

Control cost and quantum speed limit time in controlled almost-exact state transmission in open systems

Shen-Shuang Nie,¹ Feng-Hua Ren,² Run-Hong He ,¹ Jing Wu,¹ and Zhao-Ming Wang^{1,*}

¹*College of Physics and Optoelectronic Engineering, Ocean University of China, Qingdao 266100, China*

²*School of Information and Control Engineering, Qingdao University of Technology, Qingdao 266520, China*



(Received 2 June 2021; accepted 2 November 2021; published 22 November 2021)

We investigate the influence of environment noise on the control cost and the quantum speed limit time (QSLT) in the process of almost-exact state transmission (AEST) through a spin chain under pulse control. The chain is immersed in its surrounding non-Markovian, finite-temperature heat baths. We find that AEST can be realized in weak system-bath coupling, low temperature, and strong non-Markovian baths under effective external control. Correspondingly, the control cost and QSLT increase with increasing bath temperature and coupling strength. It is noticeable that non-Markovianity from the baths can be helpful to reduce the control cost and shorten the QSLT. Furthermore, we find that there exists a trade-off between the control cost and transmission fidelity and that higher fidelity requires higher cost. In addition, the minimum control cost has been found to obtain certain transmission fidelity.

DOI: [10.1103/PhysRevA.104.052424](https://doi.org/10.1103/PhysRevA.104.052424)

I. INTRODUCTION

Quantum state transfer (QST) has been widely investigated for its fundamental role to shuttle information between two quantum elements. For short-distance communication, the spin chain as a communication channel has been extensively investigated since Bose's work [1]. However, normally the transmission fidelity decreases dramatically with increasing length of the chain [2]. A lot of work has been suggested to improve the transmission fidelity. For example, the perfect state transfer (PST) can be obtained by properly engineering the coupling configuration of the chain [3–6]. High-fidelity QST in spin chain can also be achieved by applying appropriate external fields [7,8] or by applying an effective quantum control technique [9]. Experimentally, high-fidelity QST based on the Floquet-engineered method in a complex many-body system has been demonstrated [10]. In addition, conditional state transfer between quantum spin transistors in a Heisenberg spin chain has been realized [11].

In an actual system, the communication channel will have interactions with its surrounding environment. Then the decoherence occurs and as a result the transmission fidelity decreases [12,13]. For a Markovian environment, where the environmental memory effects can be safely disregarded, Lindblad equations can be used to describe the system dynamics [14,15]. When the memory effects cannot be neglected, a non-Markovian description of the system dynamics is required [16]. The non-Markovianity of the environment will bring new physics to the open system [17–19]. For example, the memory effect of the non-Markovian environment can be used to generate macroscopic entanglement between two mirrors in optomechanical system [20]. Non-Markovianity

of the environment enhances the quality of the quantum correlation teleportation process with a couple of two-level atoms embedded in a zero-temperature bosonic bath [21]. Measure for the degree of non-Markovianity is undoubtedly important, and different definitions are proposed [22–25]. One widely accepted measure is based on the decreasing monotonicity of the trace distance under the completely positive and trace-preserving operations [22]. In a Markovian process, the monotonicity indicates that the information of distinguishability always flows from the system to environment, while for non-Markovian process, the monotonicity is violated and the information of distinguishability may flow back from the environment to the system [22]. Normally it is a difficult task to solve the non-Markovian dynamics. The quantum state diffusion (QSD) equation has been proved to be a promising approach to tackle this difficulty [16,26–29]. Analytical or exact solutions for many interesting systems have been obtained [30,31]. Recently, by using the QSD approach, QST through a spin chain between two zero-temperature non-Markovian baths [16] or in finite-temperature non-Markovian heat baths [32] has been investigated. The transmission fidelity is found to decrease with increasing system-bath coupling strength, bath temperature, and bath Markovianity.

To combat the detrimental effects of the environment or the dispersion of the chain itself in the state transmission, an effective pulse control method has been proposed to realize almost-exact state transmission (AEST) in a spin chain [9,33]. The pulses are realized by a leakage elimination operator (LEO) Hamiltonian and they must satisfy some certain conditions to guarantee the AEST [33]. However, according to the second law of thermodynamics, nonideal processes are always accompanied by irreversible consumption of thermodynamic resources [34,35]. To obtain AEST via pulse control, the control cost must be considered. Studies on the fundamental lower bound [36–38] of the thermodynamic energy cost

*Corresponding author: wangzhaoming@ouc.edu.cn

provide a significant contribution to advance the molecular devices [39] and nanomachines [40], since thermal and quantum fluctuations play considerable roles on the nanometer scale [41,42]. Another fundamental bound is the maximal evolution speed of the quantum system [43]. The upper bound of state evolution speed, i.e., the quantum speed limit is established from the uncertain relationship between Heisenberg time and energy [44–46]. Quantum speed limit time (QSLT) represents the intrinsic minimal time interval for a quantum system evolving from an initial state to a target state. It has application in virtually all areas of quantum physics, such as quantum communication [47], quantum computation [48], and optimal control theory [49,50]. QSLT in open systems has also been investigated [43,51–54]. In particular, a QSLT based on p norm for an arbitrarily driven open system has been studied [43]. The memory effect of environment characterized by non-Markovianity has been found to shorten QSLT in single-qubit [43] or multiqubit [55] cases.

For open systems, the bath is usually assumed to be collective for simplicity. However, under realistic conditions such as in adiabatic quantum computation [56,57], it is more practical to assume that each qubit interacts with its individual bath. In this paper, we first investigate the AEST through a spin chain by external pulse control. Each spin encounters its individual non-Markovian and finite-temperature heat bath. Using the strategy in Ref. [33], adding a LEO Hamiltonian to the system, we find that AEST can be obtained in finite-temperature heat baths with weak system-bath coupling, strong non-Markovianity, and low temperature. Then we study the influence of the environment noise on the control cost and QSLT for the achievement of AEST. We find that non-Markovianity of the baths plays an essential role in both reducing the control cost and shortening the QSLT.

II. MODEL AND HAMILTONIAN

Considering a quantum system embedded in N -independent multimode bosonic baths, the total Hamiltonian can be written as

$$H_{\text{tot}} = H_s + H_b + H_{\text{int}}, \quad (1)$$

where H_s is the system Hamiltonian and $H_b = \sum_{j=1}^N H_b^j$ is the N -independent baths Hamiltonian with the j th bath's Hamiltonian $H_b^j = \sum_k \omega_k^j b_k^{\dagger} b_k^j$ (setting $\hbar = 1$ from now on). b_k^{\dagger} , b_k^j represent the bosonic creation and annihilation operators and ω_k^j refers to the bosonic frequency of the j th bath. The interaction Hamiltonian is

$$H_{\text{int}} = \sum_{j,k} (g_k^{j*} L_j^{\dagger} b_k^j + g_k^j L_j b_k^{j\dagger}), \quad (2)$$

where L_j is the Lindblad operator. It describes the coupling between the system and the j th bath. g_k^j is the coupling constant between the system and k th model of j th bath.

Suppose that the initial state of the j th bath is in a thermal equilibrium state at temperature T_j and the density operator reads

$$\rho_j(0) = e^{-\beta_j H_b^j} / Z_j, \quad (3)$$

where $Z_j = \text{Tr}[e^{-\beta_j H_b^j}]$ is the partition function and $\beta_j = 1/K_B T_j$. The system dynamics can be calculated by the QSD approach. The total wave function $|\Psi_{\text{tot}}(t)\rangle$ is projected to the coherent state of the bath modes $|z\rangle$ and $|w\rangle$, $|\Psi_t(z_1^*, w_1^*, \dots, z_N^*, w_N^*)\rangle = \langle z_1, w_1, \dots, z_N, w_N | \Psi_{\text{tot}}(t)\rangle$, which is known as the stochastic quantum trajectory. It obeys a linear QSD equation [30]

$$\begin{aligned} & \frac{\partial |\Psi_t\rangle}{\partial t} \\ &= \left[-iH_s + \sum_j (L_j z_t^{j*} - L_j^{\dagger} \bar{O}_z^j + L_j^{\dagger} w_t^{j*} - L_j \bar{O}_w^j) \right] |\Psi_t\rangle, \end{aligned} \quad (4)$$

where

$$z_t^{j*} = -i \sum_k \sqrt{\bar{n}_k^j + 1} g_k^{j*} z_k^{j*} e^{i\omega_k^j t}, \quad (5)$$

$$w_t^{j*} = -i \sqrt{\bar{n}_k^j} g_k^{j*} w_k^{j*} e^{-i\omega_k^j t}, \quad (6)$$

are the j th independent complex Gaussian processes. $\bar{n}_k^j = \frac{1}{\exp(\hbar\omega_k^j/K_B T_j) - 1}$ is the mean thermal occupation number of quanta in mode ω_k^j . The operators $O_{z,(w)}^j$ are defined by the $\frac{\delta |\Psi_t\rangle}{\delta z_s^{j*}, (w_s^{j*})} = O_{z,(w)}^j(t, s, z_1^*, w_1^*, \dots) |\Psi_t\rangle$ [30,58], and $\bar{O}_{z,(w)}^j = \int_0^t ds \alpha_{z,(w)}^j(t-s) O_{z,(w)}^j$. Here $\alpha_{z,(w)}^j(t-s)$ are the correlation functions.

From the consistency condition, the operators $O_{z,(w)}^j$ satisfy [58]

$$\begin{aligned} \frac{\partial O_z^j}{\partial t} &= \left[-iH_s + \sum_j (L_j z_t^{j*} - L_j^{\dagger} \bar{O}_z^j + L_j^{\dagger} w_t^{j*} - L_j \bar{O}_w^j), O_z^j \right] \\ &\quad - \sum_j \left(L_j^{\dagger} \frac{\delta \bar{O}_z^j}{\delta z_s^{j*}} + L_j \frac{\delta \bar{O}_w^j}{\delta w_s^{j*}} \right), \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial O_w^j}{\partial t} &= \left[-iH_s + \sum_j (L_j z_t^{j*} - L_j^{\dagger} \bar{O}_z^j + L_j^{\dagger} w_t^{j*} - L_j \bar{O}_w^j), O_w^j \right] \\ &\quad - \sum_j \left(L_j^{\dagger} \frac{\delta \bar{O}_z^j}{\delta w_s^{j*}} + L_j \frac{\delta \bar{O}_w^j}{\delta w_s^{j*}} \right). \end{aligned} \quad (8)$$

Now we use Eqs. (7) and (8) to derive a non-Markovian master equation. We define $P_t \equiv |\Psi_t\rangle\langle\Psi_t|$ and the reduced density matrix of the system $\rho_s = M[P_t]$. Here $M[P_t]$ is given by the ensemble average over all of the quantum trajectories. According to Refs. [58,59], the evolution equation of the system density matrix reads

$$\begin{aligned} \frac{\partial}{\partial t} \rho_s &= -i[H_s, \rho_s] + \sum_j \{ [L_j, M[P_t \bar{O}_z^{j\dagger}]] - [L_j^{\dagger}, M[\bar{O}_z^j P_t]] \\ &\quad + [L_j^{\dagger}, M[P_t \bar{O}_w^{j\dagger}]] - [L_j, M[\bar{O}_w^j P_t]] \}. \end{aligned} \quad (9)$$

The above equation is the master equation in finite-temperature baths. However, this equation is not a closed equation for ρ_s . The $\bar{O}_{z,(w)}^j$ operators generally contain noises except for some special cases. The noise-independent $\bar{O}_{z,(w)}^j(t)$ operators can be obtained by invoking a perturbation technique [60]. For weak system-bath coupling, the zero order

of the function expansion plays a leading role and we use the approximation $\bar{\mathcal{O}}_{z,(w)}^j(t, z_1^*, w_1^*, \dots, z_N^*, w_N^*) = \bar{\mathcal{O}}_{z,(w)}^j(t)$, then Eq. (9) becomes a closed master equation [32]

$$\begin{aligned} \frac{\partial}{\partial t} \rho_s = & -i[H_s, \rho_s] + \sum_j \{ [L_j, \rho_s \bar{\mathcal{O}}_z^{j\dagger}(t)] - [L_j^\dagger, \bar{\mathcal{O}}_z^j(t) \rho_s] \\ & + [L_j^\dagger, \rho_s \bar{\mathcal{O}}_w^{j\dagger}(t)] - [L_j, \bar{\mathcal{O}}_w^j(t) \rho_s] \}. \end{aligned} \quad (10)$$

In order to calculate the correlation function, we need to introduce the spectral density. Here we use the spectrum density $J(\omega_j) = \frac{\Gamma_j}{\pi} \frac{\omega_j}{1+(\omega_j/\gamma_j)^2}$, which is Ohmic type with a Lorentz-Drude cutoff [61], where Γ_j, γ_j are real parameters. Γ_j represents the coupling strength between the system and the j th bath and γ_j is the characteristic frequency of the j th bath. For weak system-bath coupling, $\Gamma_j \ll 1$, we have [32]

$$\alpha_z^j(t-s) = \Gamma_j T_j \Lambda_j(t, s) + i\Gamma_j \dot{\Lambda}_j(t, s), \quad (11)$$

$$\alpha_w^j(t-s) = \Gamma_j T_j \Lambda_j(t, s), \quad (12)$$

where $\Lambda_j(t, s) = \frac{\gamma_j}{2} e^{-\gamma_j|t-s|}$ is an Ornstein-Uhlenbeck correlation function and it decays on the environmental memory time $1/\gamma_j$. For $\gamma_j \rightarrow \infty$, it corresponds to a Markovian limit. Now the two correlation functions in Eqs. (11) and (12) satisfy

$$\frac{\partial \alpha_{z(w)}^j(t-s)}{\partial t} = -\gamma_j \alpha_{z(w)}^j(t-s). \quad (13)$$

From the above relations, the $\bar{\mathcal{O}}_{z,(w)}^j$ operators satisfy the following equations [32]:

$$\begin{aligned} \frac{\partial \bar{\mathcal{O}}_z^j}{\partial t} = & \left(\frac{\Gamma_j T_j \gamma_j}{2} - \frac{i\Gamma_j \gamma_j^2}{2} \right) L_j - \gamma_j \bar{\mathcal{O}}_z^j \\ & + \left[-iH_s - \sum_j (L_j^\dagger \bar{\mathcal{O}}_z^j + L_j \bar{\mathcal{O}}_w^j), \bar{\mathcal{O}}_z^j \right], \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial \bar{\mathcal{O}}_w^j}{\partial t} = & \frac{\Gamma_j T_j \gamma_j}{2} L_j^\dagger - \gamma_j \bar{\mathcal{O}}_w^j \\ & + \left[-iH_s - \sum_j (L_j^\dagger \bar{\mathcal{O}}_z^j + L_j \bar{\mathcal{O}}_w^j), \bar{\mathcal{O}}_w^j \right]. \end{aligned} \quad (15)$$

The master equation (10) can be numerically calculated by using Eqs. (14) and (15). In the Markovian limit, the system density matrix reduces to the Lindblad equation [32]:

$$\begin{aligned} \frac{\partial}{\partial t} \rho_s = & -i[H_s, \rho_s] \\ & + \sum_j \frac{\Gamma_j T_j}{2} [(2L_j \rho_s L_j^\dagger - L_j^\dagger L_j \rho_s - \rho_s L_j^\dagger L_j) \\ & + (2L_j^\dagger \rho_s L_j - L_j L_j^\dagger \rho_s - \rho_s L_j L_j^\dagger)]. \end{aligned} \quad (16)$$

Here we consider a one-dimensional XY spin chain as the communication channel,

$$H_s = \sum_{i=1}^{N-1} J_{i,i+1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y), \quad (17)$$

where $J_{i,i+1}$ is the coupling strength between the nearest neighbor sites i and $i+1$. σ_i^k ($k = x, y$) are the Pauli operators acting on the i th spin. We set $J_{i,i+1} = -1$, and it corresponds to a uniform chain with open boundary conditions.

Suppose initially the system is prepared at the state $|\Psi_s(0)\rangle = |1 \dots 00\rangle$, and the target task is to transfer the state $|1\rangle$ to the other end of the chain at time t with $|\Psi_s(t)\rangle = |0 \dots 01\rangle$. Here the size of the system is typically small so that each sites are close to share the same environment parameters. Therefore, we assume that the parameters of N -independent baths are taken as same, $\Gamma_i = \Gamma, \gamma_i = \gamma, T_i = T$ for $i = 1, 2, 3 \dots N$ [32]. We emphasize that even all the parameters are assumed to be equal, it is different from the one single bath with Γ, γ , and T cases. For N -independent baths model, there are $2N$ coupled equations in total for $\bar{\mathcal{O}}_{z,(w)}^j$ operators. As a comparison, there would be just one equation for a single common bath. The fidelity at time t can be defined as $F(t) = \sqrt{\langle \Psi_s(t) | \rho_s(t) | \Psi_s(t) \rangle}$, where $\rho_s(t)$ is the system density matrix in Eq. (10). Now the initial state of the whole system is

$$\rho_{\text{tot}}(0) = \rho_s(0) \otimes \rho_1(0) \otimes \dots \otimes \rho_N(0), \quad (18)$$

where $\rho_j(0)$ indicates that the j th bath is in a thermal equilibrium state at time $t = 0$.

III. AEST IN FINITE TEMPERATURE HEAT BATHS UNDER PULSE CONTROL

Dispersion effects and the environmental noise often degrade the transmission fidelity [2,15]. Pulse control technique has been used to realize the AEST [9,33], adiabatic speedup [62], adiabatic QST [63–65], and adiabatic quantum computation [59,66]. In this part, we investigate AEST through a uniform spin chain in finite-temperature heat baths by pulse control. The basic idea is letting the system go along the AEST passage by adding an LEO Hamiltonian to the system's dynamics.

At first we construct an LEO Hamiltonian as in Ref. [33]. This will be achieved by considering PST Hamiltonian (H_{PST}), whose coupling strength $J_{i,i+1} = -\sqrt{i(N-i)}$ in Eq. (17). Then the PST can be obtained, at time $t = n\pi/4$ (n is an odd integer), by the PST couplings driving [3]. The initial state of the system is still $|\Psi_s(0)\rangle$ and its evolution is denoted as $|\psi_1(t)\rangle = \exp(-iH_{\text{PST}}t)|\Psi_s(0)\rangle$. The LEO Hamiltonian can be constructed as [33]

$$H_{\text{LEO}} = c(t) |\psi_1(t)\rangle \langle \psi_1(t)|, \quad (19)$$

where $c(t)$ is the control function. The LEO Hamiltonian can be realized by a sequence of pulses and how to apply the control has been discussed in Ref. [33]. Then the total Hamiltonian is

$$H = H_{\text{tot}} + H_{\text{LEO}}. \quad (20)$$

In this paper, we consider nonperturbative control pulse which is tunable and finite both on the strength and duration. We choose the zero-area pulse sequence [64,67] and the pulse takes alternating positive and negative values. The characteristic of such sequence is its net area is always zero in a control period. For the rectangular pulses, the control function can be

taken as [68]

$$c(t) = \begin{cases} I, n\tau < t < (n+1)\tau, & n \text{ is even} \\ -I, & \text{otherwise,} \end{cases} \quad (21)$$

where I and τ are the pulse intensity and half-period, respectively. The effective control can be obtained when the pulses satisfy $I\tau = 2\pi m$ with $m = 1, 2, 3, \dots$ [68]. Similarly, for the sine function [69], $c(t) = I\sin(\omega t)$ with $\omega\tau = \pi$, the effective control condition is $J_0(I\tau/\pi) = 0$. $J_0(x)$ is the zeroth-order Bessel function of the first kind [33,62].

First we use the sine function as an example. The pulse parameters are taken as $\tau = \pi/32$ and $I = 76.96$ which satisfy $I\tau = 2.405\pi$. Note here 2.405 is the zero point of the zeroth-order Bessel function. In Fig. 1, we plot the fidelity F versus the rescaled time t/T_{tot} for different parameters Γ , γ , and T , respectively. We take $L = \sigma^- = \sum_{j=1}^N \sigma_j^-$ as an example, where σ_j^- denotes the lowering operator for the j th spin. The total evolution time is taken as $T_{\text{tot}} = \pi/4$ throughout the paper, which corresponds to a fixing actual driving time. Also AEST is realized at this time when the chain has the PST coupling [4]. The length of the chain is chosen to be $N = 7$. In Fig. 1(a), we plot the influence of the system-bath coupling strength Γ to the transmission fidelity F . $\gamma = 2$, $T = 40$. It shows that when adding the pulse, AEST can be obtained. When $\Gamma = 0.06$, $F \approx 0.96$ at $t = \pi/4$. In Fig. 1(b), we consider different parameters γ . $\Gamma = 0.04$, $T = 50$. The results show that non-Markovianity from the baths plays an essential role in boosting the fidelity. AEST can only be obtained in strong non-Markovian baths. This observation is in agreement with Ref. [25]. For example, $\gamma = 0.5$, $F = 0.999$. It is also in accordance with Ref. [9] that memory effects can boost the effectiveness of LEO control pulses in a zero-temperature bath. Clearly the control fails to boost the fidelity in Markovian baths. In Fig. 1(b), we also plot the comparison of the fidelity evolution with and without control in Markovian baths and find that the fidelity is almost equal. This can be explained by the fact that for a Markovian bath the correlation function becomes a δ function, and then gives $\bar{O}_z^j = \Gamma_j T_j L_j/2$, $\bar{O}_w^j = \Gamma_j T_j L_j^\dagger/2$. The control Hamiltonian H_{LEO} will fail to affect $\bar{O}_{z,(w)}^j$ operators, and as a result, the control loses its effectiveness [9]. Figure 1(c) plots the time evolution of the fidelity for different temperature T , $\Gamma = 0.02$, $\gamma = 2$. As expected, the fidelity is more higher in a lower temperature. AEST can even be realized for a higher temperature as long as the bath is non-Markovian. For example, $T = 150$, $F(T_{\text{tot}}) = 0.952$.

IV. TRADE-OFF BETWEEN CONTROL COST AND TRANSMISSION FIDELITY

When adding the external driving, the control cost must be considered. A family of cost function has been proposed [41] to quantify the driving cost. One of the cost function can be defined as $C_t^n \equiv v_{t,n} \int_0^t dt \|H_{\text{LEO}}(t)\|$, where $\|A\| = \sqrt{\text{tr}\{A^\dagger A\}}$ is the Hilbert-Schmidt norm [70], and in the following we set

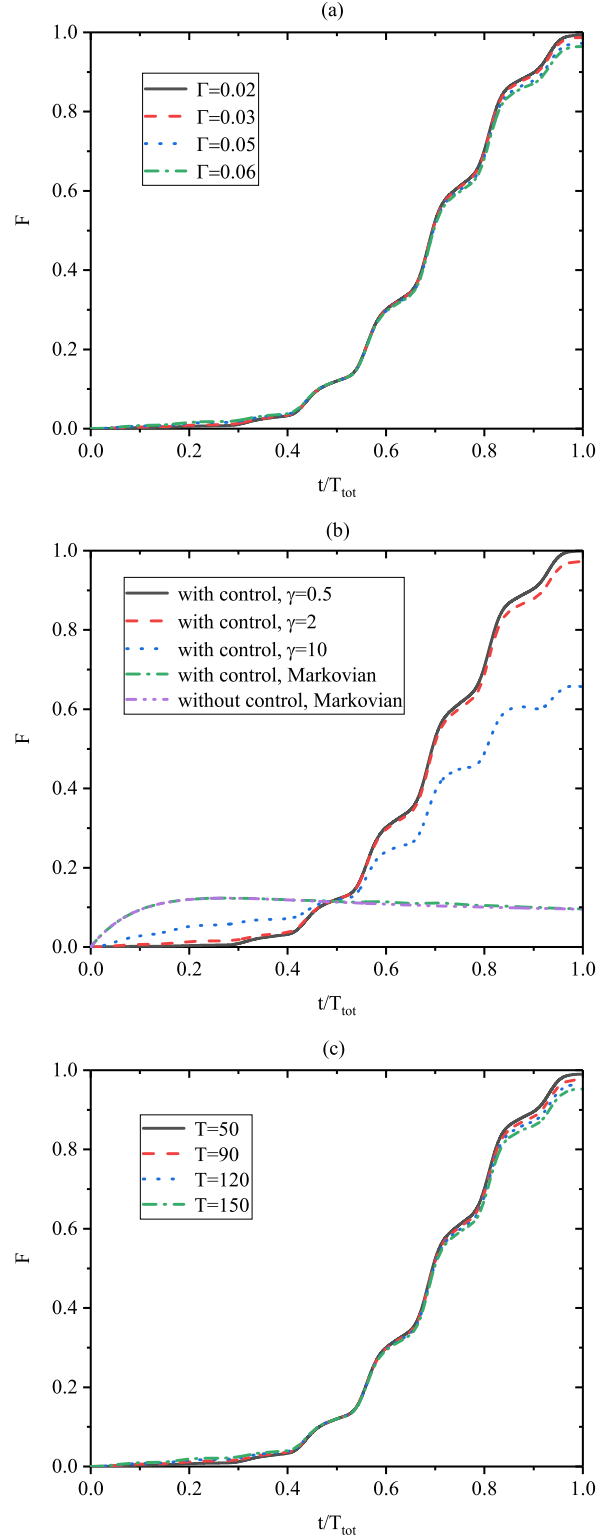


FIG. 1. Controlled AEST in $L = \sigma^-$ open system: the fidelity F as a function of rescaled time t/T_{tot} for different environmental parameters (a) Γ , $\gamma = 2$, $T = 40$; (b) γ , $\Gamma = 0.04$, $T = 50$; (c) T , $\Gamma = 0.02$, $\gamma = 2$. The total evolution time is $T_{\text{tot}} = \pi/4$ for the controlled or free evolution. The control function is $c(t) = I\sin(\omega t)$ with $\omega\tau = \pi$. $\tau = \pi/32$, $I = 76.96$, which satisfies the condition $I\tau = 2.405\pi$. The length of the chain is chosen to be $N = 7$.

$v_{t,n} = n = 1$ for simplicity [41]:

$$C_t^1 \equiv C = \int_0^\tau dt \|H_{\text{LEO}}(t)\|. \quad (22)$$

The cost C can be viewed as the additional effect of control pulse [41]. Therefore, it is easy to see that the instantaneous cost of the control pulse is

$$\partial_t C = \|H_{\text{LEO}}(t)\| = |c(t)| \sqrt{\sum_n |\langle n|\psi_1(t)\rangle|^2}. \quad (23)$$

From Eq. (23), we can see that the control cost only depends on the control function $c(t)$ itself and the sum of $|f_{1,n}(t)|^2$. Here $f_{1,n}(t) = \langle n|\psi_1(t)\rangle$ is the transition amplitude of an excitation from the initial state to the $|n\rangle$ state [1]. We take $L = \sigma^x$ as an example in this part. We point out that the types of the Lindblad operators do not change the conclusion of the paper. The environmental parameters are taken as $\gamma = 1$, $T = 30$, $\Gamma = 0.03$. We use the rectangular pulses defined in Eq. (21) and the intensities in Fig. 2 all satisfy the control condition $I\tau = 2\pi$. In Fig. 2(a), we plot the fidelity F versus the rescaled time t/T_{tot} for different control intensities ($I = 32, 64, 112, 128$). Clearly, the final fidelity increases with increasing control intensity. When $I = 32$, the final fidelity is $F = 0.971$, and when $I = 64$, $F = 0.995$. Figure 2(b) plots the corresponding instantaneous cost $\partial_t C$ as a function of the rescaled time t/T_{tot} for the same pulse intensities as plotted in Fig. 2(a). Figure 2(b) shows that a higher fidelity F corresponds to a higher control cost, so there exists a trade-off between the fidelity and the control cost. When the intensity is strong enough, the fidelity is almost one. For example, when $I = 112$, $F = 0.999$. In this case, increasing the intensity just improves the control cost and it does not boost the fidelity ($I = 128$, $F = 0.999$). Then the minimum control intensity with minimum cost can be found to obtain certain transmission fidelity when the environmental parameters are fixed. To compare the effects of different types of Lindblad operator on the control cost, in Fig. 2(c) we plot the instantaneous cost $\partial_t C$ as a function of rescaled time t/T_{tot} for $L = \sigma^z$, $L = \sigma^-$, and $L = \sigma^x$, respectively. The final fidelity is set as $F = 0.999$. In the inset of Fig. 2(c), we plot the strength of the pulse I as a function of rescaled time t/T_{tot} . Clearly different pulse intensities are required to obtain certain fidelity for different types of L . The maximum control cost corresponds to $L = \sigma^x$, $L = \sigma^z$ is the least, and $L = \sigma^-$ is in the middle.

Next we consider the control cost when transferring a quantum state with certain fidelity. In Figs. 3(a)–3(c), we plot the instantaneous cost $\partial_t C$ as a function of the rescaled time t/T_{tot} for different parameters Γ , γ , and T , respectively. The final fidelity is 0.996 under rectangular pulse control. As expected, we observe that the control cost increases with increasing coupling strength Γ , the parameter γ , and bath temperature T for a fixed final fidelity. Strong system-bath coupling and high temperature will destroy the quantumness of the system and as a result more control cost is required. Non-Markovianity from baths can be helpful to reduce the control cost notably from Fig. 3(b). A possible explanation may be that a memory environment has the property with the

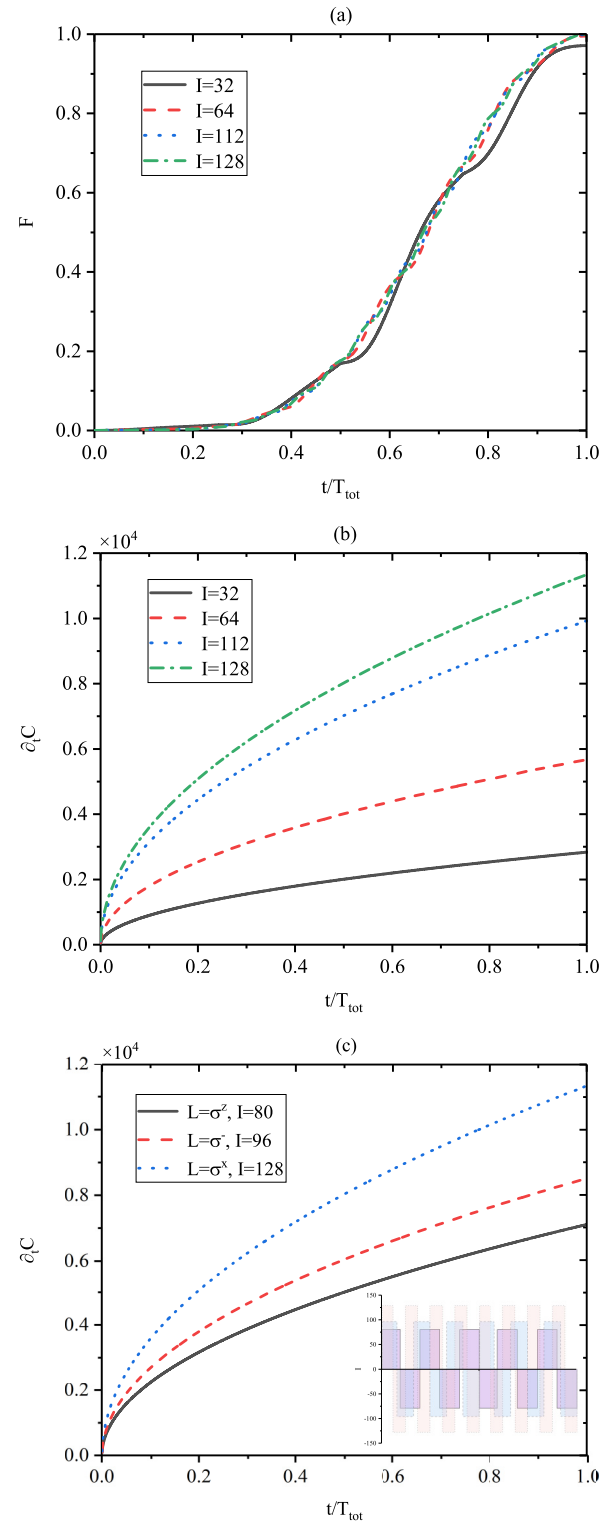


FIG. 2. (a) Controlled AEST in $L = \sigma^x$ open system: the fidelity F as a function of rescaled time t/T_{tot} for different control pulse intensities. (b) The instantaneous cost $\partial_t C$ as a function of rescaled time t/T_{tot} under different control pulses. (c) The instantaneous cost $\partial_t C$ as a function of rescaled time t/T_{tot} for different Lindblad operators. $\gamma = 1$, $T = 30$, $\Gamma = 0.03$, $N = 6$.

reverse information flow from baths to the system, with less loss of quantumness of the system.

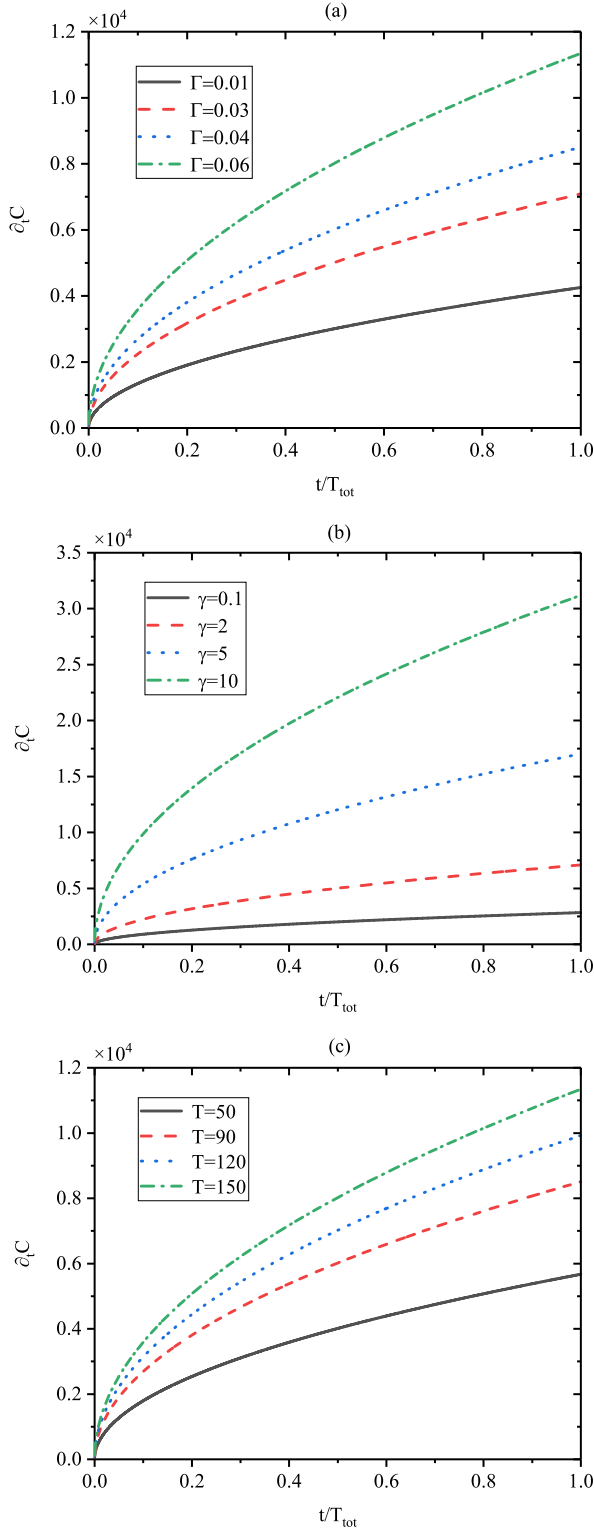


FIG. 3. The instantaneous cost $\partial_t C$ as a function of the rescaled time t/T_{tot} for different parameters Γ , γ , and T , respectively. (a) Γ , $\gamma = 2$, $T = 30$; (b) γ , $\Gamma = 0.02$, $T = 40$; (c) T , $\Gamma = 0.03$, $\gamma = 1$. The length of the chain is chosen to be $N = 6$.

V. QSLT IN CONTROLLED AEST

In quantum physics, the uncertain relationship between the Heisenberg energy and the time limits the quantum

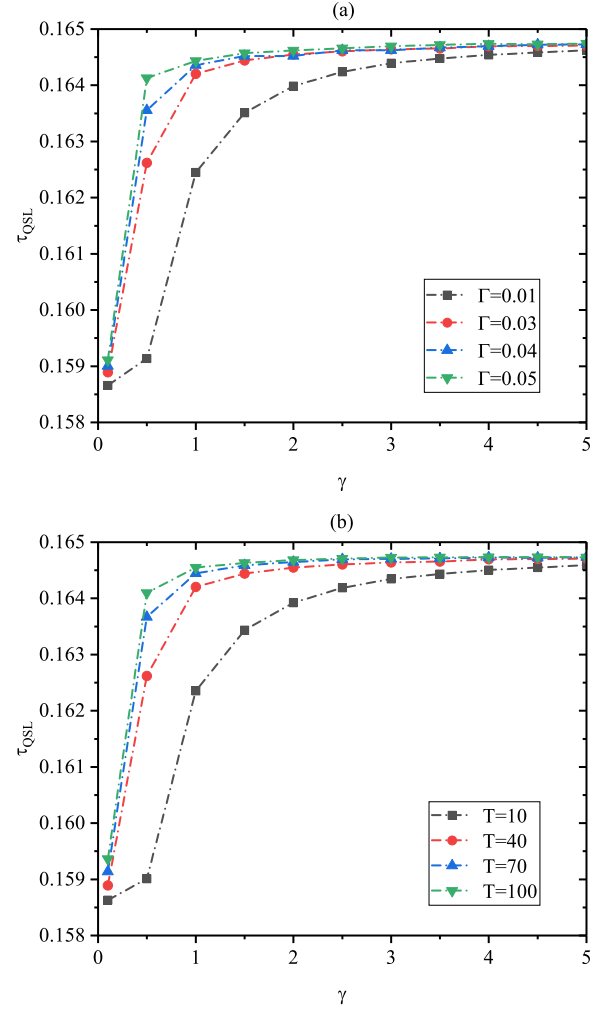


FIG. 4. The quantum speed limit time τ_{QSL} for various values of environment parameters. (a) $T = 40$; (b) $\Gamma = 0.03$. The length of the chain is chosen to be $N = 5$.

speed of performing a quantum operation or QST [54]. Recently, the QSLT in open systems has been extensively studied [51,53,71–74]. In Ref. [43], Deffner and Lutz derived a unified bound for the QSLT in an open system by using the Cauchy-Schwarz inequality. Their approach defines a geometric Bures angle, $\mathcal{L}(\rho_0, \rho_t) = \arccos(\sqrt{\langle \psi_0 | \rho_t | \psi_0 \rangle})$, which implies the “distance” between the initial and final states. Then by using the time derivative of the Bures angle and $x \leq |x|$ [75], we can arrive an inequality

$$2\cos(\mathcal{L})\sin(\mathcal{L})\dot{\mathcal{L}} \leq |\langle \psi_0 | \dot{\rho}_t | \psi_0 \rangle|. \quad (24)$$

Using Eq. (24) and the Cauchy-Schwarz inequality, we obtain

$$2\cos(\mathcal{L})\sin(\mathcal{L})\dot{\mathcal{L}} \leq \|\dot{\rho}\|. \quad (25)$$

Integrating Eq. (25) over time leads to the Mandelstam-Tamm type bound on the rate of quantum evolution, i.e., the QSLT τ_{QSL}

$$t \geq \tau_{\text{QSL}} \equiv \frac{1}{\Lambda_t} \sin^2[\mathcal{L}(\rho_0, \rho_t)]. \quad (26)$$

Here $\Lambda_t = \frac{1}{t} \int_0^t dt \|\dot{\rho}\|$ represents the average of $\|\dot{\rho}\|$ over the actual driving time duration t . There are two viewpoints about QSLT. One focuses on the actual evolution time t , and the fastest evolution occurs when the actual driving time t equals the QSLT. It has been used in the discussion of entanglement-assisted speedup of quantum evolution [76,77]. Another viewpoint is that the QSLT can be studied by first fixing the actual driving time t , which as been utilized to explore the non-Markovinity of the baths on the speed of quantum evolution [43,55].

Next, we will discuss the influence of the environmental noise on the QSLT during the process of AEST. In our paper, we fix the actual driving time $t = \pi/4$ in Eq. (26) since at this time AEST can be realized as mentioned above. Additionally, the final fidelity is taken as $F(t = \pi/4) = 0.999$ for different parameters to guarantee that the initial state and the target state can be approximately the same. The values of τ_{QSL} for different environmental parameters are presented in Fig. 4. Note that the parameter windows are taken as $\Gamma \in [0.01, 0.05]$, $T \in [10, 100]$, and $\gamma \in [0.1, 5]$ in Fig. 4 due to the weak coupling approximations and the fidelity cannot reach 0.999 for baths with strong Markovianity. Figures 4(a) and 4(b) plot τ_{QSL} versus γ for different Γ ($T = 40$) and T ($\Gamma = 0.03$), respectively. They show that from the initial state $|1 \dots 00\rangle$ to the target state $|0 \dots 01\rangle$, τ_{QSL} increases with increasing Γ or T , which means that weak system-bath couplings and low temperature will reduce the QSLT. Figures 4(a) and 4(b) also show that non-Markovinity from the baths can help to shorten the QSLT and thus increase the capacity for potential speedup. The memory effect of environment is an important element for further reduction in QSLT [55].

VI. CONCLUSIONS

Realization of AEST is a key to successfully complete quantum information processing tasks; however, the environment noise around the communication channel will always destroy the state transmission fidelity. In this paper, we have explored the QST through a spin chain under pulse control, where each spin of the chain is immersed in a non-Markovian, finite-temperature heat bath. Using QSD equation approach, we numerically calculate the dynamics of this open system. We find that AEST can be obtained in non-Markovian and low-temperature heat baths with weak system-bath coupling under a suitable pulse intensity and duration. We then consider the control cost and QSLT during the process of this AEST. We find that the cost and QSLT increase with increasing parameters Γ , γ , and T for certain fidelity. Higher transmission fidelity corresponds to higher cost. The minimum cost has been found to obtain a certain target state. Moreover, it is also found that strong non-Markovianity (long memory time) can help to reduce the control cost and shorten the QSLT. Our findings will have potential applications for the control cost and information transfer in quantum device design.

ACKNOWLEDGMENTS

We would like to thank E. Y. Sherman for helpful discussions. This paper is based upon work supported by National Natural Science Foundation of China (Grant No. 11475160) and the Natural Science Foundation of Shandong Province (Grant No. ZR2014AM023).

-
- [1] S. Bose, *Phys. Rev. Lett.* **91**, 207901 (2003).
 [2] M.-L. Hu and H.-L. Lian, *Eur. Phys. J. D* **55**, 711 (2009).
 [3] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, *Phys. Rev. Lett.* **92**, 187902 (2004).
 [4] M. Christandl, N. Datta, T. C. Dorlas, A. Ekert, A. Kay, and A. J. Landahl, *Phys. Rev. A* **71**, 032312 (2005).
 [5] A. Kay, *Phys. Rev. A* **73**, 032306 (2006).
 [6] X.-M. Zhang, Z.-W. Cui, X. Wang, and M.-H. Yung, *Phys. Rev. A* **97**, 052333 (2018).
 [7] J. Fitzsimons and J. Twamley, *Phys. Rev. Lett.* **97**, 090502 (2006).
 [8] V. Balachandran and J. Gong, *Phys. Rev. A* **77**, 012303 (2008).
 [9] Z.-M. Wang, F.-H. Ren, D.-W. Luo, Z.-Y. Yan, and L.-A. Wu, *Phys. Rev. A* **102**, 042406 (2020).
 [10] H. Zhou, X. Chen, X. Nie, J. Bian, Y. Ji, Z. Li, and X. Peng, *Sci. Bull.* **64**, 888 (2019).
 [11] O. V. Marchukov, A. G. Volosniev, M. Valiente, D. Petrosyan, and N. T. Zinner, *Nat. Commun.* **7**, 13070 (2016).
 [12] J. Jeske, N. Vogt, and J. H. Cole, *Phys. Rev. A* **88**, 062333 (2013).
 [13] X. Chen, R. Merau, and D. L. Feder, *Phys. Rev. A* **93**, 012343 (2016).
 [14] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2007).
 [15] M.-L. Hu, *Eur. Phys. J. D* **59**, 497 (2010).
 [16] F.-H. Ren, Z.-M. Wang, and Y.-J. Gu, *Quantum Inf. Process.* **18**, 193 (2019).
 [17] H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, *Rev. Mod. Phys.* **88**, 021002 (2016).
 [18] I. de Vega and D. Alonso, *Rev. Mod. Phys.* **89**, 015001 (2017).
 [19] S. S. Roy, U. Mishra, and D. Rakshit, *Europhys. Lett.* **129**, 30005 (2020).
 [20] X. Zhao, *Opt. Express* **27**, 29082 (2019).
 [21] A. Motavallibashi, H. Mohammadi, and A. Akhound, *J. Opt. Soc. Am. B* **38**, 1200 (2021).
 [22] H.-P. Breuer, E.-M. Laine, and J. Piilo, *Phys. Rev. Lett.* **103**, 210401 (2009).
 [23] D. Chruściński and S. Maniscalco, *Phys. Rev. Lett.* **112**, 120404 (2014).
 [24] R. Sampaio, S. Suomela, R. Schmidt, and T. Ala-Nissila, *Phys. Rev. A* **95**, 022120 (2017).
 [25] N. Mirkin, P. Poggi, and D. Wisniacki, *Phys. Rev. A* **99**, 062327 (2019).
 [26] W. Shi, X. Zhao, and T. Yu, *Phys. Rev. A* **87**, 052127 (2013).
 [27] T. Yu, L. Diósi, N. Gisin, and W. T. Strunz, *Phys. Rev. A* **60**, 91 (1999).
 [28] C.-F. Li, G.-C. Guo, and J. Piilo, *Europhys. Lett.* **128**, 30001 (2020).
 [29] T. Nakajima, M. R. Delbecq, T. Otsuka, S. Amaha, J. Yoneda, A. Noiri, K. Takeda, G. Allison, A. Ludwig, A. D. Wieck,

- X. Hu, F. Nori, and S. Tarucha, *Nat. Commun.* **9**, 2133 (2018).
- [30] L. Diósi, N. Gisin, and W. T. Strunz, *Phys. Rev. A* **58**, 1699 (1998).
- [31] W. T. Strunz, L. Diósi, and N. Gisin, *Phys. Rev. Lett.* **82**, 1801 (1999).
- [32] Z.-M. Wang, F.-H. Ren, D.-W. Luo, Z.-Y. Yan, and L.-A. Wu, *J. Phys. A: Math. Theor.* **54**, 155303 (2021).
- [33] Z.-M. Wang, M. S. Sarandy, and L.-A. Wu, *Phys. Rev. A* **102**, 022601 (2020).
- [34] S. Campbell and S. Deffner, *Phys. Rev. Lett.* **118**, 100601 (2017).
- [35] R. Landauer, *IBM J. Res. Dev.* **5**, 183 (1961).
- [36] T. Sagawa and M. Ueda, *Phys. Rev. Lett.* **100**, 080403 (2008).
- [37] O. J. E. Maroney, *Phys. Rev. E* **79**, 031105 (2009).
- [38] B. Piechocinska, *Phys. Rev. A* **61**, 062314 (2000).
- [39] C. Joachim, J. K. Gimzewski, and A. Aviram, *Nature (London)* **408**, 541 (2000).
- [40] V. Serreli, C. F. Lee, E. R. Kay, and D. A. Leigh, *Nature (London)* **445**, 523 (2007).
- [41] Y. Zheng, S. Campbell, G. De Chiara, and D. Poletti, *Phys. Rev. A* **94**, 042132 (2016).
- [42] G. Xiao and J. Gong, *Phys. Rev. E* **90**, 052132 (2014).
- [43] S. Deffner and E. Lutz, *Phys. Rev. Lett.* **111**, 010402 (2013).
- [44] P. Pfeifer, *Phys. Rev. Lett.* **70**, 3365 (1993).
- [45] N. Margolus and L. B. Levitin, *Physica D (Amsterdam, Neth.)* **120**, 188 (1998).
- [46] V. Giovannetti, S. Lloyd, and L. Maccone, *J. Opt. B* **6**, S807 (2004).
- [47] J. D. Bekenstein, *Phys. Rev. Lett.* **46**, 623 (1981).
- [48] S. Lloyd, *Nature (London)* **406**, 1047 (2000).
- [49] T. Caneva, M. Murphy, T. Calarco, R. Fazio, S. Montangero, V. Giovannetti, and G. E. Santoro, *Phys. Rev. Lett.* **103**, 240501 (2009).
- [50] P. V. Pyshkin, E. Y. Sherman, and L.-A. Wu, *Phys. Rev. A* **100**, 063401 (2019).
- [51] Y.-J. Zhang, W. Han, Y.-J. Xia, J.-P. Cao, and H. Fan, *Sci. Rep.* **4**, 4890 (2014).
- [52] A. del Campo, I. L. Egusquiza, M. B. Plenio, and S. F. Huelga, *Phys. Rev. Lett.* **110**, 050403 (2013).
- [53] M. M. Taddei, B. M. Escher, L. Davidovich, and R. L. de Matos Filho, *Phys. Rev. Lett.* **110**, 050402 (2013).
- [54] S. Deffner and E. Lutz, *J. Phys. A: Math. Theor.* **46**, 335302 (2013).
- [55] C. Liu, Z.-Y. Xu, and S. Zhu, *Phys. Rev. A* **91**, 022102 (2015).
- [56] X.-Y. Zhao, *New J. Phys.* **14**, 123016 (2012).
- [57] T. Albash and D. A. Lidar, *Phys. Rev. A* **91**, 062320 (2015).
- [58] T. Yu, *Phys. Rev. A* **69**, 062107 (2004).
- [59] F.-H. Ren, Z.-M. Wang, and L.-A. Wu, *Phys. Rev. A* **102**, 062603 (2020).
- [60] W. T. Strunz, L. Diósi, N. Gisin, and T. Yu, *Phys. Rev. Lett.* **83**, 4909 (1999).
- [61] C. Meier and D. J. Tannor, *J. Chem. Phys.* **111**, 3365 (1999).
- [62] Z.-M. Wang, D.-W. Luo, M. S. Byrd, L.-A. Wu, T. Yu, and B. Shao, *Phys. Rev. A* **98**, 062118 (2018).
- [63] Y.-H. Chen, Y. Xia, Q.-Q. Chen, and J. Song, *Phys. Rev. A* **89**, 033856 (2014).
- [64] Y.-F. Chen, L.-C. Zhang, Y.-J. Gu, and Z.-M. Wang, *Phys. Lett. A* **382**, 2795 (2018).
- [65] B.-H. Huang, Y.-H. Kang, Y.-H. Chen, Z.-C. Shi, J. Song, and Y. Xia, *Phys. Rev. A* **97**, 012333 (2018).
- [66] H. Wang and L.-A. Wu, *Sci. Rep.* **6**, 22307 (2016).
- [67] L.-C. Zhang, F.-H. Ren, Y.-F. Chen, R.-H. He, Y.-J. Gu, and Z.-M. Wang, *Europhys. Lett.* **125**, 10010 (2019).
- [68] P. V. Pyshkin, D.-W. Luo, J. Jing, J. Q. You, and L.-A. Wu, *Sci. Rep.* **6**, 37781 (2016).
- [69] R. Wang, F.-H. Ren, Y.-J. Gu, and Z.-M. Wang, *Quantum Inf. Process.* **19**, 280 (2020).
- [70] A. C. Santos and M. S. Sarandy, *Sci. Rep.* **5**, 15775 (2015).
- [71] A. D. Cimmarusti, Z. Yan, B. D. Patterson, L. P. Corcos, L. A. Orozco, and S. Deffner, *Phys. Rev. Lett.* **114**, 233602 (2015).
- [72] Z.-Y. Xu and S.-Q. Zhu, *Chin. Phys. Lett.* **31**, 020301 (2014).
- [73] Z.-Y. Xu, S. Luo, W.-L. Yang, C. Liu, and S. Zhu, *Phys. Rev. A* **89**, 012307 (2014).
- [74] V. Mukherjee, A. Carlini, A. Mari, T. Caneva, S. Montangero, T. Calarco, R. Fazio, and V. Giovannetti, *Phys. Rev. A* **88**, 062326 (2013).
- [75] N. Mirkin, F. Toscano, and D. A. Wisniacki, *Phys. Rev. A* **94**, 052125 (2016).
- [76] J. Batle, M. Casas, A. Plastino, and A. R. Plastino, *Phys. Rev. A* **72**, 032337 (2005).
- [77] F. Frowis, *Phys. Rev. A* **85**, 052127 (2012).