Role of quadrature squeezing in continuous-variable quantum teleportation

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Quantum teleportation (QT) lies at the heart of modern day quantum information science and technology. Despite extensive studies over past two decades, obtaining the necessary and/or sufficient criterion for QT with continuous-variable (CV) resources, besides entanglement, still remains an open concern. In this backdrop, here we analyze the role of a purely quantum optical (QO) attribute, known as quadrature squeezing, in CV teleportation. We first provide an analytic proof that for Gaussian resources quadrature squeezing is necessary for QT. However, for non-Gaussian resources we show a clear distinction between the pure and the mix states. For the pure states, quadrature squeezing appears to be necessary for QT, in the sense that there is no QT without quadrature squeezing. However, in the case of mix states we observe otherwise, i.e., QT could be achieved even without quadrature squeezing. Our results present the exotic character of the QO attributes of the CV resources and necessitate a deeper search for the necessary and/or sufficient criterion for CV QT.

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I. INTRODUCTION

Quantum teleportation (QT) [1,2], an information processing task that exploits the nonlocal character of the resources, plays an important role in modern day quantum information science [3]. In this protocol, two or more interested parties can share information between them at the cost of their shared entanglement. Soon after the first description for the discrete-variable (DV) systems by Bennett *et al.* [4], the concept of teleportation had been extended to the continuous-variable (CV) systems by Braunstein and Kimble [5] with subsequent experimental realizations by many [6–12]. Recent advances have further led to the hybrid systems that take into account DV as well as CV components [13].

In the past two decades there has been extensive analysis of various information-theoretic attributes that could play a critical role in ensuring quantum teleportation with CV systems that lie beyond the scope of the classical description [14–28]. While entanglement is necessary to obtain quantum teleportation [29,30], use of non-Gaussian resources proved to be beneficial in improving the performance as well as lowering the requirements to obtain a certain success probability over the Gaussian resources [14–17,19–21]. However, the fact that non-Gaussianity alone does not suffice for QT [18,22–24,27] leads to a deeper search for condition(s) that could be necessary and/or sufficient for QT, besides entanglement.

Various authors have pointed out the Einstein-Podolsky-Rosen (EPR) correlation as a very important ingredient in obtaining QT with CV systems [17,26]. Moreover, it appears to be a better witness of CV QT compared to other correlations [28]. On the contrary, Lee *et al.* [20] and Wang *et al.* [25] presented a practical example where EPR correlation is not necessary for QT. In our previous work, we further pointed out examples where EPR correlation fails to suffice for QT, leading to the conclusion that EPR correlation is neither necessary nor sufficient for CV QT in general [31]. This leaves the quest for a necessary and/or sufficient criterion for QT with CV systems, beyond entanglement, *open*.

It may be noted that so far the studies on QT in the literature have been centered around the information-theoretic aspects of the CV resources. Against this backdrop, here we analyze the role of a purely quantum optical characteristic of the resource states, namely, the U(2)-invariant quadrature squeezing [32], in the context of teleportation. To that end, for Gaussian resources, we first provide an analytic proof, by considering a generic variance matrix, that quadrature squeezing is necessary to obtain teleportation beyond the scope of classical theory. However, in the case of resource states possessing a non-Gaussian Wigner distribution, traditionally known as the non-Gaussian states, we show a clear distinction between the pure and the mix states.

As examples of non-Gaussian pure resources, we consider the beam splitter (BS) output states with single-mode nonclassical input states. As the specific input we consider two different classes of states, in particular (a) the states involving photon addition and/or subtraction along with quadrature squeezing [33,34] and (b) the states obtained by symmetric (even) and antisymmetric (odd) superpositions of coherent states [35]. Our analytic results for these states show that quadrature squeezing is necessary for QT, in the sense that there is no QT in the absence of quadrature squeezing. On the other hand, as the mixed states we consider a simple decoherence model where these pure input states contain additional thermal noise. Here we observe that quadrature squeezing no longer appears to be a necessary condition as over a reasonable parameter region, one can obtain QT without quadrature squeezing.

The present paper is organized as follows. In Sec. II we provide a simple and elegant analytic proof that for Gaussian

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states, quadrature squeezing is necessary for QT. In Sec. III we present our observation for the non-Gaussian states. Here we discuss our results on pure and mixed input states separately. In Sec. IV we summarize and discuss the various subtleties of the present work.

II. QT WITH GAUSSIAN RESOURCES AND SQUEEZING

We prove the necessity of quadrature squeezing for Gaussian states by showing the one-way equivalence between entanglement and quadrature squeezing. Let us consider a two-mode Gaussian state described the variance matrix in the standard form [32,36,37]

$$V_{ab} = \begin{pmatrix} \eta & 0 & c_1 & 0 \\ 0 & \eta & 0 & c_2 \\ c_1 & 0 & \zeta & 0 \\ 0 & c_2 & 0 & \zeta \end{pmatrix}, \tag{1}$$

subject to the Heisenberg uncertainty relation $V_{ab} + \frac{i}{2}\Omega \geqslant 0$, where $\Omega = \Omega_1 \oplus \Omega_2$ such that $\Omega_i = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $(i = 1, 2)$ is the 2×2 symplectic metric corresponding to a single-mode optical field and $\eta, \zeta \geqslant 0$. Without any loss of generality we consider $c_1 \geqslant 0$ and $|c_1| \geqslant |c_2|$.

The inseparability of a bipartite state manifests as the negativity under partial transposition. For a two-mode Gaussian state this corresponds to the minimum symplectic eigenvalue being less than unity [38], i.e., $\nu_{\min} < \frac{1}{2}$. In the case of a two-mode Gaussian state with the standard variance matrix (1), the condition of $\nu_{\min} < \frac{1}{2}$ is essentially

$$4(\eta+\zeta)^2 > (1+4c_1c_2)^2 + 8\eta\zeta\left(1 + 2\eta\zeta - 2\overline{c_1^2 + c_2^2}\right). \tag{2}$$

On the other hand, the condition of quadrature squeezing is given as the minimum eigenvalue of the variance matrix being less than half, i.e., $\lambda_{\min} < \frac{1}{2}$ [36]. In the case of the Gaussian state of interest (1), the minimum eigenvalue is given as $\lambda_{\min} = [(\eta+\zeta) - \sqrt{(\eta-\zeta)^2 + 4c_1^2}]/2$.

We start with the following equation for any Gaussian state of the form given in Eq. (1): $2(\eta\zeta - c_1^2)(c_1^2 - c_2^2) + c_1(c_1 + c_2) \geqslant 0$. The zero on the right-hand side is obtained for $c_1 = c_2$, which describes a separable state. In the case of an inseparable state the left-hand-side expression is always greater than zero. Let us consider that the Gaussian state (1) shows a negative partial transpose (NPT) (2), i.e., $4(\eta + \zeta)^2 = (1 + 4c_1c_2)^2 + 8\eta\zeta(1 + 2\eta\zeta - 2\overline{c_1^2 + c_2^2}) + \delta$ $(\delta > 0)$. It could then be shown in a straightforward calculation that

$$\begin{aligned}
&2\left(\eta\zeta - c_1^2\right)\left(c_1^2 - c_2^2\right) + c_1(c_1 + c_2) > 0,\\
&4(\eta+\zeta)^2 - \delta > \left[1 + 4\left(\eta\zeta - c_1^2\right)\right]^2,\\
&2(\eta+\zeta) > 1 + 4\left(\eta\zeta - c_1^2\right),\\
&\eta + \zeta - 1 < \sqrt{(\eta-\zeta)^2 + 4c_1^2},\\
&\frac{(\eta+\zeta) - \sqrt{(\eta-\zeta)^2 + 4c_1^2}}{2} < \frac{1}{2}, \text{ or } \lambda_{\min} < \frac{1}{2}.
\end{aligned}$$

This completes the proof that for a two-mode Gaussian state, entanglement necessarily implies quadrature squeezing; however, the converse is not true since squeezing could be observed in a separable state as well.

One may wonder whether the same conclusion holds true for any Gaussian state. It must be noted that the variance matrix for any general Gaussian state could be obtained by local transformations, i.e., $\mathrm{Sp}(2, \mathbf{R}) \oplus \mathrm{Sp}(2, \mathbf{R})$ operations applied in the standard form (1). Since entanglement is independent of the local operations, the NPT criterion for the standard form will hold for any other form as well. On the other hand, the squeezing condition ($\lambda_{\min} < \frac{1}{2}$) is invariant under general U(2) operations which form larger set than the symplectic operations. Thus, under the local operations (as mentioned above), the conclusion remains invariant. As a consequence, in conjunction with the earlier result that entanglement is necessary for QT [29,30], it trivially follows that quadrature squeezing is necessary for QT in the case of Gaussian entangled resources.

Next we consider the case of non-Gaussian entangled resources.

III. SQUEEZING VS QT WITH NON-GAUSSIAN RESOURCES

A. Pure states

In the Braunstein-Kimble (BK) protocol for CV QT, since the practical entangled resources are not maximal, even for a pure state input $|\psi_{\text{in}}\rangle$, the output state ρ_{out} becomes mixed and an imperfect copy of the input. Consequently, the success probability of teleporting an input pure state in the BK protocol is measured by the corresponding fidelity $F = \mathrm{Tr}[|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|\rho_{\text{out}}] = \langle\psi_{\text{in}}|\rho_{\text{out}}|\psi_{\text{in}}\rangle$. Evaluation of this fidelity becomes particularly simple in terms of the characteristic function as

$$F = \int \frac{d^2z}{\pi}\chi_{\text{in}}(-z)\chi_{\text{in}}(z)\chi_{ab}(z, z^*), \tag{3}$$

where χ_{in} (χ_{ab}) is the characteristic function corresponding to the input state (two-mode resource state). In the case of an unknown coherent input state, the expression of fidelity (3) further reduces to

$$F_{\text{ch}} = \int \frac{d^2z}{\pi} e^{-|z|^2}\chi_{ab}(z, z^*). \tag{4}$$

The maximum fidelity of teleportation of a coherent state attainable by a separable state in the BK protocol is $\frac{1}{2}$ [29,30]. Evidently, $F > \frac{1}{2}$ is considered as QT.

Now we consider the class of non-Gaussian entangled pure states which are generated at the output of a 50:50 passive BS where one of the input modes is fed with single-mode nonclassical states while the other port is left in vacuum. As specific input states we consider (a) states involving multiple nonclassicality inducing operations (MNIOs) and (b) the even and odd coherent superposition states known as the cat states [39–41]. The MNIO states could be seen as the ones generated from vacuum by double photon addition or subtraction in conjunction with quadrature squeezing in different orders. These states are mathematically defined as

$$|\psi_{(2+)}\rangle = \frac{1}{\sqrt{N^{(2+)}}}a^{\dagger 2}S_a(r)|0\rangle,$$

$$|\psi_{(2-)}\rangle = \frac{1}{\sqrt{N^{(2-)}}}a^2S_a(r)|0\rangle,$$

$$|\psi_{(2)}\rangle = S_a(r)|2\rangle,$$
$$|\psi_{(\alpha\pm)}\rangle = \frac{1}{\sqrt{N_\pm}}(|\alpha\rangle \pm |-\alpha\rangle), \tag{5}$$

where $S_a(r) = \exp[r(a^{\dagger 2} - a^2)/2]$ and a stands for the specific mode. The normalization constants are given as $N^{(2+)} = \mu^2(3\mu^2 - 1)$, $N^{(2-)} = \nu^2(3\mu^2 - 2)$, and $N_\pm = 2(1 \pm e^{-2\alpha^2})$, where $\mu = \cosh r$ and $\nu = \sinh r$. The BS action, i.e., U_{BS}, could be realized as a map between the input and the output mode operators as described by

$$U_{\mathrm{BS}}^\dagger \begin{pmatrix} a_{\mathrm{in}} \\ b_{\mathrm{in}} \end{pmatrix} U_{\mathrm{BS}} \rightarrow \begin{pmatrix} a_{\mathrm{out}} \\ b_{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a_{\mathrm{in}} \\ b_{\mathrm{in}} \end{pmatrix}. \tag{6}$$

Entanglement and teleportation characteristics of these states have already been studied in detail [42,43]. Here we borrow these results on the teleportation fidelity for the sake of completeness.

Since all the resource states are obtained with a passive 50:50 BS from input nonclassical states, here we first prove the following lemma that we will use throughout the rest of the paper.

Lemma. Beam-splitter output states are quadrature squeezed only if the input is quadrature squeezed.

Proof. Let us consider the standard form of a single-mode variance matrix $V_{\mathrm{single}} = \mathrm{diag}\{\eta, \zeta\}$ $(\eta, \zeta \geqslant \frac{1}{2})$, while the variance matrix for the vacuum is given by $V_{\mathrm{vac}} = \mathrm{diag}\{\frac{1}{2}, \frac{1}{2}\}$. For the sake of simplicity, without any loss of generality, let us consider $\eta \geqslant \zeta$. Under the action of the BS (6), the output two-mode variance matrix becomes

$$V_{\mathrm{out}} = \begin{pmatrix} \frac{\eta+\frac{1}{2}}{2} & 0 & \frac{-\eta+\frac{1}{2}}{2} & 0 \\ 0 & \frac{\zeta+\frac{1}{2}}{2} & 0 & \frac{-\zeta+\frac{1}{2}}{2} \\ \frac{-\eta+\frac{1}{2}}{2} & 0 & \frac{\eta+\frac{1}{2}}{2} & 0 \\ 0 & \frac{-\zeta+\frac{1}{2}}{2} & 0 & \frac{\zeta+\frac{1}{2}}{2} \end{pmatrix}. \tag{7}$$

In a very straightforward calculation the minimum eigenvalue for V_{out} could be shown to given as $\lambda_{\min} = \min\{\frac{1}{2}, \eta, \zeta\}$. Evidently, the output state is quadrature squeezed $\lambda_{\min} < \frac{1}{2}$ only if $\zeta < \frac{1}{2}$ (as $\eta \geqslant \zeta$), i.e., the input states is quadrature squeezed. This completes the proof. Here we would like to emphasize the fact that since the quadrature squeezing criterion is U(n) invariant, any local unitary transformation on the input single-mode variance matrix would leave the conclusion unchanged.

In Fig. 1 we plot a comparison between the teleportation fidelity of a coherent state and the quadrature squeezing for the BS output resource states with an input MNIO class of states. As evident from the figure, the squeezing parameter strength required to yield quantum teleportation is higher than the respective strength required for quadrature squeezing. In other words, there is no quantum teleportation in the absence of quadrature squeezing.

Similarly, in Fig. 2 we plot a comparison between teleportation and quadrature squeezing for the BS output resources with even and odd coherent states as input. It is evident from the figure that there is a clear distinction between the even and the odd coherent superposition. While the even superposition ($|\psi_{(\alpha+)}\rangle$) always leads to QT, the odd superposition ($|\psi_{(\alpha-)}\rangle$) never leads to QT. From the respective squeezing character it is quite clear that there is no QT in the absence of quadrature squeezing.

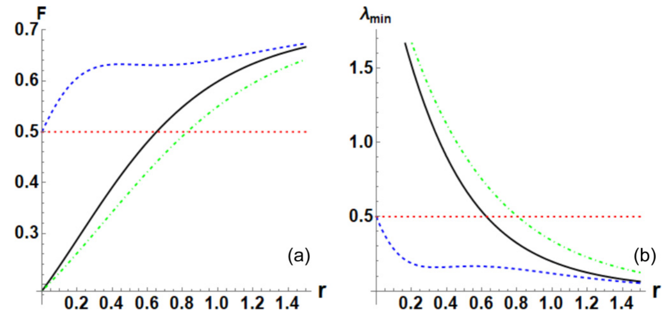

FIG. 1. Plot of (a) teleportation fidelity F and (b) lowest eigenvalue of the variance matrix $\lambda_{\min}$ with squeezing strength r for the MNIO class of input states. Different curves correspond to input $|\psi_{(2+)}\rangle$ (black solid line), $|\psi_{(2-)}\rangle$ (blue dashed line), and $|\psi_{(2)}\rangle$ (green dash-dotted line). Horizontal curves (red dotted line) correspond to the values (a) $F = \frac{1}{2}$ and (b) $\lambda_{\min} = \frac{1}{2}$. Plotted quantities are dimensionless.

B. Mixed states

Let us now consider the case of mixed states. As examples, we focus on the aforementioned pure states suffering from source error yielding additional thermal noise. Such noises can appear for several reasons such as imperfect cavity and spurious noise (naturally appearing in any experimental setup). Speaking mathematically, these noisy states are described as

$$\rho_{(2+)}(\epsilon) = (1-\epsilon)|\psi_{(2+)}\rangle\langle\psi_{(2+)}| + \epsilon\rho_{\mathrm{th}}(\bar{n}),$$
$$\rho_{(2+)}(\epsilon) = (1-\epsilon)|\psi_{(2-)}\rangle\langle\psi_{(2-)}| + \epsilon\rho_{\mathrm{th}}(\bar{n}),$$
$$\rho_{(2)}(\epsilon) = (1-\epsilon)|\psi_{(2)}\rangle\langle\psi_{(2)}| + \epsilon\rho_{\mathrm{th}}(\bar{n}),$$
$$\rho_{(\alpha\pm)}(\epsilon) = (1-\epsilon)|\psi_{(\alpha\pm)}\rangle\langle\psi_{(\alpha\pm)}| + \epsilon\rho_{\mathrm{th}}(\bar{n}), \tag{8}$$

where $\rho_{\mathrm{th}}(\bar{n}) = \frac{1}{\bar{n}}\sum_k (\frac{\bar{n}}{1+\bar{n}})^k |k\rangle\langle k|$, $\bar{n}$ represents the average number of thermal photons, and ϵ stands for the noise strength.

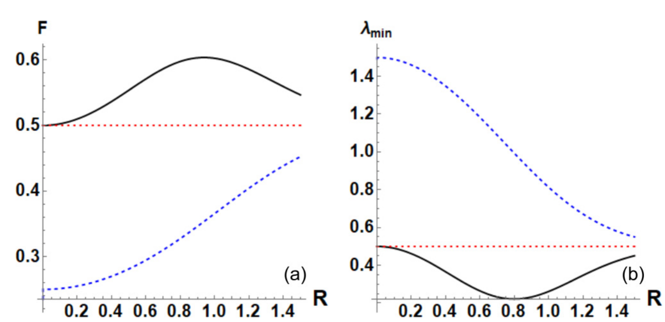

FIG. 2. Plot of (a) teleportation fidelity F and (b) lowest eigenvalue of the variance matrix $\lambda_{\min}$ with squeezing strength r for input even and odd coherent superposition states. Different curves correspond to input $|\psi_{(\alpha+)}\rangle$ (black solid line) and $|\psi_{(\alpha-)}\rangle$ (blue dashed line). Horizontal curves (red dotted line) correspond to the values (a) $F = \frac{1}{2}$ and (b) $\lambda_{\min} = \frac{1}{2}$. Plotted quantities are dimensionless.

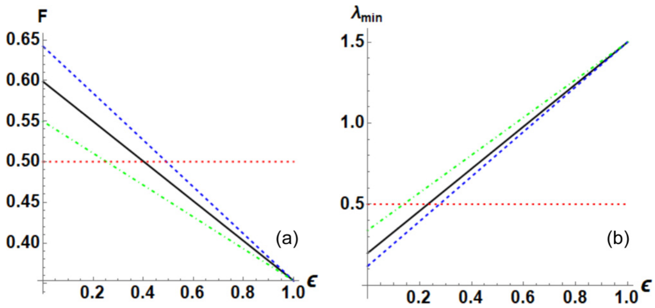

FIG. 3. Plot of (a) teleportation fidelity F and (b) lowest eigenvalue of the variance matrix $\lambda_{\min}$ with noise strength ϵ for the noisy MNIO states. Different curves correspond to input $\rho_{(2+)}(\epsilon)$ (black solid line), $\rho_{(2-)}(\epsilon)$ (blue dashed line), and $\rho_{(2)}(\epsilon)$ (green dash-dotted line). Horizontal curves (red dotted line) correspond to the values (a) $F = \frac{1}{2}$ and (b) $\lambda_{\min} = \frac{1}{2}$. The other parameters are $r = 1.5$ and $\bar{n} = 1.0$. Plotted quantities are dimensionless.

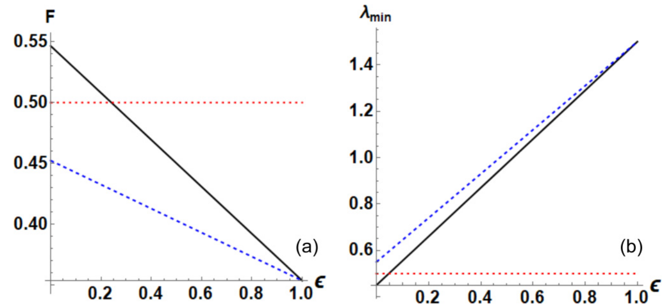

FIG. 4. Plot of (a) teleportation fidelity F and (b) lowest eigenvalue of the variance matrix $\lambda_{\min}$ with noise strength ϵ for input noisy even and odd coherent superposition states. Different curves correspond to input $\rho_{(\alpha+)}(\epsilon)$ (black solid line) and $\rho_{(\alpha-)}(\epsilon)$ (blue dashed line). Horizontal curves (red dotted line) correspond to the values (a) $F = \frac{1}{2}$ and (b) $\lambda_{\min} = \frac{1}{2}$. The other parameters are $r = 1.5$ and $\bar{n} = 1.0$. Plotted quantities are dimensionless.

In Fig. 3 we plot a comparison between teleportation and squeezing for the input noisy MNIO class of states. It is evident from the figure that as the noise strength increases, the difference between the QT and quadrature squeezing becomes vivid. While the resource states lose quadrature squeezing ($\lambda_{\min} > \frac{1}{2}$) for a small noise strength, they still yield QT. This indicates that in the case of mixed states, QT could be obtained even without quadrature squeezing.

Similarly, in Fig. 4 we show the relative character of QT and quadrature squeezing under the effect of noise. We observe that, very similar to the case of noisy MNIO states (Fig. 3), here also over a reasonable parameter region QT exists without quadrature squeezing for the even coherent superposition. It may also be noted that the odd coherent superposition does not possess any squeezing character under any condition.

IV. CONCLUSION

To summarize, in this paper we have analyzed a quantum optical character, namely, the U(2)-invariant quadrature squeezing in the context of CV teleportation. To that end, we analytically proved that for Gaussian resources, in general, quadrature squeezing is necessary for QT. On the other hand, in the case of non-Gaussian resources we observed a clear distinction between the pure and the mixed states. We have elaborated our results by considering a class of states, in particular, the states obtained at the output of a 50:50 BS with single-mode nonclassical input states. Our analytic result showed that in the case of pure states quadrature squeezing is necessary for QT, in the sense that there is no QT without quadrature squeezing. However, in the case of mixed states, we have observed otherwise. In our simple noise model where the input pure states contain additional thermal noise, there is a finite parameter region over which QT exists even without quadrature squeezing.

It may be noted that, so far, the characterizations of the CV resources, in the context of QT, have been mostly centered around the information-theoretic aspects. With this background, here we have focused on the respective quantum optical characterization. Although the attribute, namely, quadrature squeezing, appears to be necessary for QT in the case of all Gaussian states and the non-Gaussian pure states, it fails in the case of non-Gaussian mixed states. This indicates the imminent role of non-Gaussianity arising solely due to classical mixing in the context of CV QT.

One may further consider similar attributes such as the sum squeezing and the difference squeezing that might be an integral component of QT with non-Gaussian optical resources [25]. However, it must be noted that both the sum squeezing and the difference squeezing could be obtained from U(2)-invariant quadrature squeezing and vice versa by a suitable transformation (at the frequency level) [44]. The present analysis of the U(2)-invariant quadrature squeezing has revealed the exotic character of the quantum optical attributes of the CV resources and necessitates a deeper search for the physical attributes, beyond entanglement, that might be crucial in surpassing the classical limit of teleportation.

Nonetheless, it might also be worth analyzing the role of quadrature squeezing for important states such as binomial code [45] or the Gottesman-Kitaev-Preskill code [46] in the context of quantum teleportation, a one-way quantum computation [47]. Recent work on teleportation and error correction of such codes [48] further increases the scope of the present results to the cluster states, a useful resource in achieving quantum computation through quantum teleportation [49].

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