Letter | Editors' Suggestion

Losses in interacting quantum gases: Ultraviolet divergence and its regularization

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We investigate the effect of losses on an interacting quantum gas. We show that, for gases in dimensions higher than one, assuming together a vanishing correlation time of the reservoir where dissipation occurs, and contact interactions lead to a divergence of the energy increase rate. This divergence is a combined effect of the contact interactions, which impart arbitrary large momenta to the atoms, and the infinite energy width of the reservoir associated with its vanishing correlation time. We show how the divergence is regularized when taking into account the finite energy width of the reservoir, and, for a large energy width, we give an expression for the energy increase rate that involves the contact parameter. We then consider the specific case of a weakly interacting Bose-Einstein condensate, that we describe using the Bogoliubov theory. Assuming slow losses so that the gas is at any time described by a thermal equilibrium, we compute the time evolution of the temperature of the gas. Using a Bogoliubov analysis, we also consider the case where the regularization of the divergence is due to the finite range of the interaction between atoms.

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The effect of the coupling of a many-body quantum system to an environment has attracted a lot of attention in the last years, in the context of cold-atom experiments. Engineered coupling was proposed to realize particular many-body states [1,2], including strongly correlated phases or highly entangled states [3]. It can also be used as a resource for quantum computation [4]. A particular coupling to an environment, that has received a lot of attention recently, is realized when the gas suffers from losses. Losses can produce highly correlated phases [5–8], induce the Zenon effect [9–11], drive phase transitions [12], lead to nonthermal states [13–16], and produce cooling [17–19]. In all the works mentioned above, the coupling to the environment is described assuming that the correlation time of the environment is much smaller than any characteristic evolution time of the system. Then the time evolution of the system obeys a universal Lindblad equation (see the review in Ref. [20]) describing the coupling to an environment of vanishing correlation time. In this Letter, we show that this approximation is not always correct.

For a homogeneous single-atom loss process, the universal Lindblad equation reads, for a gas in the continuous space,

$$\frac{d\rho}{dt} = -(i/\hbar)[H_0, \rho] + \Gamma \int d^d\mathbf{r} \left\{ -\frac{1}{2}\{\psi_\mathbf{r}^+\psi_\mathbf{r}, \rho\} + \psi_\mathbf{r}\rho\psi_\mathbf{r}^+ \right\}, \quad (1)$$

where H_0 is the Hamiltonian of the quantum gas, ρ is its density matrix, d is the dimension of the system, ψ_r annihilates an atom at position $\mathbf{r}$, and Γ is the loss rate. For simplicity, we consider here a single-component gas. Equation (1) is universal in the sense that the loss process is characterized by a single parameter Γ, where the details of the reservoir are irrelevant.

The evolution under the above Lindblad equation is simple if one assumes the state of the gas is uncorrelated, for instance, within a mean-field approximation: The population of each single-particle state decreases exponentially [9]. However, interactions between atoms introduce correlations, which highly complicates the calculation of the effect of losses. In cold-atom experiments, the range of the interaction potential between atoms is typically much smaller than all length scales in the problem. Then the effect of interactions is well modeled by a contact interaction term. This description of interactions is also a universal model: Details of the interaction potential are irrelevant and interactions are described by a single parameter, the scattering length. In this Letter, we show that the combination of the above two universal models leads to unphysical predictions in dimensions higher than one: For a gas with contact interactions evolving under Eq. (1), the increase rate of the energy diverges.

The divergence of the energy increase rate originates from the following process. The contact interaction in the gas is responsible for singularities of the many-body wave function when two atoms meet [21], leading, in dimension higher than one, to a diverging kinetic energy. This divergence is counterbalanced by the interaction energy such that the total energy is finite. The Lindblad dynamics of Eq. (1) assumes that loss events are instantaneous with respect to the gas dynamics: Within the quantum trajectory description equivalent to the Lindblad dynamics [20], a loss event corresponds to the instantaneous action of the jump operator $\psi_\mathbf{r}$. Thus, just after a loss event has occurred, the many-body wave function of the remaining atoms is equal to its value just before the loss event. This wave function presents a singularity when the position of an atom approaches the position of the lost atom. The divergence of the kinetic energy associated with this singularity is no longer counterbalanced by the interaction

energy: It amounts to an infinite value of the energy in the system. Note that the infinitely large increase of the energy is made possible by the infinite energy available in the reservoir involved in the loss process: The vanishing correlation time is associated with an infinite energy width.

Several mechanisms could lead to a regularization of the above divergence. First, the finite range of the interaction between the atoms will introduce a cutoff that prevents the divergence of the kinetic energy. Second, the reservoir has in practice a finite energy width which limits the maximum energy a loss event can deposit in the system. In this Letter, we consider both regularizations, with an emphasis on the effect of the finite reservoir energy width.

We first propose a model for the loss mechanism, with a finite energy width $E_{\rm res}$. Using an analysis of the two-atom case, we then derive the expected value of the energy density increase rate for a gas with contact interactions, valid for large $E_{\rm res}$. We find a general expression that involves the contact parameter. Although our derivation concentrates on the bosonic case for simplicity of notations, our results are general. To compute the evolution of the system beyond this limit of large $E_{\rm res}$, one needs a many-body model of the system that includes correlations between atoms introduced by interactions. We will concentrate on the case of a weakly interacting Bose-Einstein condensate and we use the Bogoliubov description. Within this framework, we compute the evolution of the energy. Assuming a loss rate much smaller than the relaxation rate of the gas, the system can be described locally at any time by a thermal equilibrium state. We compute the expected evolution of the temperature under the effect of losses. Within the Bogoliubov treatment, we also consider the case where the regularization comes from the finite range of the interactions.

Model for the loss process. We consider a gas made of particles of mass m in dimension $d = 1$ [one dimension (1D)], $d = 2$ (2D), or $d = 3$ (3D), and we use periodic boundary conditions in a box of size L^d. A homogeneous one-body loss process occurs if, at each point, the atoms are coupled to a continuum. In a gas confined in 1D or 2D, the frozen dimension(s) could serve as the continuum, if the atoms are coupled to an untrapped state. In 3D, the loss mechanism could be the deexcitation of the atoms, if the latter are in a metastable state, in which case the momentum of the emitted photon provides the continuum for the loss mechanism. Here, instead we will consider a simpler, yet equivalent, model [22], where the loss mechanism is induced by a noisy coupling to an untrapped internal state, with the different Fourier components playing the role of the continuum. This would correspond to the effect of a noisy magnetic field for magnetically trapped atoms [19]. More precisely, we consider a coupling to the reservoir which writes

$$V = \int d^d\mathbf{r}\Omega(t)\psi_{\mathbf{r}}b_{\mathbf{r}}^+ + \text{H.c.} = \sum_{\mathbf{p}}\Omega(t)\Psi_{\mathbf{p}}B_{\mathbf{p}}^+ + \text{H.c.}, \quad (2)$$

where $\psi_{\mathbf{r}}$ ($b_{\mathbf{r}}$) annihilates an atom of the system (of the reservoir) at position $\mathbf{r}$, $\Psi_{\mathbf{p}}$ ($B_{\mathbf{p}}$) annihilates an atom of the system (of the reservoir), of momentum $\mathbf{p}$, H.c. is the abbreviation for "Hermitian conjugate," and $\Omega(t)$ is a noisy function. $\mathbf{p}$ takes discrete values whose coordinates are multiples of $2\pi\hbar/L$, and we note $p = |\mathbf{p}|$. We define the energy-dependent rate $\Gamma(E)$ from the spectral density of $\Omega(t)$ according to

$$\Gamma(E) = \frac{1}{\hbar^2}\int d\tau e^{-iE\tau/\hbar}\langle\Omega^*(\tau)\Omega(0)\rangle, \quad (3)$$

and we note $\Gamma_0 = \Gamma(0)$. We assume a Gaussian correlation function such that $\Gamma(E) = \Gamma_0 e^{-E^2/(2E_{\rm res}^2)}$, where $E_{\rm res}$ is the energy width of the loss process, corresponding to a correlation time $\hbar/E_{\rm res}$. The energy of the state of momentum $\mathbf{p}$ in the reservoir is $p^2/(2m)$, where m is the mass of the atoms, up to a constant term that could be compensated by a shift in E of $\Gamma(E)$ and that we take equal to zero. If there would be a single atom in the system, its loss rate, obtained within a Born-Markov approximation [22], would be Γ_0. The Lindblad equation (1) is obtained by making $E_{\rm res} \to \infty$ at a fixed value of Γ_0: $E_{\rm res}$ then no longer plays a role and the parameter Γ_0 entirely characterizes the loss process. However, as shown below, in the presence of contact interactions between atoms, such an approximation leads to a divergence of the energy increase rate in dimension $d > 1$. In this Letter, we consider a finite value for $E_{\rm res}$.

Two-atom case. Let us first investigate the behavior expected for a system comprising initially two atoms. In addition to the kinetic energy term, the Hamiltonian contains a contact interaction term. We go in the center-of-mass frame so that the total momentum is vanishing and we consider a state of energy E_0. The two-atom wave function writes $\varphi(\mathbf{r}_1, \mathbf{r}_2) = \varphi(\mathbf{r}_1 - \mathbf{r}_2)$, with $\iint d^d\mathbf{r}_1 d^d\mathbf{r}_2|\varphi(\mathbf{r}_1 - \mathbf{r}_2)|^2 = 1$. For simplicity of notation, we will consider identical bosonic atoms and use a second quantization representation, such that this state reads $|\varphi\rangle = (1/2)\sum_{\mathbf{p}}\varphi(\mathbf{p})\Psi_{\mathbf{p}}^+\Psi_{-\mathbf{p}}^+|0\rangle$, where, for $\mathbf{p} \neq \mathbf{0}$, $\varphi(\mathbf{p}) = \sqrt{2}\int d^d\mathbf{r}e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}\varphi(\mathbf{r})$. The contact interaction imposes the short-distance behavior $\varphi(r) \simeq u_0(|r| - a_{\rm 1D})$ in 1D, $\varphi(\mathbf{r}) \simeq u_0\ln(|\mathbf{r}|)/a_{\rm 2D})$ in 2D, and $\varphi(\mathbf{r}) \simeq u_0(1/|\mathbf{r}| - 1/a_{\rm 3D})$ in 3D, where the parameter u_0 depends on d, E_0, and L. In momentum space, this asymptotic form leads to the large p behavior [21,24],

$$|\varphi(\mathbf{p})|^2 \underset{|\mathbf{p}|\to\infty}{\simeq} \alpha_d\frac{\hbar^4|u_0|^2}{p^4}, \quad (4)$$

where $\alpha_d = 8$ in 1D, $\alpha_d = 8\pi^2$ in 2D, and $32\pi^2$ in 3D.

The two-atom state $|\varphi\rangle$ is coupled by V to the states $|\mathbf{p}\rangle = \Psi_{\mathbf{p}}^+B_{-\mathbf{p}}^+|0\rangle$ whose energy, equal to the sum of the kinetic energies of the lost atom and the remaining atom, is p^2/m. For weak enough Γ_0, one can use the Born-Markov approximation to compute the loss rate towards the state $|\mathbf{p}\rangle$ [22]. Using $\langle\mathbf{p}|V|\varphi\rangle = \Omega(t)\varphi(\mathbf{p})$, one finds a loss rate

$$\gamma(p) = |\varphi(\mathbf{p})|^2\Gamma(p^2/m - E_0). \quad (5)$$

We can then compute the initial rate of change of the energy for the trapped atoms: $dE/dt = -\Gamma E_0 + \sum_{\mathbf{p}}p^2/(2m)|\varphi(\mathbf{p})|^2\Gamma(p^2/m - E_0)$. Here, $\Gamma = \sum_{\mathbf{p}}\gamma(\mathbf{p})$ is the total loss rate. We will assume that $E_{\rm res}$ is large enough so that there exists a momentum p_0 such that $E_0 \ll p_0^2/m \ll E_{\rm res}$. The first inequality ensures that the finite energy width of the reservoir does not affect the loss events towards states of momenta smaller than p_0. The second inequality ensures that, for final states of momenta larger than p_0, $|\varphi(\mathbf{p})|^2$ takes its large p asymptotic behavior given Eq. (4). Then, the contribution to dE/dt of the decay processes towards the momentum states

of the remaining atom of momentum larger than p_0 is

$$dE/dt)_{|\mathbf{p}|>p_0} = \Gamma_0 \frac{\alpha_d \hbar^4 L^d |u_0|^2}{(2\pi\hbar)^d m} \mathcal{B}, \tag{6}$$

where

$$\mathcal{B} = \begin{cases} \int_{p_0}^{\infty} dp \Gamma(p^2/m)/(p^2\Gamma_0) \simeq 1/p_0 & \text{in 1D,} \\ \pi \int_{p_0}^{\infty} \frac{dp}{p} \Gamma(p^2/m)/\Gamma_0 \simeq \frac{\pi}{4} \ln(\sqrt{mE_{\rm res}}/p_0) & \text{in 2D,} \\ 2\pi \int_{p_0}^{\infty} dp \Gamma(p^2/m)/\Gamma_0 \simeq \nu\sqrt{mE_{\rm res}} & \text{in 3D,} \end{cases} \tag{7}$$

where $\nu = 6.769\ldots$. In 1D, the result no longer depends on $E_{\rm res}$: The energy change rate has a well-defined finite value when $E_{\rm res} \to \infty$. In the following, we consider only gases in dimension $d > 1$. Then $\mathcal{B}$ presents a UV divergence when $E_{\rm res} \to \infty$, which leads to the diverging energy change rate announced in the Introduction. The finite value of $E_{\rm res}$ regularizes this divergence. For large enough $E_{\rm res}$, however, $\mathcal{B}$ is large enough such that Eq. (6) gives the main contribution to dE/dt and below we assume dE/dt is simply given by Eq. (6). Note finally that this two-atom result could have been derived for two different atoms, such as two different fermions, providing the losses affect both atoms in the same way.

Many-body case: Role of the contact. The results above can be generalized to many-body systems containing N atoms since, for large enough $E_{\rm res}$, the physics will be dominated by the two-body physics presented above. More precisely, one expects the above results to hold provided one does a sum over the pairs of atoms. The relevant quantity will be the contact C, which quantifies the number of pairs in the gas [21,24,25]. The contact is defined by the amplitude of the $1/p^4$ tails of the momentum distribution. More precisely, $C = \lim_{p\to\infty} W(p)p^4$, where the momentum distribution is normalized to $\int d^d\mathbf{p} W(\mathbf{p}) = N$. In the two-atom case discussed above, $W(p) = |\varphi(\mathbf{p})|^2 L^d/(2\pi\hbar)^d$ such that $C = \hbar^{4-d}\alpha_d |u_0|^2 L^d/(2\pi)^d$. Thus, Eq. (6) generalizes to a many-body system as

$$dE/dt = \Gamma_0 \frac{C}{m} \mathcal{B}. \tag{8}$$

We emphasize the broad applicability of this expression: It is valid both in 2D and 3D, and for fermions or bosons. Its validity domain is, however, restricted to very large $E_{\rm res}$. To go beyond this approximation, and to estimate its applicability range, one should know the details of the many-body physics. In the following, we do the calculation in the case of a weakly interacting Bose gas described by the Bogoliubov theory.

Exact treatment for a gas described by Bogoliubov. In this section we suppose the gas is a Bose condensed gas of density n. Beyond-mean-field physics is captured, to first approximation, by the Bogoliubov theory. In this theory, the Hamiltonian reduces to

$$H_{\rm BG} = e_0 L^d + \sum_{\mathbf{p}\neq\mathbf{0}} \epsilon_p a_{\mathbf{p}}^+ a_{\mathbf{p}}, \tag{9}$$

where $a_{\mathbf{p}}^+$ creates a Bogoliuov excitation of momentum $\mathbf{p}$ whose energy is $\epsilon_p = \sqrt{p^2/(2m)[p^2/(2m) + 2gn]}$, and e_0 is the ground-state energy density. Bogoliubov operators are bosonic operators which fulfill $[a_{\mathbf{p}}, a_{\mathbf{p}}^+] = 1$, and they are related to the atomic operators by the Bogoliubov transform

$$\begin{cases} \Psi_{\mathbf{p}} = u_p a_{\mathbf{p}} + v_p a_{-\mathbf{p}}^+, \\ \Psi_{-\mathbf{p}}^+ = v_p a_{\mathbf{p}} + u_p a_{-\mathbf{p}}^+, \end{cases} \tag{10}$$

where $u_p^2 - v_p^2 = 1$, $v_p^2 = (f_p + f_p^{-1} - 2)/4$, and $f_p = p^2/(2m\epsilon_p)$. We set $u_0 = 1$ and $v_0 = 0$ such that the above equation also holds for $\mathbf{p} = \mathbf{0}$. Note that we use the symmetry-breaking Bogoliubov approach that does not converse the atom number [26].

Using the Bogoliubov transformation, the coupling to the reservoir, given Eq. (2), reads

$$V = \sum_{\mathbf{p}} a_{\mathbf{p}} (u_p \Omega(t) B_{\mathbf{p}}^+ + v_p \Omega^*(t) B_{-\mathbf{p}}) + \text{H.c.} \tag{11}$$

We compute the master equation describing the time evolution of the density matrix of the system, $\hat{\rho}$: Using second-order perturbation theory, omitting fast oscillating terms, whose effect averages out, and making the Born-Markov approximation, we obtain [22]

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = &-(i/\hbar)[H_0, \hat{\rho}] \\ &- \sum_{\mathbf{p}} \left\{ \Gamma\left(\frac{p^2}{2m} - gn - \epsilon_p\right) u_p^2 \left(\frac{1}{2}\{a_{\mathbf{p}}^+ a_{\mathbf{p}}, \rho\} - a_{\mathbf{p}} \rho a_{\mathbf{p}}^+\right) \right. \\ &\left. + \Gamma\left(\frac{p^2}{2m} - gn + \epsilon_p\right) v_p^2 \left(\frac{1}{2}\{a_{\mathbf{p}} a_{\mathbf{p}}^+, \rho\} - a_{\mathbf{p}}^+ \rho a_{\mathbf{p}}\right) \right\}, \end{aligned} \tag{12}$$

where the function $\Gamma(E)$ is given in Eq. (3). For noninteracting atoms, $v_p = 0$, $u_p = 1$, $gn = 0$, and $\epsilon_p = p^2/(2m)$, such that the above equation reduces to Eq. (1), as expected. Correlations between atoms introduced by the interactions are responsible for the anomalous terms in v_p^2.

The first effect of losses is to decrease the density n. The difference between dn/dt and $-\Gamma_0 n$ is of the order of the density of atoms in the modes of wave vector $p > \sqrt{mE_{\rm res}}$. We assume that $E_{\rm res}$ is large enough so that we can make the approximation $dn/dt \simeq -\Gamma_0 n$. Let us now investigate the evolution of the energy $E = \langle H_0 \rangle$. We use the Bogoliuobv approximation $H_0 \simeq H_{\rm BG}$, where $H_{\rm BG}$ is given in Eq. (9), such that

$$\frac{dE}{dt} = -\Gamma_0 nA + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \left(\frac{d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle}{dt}\right)_{\rm BG}, \tag{13}$$

where $A = L^d de_0/dn + \sum_{\mathbf{p}} \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle d\epsilon_{\mathbf{p}}/dn$ and $(d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle/dt)_{\rm BG}$ is the evolution of $\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle$ within the Bogoliubov approximation. Inverting the Bogoliubov transform Eq. (10), we find that $a_{\mathbf{p}}^+$ and $a_{\mathbf{p}}$ depend explicitly on time, via the dependence of u_p and v_p on n. However, we assume that losses are slow enough so that one has the following adiabatic: $\langle d(a_{\mathbf{p}}^+ a_{\mathbf{p}})/dt\rangle \simeq 0$ [22]. Then $(d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle/dt)_{\rm BG}$ reduces to $(d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle/dt)_{\rm BG} = \text{Tr}(a_{\mathbf{p}}^+ a_{\mathbf{p}} d\hat{\rho}/dt)$ and injecting

Eq. (12) we obtain

$$\left(\frac{d\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle}{dt}\right)_{\mathrm{BG}} = -\Gamma\left(\frac{p^2}{2m} - gn - \epsilon_p\right)u_{\mathbf{p}}^2\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle + \Gamma\left(\frac{p^2}{2m} - gn + \epsilon_p\right)v_p^2\left(1 + \langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle\right). \tag{14}$$

In the case of a reservoir of infinite energy width, for which $\Gamma(E) = \Gamma_0$ for any E, the above equation reduces to $(d\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle/dt)_{\mathrm{BG}} = \Gamma_0(-\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle + v_p^2)$. We recover here the results derived for 1D Bose gases in the quasicondensate regime [27], although those results were derived very differently. In particular, since $v_p^2 \simeq (mgn)^2/\mathrm{p}^4$ at large p, we find that $\langle a_p^{+}a_p\rangle$ develops $1/p^4$ tails. In dimension 1, such tails are responsible for a failure of Tan's relation [16]. In dimensions 2 and 3, such tails lead to the unphysical result that dE/dt diverges. Proper physical results are obtained in higher dimensions only taking into account the finite energy width of the reservoir. For very large E_{res}, dE/dt is dominated by the second term of the right-hand side of Eq. (13), itself dominated by the large p terms for which $v_p^2 \simeq (mgn)^2/p^4$, $\epsilon_p \simeq p^2/(2m)$, and $\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle \simeq 0$. Evaluating the sum, and using the fact that the contact within Bogoliubov theory is $C = L^d(mgn)^2/(2\pi\hbar)^d$, we recover Eq. (8).

The system is ergodic in dimension $d > 1$: Beyond Bogoliubov terms in H_0 include couplings between Bogoliubov modes which, in the absence of losses and as long as local observables are concerned, ensure relaxation towards a thermal state. Here, we assume that Γ_0 is much smaller than the relaxation rate so that $\hat{\rho}$ relaxes at any time to the density matrix of a thermal state. The latter is characterized by the atomic density n and the energy density, or equivalently by n and the temperature T. The energy of the gas fulfills $E = E_{\mathrm{th}}(n, T)$, where $E_{\mathrm{th}}(n, T)$ is evaluated injecting the occupation factors $\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle = (e^{\epsilon_p/(k_BT)} - 1)^{-1}$ into Eq. (9). The time evolution of the gas is entirely characterized by the functions $n(t) = n_0e^{-\Gamma_0 t}$ and $T(t)$. In order to compute $T(t)$, we evaluate dT/dt in the following way. Since E is conserved by the thermalization process, the calculation of dE/dt with Eqs. (13) and (14) is valid, providing one injects $\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle = (e^{\epsilon_p/(k_BT)} - 1)^{-1}$ in the right-hand side of Eq. (14). Once dE/dt has been computed one can compute dT/dt using $dE/dt = -\Gamma_0 n(\partial E_{\mathrm{th}}/\partial n)_T + dT/dt(\partial E_{\mathrm{th}}/\partial T)_n$. Calculations are detailed in the Supplemental Material.

In Fig. 1 we present the time evolution of the temperature of the system, for different values of E_{res}. We find that the ratio $k_BT/(gn)$ is a growing function of time. This contrasts with the prediction obtained for phonons, which are the Bogoliubov modes of momentum $p \ll \sqrt{mgn}$: In the absence of rethermalization between Bogoliubov modes, and for $E_{\mathrm{res}} \gg gn$, one expects that, for phonons, $k_BT/(gn)$ takes the asymptotic value $k_BT/(gn) = 1$ [13,28]. The growth of k_BT/gn is due to the contribution of high-p Bogoliubov modes. The growing rate increases with E_{res}, as expected: We expect that dT/dt diverges as E_{res} goes to infinity.

Regularization by a finite interaction range. The Bogoliubov analysis can serve also to describe the regularization of the UV divergence due to the finite interaction range.

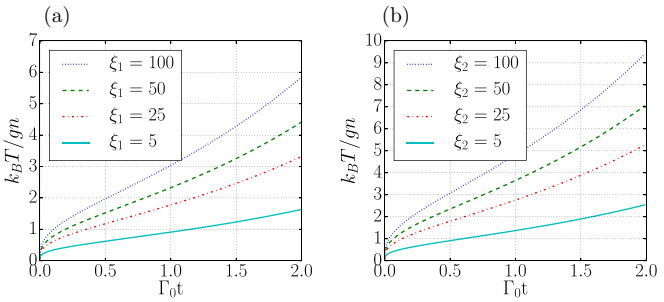

FIG. 1. Evolution of the temperature of a 3D weakly interacting Bose-Einstein condensate under the effect of losses. Temperature is rescaled to the time-dependent chemical potential $\mu \simeq gn \simeq gn_0e^{-\Gamma_0 t}$, where n is the atomic density, equal to n_0 at $t = 0$, g the interaction strength, and Γ_0 is the single-atom loss rate. The initial value is $k_BT(0)/(gn_0) = 0.1$. In (a), the interactions are contact interactions but the reservoir has a finite energy width E_{res}, parametrized by the dimensionless parameter $\xi_1 = \sqrt{E_{\mathrm{res}}/(gn_0)}$. In (b), we assume a reservoir of infinite energy width, or equivalently of vanishing correlation time, but the interactions have a finite range σ (see text) and $\xi_2 = \hbar/(\sigma\sqrt{mgn_0})$.

We consider here a two-body interaction potential $V(r) = ge^{-r^2/(2\sigma^2)}/[(2\pi)^{3/2}\sigma^3]$, where r is the distance between the two atoms and σ is the interaction range. The Bogoliubov transform given in Eq. (10) is still valid, using the Bogoliubov spectrum [29] $\epsilon_{\mathbf{p}} = \sqrt{p^2/(2m)[p^2/(2m) + 2gne^{-p^2\sigma^2/(2\hbar^2)}]}$. One can then compute the effect of losses as above. In the limit of infinite E_{res}, the divergence of dE/dt is regularized by the finite interaction range σ. For very small σ, dE/dt is dominated by the large p term of the sum in Eq. (13), for which $v_p^2 \simeq (mgn)^2(e^{-p^2\sigma^2/\hbar^2})/p^4$. Evaluating the sum, we recover Eq. (8) with $\mathcal{B} = \pi^{3/2}\hbar/\sigma$. As above, from the calculation of dE/dt due to losses, we compute the time evolution of the temperature in the system. Figure 1 shows the time evolution of the temperature, for different values of σ. The evolution of $k_BT/(gn)$ is qualitatively similar to what is observed for contact interactions but with a reservoir of finite energy width: $\hbar^2/(m\sigma^2)$ here plays the role of E_{res}.

Conclusion. Remarkably, although losses are ubiquitous in experiments, the descriptions and the understanding of their effects are still in their infancy. Before this Letter, the effect of losses has been studied using the universal Lindblad equation Eq. (1). However, studies were made either in 1D, in which case the divergence of the energy increase rate does not exist, or for a gas confined in the lowest band of a lattice, in which case the lattice period provides a cutoff that prevents the divergence, or using a mean-field approximation that neglects correlations between atoms. In contrast this Letter provides a prediction for the effect of losses on an interacting quantum gas in higher dimensions and in the continuum. Predictions of this Letter could be tested experimentally using an engineered noisy coupling to an untrapped state [19] whose energy width E_{res} can be varied. Alternatively, the measurement of the temperature evolution could be used to infer the energy width of the reservoir. This Letter raises many questions. How can we extend the results obtained with Bogoliubov to a quasicondensate describing 2D gases at the

thermodynamic limit? How can the results presented here be extended to two-body or three-body losses? How can we extend the calculations done in this Letter to other models of quantum gases, such as two-component fermionic gases? Our work questions the fundamental relation giving the time evolution of the density under the effect of losses. Here, we assumed a reservoir energy width large enough so that dn/dt remains close to $-\Gamma_0 n$. However, one expects a small correction involving two-body processes, whose calculation deserves investigation.

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