [72]. So it seems safe to conjecture that S should remain locally distinguishable under OPLMs. However, we will show that this is not the case, in general.

In particular, we present examples of orthogonal sets from $\mathbb{C}^2 \otimes \mathbb{C}^4$, $\mathbb{C}^4 \otimes \mathbb{C}^4$, $\mathbb{C}^5 \otimes \mathbb{C}^5$, and $\mathbb{C}^5 \otimes \mathbb{C}^5 \otimes \mathbb{C}^5$ with the following properties:

- (i) Locally distinguishable and without local redundancy. Note that the local redundancy question does not arise in $\mathbb{C}^5 \otimes \mathbb{C}^5$.
- (ii) There exists an OPLM that converts the set with certainty into a locally indistinguishable orthogonal set such that the cardinality remains unchanged.

We will also show that not all orthogonal sets have the above two properties. Therefore, those that do not are genuinely local. Our result can be viewed as activation of nonlocality by local measurements in the scenario of quantum state discrimination by LOCC. The activation is genuine for the sets do not suffer from local redundancy.

A simple application of our result is the local hiding of locally available information. This can be understood as follows. We know the information encoded in a locally distinguishable set is always locally available. Now suppose it exhibits activable nonlocality. Then it can be converted into a locally indistinguishable orthogonal set of the same cardinality by LOCC with certainty. So the information is no longer completely available locally. This local hiding of information is irreversible, and to retrieve it in its entirety, one must now use entanglement.

Let us first recall a couple of fundamental results in local distinguishability. We will need them frequently in our proofs.

Theorem 1 [21]. Two multipartite orthogonal pure states are locally distinguishable.

The next tells us which sets of orthogonal pure states in $\mathbb{C}^2 \otimes \mathbb{C}^2$ are locally distinguishable and which are not.

Theorem 2 [25]. (a) Three orthogonal pure states in $\mathbb{C}^2 \otimes \mathbb{C}^2$ are locally distinguishable iff at least two of those states are product states. (b) Four orthogonal states in $\mathbb{C}^2 \otimes \mathbb{C}^2$ are locally distinguishable if and only if all of them are product states.

We begin by considering the simple cases.

Proposition 1. Two multipartite orthogonal pure states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ remain locally distinguishable under OPLMs.

By Theorem 1, the states are locally distinguishable. Therefore, an outcome of an OPLM either distinguishes them or converts them into another orthogonal set, which must also contain two pure states in which case Theorem 1 applies.

Next, consider a locally distinguishable set from $\mathbb{C}^2 \otimes \mathbb{C}^2$.

Proposition 2. Let S be an orthogonal set of locally distinguishable states $|\varphi_1\rangle, \dots, |\varphi_n\rangle$ in $\mathbb{C}^2 \otimes \mathbb{C}^2$, where $2 \leq n \leq 4$. Then, under an OPLM, the transformed set remains locally distinguishable.

Because of Proposition 1, it suffices to consider only the cases: $n = 3, 4$. Let $n = 3$. As the states are locally distinguishable, at least two of them must be product states [Theorem 2(a)]. First, suppose that all are product states. Then, for any given outcome of an OPLM, the new orthogonal states must also be product states because LOCC cannot convert product states into entangled states. Then, according to Theorem 2(a), they must be locally distinguishable. Now suppose two are product states, and the other is entangled. By a similar argument, the new orthogonal set consists of either two product states and an entangled state, or three product states. In both cases, Theorem 2(a) tells us they are locally distinguishable. Now consider $n = 4$. By Theorem 2(b), all of them must be product states because they are locally distinguishable, and the previous arguments carry over.

The above results suggest that, perhaps, locally distinguishable states do not give up their local distinguishability under OPLMs. Although compelling, this turns out not to be the case in the higher dimensions.

Orthogonal sets with genuine activable nonlocality. Note that the cardinality of a set of pure states with activable nonlocality is at least three (follows from Proposition 1). We now discuss the examples.

Let $\{|0\rangle, |1\rangle, \dots, |d-1\rangle\}$ be an orthonormal basis in \mathbb{C}^d , where $d \geq 2$. Then, rank-2 and rank-3 projection operators (projectors) are defined as

$$P_{ij} = |i\rangle\langle i| + |j\rangle\langle j|, i \neq j,$$

$$P_{ijk} = |i\rangle\langle i| + |j\rangle\langle j| + |k\rangle\langle k|, i \neq j \neq k,$$

respectively, where $i, j, k \in \{0, 1, \dots, d-1\}$. For example, $P_{01} = |0\rangle\langle 0| + |1\rangle\langle 1|$ and $P_{012} = |0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|$.

Example 1. $\mathbb{C}^2 \otimes \mathbb{C}^4$: We assume that Alice holds a qubit and Bob holds a pair of qubits. We represent the orthonormal basis corresponding to Bob's state space as follows: $|00\rangle \equiv |0\rangle, |01\rangle \equiv |1\rangle, |10\rangle \equiv |2\rangle$, and $|11\rangle \equiv |3\rangle$. Consider the following three orthogonal states (unnormalized):

$$\begin{aligned} |\psi_1\rangle &\equiv |00\rangle + |02\rangle + |11\rangle - |13\rangle, \\ |\psi_2\rangle &\equiv |00\rangle - |02\rangle - |11\rangle - |13\rangle, \\ |\psi_3\rangle &\equiv |01\rangle - |12\rangle - |10\rangle - |03\rangle. \end{aligned} \quad (1)$$

It is easy to see (and show) that the set does not have local redundancy. In particular, not all pairs remain orthogonal if we discard any of Bob's qubits. To show the states (1) are locally distinguishable we proceed as follows. First, Alice performs a measurement on her qubit in the $\{|0\rangle, |1\rangle\}$ basis and tells Bob the result. Now, each of Alice's outcome results in a set of three orthogonal states for Bob to distinguish. If Alice gets "0", Bob distinguishes the states $|0\rangle \pm |2\rangle$ and $|1\rangle - |3\rangle$, and if Alice gets "1", Bob distinguishes $|1\rangle \mp |3\rangle$ and $|0\rangle + |2\rangle$.

We now prove the second property. First, Bob performs a binary measurement defined by the orthogonal projectors P_{01} and P_{23} and informs Alice of the outcome. If Bob gets P_{01} they are left with one of $|00\rangle \pm |11\rangle$ and $|01\rangle - |10\rangle$. Or, if Bob gets P_{23} they are left with one of $|02\rangle \mp |13\rangle$ and $|12\rangle + |03\rangle$. So, in each case, they are left with one of three mutually orthogonal pure entangled states that can be embedded in a $\mathbb{C}^2 \otimes \mathbb{C}^2$ space. But, according to Theorem 2(a), each set is locally indistinguishable. So the states given by (1) can always be locally converted into another set of three orthogonal states that cannot be locally distinguished. This completes the proof.

The following example is built on the previous one, *mutatis mutandis*. But it is interesting in its own right.

Example 2. $\mathbb{C}^4 \otimes \mathbb{C}^4$: We assume that Alice and Bob each holds a pair of qubits. The orthonormal basis corresponding to each local state space is represented as follows: $|00\rangle \equiv |0\rangle, |01\rangle \equiv |1\rangle, |10\rangle \equiv |2\rangle$, and $|11\rangle \equiv |3\rangle$. Now, consider the following orthogonal states (unnormalized):

$$\begin{aligned} |\psi_1\rangle &\equiv |00\rangle + |02\rangle + |31\rangle - |33\rangle, \\ |\psi_2\rangle &\equiv |00\rangle - |02\rangle - |31\rangle - |33\rangle, \\ |\psi_3\rangle &\equiv |01\rangle - |32\rangle - |30\rangle - |03\rangle, \\ |\psi_4\rangle &\equiv |10\rangle + |12\rangle + |21\rangle - |23\rangle, \\ |\psi_5\rangle &\equiv |10\rangle - |12\rangle - |21\rangle - |23\rangle, \\ |\psi_6\rangle &\equiv |11\rangle - |22\rangle - |20\rangle - |13\rangle. \end{aligned} \quad (2)$$

It is a tedious but straightforward exercise to show that the above set does not have local redundancy. We now show that the states are locally distinguishable. First, Alice performs a binary measurement defined by the orthogonal projectors P_{03} and P_{12} on her qubits. If she gets the first outcome, they end up with one of the first three states $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$. Now, these three states are in one-to-one correspondence with those given by (1). This follows by inspection. Hence, $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$ are locally distinguishable. Now, if she gets the second outcome, they end up with one of the three states $|\psi_4\rangle, |\psi_5\rangle, |\psi_6\rangle$. Once again, these are in one-to-one correspondence with those given by (1). Hence they are locally distinguishable. So the whole set is locally distinguishable.

Now we prove the second property. First, Bob performs a binary measurement defined by the orthogonal projectors P_{01} and P_{23} and informs Alice of the outcome. If Bob gets P_{01} they are left with one of the six orthogonal states: $|00\rangle \pm |31\rangle, |01\rangle - |30\rangle, |10\rangle \pm |21\rangle$, and $|11\rangle - |20\rangle$. Or, if Bob gets P_{23} they are left with one of another six orthogonal states: $|02\rangle \mp |33\rangle, |32\rangle + |03\rangle, |12\rangle \mp |23\rangle$, and $|22\rangle + |13\rangle$. Now, in each case, the corresponding set contains locally indistinguishable triplets. For example, the first set contains the triplets $\{|00\rangle \pm$

$|\mathbf{31}\rangle, |\mathbf{01}\rangle - |\mathbf{30}\rangle\}$ and $\{|\mathbf{10}\rangle \pm |\mathbf{21}\rangle, |\mathbf{11}\rangle - |\mathbf{20}\rangle\}$. Each triplet is locally indistinguishable by Theorem 2(a) as the corresponding states can be embedded in a suitable $\mathbb{C}^2 \otimes \mathbb{C}^2$ space. Hence, the whole set must be locally indistinguishable. A similar argument holds for the second case. So the states given by (2) can always be locally converted into another set of six orthogonal states that cannot be perfectly distinguished with certainty by LOCC. This completes the proof.

By now, the basic idea behind the LOCC protocols proving the desired properties is clear. In the following example, each local dimension is prime and, therefore, there cannot be local redundancy.

Example 3. $\mathbb{C}^5 \otimes \mathbb{C}^5$: Consider the set of three orthogonal states (unnormalized):

$$\begin{aligned} |\phi_1\rangle &\equiv |00\rangle + |11\rangle + |22\rangle + |33\rangle + |44\rangle, \\ |\phi_2\rangle &\equiv |00\rangle - |11\rangle - |22\rangle - \omega|33\rangle - \omega^2|44\rangle, \\ |\phi_3\rangle &\equiv |01\rangle + |23\rangle, \end{aligned} \quad (3)$$

where ω, ω^2 are cubic roots of unity. First we show the states are locally distinguishable. Alice performs a binary measurement defined by the orthogonal projectors P_{02} and P_{134} on her system. If she gets the first outcome, they are left to distinguish $|00\rangle \pm |22\rangle, |01\rangle + |23\rangle$. Now, Bob performs a binary measurement defined by P_{02} and P_{134} . If he gets the first outcome they are left to distinguish the orthogonal pair $|00\rangle \pm |22\rangle$ that we know can be locally distinguished (Theorem 1). If he gets the second outcome they are left only with $|01\rangle + |23\rangle$, so the task is completed. Now, if Alice gets the second outcome P_{134} in the first round, they are left to distinguish a pair of orthogonal states $|11\rangle + |33\rangle + |44\rangle$ and $|11\rangle + \omega|33\rangle + \omega^2|44\rangle$ and Theorem 1 applies. So we have shown that the states (3) are locally distinguishable.

Now we prove the second property. Alice performs a binary measurement defined by the orthogonal projectors P_{01} and P_{234} . There are only two possible outcomes. If she gets P_{01} they are left with one of the three orthogonal states $|00\rangle \pm |11\rangle, |01\rangle$. From Theorem 2(a) it follows that they are locally indistinguishable. On the other hand, if she gets P_{234} they are left with one of the three orthogonal states $|22\rangle + |33\rangle + |44\rangle, |22\rangle + \omega|33\rangle + \omega^2|44\rangle$, and $|23\rangle$. But we know such a set cannot be perfectly distinguished with certainty by LOCC [26]. This completes the proof.

We now give an example from a multipartite system. Recall that local (in)distinguishability for a k -partite system, where $k \geq 3$, is defined where all the k parties are separated from each other. However, if a given set is locally indistinguishable in some k' -partite configuration, where $k' \geq 2$, it must be locally indistinguishable. But note that the converse is false. One can find orthogonal states in a tripartite system ABC that are locally distinguishable across all the bipartitions A|BC, B|CA, and C|AB but not in A|B|C [20].

Example 4. $\mathbb{C}^5 \otimes \mathbb{C}^5 \otimes \mathbb{C}^5$: Consider the orthogonal states:

$$\begin{aligned} |\phi_1\rangle &\equiv |000\rangle + |111\rangle + |222\rangle + |333\rangle + |444\rangle, \\ |\phi_2\rangle &\equiv |000\rangle - |111\rangle - |222\rangle - \omega|333\rangle - \omega^2|444\rangle, \\ |\phi_3\rangle &\equiv |011\rangle + |233\rangle, \end{aligned} \quad (4)$$

The above states are a three-party generalization of those in Example 3. The proof is similar.

Discussions. The examples that we discussed provide an idea of constructions in other state spaces. The first and the second example may be suitably generalized in $\mathbb{C}^{2n} \otimes \mathbb{C}^{4n}$ and $\mathbb{C}^{4n} \otimes \mathbb{C}^{4n}$, respectively, for $n \geq 2$. However, proper care should always be taken to ensure there is no local redundancy involved when the dimensions of the local subsystems are composite. The third example could help us to find examples in other spaces where the local dimensions are prime. However, we do not know if they could be found in state spaces such as $\mathbb{C}^2 \otimes \mathbb{C}^3$ and $\mathbb{C}^3 \otimes \mathbb{C}^3$. We also showed that sets with genuine activable nonlocality do not exist in $\mathbb{C}^2 \otimes \mathbb{C}^2$. Whether there are other state spaces also where they do not exist is an interesting question.

Is there an upper bound on the size of sets with genuine activable nonlocality in a given state space? We do not have any particularly helpful intuition here. But we believe other methods for obtaining such sets could help. So how big they could be in a given state space remains an open question. Finally, is it possible to have activable nonlocality without entanglement? In particular, does there exist an orthogonal product set with genuine activable nonlocality? We suspect not. In fact, it would be surprising if it does.

Conclusions. In this paper, we showed that quantum nonlocality can be genuinely activated in the scenario of quantum state discrimination by LOCC. In particular, we considered orthogonal sets of pure states that are locally distinguishable and without local redundancy. We gave several examples where such a set can be locally converted, with certainty, into another orthogonal set, which is locally indistinguishable. That is, a state chosen from an activable set can be locally converted, with certainty, into another state, the identity of which can be determined by a global measurement but not by LOCC. We also discussed a potential application, namely, local hiding of the entire locally available information. The information, once hidden, is no longer locally available in full and to access it, one must use entanglement. We also discussed interesting open questions.

The notion of activation of quantum nonlocality is known to hold in the context of Bell nonlocality [73–77]. Our result shows that activation of nonlocality also appears meaningfully in local quantum state discrimination. So whether the activation phenomenon can also be observed in other manifestations of nonlocality (for example, Ref. [78]) is an intriguing question.

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- [69] In a particular round, the local measurements either identify the state correctly at which point the protocol ends or failing to do so, preserve orthogonality of the postmeasurement states, possibly eliminating one or more from the contention, so that the protocol can continue.
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