





Efficient linear-optical generation of a multipartite W state

Paweł Błasiak ^{1,2,*}, Ewa Borsuk ^{1,†}, Marcin Markiewicz ^{3,‡} and Yong-Su Kim ^{4,5,§}

¹*Institute of Nuclear Physics, Polish Academy of Sciences, PL-31342 Kraków, Poland*

²*City, University of London, London EC1V 0HB, United Kingdom*

³*International Centre for Theory of Quantum Technologies, University of Gdańsk, PL-80308 Gdańsk, Poland*

⁴*Center for Quantum Information, Korea Institute of Science and Technology (KIST), Seoul 02792, Republic of Korea*

⁵*Division of Nano & Information Technology, KIST School, Korea University of Science and Technology, Seoul 02792, Republic of Korea*



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A scheme is presented for generation of a multipartite W state for an arbitrary number of qubits. Based on a recent proposal of entanglement without touching, it serves to demonstrate the potential of particle indistinguishability as a useful resource of entanglement for practical applications. The devised scheme is efficient in design, meaning that it is built with linear optics without the need for auxiliary particles nor measurements. Yet, the success probability is shown to be highly competitive compared with the existing proposals (i.e., decreases polynomially with the number of qubits) and remains insensitive to particle statistics (i.e., has the same efficiency for bosons and fermions).

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I. INTRODUCTION

Entanglement is a key quantum information resource which provides a basis for all modern developments in quantum technologies and remains a central theme in quantum foundations research [1–9]. Flexibility of entanglement generation is thus crucially important for practical implementations which require manipulation of an increasing number of qubits prepared in a desired state. Needless to say the challenges lie on both the experimental as well as the theoretical sides, with various factors deciding the usefulness of a given proposal. It is always a subtle interplay between the scaling properties for the efficiency of state generation and the resources measured by the complexity of the experimental design.

Here we focus on a prominent example of a multipartite entangled state, the W state, which for N qubits take the form,

$$W_N = \frac{1}{\sqrt{N}} (|\uparrow\downarrow\downarrow \dots \downarrow\rangle + |\downarrow\uparrow\downarrow \dots \downarrow\rangle + \dots + |\downarrow \dots \downarrow\downarrow\uparrow\rangle), \quad (1)$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}$ is a computational basis. A remarkable property of this state is that the amount of entanglement shared between any of the qubits and all the rest is optimal, in a sense that entanglement is robust to the loss of one or more qubits in the system [10] (this should be compared with a multipartite GHZ state for which the loss of a single qubit forfeits any entanglement between all the rest). Notably, the W state belongs to a separate entanglement class under SLOCCs [11]. It also shows stronger nonlocal properties [12].

This makes the multipartite W states an interesting quantum resource, with a promise for practical applications in quantum information [13–15]. There have been a few implementations of the W state [16–20], but their complexity which grows with the number of qubits presents a significant challenge. For most theoretical proposals [21–23] the efficiency drops exponentially with N , with the recent exception in Ref. [24] (see also Ref. [25] for fermions).

In this paper, we develop a scheme for generation of the multipartite W state which can successfully compete with all known proposals, including the quantum erasure proposal in Ref. [24]. Our proposition is based on a *no-touching* paradigm for entanglement generation in linear optical circuits explicitly discussed in Ref. [26] (see also Refs. [27,28] for early indications of this idea and Refs. [29–31] for some particular realizations). This idea originates from the foundational question about the possibility of entanglement extraction from pure particle indistinguishability without any component from particle interactions. The problem has led to a special class of linear optical designs in which the particles traversing the circuit never touch one another over the entire evolution. Apparently, such experimental schemes provide an efficient platform for entanglement generation, which in the present paper is illustrated on the example of the multipartite W state. We also note that the no-touching designs are insensitive to particle statistics, that is, they are equally suitable for bosons and fermions.

The paper starts with some brief remarks on various factors relevant for the assessment of experimental proposals for state generation. Then we proceed with a detailed description of the no-touching scheme for generation of the multipartite W state for an arbitrary number of qubits. As the basis for our construction we use the dual-rail encoding of qubits and then give the corresponding design for polarization encoding. We also comment on experimental feasibility of the proposed schemes. The paper concludes with a discussion in which the

*pawel.blasiak@ifj.edu.pl

†ewa.borsuk@ifj.edu.pl

‡marcinm495@gmail.com

§yong-su.kim@kist.re.kr

efficiency of our design is compared with other proposals in the literature.

II. REMARKS ON EFFICIENCY COMPARISON

A standard tool for comparison of experimental designs is the efficiency denoted here by Eff_N , i.e., the success probability of obtaining the desired result (in our case it is a given state). The main factor reducing the efficiency comes from post-selection which occurs in many different guises in virtually every experimental design. A typical example are event-ready techniques for entanglement generation (e.g., entanglement swapping [32–34]) or more generally techniques based on coincidence counts (e.g., [35,36]). Then an evident figure of merit for multiqubit entanglement generation schemes is the scaling of the efficiency Eff_N with the increasing number of qubits N .

However, such an analysis blatantly ignores other details of the experimental design which may easily compromise the utility of a given proposal. A more thorough discussion should take into account the complexity as well as the resources required to implement a given proposal. Here we briefly point out some relevant factors in experimental designs which may have an impact that affects the overall assessment.

Suppose we consider a scheme that generates a given quantum state of N qubits which, for simplicity, are encoded in particle degrees of freedom (e.g., polarization, spin, or dual-rail encoding). It means that N particles carry the desired state ψ_{out} at the output. The efficiency Eff_N describes how often this state is produced, as a result of some well-defined post-selection procedure. Then, even if the complexity of linear operations in the circuit is not taken into account, the following features of the design still remain relevant:

- (a) *How many particles does the scheme start with? What input state ψ_{in} are they prepared in?*
- (b) *Are intermediate measurements required in the design? Does the scheme involve feed-forward?*

Re. (a). Every state generation scheme requires some initial multiparticle state ψ_{in} to start with, and some of them are easier to prepare than others. In particular, the fewer particles the better. Therefore every auxiliary particle in the input, on top of what is gained in the output, should count as an extra cost. Furthermore, if our goal is generation of entanglement, then any scheme starting with separable states ψ_{in} , like single-particle inputs $|1\rangle^{\otimes N}$, should be considered less demanding compared to those requiring entanglement to start with.

Re. (b). Measurements are a different kind of resource which introduces nonlinearity into the system. Thus the number of intermediate measurements adds to the overall cost of a given design. Even if the problems with detection inefficiency are ignored, the measurements introduce a stochastic element to the procedure. This either results in extra post-selection (cf. event-ready techniques [32,33]) or requires active correction using feed-forward. Needless to say that the latter presents a considerable technical challenge [34].

Clearly, the analysis solely based on the scaling of the efficiency Eff_N with the increasing number of particles N

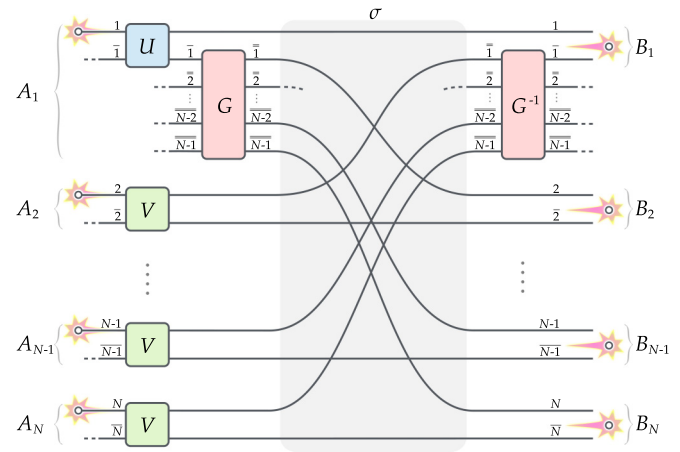


FIG. 1. **Generation of the W_N state in the no-touching scheme.** N independent particles injected into the circuit undergo a sequence of transformations: local unitaries $U \circ G, V, \dots, V$ in the respective subsystems A_1, \dots, A_N followed by permutation of the paths σ and then local unitary G^{-1} in subsystem A_1 . Post-selection on a single particle (coincidence) in each target qubit B_1, \dots, B_N generates the W_N state for the particular choice of unitaries in Eqs. (2) and (3) fulfilling condition Eq. (16).

neglects all other relevant factors like those mentioned above. However, those other resources required in the design significantly contribute to the overall complexity (or cost) deciding the experimental utility of a given proposal. This makes it difficult to compare designs utilizing different resources, since in general it is not clear how to weigh between their costs. There is, however, one exception: From two designs having a comparable efficiency Eff_N , the one with no extra resources seems to be a better choice. This principle will allow one to appreciate the performance of the W state generation scheme constructed in the following section.

III. NO-TOUCHING DESIGN FOR W STATE

A. Preliminaries

We follow the discussion of entanglement generation in the no-touching scheme proposed in Ref. [26]. Let us consider an optical scheme with $3N - 2$ paths (or modes) grouped into N subsystems A_1, \dots, A_N with paths labeled as follows:

$$\begin{aligned} \text{System } A_1 &= \{1, \bar{1}\} \cup \{\bar{2}, \bar{3}, \dots, \bar{N-1}\}, \\ \text{System } A_2 &= \{2, \bar{2}\}, \\ &\dots \quad \dots \\ \text{System } A_N &= \{N, \bar{N}\}. \end{aligned}$$

See Fig. 1 for illustration. For better clarity we denote with a double bar the modes that will play an auxiliary role in subsystem A_1 . Further, we assume that *target* qubits B_1, \dots, B_N are encoded in pairs of paths in the output (so called *dual-rail* qubits) for which we choose:

$$\text{Qubit } B_k = \{k, \bar{k}\} \quad \text{for } k = 1, \dots, N.$$

This means that the computational basis for qubit B_k is identified with a single particle in the respective path $|\uparrow\}_k \equiv a_k^\dagger |0\rangle$

and $|\downarrow\rangle_k \equiv a_k^\dagger|0\rangle$, where $|0\rangle$ is the vacuum. Thus the general state of the qubit is encoded with a single particle in B_k as $\alpha|\uparrow\rangle_k + \beta|\downarrow\rangle_k \equiv (\alpha a_k^\dagger + \beta a_k^\dagger)|0\rangle$.

The protocol consists of the following stages (Fig. 1).

- (1) Start with N independent particles with a single particle injected into each subsystem A_1, \dots, A_N .
- (2) Local unitary on each subsystem A_1, \dots, A_N .
- (3) Permutation of the paths σ ,
- (4) Local unitary in the output on subsystem A_1 .
- (5) Post-selection on a single particle (i.e., coincidence count) in the output qubits B_1, \dots, B_N .

Note that such a design guarantees that the particles do not *touch* one another over the entire evolution as discussed in Ref. [26]. For the discussion of its applicability to nonlocality tests see Ref. [37]. One particular property of such schemes is that their efficiency Eff_N is insensitive to particle statistics (i.e., the success probability is equal for bosons and fermions). In the following, we describe the no-touching protocol generating the W_N state Eq. (1) in the target qubits B_1, \dots, B_N .

B. Details of the protocol

Let us be precise about the no-touching protocol given in Fig. 1 and define three unitary transformations U , V , and G in the following form:

$$U = \begin{pmatrix} \alpha & \beta \\ \beta & -\alpha \end{pmatrix}, \quad V = \begin{pmatrix} \delta & \varepsilon \\ \varepsilon & -\delta \end{pmatrix}, \quad (2)$$

and

$$G = \frac{1}{\sqrt{N-1}} \begin{pmatrix} 1 & \gamma_{1,2} & \dots & \gamma_{1,N-1} \\ 1 & \gamma_{2,2} & \dots & \gamma_{2,N-1} \\ \dots & \dots & \dots & \dots \\ 1 & \gamma_{N-1,2} & \dots & \gamma_{N-1,N-1} \end{pmatrix}. \quad (3)$$

For simplicity we will assume that α , β , δ , and ε are real. Since U and V are unitary, we need to have $\beta^2 = 1 - \alpha^2$ and $\varepsilon^2 = 1 - \delta^2$. Numbers $\gamma_{k,i}$ are chosen arbitrarily so that G is unitary, and they will not play any role in our argument. Permutation of the modes depicted in Fig. 1 is defined as follows:

$$\sigma : \begin{cases} 1 \rightarrow 1, \\ \bar{k} \rightarrow k+1 & \text{for } k = 1, \dots, N-1, \\ k \rightarrow \bar{k}-1 & \text{for } k = 2, \dots, N, \\ \bar{k} \rightarrow \bar{k} & \text{for } k = 2, \dots, N. \end{cases} \quad (4)$$

In this way we have specified all components of the protocol. For further convenience, let us write out all unitary transformations that will be used in our analysis:

$$a_1^\dagger \xrightarrow{U} \alpha a_1^\dagger + \beta a_N^\dagger, \quad (5)$$

$$a_k^\dagger \xrightarrow{V} \delta a_k^\dagger + \varepsilon a_k^\dagger \quad \text{for } k = 2, 3, \dots, N, \quad (6)$$

$$a_1^\dagger \xrightarrow{G} \frac{1}{\sqrt{N-1}} (a_1^\dagger + a_2^\dagger + \dots + a_{N-1}^\dagger), \quad (7)$$

$$a_k^\dagger \xrightarrow{G^{-1}} \frac{1}{\sqrt{N-1}} (a_1^\dagger + \gamma_{k,2}^* a_2^\dagger + \dots + \gamma_{k,N-1}^* a_{N-1}^\dagger) \quad \text{for } k = 1, 3, \dots, N-1. \quad (8)$$

Now, we are ready to trace the evolution of the input state of N independent particles injected into the circuit in Fig. 1 which ends with post-selection in the output. Postponing the questions of experimental difficulties to the next section, like the phase stability which should be maintained during the evolution, we may write

$$a_1^\dagger a_2^\dagger \dots a_N^\dagger |0\rangle \xrightarrow[\text{Eqs. (5)-(7)}]{U, G, V, \dots, V} \left(\alpha a_1^\dagger + \frac{\beta}{\sqrt{N-1}} (a_1^\dagger + \dots + a_{N-1}^\dagger) \right) (\delta a_2^\dagger + \varepsilon a_2^\dagger) \dots (\delta a_N^\dagger + \varepsilon a_N^\dagger) |0\rangle \quad (9)$$

$$\xrightarrow[\text{Eq. (4)}]{\sigma} \left(\alpha a_1^\dagger + \frac{\beta}{\sqrt{N-1}} (a_2^\dagger + \dots + a_N^\dagger) \right) (\delta a_1^\dagger + \varepsilon a_2^\dagger) \dots (\delta a_{N-1}^\dagger + \varepsilon a_N^\dagger) |0\rangle \quad (10)$$

$$\xrightarrow[\text{Eq. (8)}]{G^{-1}} \left(\alpha a_1^\dagger + \frac{\beta}{\sqrt{N-1}} (a_2^\dagger + \dots + a_N^\dagger) \right) \quad (11)$$

$$\left(\frac{\delta}{\sqrt{N-1}} (a_1^\dagger + \gamma_{1,2}^* a_2^\dagger + \dots + \gamma_{1,N-1}^* a_{N-1}^\dagger) + \varepsilon a_2^\dagger \right) \quad (12)$$

...

$$\left(\frac{\delta}{\sqrt{N-1}} (a_1^\dagger + \gamma_{N-1,2}^* a_2^\dagger + \dots + \gamma_{N-1,N-1}^* a_{N-1}^\dagger) + \varepsilon a_N^\dagger \right) |0\rangle \quad (13)$$

$$\xrightarrow[\text{post-selection}]{\rightsquigarrow} \left(\alpha \varepsilon^{N-1} a_1^\dagger a_2^\dagger \dots a_N^\dagger + \frac{\beta \delta \varepsilon^{N-2}}{N-1} \sum_{k=2}^N a_k^\dagger a_2^\dagger \dots a_{k-1}^\dagger a_1^\dagger a_{k+1}^\dagger \dots a_N^\dagger \right) |0\rangle \quad (14)$$

$$\stackrel{\text{Eq. (16)}}{=} \sqrt{\frac{\delta^2 (1 - \delta^2)^{N-1}}{\delta^2 + (N-1)^2 (1 - \delta^2)}} (|\uparrow\downarrow\downarrow \dots \downarrow\rangle + |\downarrow\uparrow\downarrow \dots \downarrow\rangle + \dots + |\downarrow \dots \downarrow\uparrow\downarrow\rangle + |\downarrow \dots \downarrow\downarrow\uparrow\rangle). \quad (15)$$

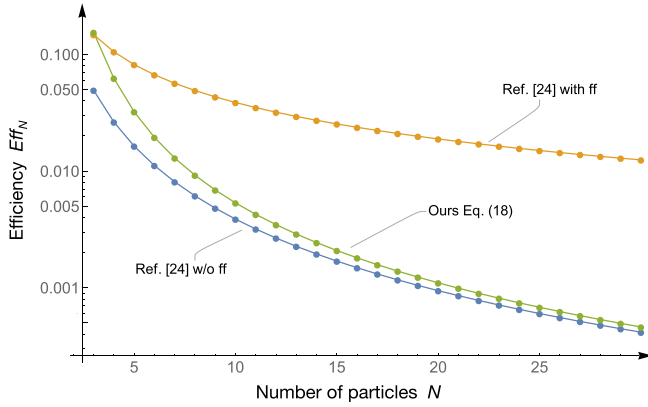


FIG. 2. **Optimal efficiency of the scheme.** The success probability for generation of the W_N state in our scheme Eq. (18) compared with the scheme with auxiliary particle and quantum erasure described in Ref. [24] [without and with active feed-forward; see Eqs. (12) and Eq. (14) therein].

Note that the product structure in Eqs. (11)–(13) allows for quick identification of terms that remain after post-selection. Since we are interested only in cases with a single particle in each subsystem $B_k = \{k, \bar{k}\}$, then for each creator in the first bracket Eq. (11) there is only one choice of creators in the remaining brackets Eq. (12) and (13) which fulfills the post-selection condition: a single particle (creator) in each subsystem B_k (notice that the modes with double bar can be dropped altogether). What is left are only the terms in Eq. (14). The last equality in Eq. (15) holds for bosons when all coefficients are equal,

$$\alpha \varepsilon^{N-1} = \frac{\beta \delta \varepsilon^{N-2}}{N-1}, \quad (16)$$

which holds for the choice $\alpha^2 = \frac{\delta^2}{\delta^2 + (N-1)^2(1-\delta^2)}$. Thus we obtain the W_N state Eq. (1) in the output. We remark that for fermions all terms except the first one in Eq. (15) get the “−” sign which can be easily corrected by phase shift $e^{i\pi}$ in the first path (either in the input or output).

Note that the expression in Eq. (15) is unnormalized due to post-selection which projects on the subspace with a single particle in each channel B_1, \dots, B_N . From the normalization we can read out the success probability (efficiency) of the process which is equal to

$$\text{Eff}_N(\delta) = \frac{N \delta^2 (1 - \delta^2)^{N-1}}{\delta^2 + (N-1)^2 (1 - \delta^2)}. \quad (17)$$

Since there is one free parameter left in the protocol we can optimize over δ and obtain the maximal efficiency:

$$\text{Eff}_N = \max_{\delta} \text{Eff}_N(\delta) \sim \frac{e^{-1}}{N^2} + \frac{7e^{-1}}{2N^3} + o\left(\frac{1}{N^4}\right). \quad (18)$$

The result is plotted in Fig. 2 and compared with the efficiency of the quantum erasure scheme in Ref. [24]. See Appendix for explicit calculation. Let us note that the obtained efficiency is insensitive to particle statistics, which is the same for bosons and fermions. This follows from two observations: (i) in the passive linear optics regime the unitary evolution is governed by the same set of equations Eqs. (5)–(8) regardless

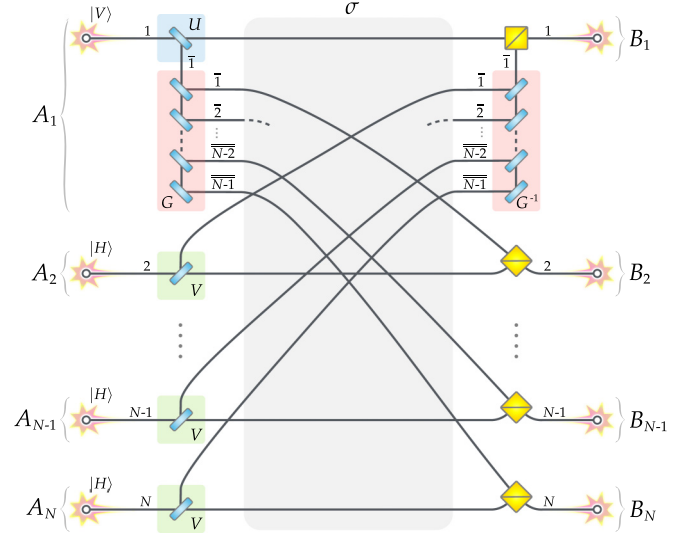


FIG. 3. **Optical scheme with photon polarization.** Rewriting of the proposal in Fig. 1 into photon polarization qubits. In that case, each pair of paths $\{k, \bar{k}\}$ in dual-rail encoding gets replaced with two polarization modes $\{|H\rangle_k, |V\rangle_k\}$, with spatial merging obtained by polarizing beam splitters (PBS) in the output. Then the unitaries U, V, G , and G^{-1} are implemented by appropriate choice of beam splitters (BS). The analysis of this circuit parallels the dual-rail case in Fig. 1 and the resulting efficiencies are clearly the same.

of the statistics [38], and (ii) in the post-selected sector the same terms survive for both types of particles (due to the no-touching feature of the scheme [26]).

We remark that the above discussion based on dual-rail encoding of qubits gives a generic pattern which directly translates into other realizations. For illustration, in Fig. 3 we give analogous design for photon polarization qubits using the scheme in Fig. 1 as a template.

C. Experimental feasibility

Our N particle W state generation protocol requires N identical single photons and the phase stabilized linear optical networks. We remark that such technical demands can be fulfilled with the current quantum photonics technology. For example, it has been shown that tens of identical single photons can be generated either via spontaneous parametric down-conversion (SPDC) [35] or from a quantum dot [39]. The temporal multiplexing technique with SPDC also provides a promising avenue to scale up the number of identical single photon generation [40]. The phase stabilized linear optical networks can be achieved with the active feedback control. For instance, a huge phase stabilized interferometer with an arm length of a few hundreds of kilometers has been implemented in the context of twin-field quantum key distribution [41]. We also note that the integrated quantum photonics can provide an efficient way to implement complicated linear optical networks with the excellent phase stability [9]. Therefore, our cost efficient W state generation protocol can be implemented with tens of photons with the current quantum photonics technology.

IV. DISCUSSION

The presented protocol for generation of the W state is cost effective. It only requires linear optics and post-selection to get the N particle W state from N independent particles in the input. The efficiency of state generation scales polynomially $\text{Eff}_N \sim 1/N^2$ with the number of particles. This leaves behind most proposals in the literature which use quantum fusion techniques [16–20] for the iterative construction of the larger W_N states from smaller ones $W_{M<N}$, which inherently suffer from the exponential decrease of efficiency \mathcal{O}^{-N} . Our proposal clearly benefits from the direct construction of the state rather than the use of iterative techniques. It is also more economical with the respect of resources required for the experimental implementation.

Interestingly, there is a recent proposal of multipartite W state generation based on quantum erasure [24] (see also Ref. [25] for fermions). It is to our knowledge the most efficient protocol whose efficiency scales polynomially like $1/N^2$, with a potential for further improvement to $1/N$ by feed-forward with active state correction on each qubit (see Fig 2). Note, however, that this scheme requires one auxiliary particle in the input on which appropriate measurement is made, i.e., $N + 1$ particles are needed to produce the N particle W_N state. Clearly, this is an additional cost both in terms of particles and measurements that a fair comparison should take into account. Neither of those are required in our design and yet it performs slightly better (cf. Appendix). As noted, even if post-processing of each qubit based on active feed-forward in the quantum erasure scheme in Ref. [24] increases the efficiency, this comes with a considerable extra cost which scales linearly with the number of particles N (and thus compromising the potential gain). Needless to say that active feed-forward poses a nontrivial experimental challenge.

Finally, we remark that the presented approach is insensitive to particle statistics, i.e., works with equal efficiency for fermions, bosons, or anyons. This is a generic property of the no-touching designs [26]. Moreover, it can be safely used for Bell nonlocality tests, since the post-selection scheme follows the all-but-one principle discussed in Ref. [37], which assures no post-selection loophole. Therefore our protocol can be safely utilized as a part of device-independent protocols in quantum technological applications.

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APPENDIX: MAXIMAL EFFICIENCY Eff_N IN EQ. (18)

Finding the maximum in Eq. (17) is straightforward. It boils down to differentiating and choosing the solution such that $|\delta| \leq 1$. Thus we get

$$\frac{\partial \text{Eff}_N(\delta)}{\partial \delta} = 0 \Rightarrow \delta_{\max}^2 = \frac{1 - N + \sqrt{\frac{N^3 - 6N^2 + 13N - 8}{N}}}{4 - 2N}, \quad (\text{A1})$$

and hence Eq. (18) takes the form,

$$\text{Eff}_N = \text{Eff}_N(\delta_{\max}) = \frac{N \delta_{\max}^2 (1 - \delta_{\max}^2)^{N-1}}{\delta_{\max}^2 + (N-1)^2 (1 - \delta_{\max}^2)}. \quad (\text{A2})$$

This result is plotted in Fig. 2.

For completeness, we also give asymptotic expansion of the above expression which reads

$$\text{Eff}_N \sim \frac{e^{-1}}{N^2} + \frac{7e^{-1}}{2N^3} + o\left(\frac{1}{N^4}\right). \quad (\text{A3})$$

This should be compared with the corresponding efficiency for the recent proposal with quantum erasure in Ref. [5] (to our knowledge the most efficient proposal for W state generation in the literature). Equation (14) therein gives

$$\text{Eff}_N^{(\text{Kim } et al.)} \sim \frac{e^{-1}}{N^2} + \frac{e^{-1}}{2N^3} + o\left(\frac{1}{N^4}\right). \quad (\text{A4})$$

We note that although the use of active feed-forward in the scheme of Kim *et al.* [24] increases the efficiency by factor N , it comes at an incomparable cost which scales with the number of particles N , too.

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- [1] J. S. Bell, *Speakable and Unsayable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
 - [2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
 - [3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
 - [4] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, *Rev. Mod. Phys.* **86**, 419 (2014).
 - [5] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Quantum cryptography, *Rev. Mod. Phys.* **74**, 145 (2002).
 - [6] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, Quantum computers, *Nature (London)* **464**, 45 (2010).
 - [7] A. Aspuru-Guzik and P. Walther, Photonic quantum simulators, *Nat. Phys.* **8**, 285 (2012).
 - [8] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Multiphoton entanglement and interferometry, *Rev. Mod. Phys.* **84**, 777 (2012).
 - [9] J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, Integrated photonic quantum technologies, *Nat. Photonics* **14**, 273 (2020).

- [10] M. Koashi, V. Bužek, and N. Imoto, Entangled webs: Tight bound for symmetric sharing of entanglement, *Phys. Rev. A* **62**, 050302(R) (2000).
- [11] W. Dür, G. Vidal, and J. I. Cirac, Three qubits can be entangled in two inequivalent ways, *Phys. Rev. A* **62**, 062314 (2000).
- [12] A. Sen(De), U. Sen, M. Wieśniak, D. Kaszlikowski, and M. Żukowski, Multiqubit W states lead to stronger nonclassicality than Greenberger-Horne-Zeilinger states, *Phys. Rev. A* **68**, 062306 (2003).
- [13] P. Agrawal and A. Pati, Perfect teleportation and superdense coding with W states, *Phys. Rev. A* **74**, 062320 (2006).
- [14] C. Zhu, F. Xu, and C. Pei, W -state analyzer and multi-party measurement-device-independent quantum key distribution, *Sci. Rep.* **5**, 17449 (2015).
- [15] V. Lipinska, G. Murta, and S. Wehner, Anonymous transmission in a noisy quantum network using the W state, *Phys. Rev. A* **98**, 052320 (2018).
- [16] M. Eibl, N. Kiesel, M. Bourennane, C. Kurtsiefer, and H. Weinfurter, Experimental Realization of a Three-Qubit Entangled W State, *Phys. Rev. Lett.* **92**, 077901 (2004).
- [17] H. Mikami, Y. Li, K. Fukuoka, and T. Kobayashi, New High-Efficiency Source of a Three-Photon W State and its Full Characterization Using Quantum State Tomography, *Phys. Rev. Lett.* **95**, 150404 (2005).
- [18] T. Tashima, T. Wakatsuki, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, Local Transformation of Two Einstein-Podolsky-Rosen Photon Pairs into a Three-Photon W State, *Phys. Rev. Lett.* **102**, 130502 (2009).
- [19] T. Tashima, T. Kitano, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, Demonstration of Local Expansion Toward Large-Scale Entangled Webs, *Phys. Rev. Lett.* **105**, 210503 (2010).
- [20] B. Fang, M. Menotti, M. Liscidini, J. E. Sipe, and V. O. Lorenz, Three-Photon Discrete-Energy-Entangled W State in an Optical Fiber, *Phys. Rev. Lett.* **123**, 070508 (2019).
- [21] X.-P. Zang, M. Yang, F. Ozaydin, W. Song, and Z.-L. Cao, Deterministic generation of large scale atomic W states, *Opt. Express* **24**, 12293 (2016).
- [22] N.-N. Wu and M. Jiang, A highly efficient scheme for joint remote preparation of multi-qubit W state with minimum quantum resource, *Quantum Inf. Process.* **17**, 340 (2018).
- [23] S. Bugu, F. Ozaydin, T. Ferrus, and T. Koder, Preparing multipartite entangled spin qubits via pauli spin blockade, *Sci. Rep.* **10**, 3481 (2020).
- [24] Y.-S. Kim, Y.-W. Cho, H.-T. Lim, and S.-W. Han, Efficient linear optical generation of a multipartite W state via a quantum eraser, *Phys. Rev. A* **101**, 022337 (2020).
- [25] B. Bellomo, R. Lo Franco, and G. Compagno, N identical particles and one particle to entangle them all, *Phys. Rev. A* **96**, 022319 (2017).
- [26] P. Blasiak and M. Markiewicz, Entangling three qubits without ever touching, *Sci. Rep.* **9**, 20131 (2019).
- [27] B. Yurke and D. Stoler, Bell's-inequality experiments using independent-particle sources, *Phys. Rev. A* **46**, 2229 (1992).
- [28] B. Yurke and D. Stoler, Einstein-Podolsky-Rosen Effects from Independent Particle Sources, *Phys. Rev. Lett.* **68**, 1251 (1992).
- [29] I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu, and V. Umansky, Interference between two indistinguishable electrons from independent sources, *Nature (London)* **448**, 333 (2007).
- [30] Y.-S. Kim, T. Pramanik, Y.-W. Cho, M. Yang, S.-W. Han, S.-Y. Lee, M.-S. Kang, and S. Moon, Informationally symmetrical Bell state preparation and measurement, *Opt. Express* **26**, 29539 (2018).
- [31] L. Ju, M. Yang, N. Paunković, W.-J. Chu, and Z.-L. Cao, Creating photonic GHZ and W states via quantum walk, *Quantum Inf. Process.* **18**, 176 (2019).
- [32] M. Żukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, "Event-Ready-Detectors" Bell Experiment via Entanglement Swapping, *Phys. Rev. Lett.* **71**, 4287 (1993).
- [33] J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Experimental Entanglement Swapping: Entangling Photons That Never Interacted, *Phys. Rev. Lett.* **80**, 3891 (1998).
- [34] X.-S. Ma, T. Herbst, Thomas Scheidl, D. Wang, S. Kropatschek, W. Naylor, B. Wittmann, A. Mech, J. Kofler, E. Anisimova, V. Makarov, T. Jennewein, R. Ursin, and A. Zeilinger, Quantum teleportation over 143 kilometres using active feed-forward, *Nature (London)* **489**, 269 (2012).
- [35] H.-S. Zhong, Y. Li, W. Li, L.-C. Peng, Z.-E. Su, Y. Hu, Y.-M. He, X. Ding, W. Zhang, H. Li, L. Zhang, Z. Wang, L. You, X.-L. Wang, X. Jiang, L. Li, Y.-A. Chen, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, 12-Photon entanglement and Scalable Scattershot Boson Sampling with Optimal Entangled-Photon Pairs from Parametric Down-Conversion, *Phys. Rev. Lett.* **121**, 250505 (2018).
- [36] M. Krenn, A. Hochrainer, M. Lahiri, and A. Zeilinger, Entanglement by Path Identity, *Phys. Rev. Lett.* **118**, 080401 (2017).
- [37] P. Blasiak, E. Borsuk, and M. Markiewicz, On safe post-selection for Bell nonlocality: Causal diagram approach, *arXiv:2012.7285* (2021).
- [38] M. C. Tichy, Interference of identical particles from entanglement to boson-sampling, *J. Phys. B: At. Mol. Opt. Phys.* **47**, 103001 (2014).
- [39] H. Wang, J. Qin, X. Ding, M.-C. Chen, S. Chen, X. You, Y.-M. He, X. Jiang, L. You, Z. Wang, C. Schneider, J. J. Renema, S. Hofling, C.-Y. Lu, and J.-W. Pan, Boson Sampling with 20 Input Photons and a 60-Mode Interferometer in a 10^{14} -Dimensional Hilbert Space Demonstration, *Phys. Rev. Lett.* **123**, 250503 (2019).
- [40] F. Kaneda and P. G. Kwiat, High-efficiency single-photon generation via large-scale active time multiplexing, *Sci. Adv.* **5**, eaaw8586 (2019).
- [41] S. Wang, D.-Y. He, Z.-Q. Yin, F.-Y. Lu, C.-H. Cui, W. Chen, Z. Zhou, G.-C. Guo, and Z.-F. Han, Beating the Fundamental Rate-Distance Limit in a Proof-Of-Principle Quantum Key Distribution System, *Phys. Rev. X* **9**, 021046 (2019).