Letter

Asymmetry in emission of photons with left- and right-hand circular polarizations in two-photon decay

Konstantin N. Lyashchenko,[1,*] Victoria A. Knyazeva,[2] Oleg Yu. Andreev,[2,3] and Deyang Yu[1,4]

[1]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

[2]St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg 199034, Russia

[3]Petersburg Nuclear Physics Institute named by B.P. Konstantinov of National Research Centre Kurchatov Institute, Gatchina, Leningrad District 188300, Russia

[4]University of Chinese Academy of Sciences, Beijing 100049, China

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Two-photon decay of the $2s$ state in H-like ions is investigated. We report that asymmetry in the emission of photons with left- and right-hand circular polarizations can be observed in this transition if the initial state has a certain polarization. This asymmetry can be used to measure the polarization of ion beams.

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Experiments with spin-polarized particles have become of importance in atomic physics. These experiments can be used for tests of fundamental symmetries [1–5]. This includes investigation of the parity nonconservation and search for the electric dipole moment of electron. In particular, the parity nonconservation effects have been experimentally investigated in neutral atoms [6,7]. However, these effects are greatly enhanced in highly charged ions. A number of experiments have been proposed to observe the parity nonconservation effects in highly charged ions [8], requiring polarized highly charged ion beams being in an excited state. Experimental investigation of the parity nonconservation effects in a highly charged ion is limited by the absence of polarized highly charged ion beams and the ability to measure the circular polarization of the emitted photons. The production of polarized highly charged ions is still a challenge. Several methods were proposed for obtaining polarized beams of heavy highly charged ions in the storage rings [2,5,9]. When the polarizations of the ion beams are obtained, it will be an important task to measure and control the polarization of the beam [10].

In the present article we show that two-photon transitions in highly charged ions can be used to measure the ion-beam polarization. The polarization can be observed due to the asymmetry in the emission of the left- and right-hand photons, arising if the initial state of the ion is polarized.

In the one-photon decay, the emission of the left- and right-hand photons, depending on the polarizations of the initial and final states of the ion, is easy to see. In principle, it also could be used to measure the polarization degree. However, in the case of heavy ions, the experimental study and use of this dependence are limited by the absence of detectors for polarized high-energy photons. The same goes for proposals with two-photon cascade transitions [11]. In two-photon noncascade decay, this difficulty can be circumvented.

The two-photon decay has been studied since the works of Göppert-Mayer [12] and Breit and Teller [13]. The H-like ions are the simplest system for the theoretical study of two-photon decays. In particular, the total and differential transition probabilities for the two-photon decay of the $2s$ state were investigated in works [12–30]. The polarization properties of photons in the two-photon transitions were studied by Manakov *et al.* [31], where only unpolarized electrons were considered. However, for observing the asymmetry between the left- and right-hand photons it is crucial that the initial or final electron state is polarized.

The two-photon decay of the $2s$ state of an H-like ion can be schematically depicted as

$$2s \to 1s + \gamma_{k_1} + \gamma_{k_2}, \tag{1}$$

where γ_{k_1} and γ_{k_2} denote the two emitted photons with the momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. Let us assume that the initial $2s$ state of the H-like ion has a certain projection of the total angular momentum, where the quantization axis is determined by vector $\boldsymbol{\xi}$ directed so that the projection of the momentum onto the vector $\boldsymbol{\xi}$ is $+1/2$. The final state is characterized by two vectors of the photon momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. We do not observe the projection of the total angular momentum of the final $1s$ state.

The condition for the appearance of the asymmetry in the emission of the left- and right-hand photons can already be seen in the geometry of the process depicted in Fig. 1. If the vector $\boldsymbol{\xi}$ is orthogonal to the plane formed by the vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, the inversion operation for this system is reduced to rotation about the origin because $\boldsymbol{\xi}$ is a pseudovector and $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the true vectors. In this case, the asymmetry does not take place. However, if the vector $\boldsymbol{\xi}$ is not orthogonal to this plane, the inversion operation cannot be reduced to rotation. Hence, if we observe the circular polarizations of the photons, then the probabilities of emission of the left- and right-hand photons can be different. This effect can be expressed in another way: If the circular polarizations of emitted photons are fixed then the initial states with different projections of the

*k.n.lyashenko@impcas.ac.cn

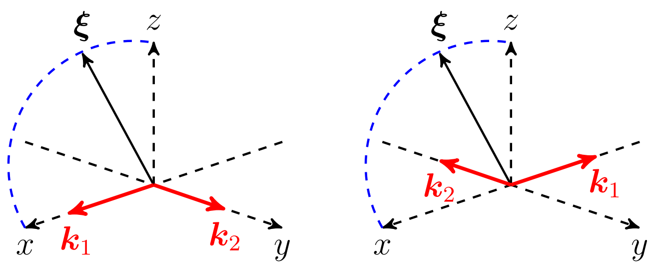

FIG. 1. Geometry of the two-photon decay. The vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the momenta of the emitted photons. The vector $\boldsymbol{\xi}$ defines the axis of the quantization of the electron total angular momentum. The left and right pictures show these vectors before and after the inversion operation.

total angular momentum can give different transition probabilities. Below we investigate the asymmetry in the emission of the photons with left- and right-hand circular polarizations and discuss the possible implementations of this effect.

The Feynman graphs corresponding to the two-photon decay are presented in Fig. 2. The amplitude of the process can be written as

$$U = \sum_{n_n, \varkappa_n, m_n} \left[\frac{A_{fn}^{(k_2\lambda_2)} A_{ni}^{(k_1\lambda_1)}}{\varepsilon_i - \omega_1 - \varepsilon_n} + \frac{A_{fn}^{(k_1\lambda_1)} A_{ni}^{(k_2\lambda_2)}}{\varepsilon_f + \omega_1 - \varepsilon_n} \right], \tag{2}$$

where indices i and f denote initial $2s$ and final $1s$ states of the ion with the energies ε_i and ε_f, respectively. Index $n = (n_n, \varkappa_n, m_n)$ denotes the intermediate state with the principal quantum number n_n, the Dirac angular quantum number $\varkappa_n$, and the projection of the total angular momentum m_n. Summation over n runs over the complete Dirac spectrum. Relativistic units are used throughout this work. In Eq. (2) $A_{ni}^{(k\lambda)}$ denotes the photon-emission matrix element for the photon with the energy-momentum four-vector $k^\mu = (\omega, \boldsymbol{k})$ and polarization λ.

The differential transition probability with summation over the projections of the final state (m_f) and the certain projection of the initial state (m_i) is connected with the amplitude as

$$M_{m_i}^{\lambda_1\lambda_2} = \sum_{m_f} \frac{dW}{d\omega_1 d\Omega_1 d\Omega_2} = \sum_{m_f} \frac{\omega_1^2 \omega_2^2}{(2\pi)^5} |U|^2. \tag{3}$$

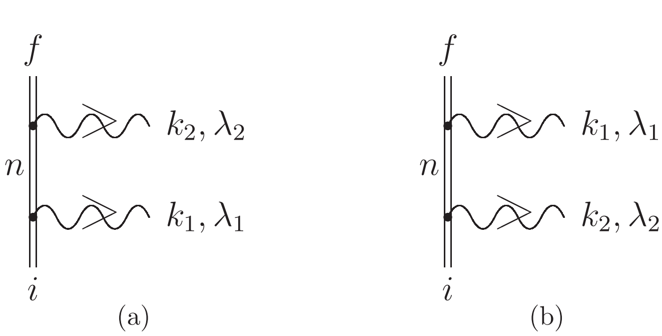

FIG. 2. Feynman graphs corresponding to the direct (a) and exchange (b) terms of the two-photon decay. The letters i and f denote the initial and final states of the electron. The wavy lines describe the emitted photons with momenta k_1, k_2 and polarizations λ_1, λ_2.

Since the sum of the energies of the emitted photons is determined by the energy conservation law $\omega_1 + \omega_2 = \varepsilon_i - \varepsilon_f$, it is convenient to introduce the parameter $x = \frac{\omega_1}{\omega_1 + \omega_2}$.

To analytically demonstrate the presence of the asymmetry effect in the two-photon decay we need to investigate the dependence of the differential transition probability $M_{m_i}^{\lambda_1\lambda_2}$ on the photon emission angles. For this purpose, we can consider the simplest case of the E1E1 emission. Moreover, it is sufficient to consider either the direct or the exchange term of the amplitude with the fixed $\varkappa_n$ of the intermediate state. Below we consider the direct term [the first term in Eq. (2)] with $\varkappa_n = 1$.

In the case of the linear polarizations it is easy to show that $M_{m_i}^{\lambda_1\lambda_2} = M_{-m_i}^{\lambda_1\lambda_2}$ for arbitrary $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. In the case of the circular polarization the situation is different: The expression for $M_{m_i}^{\lambda_1\lambda_2}$ with $\lambda_1 = \lambda_2$ has the following form:

$$\begin{pmatrix} M_{1/2}^{++} \\ M_{-1/2}^{++} \end{pmatrix} = \begin{pmatrix} M_{-1/2}^{--} \\ M_{1/2}^{--} \end{pmatrix} \sim \begin{pmatrix} \sin^2\left(\frac{\theta_1}{2}\right) \\ \cos^2\left(\frac{\theta_1}{2}\right) \end{pmatrix} (1 - \cos\theta_1 \cos\theta_2 \\ - \sin\theta_1 \sin\theta_2 \cos(\varphi_2 - \varphi_1)), \tag{4}$$

where θ_j and φ_j are spherical angles of the wave vector $\boldsymbol{k}_j$; $\lambda_j =$ "+" and $\lambda_j =$ " − " denote right- and left-hand photons, respectively ($j = 1, 2$). It follows from Eq. (4) that, in general, $M_{m_i}^{\lambda_1\lambda_2} \neq M_{-m_i}^{\lambda_1\lambda_2}$ which explains the presence of the asymmetry effect: the difference in the transition probabilities for different m_i and fixed λ_1 and λ_2.

It is convenient to describe the asymmetry effect by the angle (α) between the normal to the plane formed by the vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ and the quantization axis. The asymmetry effect is absent if $\alpha = 0$ (i.e., $\theta_1 = \theta_2 = \pi/2$). Indeed, in this case Eq. (4) transforms to

$$M_{m_i}^{++} = M_{-m_i}^{++} = M_{m_i}^{--} = M_{-m_i}^{--} \\ \sim 1 - \cos(\varphi_2 - \varphi_1), \tag{5}$$

and we do not see the asymmetry effect.

In order to obtain an expression for the transition amplitude as a function of the angle α, we direct z axis along the normal to the $(\boldsymbol{k}_1, \boldsymbol{k}_2)$ plane. Then the axis of quantization $\boldsymbol{\xi}$ is determined by the angle α and azimuthal angle β. For $\beta = 0$ and arbitrary angle α Eq. (5) takes the following form:

$$M_{m_i}^{++}(\alpha) = M_{-m_i}^{--}(\alpha) \\ \sim (1 - \cos(\varphi_2 - \varphi_1))\big(1 + (-1)^{1/2 + m_i} \sin\alpha\big), \tag{6}$$

where the dependence on the projection of the initial state m_i is explicitly presented. We introduce the asymmetry parameter Ξ as

$$\Xi = \frac{M_{-1/2}^{++}(\alpha) - M_{1/2}^{++}(\alpha)}{\frac{1}{2}\big(M_{-1/2}^{++}(\alpha) + M_{1/2}^{++}(\alpha)\big)} \\ = \frac{M_{1/2}^{--}(\alpha) - M_{1/2}^{++}(\alpha)}{\frac{1}{2}\big(M_{1/2}^{++}(\alpha) + M_{1/2}^{--}(\alpha)\big)}. \tag{7}$$

From Eq. (6) it follows that the effect of asymmetry is present in the direct term and that $\Xi \sim \sin\alpha$. The analogous result is obtained for the exchange term. The interference between these terms also makes a contribution to the asymmetry and leads to its strong dependence on the energies of the emitted photons.

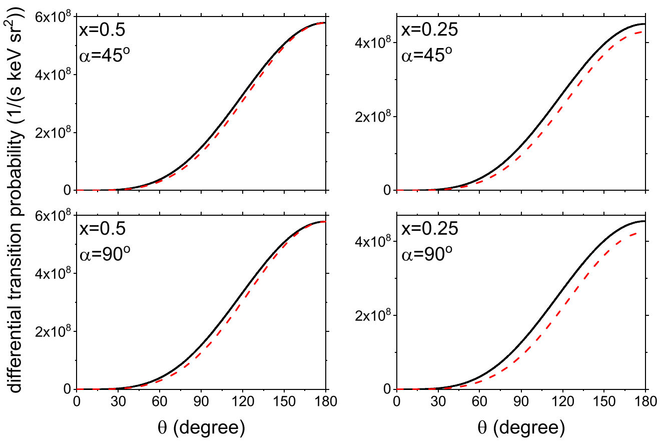

FIG. 3. The differential transition probability for uranium ion as a function of the angle between the momenta of the emitted photons. The left and right columns correspond to $x = 0.5$ and $x = 0.25$, respectively. The upper and lower rows demonstrate differential transition probability for two different angles α: 45° and 90°. The black solid curves present the results for initial projection $m_i = -1/2$ and photon polarizations $\lambda_1 = \lambda_2 =$ "+" (or $m_i = +1/2$, $\lambda_1 = \lambda_2 =$ " − "). The red dashed curves gives the results for $m_i = +1/2$ and $\lambda_1 = \lambda_2 =$ "+" (or $m_i = -1/2$, $\lambda_1 = \lambda_2 =$ " − ").

The main results obtained from this analysis are confirmed by the exact numerical calculation, where $M_{m_i}^{\lambda_1\lambda_2}(\alpha)$ takes into account both the direct and exchange terms in the amplitude as well as the higher multipoles of the emitted photons and full summation over the Dirac spectrum. The summation over the Dirac spectrum was performed using the finite basis set constructed from B splines [32,33]. The summation over the angular quantum numbers of the intermediate electron states was limited to terms with $l_n \leqslant 4$, where l_n is the orbital angular momentum. The atomic nucleus was described by a homogeneously charged sphere. Partial wave expansion was employed for the photon wave functions.

We performed the calculation of the differential transition probability for certain projections of the total angular momentum of the initial state and certain polarizations of the emitted photons. Below we consider the tilted quantization axis with azimuth angle $\beta = 0$ and two polar angles $\alpha = 45°$ and 90°.

In Fig. 3 we present the differential transition probability $M_{m_i}^{\lambda_1\lambda_2}(\alpha)$ as a function of the angle θ between the emitted photon momenta for uranium ion and $x = 0.5$ and $x = 0.25$. The difference between the black solid and red dashed curves shows explicitly the effect of asymmetry: the difference in contributions from different projections of the initial state with the fixed photon polarizations $\lambda_1 = \lambda_2 =$ "+". From Eq. (7) it follows that the same difference takes place between the left- and right-hand photon emissions for the fixed initial projection m_i. The effect of asymmetry is more noticeable for $\alpha = 90°$ than for 45° what is in agreement with an estimate (6).

In Fig. 4 we present the asymmetry parameter Ξ for uranium ion as a function of θ. One can see that Ξ strongly increases for small θ. However, the transition probability becomes small in this region of angles.

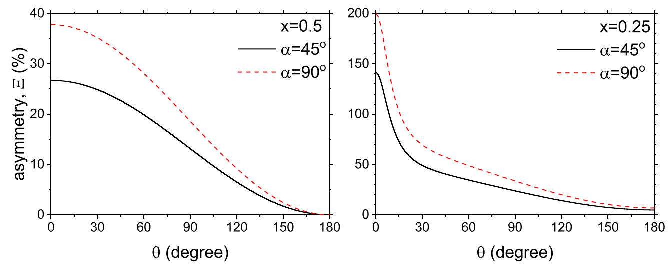

FIG. 4. The parameter of the asymmetry Ξ [see Eq. (7)] for uranium ion as a function of the angle between the momenta of the emitted photons. The left and right plots correspond to $x = 0.5$ and $x = 0.25$, respectively.

In the case of the considered $2s \rightarrow 1s$ transition, the asymmetry is a relativistic effect. In Fig. 5 we present the asymmetry parameter as a function of the atomic number (Z) for $x = 0.25$ and $\alpha = \theta = 90°$. We can see that the asymmetry parameter $\Xi \sim Z^2$ being equal to 0.1%, 34%, and 51% for boron, uranium, and oganesson ions, respectively. In this figure we also present the result of calculation where only the E1E1 emission is taken into account. Despite the fact that higher multipoles make a minor contribution to the transition probability, they make a significant contribution to the asymmetry.

The asymmetry effect can have important implementations. For the described geometry the left- and right-hand photons are emitted differently by ions with different polarizations that can be used in the measurement of the ion-beam polarizations. The detection of circularly polarized high-energy photons is connected with technical difficulties. It can be circumvented utilizing the following features of the two-photon decay. First, the asymmetry effect takes place even if we sum over the polarization of one of the photons. Second, the photon emission spectrum is continuous. Thus, we propose to measure the ion-beam polarization by measuring the polarization of only the low-energy photon, leaving the polarization of the high-energy photon unresolved. This implies the application of the experimental photon-photon coincidence technique in the case of highly charged ions [34]. In the work of Dunford [35], it was proposed to measure the asymmetry of the emission of one polarized photon in a two-photon transition, caused by the parity-violating weak interaction of electrons with the atomic nucleus in He-like ions.

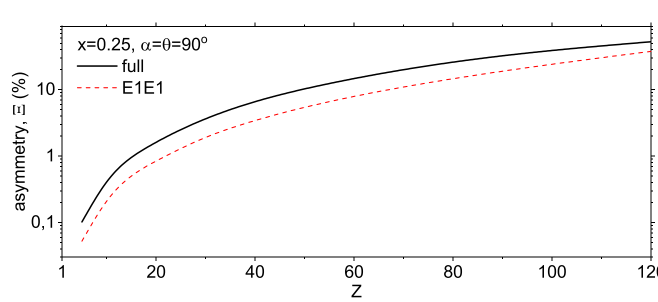

FIG. 5. The parameter of the asymmetry Ξ as a function of the atomic number of the ion Z for $x = 0.25$, $\theta = 90°$, and $\alpha = 90°$.

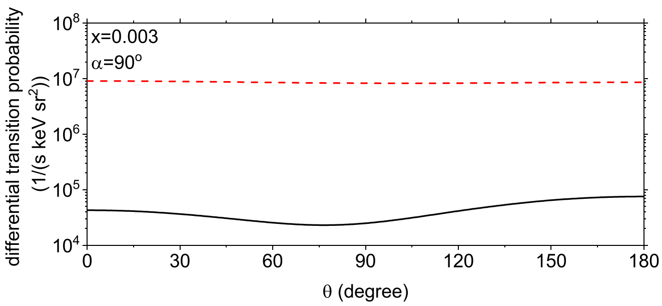

FIG. 6. The differential transition probability $M_{m_i}^{\lambda_1}(\alpha)$ [see Eq. (8)] for uranium ion as a function of the angle between the momenta of the emitted photons. The black solid curve presents the results for angle $\alpha = 90^\circ$, initial projection $m_i = +1/2$, $x = 0.003$ ($\omega_1 = 306$ eV), and photon polarization $\lambda_1 =$ "+". The red dashed curve shows the same, but for $\lambda_1 =$ " − ".

We introduce the differential transition probability where only one emitted photon has certain polarization,

$$M_{m_i}^{\lambda_1}(\alpha) = \sum_{\lambda_2} M_{m_i}^{\lambda_1\lambda_2}(\alpha). \quad (8)$$

In Fig. 6 we compare differential transition probabilities $M_{1/2}^{+}(90^\circ)$ and $M_{1/2}^{-}(90^\circ)$ for uranium ion as a function of θ for $x = 0.003$ ($\omega_1 = 306$ eV). The figure shows a considerable difference in the corresponding transition probabilities.

We assume that the experiment measures the relative number of photon pairs n^- and $n^+ = 1 - n^-$, in which the low-energy photons have left- and right-hand polarizations, respectively. They are connected with the relative number of ions (n_{m_i}) with certain projection m_i as

$$n^\pm = n_{1/2}\frac{M_{1/2}^{\pm}}{M_{1/2}^{+} + M_{1/2}^{-}} + n_{-1/2}\frac{M_{-1/2}^{\pm}}{M_{-1/2}^{+} + M_{-1/2}^{-}}, \quad (9)$$

where $M_{m_i}^{\lambda} \equiv M_{m_i}^{\lambda}(\alpha)$. Then the degree of the ion-beam polarization expresses as

$$\lambda^{\text{ion}} = \frac{n_{1/2} - n_{-1/2}}{n_{1/2} + n_{-1/2}} = (n^+ - n^-)\frac{M_{1/2}^{+} + M_{1/2}^{-}}{M_{1/2}^{+} - M_{1/2}^{-}}, \quad (10)$$

where we used $M_{m_i}^{-} = M_{-m_i}^{+}$.

We note that the asymmetry effect can be found in the reverse process where it is referred to as circular dichroism. The final 2*s* state can obtain polarization if the H-like ion initially being in the unpolarized ground state is excited by two photons one or two of which have certain circular polarizations. Compared to the one-photon excitation, the two- (or more) photon excitation requires polarized photons with much lower energies. In principle, this effect can be used to obtain polarized ion beams. In the case of heavy highly charged ions, the application of this method leads to technical difficulties, such as the need for high intensity or high energy lasers. Hence, it can be limited to the light ions and the heavy many-electron ions.

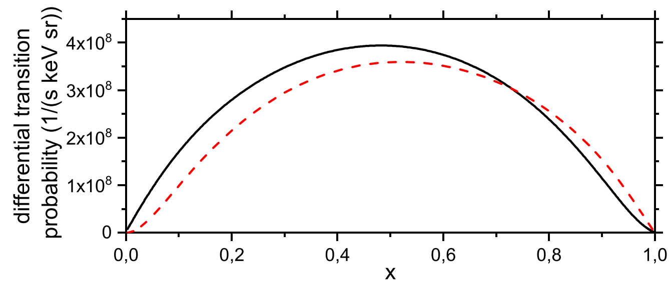

FIG. 7. The differential transition probability [given by Eq. (11)] as a function of x for uranium ion and $\alpha = 90^\circ$. The black solid curve presents the results for initial projection $m_i = -1/2$ and photon polarizations $\lambda_1 = \lambda_2 =$ "+" (or $m_i = +1/2$, $\lambda_1 = \lambda_2 =$ " − "). The red dashed curve gives the results for $m_i = +1/2$ and $\lambda_1 = \lambda_2 =$ "+" (or $m_i = -1/2$, $\lambda_1 = \lambda_2$" − ").

Finally, we would like to show how the asymmetry manifests itself in the energy distribution. In Fig. 7 the differential transition probability,

$$\frac{dW_{m_i}^{\lambda_1\lambda_2}}{d\omega_1 d\Omega_1 \sin\theta_2 d\theta_2} = \int_0^\pi d\varphi_2\, M_{m_i}^{\lambda_1\lambda_2}(\alpha)\sin\varphi_2, \quad (11)$$

as a function of x is presented for $\lambda_1 = \lambda_2 =$ "+". We consider the geometry, where the first photon is emitted along the x axis, the second photon is emitted in the xy plane and integration over the azimuth angle of the second photon φ_2 is performed. The difference between the black solid and dashed red curves demonstrates the effect of asymmetry. If we set $\alpha = 0$, then both curves would coincide and would be symmetric about the center of the allowed photon energy range. At a nonzero angle α, the asymmetry takes place. However, the sum of these curves is symmetric about the center and gives the energy distribution for unpolarized ions. The total transition probability can be obtained as

$$W = 8\pi^2 \sum_{\lambda_1\lambda_2}\int_0^{\frac{1}{2}(\omega_1+\omega_2)} d\omega_1 \frac{dW_{m_i}^{\lambda_1\lambda_2}}{d\omega_1 d\Omega_1 \sin\theta_2 d\theta_2}. \quad (12)$$

We note that W depends neither on the angle α nor on the projection m_i.

In summary, we have investigated the asymmetry of the emission of the left- and right-hand photons in the two-photon transition, which arises in the special geometry of the process. This effect can be used for the measurement of the ion-beam polarization. The observation of the described effect of asymmetry in multiphoton processes, as well as in systems with a larger number of electrons, such as many-electron ions, atoms, and molecules, requires additional study.

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