

## Reply to “Comment on ‘Nonlocality claims are inconsistent with Hilbert-space quantum mechanics’ ”

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In *Phys. Rev. A* **101**, 022117 (2020), it was argued that Bell inequalities are based on classical not quantum physics, and, hence, their violation in experiments provides no support for the claimed existence of peculiar nonlocal and superluminal influences in the real (quantum) world. This Reply to Lambare’s Comment, *Phys. Rev. A* **104**, 066201 (2021), on that paper seeks to clarify some issues related to the correct use of Hilbert space quantum mechanics for identifying the microscopic causes of later macroscopic measurement outcomes, a matter not properly addressed by Bell, who used classical hidden variables in place of the Hilbert subspaces (equivalently, their projectors) employed by von Neumann in *Mathematical Foundations of Quantum Mechanics*.

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### I. INTRODUCTION

Placing Ref. [1] in a historical context will assist in responding to Lambare’s Comment [2]. As its title suggests, the argument in Ref. [1] is based upon *Hilbert space quantum mechanics* (HSQM), that is, to say the basic framework laid down by von Neumann (who invented the term “Hilbert space”) in Ref. [3]: a complex vector space with an inner product—we may assume it to be finite dimensional for the present discussion—with physical properties represented by its subspaces or by the corresponding projectors (orthogonal projection operators) onto these subspaces. For example, in the case of a harmonic oscillator, the quantum *property* that the energy is not greater than  $2\hbar\omega$  is represented by the subspace spanned by its two lowest eigenstates or by the rank-2 projector onto this subspace. Since von Neumann [3] considered Born’s probabilistic interpretation an essential component of quantum theory, stochastic (probabilistic) time development can also be considered a part of HSQM.

The *consistent histories* (CH) interpretation of quantum theory<sup>1</sup> used in [1] and this Reply, is based firmly upon HSQM, but includes additional ideas, notably that of a *quantum history*: a sequence of quantum properties at a succession of times. It allows a probabilistic description of quantum time development analogous to a classical stochastic process with no need to refer to measurements. From the CH perspective, measurements are simply particular instances of quantum processes governed by fundamental quantum principles that make no reference to measurements.

The central feature that distinguishes HSQM from classical physics is that two projectors  $P$  and  $Q$  representing different properties *need not commute*:  $PQ$  can be different from  $QP$ .

Bell when deriving his famous inequalities [7] ignored quantum noncommutation and assumed that microscopic quantum properties could be represented by *classical*, i.e., *commuting* hidden variables, something von Neumann had rejected. When experiments related to Bohm’s version, Ch. 22 of [8], of the Einstein-Podolsky-Rosen paradox [9]—we abbreviate this as EPRB—were carried out to test the Clauser, Horne, Shimony, and Holt (CHSH) version [10] of a Bell inequality, the results agreed with HSQM and violated the inequality. Bell, followed by many others, ascribed this disagreement to the presence in the quantum world of mysterious nonlocal influences. While various objections were raised at that time and later, it was the introduction of the CH approach in the 1980s and its later development, continuing for some years after Bell’s untimely death, that has allowed a detailed understanding of the flaws in Bell’s approach, and how to replace it with a correct quantum analysis that eliminated the need for “spooky nonlocality.” The most recent and detailed presentation is in Ref. [1].

It is hoped that this Reply to Lambare’s Comment will assist other readers in better understanding some of the material in Ref. [1], in particular, how to identify microscopic *causes* of later macroscopic measurement outcomes. Both Comment and Reply employ the same spin-half notation for EPRB. In particular,  $A$  and  $B$ , which take values of  $\pm 1$ , represent the macroscopic outcomes of measurements by Alice and Bob using measurement settings  $a$  and  $b$ ; e.g.,  $a = x$  means that Alice measures  $S_x$ . Section II is a very brief introduction to some simple ideas about *causes*; Sec. III is a short discussion of how to use them for discussing spin measurements; and Sec. IV is a reply to various issues raised in the Comment. It is followed by a brief conclusion in Sec. V.

### II. CAUSES

The intuitive idea of a *causal relationship* between events  $F$  and  $G$  at two times  $t_1 < t_2$  in a probabilistic theory can be thought of in the following way. They are *statistically*

<sup>\*</sup>grif@cmu.edu<sup>1</sup>See Ref. [4] for an overview. Both Refs. [5] and [1] provide short summaries of essential ideas, which are spelled out in much greater detail in Ref. [6].

*independent* provided the joint probability distribution factors,

$$\Pr(F, G) = \Pr(F)\Pr(G), \quad (1)$$

and, otherwise, they are *correlated*, which means that

$$\Pr(G|F) = \Pr(F, G)/\Pr(F) \neq \Pr(G), \quad (2)$$

so the probability of the later event  $G$  depends in some way upon whether  $F$  did or did not occur earlier, thus, suggesting that  $F$  may somehow have influenced  $G$ . The strongest possible correlation, which would characterize a *complete* or *ideal* cause, is:

$$\Pr(G|F) = 1, \quad \Pr(F|G) = 1. \quad (3)$$

That is, if  $F$  occurs, one can be sure  $G$  will occur later, and if  $G$  occurs, it was surely preceded by  $F$ . The complete *absence* of a cause is when  $F$  and  $G$  are statistically independent (1).

But there are situations in which (3) holds, and yet one would not say that  $F$  caused  $G$ . In particular, if there is an event  $E$  at a time earlier than either  $F$  or  $G$ , and it is an ideal cause of both  $F$  and  $G$  in the sense that  $\Pr(F|E) = 1 = \Pr(E|F)$  and  $\Pr(G|E) = 1 = \Pr(E|G)$ ,  $E$  could be the *common cause* of both  $F$  and  $G$ , neither of which is the cause of the other. For example, let  $E$  be a signal Charlie sends to both Alice and Bob, who receive it as  $F$  and  $G$ , respectively. Obviously what Alice receives is not the cause of what Bob receives, even if it arrives earlier, and vice versa.

These brief remarks are intended to help orient the following discussion, and in no sense constitute a complete theory of causes. Conditional probabilities might be less than 1;  $E$ ,  $F$ , and  $G$  could be quantities taking on a number of different values, etc. Of particular importance is the fact that *causes cannot be discussed in this manner without a well-defined probabilistic model*. In the CH approach, a probabilistic sample space or *framework* is a collection of commuting projectors that sum to the identity. If some projectors in one framework do not commute with projectors in a second framework, the two frameworks are *incompatible*, and probabilistic reasoning based on one cannot be combined with that based on the other, an instance of the *single framework rule*. Further details will be found in Refs. [4,11,12].

### III. MEASUREMENTS

Consider a laboratory setup in which a detector detects a particle, perhaps an  $\alpha$  particle or  $\gamma$  ray, coming from the decay of a radioactive source. The experimenter might interpret the detector click as *caused* by the particle passing through a small hole of diameter 2 mm in a thick metal collimator placed just in front of the detector. Observing that the detector never triggers if the hole is blocked, and always detects particles deliberately sent through the hole in separate calibration runs, supports this notion of a cause. But a textbook discussion using unitary time evolution until the detector clicks, completing a measurement, runs into the difficulty that Schrödinger’s equation applied to the spherical wave of the  $\alpha$  particle just after the decay cannot shrink it to a narrow wave packet that can pass through the collimator hole *before* it reaches the detector. The same problem is present in the case of a  $\gamma$  ray.

The CH approach to such a situation is best explained using the much simpler case of measuring the spin of a spin-half particle. Let the particle be prepared in some spin state at time  $t_0$  and then travel undisturbed through a region with no magnetic field until a time  $t_1$  just before it interacts with Alice’s spin measurement apparatus with setting  $a$ , resulting in a macroscopic outcome  $A = \pm 1$  at a later time  $t_2$ . If  $a = z$ , so a spin  $S_z = +1/2$ , in units of  $\hbar$ , at  $t_1$  leads to the later outcome  $A = +1$ , and  $S_z = -1/2$  to  $A = -1$ , is Alice justified in identifying  $S_z$  at  $t_1$  as the *cause* of the later  $A$  outcome? We assume she has checked her apparatus using calibration runs in which particles with known values of  $S_z$  sent into it result in the corresponding  $A$  outcomes.

Let  $\mathcal{F}_z$  be a probabilistic sample space, in CH terminology a *framework*, of four quantum histories with  $S_z = \pm 1/2$  at  $t_1$  followed by  $A = \pm 1$  at  $t_2$ , and probabilities assigned using the Born rule. Since  $S_z$  at  $t_1$  is perfectly correlated with  $A$  at  $t_2$ , it can be considered the microscopic *cause* of the later measurement outcome. This conclusion is not altered if the  $\mathcal{F}_z$  framework is refined to make the framework  $\mathcal{E}_{xz}$ , a collection of eight histories obtained by adding to each of the histories in  $\mathcal{F}_z$  the two possibilities  $S_x = \pm 1/2$  at time  $t_0$  when the particle was prepared. The CH analysis using the  $\mathcal{E}_{xz}$  framework again leads to the conclusion that the  $S_z$  value at  $t_1$  is perfectly correlated with  $A$  at  $t_2$  and is, thus, the cause of the latter.

However, there is an alternative framework  $\mathcal{E}_{xx}$  in which the  $S_z$  values  $\mathcal{E}_{xz}$  at  $t_1$  are replaced with  $S_x$  values: So one has  $S_x$  values at both  $t_0$  and  $t_1$ , followed by  $A$  values at  $t_2$ . (Note that we are continuing to examine the case in which Alice’s measurement setting is  $a = z$ .) The  $\mathcal{E}_{xx}$  framework is incompatible with the  $\mathcal{E}_{xz}$  framework since  $S_x$  and  $S_z$  at  $t_1$  do not commute. Using  $\mathcal{E}_{xx}$ , one can show that the  $S_x$  values  $\pm 1/2$  at  $t_1$  are statistically independent of, so cannot be thought of causing, the later outcome  $A$ . (Note that the  $\mathcal{E}_{xx}$  framework is the one students are taught in textbook quantum mechanics: unitary time evolution up until the measurement begins, followed by a mysterious “collapse.” It is of little help in identifying the microscopic causes of laboratory measurement outcomes.)

The choice of *which* framework to use is made by the physicist when analyzing experimental data in order to understand its physical significance and has no influence on the actual physical process. For experimental runs with an initial  $S_x$  preparation and measurement setting  $a = z$ , Alice can use either  $\mathcal{E}_{xz}$  or  $\mathcal{E}_{xx}$ . But it is only the former that allows her to identify a *cause* for the later  $A$  outcome. And since the two frameworks are incompatible, the corresponding conclusions cannot be combined. Thus, given an initial  $S_x = -1/2$  at  $t_0$  and a final  $A = +1$  at  $t_2$ , Alice can use either  $\mathcal{E}_{xx}$  to infer  $S_x = -1/2$  at  $t_1$  or  $\mathcal{E}_{xz}$  to infer  $S_z = +1/2$  at  $t_1$ . But it makes no sense to combine them, in violation of the single framework rule, to conclude that *both*  $S_x = -1/2$  and  $S_z = +1/2$  were simultaneously true at  $t_1$ . There is no projector in the spin-half Hilbert space that can represent such a combination.

If at any time prior to  $t_1$  Alice changes the  $a = z$  setting of her apparatus to  $a = y$  in order to measure  $S_y$ , she can use an  $\mathcal{F}_y$  framework,  $S_y$  values at  $t_1$  followed by  $A$  values at  $t_2$ , in order to infer an earlier  $S_y$  value from the later outcome  $A$ . Setting  $a = y$  rather than  $a = z$  does not somehow “bring into existence” an  $S_y$  value at  $t_1$ ; instead it allows Alice to learn a particular feature about the past; something she could

not have learned using the alternative  $a = z$  setting. Classical intuition might say that because in a particular run Alice is at liberty to choose either an  $a = z$  or an  $a = y$  setting, therefore, in this run, the particle had both  $S_z$  and  $S_y$  values at  $t_1$ . Such classical intuition applied in a quantum context can be badly misleading. And just as Alice’s choice cannot influence the earlier state of the particle, the earlier state of the particle cannot influence Alice’s choice. Her choice could be made by flipping a coin (or a quantum coin, Sec. 19.2 of Ref. [6]). When properly understood, quantum mechanics is a *local* theory.

#### IV. REPLY TO COMMENT

Various criticisms of Ref. [1] are found in different sections of the Comment. This Reply responds to what appear to be the main objections without attempting to respond to every statement in detail. Note that numbered equation  $N$  in the Comment is referred to below as ‘Eq. ( $N$ )’, whereas equations in this Reply are referenced without the preceding ‘Eq.’.

##### A. Different routes to Clauser, Horne, Shimony, and Holt

In his Sec. III, Lambare points out that while a particular Bell inequality derivation—it is primarily Clauser, Horne, Shimony, and Holt (CHSH) that is in view—might fail due to faulty premisses or bad reasoning, this does not exclude the possibility that a *different* derivation could lead to the *same* inequality. This is true but of little consequence for quantum physics since such alternative derivations only lead to what most physicists believe to be the wrong answer: Inequalities that disagree with the outcomes of numerous experiments, whose results confirm the correctness of the correlations calculated using HSQM. If a student turns in the wrong answer on an examination it may be difficult to locate the mistake in his reasoning, but the answer is still wrong. In Sec. III, Lambare notes that ignoring noncommutation as pointed out by Khrennikov is one possible mistake. Another is the assumption of a joint probability distribution that does not exist in HSQM because it involves noncommuting projectors. The argument in Eq. (4) of the Comment, involving Bell’s notion of local causality (BLC) and the statistical independence of measurement choices, fails because BLC is inconsistent with quantum physics as discussed in Sec. IV B below.

In Sec. II of the Comment, Lambare states that a combination of *Parameter Independence* and *Outcome Independence*, terms used by Jarrett and Shimony, lead to the CHSH inequality, so it is may be useful to locate the source of this mistake. “Parameter Independence” means that the probability of Alice’s outcome  $A$  does not depend upon Bob’s choice of measurement setting  $b$ ; likewise  $B$  is statistically independent of  $a$ . Hence choosing the parameter  $b$  does not allow Bob to signal Alice, and choosing  $a$  does not allow Alice to signal Bob. In Ref. [1] this absence of signaling is a *consequence* of a correct quantum analysis, not an independent assumption, as some of the remarks in the Comment might seem to suggest. Thus quantum theory confirms Parameter Independence. “Outcome Independence,” on the other hand, means that it is possible to identify a *common cause* for the correlations of the macroscopic outcomes in the EPRB situation. Since a

proper quantum analysis *does* identify such a common cause, see Sec. V of Ref. [1] and Sec. IV B below, it is the incorrect assumption that the common cause must be *classical* that leads to the incorrect CHSH inequality.

##### B. Bell’s local causality and quantum common causes

In his Sec. IV, Lambare agrees with Ref. [1] that the *factorization condition* Eq. (6) in the Comment and Eq. (24) in Ref. [1], a central assumption in many derivations of Bell inequalities, is based on classical rather than quantum physics, indicating, at least, one reason why these inequalities are violated in the real world. But then he asserts there is another way of arriving at a contradiction between quantum theory and locality: Bell’s notion of local causality leads to a formula,

$$\Pr(A, B|a, b, \lambda) = \Pr(A|a, \lambda)\Pr(B|b, \lambda), \quad (4)$$

which is Eq. (7) in the Comment. Lambare goes on to claim that this leads to a contradiction, Eq. (9), when in his Eq. (8),  $\lambda$  is equated to the quantum singlet state  $|\psi\rangle$  of two spin-half particles for outcomes  $A = B = 1$  and measurement settings  $b = a$ .

In response, it is worth noting that Bell’s starting point was a very plausible *intuitive* notion of *local causality*: If a cause can be found for an event  $G$  occurring at a particular location at a particular time, it is plausible that one should be able to identify a cause  $F$  in its recent past and nearby in space. When  $G$  is Alice’s outcome  $A$  in the EPRB scenario, the argument in Ref. [1] shows that this intuition is correct: The local cause is the microscopic property of the particle just before it reaches her apparatus. See, in addition, the discussion above in Sec. III.

Next note that (4), Eq. (7) in the Comment, is satisfied in the EPRB situation discussed in Sec. V C of Ref. [1] when  $\lambda$  has an appropriate dependence upon  $a$  and  $b$ , something not excluded in (4) the way it is written. In discussions based on classical hidden variables it is customary to supplement (4) with two additional conditions,

$$\Pr(a, b) = \Pr(a)\Pr(b), \quad (5)$$

$$\Pr(\lambda|a, b) = \Pr(\lambda), \quad (6)$$

where (6) is the same as Eq. (10) in the Comment. Both (5) and (6) are assumed in the Comment, although not stated explicitly, in the course of moving from Eqs. (7) to (8) before arriving at Eq. (9), which demonstrates that this line of reasoning leads to an incorrect conclusion.

The fundamental difficulty with Lambare’s analysis is the assumption that one can identify a *single* quantum cause  $\lambda$  for the outcomes of measurements using *different* measurement settings  $a$  and  $b$ . That this reasoning is incompatible with quantum physics when different settings correspond to measuring incompatible spin components was discussed above in Sec. III. Thus, Bell’s mistake was not in his intuitive idea of a local cause, confirmed by a consistent quantum analysis, but its mathematical embodiment using classical reasoning incompatible with HSQM. Equating this *single*  $\lambda$  to an initial quantum state  $|\psi\rangle$  may help conceal but does nothing to remedy this fundamental difficulty.

The above remarks should, in addition, suffice to address the concerns about *quantum common causes* in Sec. V of the Comment as they indicate the flaw in its Eq. (10), identical to (6) above. A discussion of *quantum causes*, including *quantum common causes*, must be based upon HSQM *not* classical physics.

## V. CONCLUSION

It is hoped this Reply has adequately addressed the issues raised in the Comment in a way that will help readers understand why one cannot discuss quantum causes, not to mention other aspects of quantum theory, by simply employing the tools of classical physics. Bell's inequalities have served a very useful function in quantum foundations by showing how classical ideas, which may seem plausible but disagree with von Neumann's Hilbert space formulation of the theory, can lead to results in disagreement with experiments. The proper way to honor the memory of one of the outstanding figures of twentieth century physics, whose influence on quantum foundations studies can hardly be overestimated, is not by continuing to insist on ideas which later developments have shown to be inadequate, but instead follow Bell's example of careful serious criticism of sloppy thinking and arm-waving explanations, which he wanted to replace with clear concepts

and consistent mathematics in order to produce a structure worthy of being considered a key part of theoretical physics. A good example of what he was looking for, but obviously had not found, can be seen in his devastating critique in one of his last papers, *Against Measurement* [13], of various text-book discussions of measurement that he considered totally inadequate. We, of course, cannot know what his reaction might have been to the current CH formulation of the measurement process. But surely he would have begun with a careful reading of the relevant publications before going on to perhaps identify serious flaws, adopt some of the ideas, make significant improvements, or replace the whole CH approach with something better. This attitude, applied to CH and other quantum interpretations, is very much needed if the foundations community is to emerge from its current disagreements and disarray and arrive at a coherent understanding of quantum theory, showing that we have actually made significant progress since the golden years of 1925–1926, the centenary of which is fast approaching.

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