# Surface plasmon polaritons in metal films on anisotropic and bianisotropic substrates

A. N. Darinskii
*Institute of Crystallography FSRC "Crystallography and Photonics," Russian Academy of Sciences, Leninskii Prospect 59, Moscow 119333, Russia*

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The existence of nonradiative surface plasmon polaritons (SPPs) is theoretically studied in an isotropic metal film surrounded by optically anisotropic half-infinite dielectrics. By using general properties of the impedance matrices of half-infinite dielectrics and metal films the maximum number of SPPs has been established at a given value of the tangential wave number. It has been proved that, if the dielectrics are nonbianisotropic and magneto-optically inactive but uniaxial or biaxial and oriented arbitrarily, then at most two SPPs can exist independently of the frequency dispersion of the dielectric permittivity and magnetic permeability. At most four SPPs emerge when both the dielectric media are bianisotropic and/or magneto-optically active but the maximum number of SPPs reduces to 3 provided that one of the dielectrics does not exhibit either bianisotropy or magneto-optical activity. A metal film inserted into an infinite bianisotropic and/or magneto-optically active medium can also guide at most three SPPs.

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## I. INTRODUCTION

Surface plasmon polaritons (SPPs) exist in a variety of structures. Theoretical and experimental investigations allowed the establishment of fundamental properties of the SPP propagation on flat and randomly rough metal-dielectric interfaces, in metal films sandwiched between dielectrics and in dielectric film enclosed between metals, in waveguides of different types, and in periodic gratings, grooves, and wedges created on the surface of metals [1–8]. Recently there has appeared an interest in studying SPPs at the boundary of metals having unusual properties and in the possibility of generating SPPs with specific characteristics of the wave field [9–14]. Scientific activity is stimulated by applications of SPPs in chemical and biological sensors, in photovoltaic cells in order to enhance their effectiveness, in super-resolution microscopy and surface-enhanced Raman scattering [15–19].

Given material constants and the geometric parameters of a structure, SPPs may or may not exist, their number can be different and, apart from that, SPPs can be radiative (leaky) and nonradiative [1–6,8,20–30]. The former emerge in a frequency range where bulk electromagnetic waves exist in dielectrics and, correspondingly, they radiate electromagnetic waves into the interior of dielectrics. In contrast, the frequencies of nonradiative SPPs fall outside the interval of bulk waves in external dielectrics, so that there is no radiation.

We are concerned with nonradiative SPPs guided by a metal film. Such SPPs were comprehensively studied by explicit analytic computations in the case of isotropic dielectrics and optically anisotropic ones at orientations supporting TE and TM modes [20–22], at particular orientations of uniaxial and biaxial dielectrics not allowing TE and TM modes [30]. It was found that not more than two SPPs come about.

The present paper considers nonradiative SPPs in metal films embedded between arbitrarily oriented uniaxial or biaxial dielectrics as well as between bianisotropic and/or magneto-optically active dielectrics. Our goal is to establish the admissible maximum number of SPPs. This task cannot be accomplished by explicit analytic computations analogous to those which were carried out in [20–22,30] for nonradiative SPPs guided by films as well as in [23–29] for nonradiative SPPs on the dielectric–half-infinite metal and radiative SPPs because it is not possible to even derive the dispersion equation in a closed analytic form in the cases we are interested in. Numerical computations do not help either to solve such a general problem. The posed problem proves to be solvable by using an analytical method which is based on properties of the impedance matrices of half-infinite media and metal films, allowing one to avoid the necessity of deriving explicitly dispersion equations. Note that similar methods have already been applied to surface electromagnetic waves in homogeneous dielectrics and superlattices [31–33], and to surface acoustic waves in superlattices [34–38]. In general, the idea of using properties of the impedance matrices, which follow from fundamental physical principles, in order to analyze the existence of surface waves has been put forward in [39,40] in connection with the theory of surface acoustic waves in half-infinite homogeneous anisotropic solids.

Our paper is organized as follows. Section II introduces the impedance matrices of half-infinite dielectric materials and metal films. The proofs of the statements concerning the existence of nonradiative SPPs are given in Sec. III, and Sec. IV discusses the results obtained. Some important relations are included in Appendices A–C. Numerical examples are given in Appendix D.

## II. IMPEDANCE MATRICES

The matrices which we call impedances have somewhat different properties in nonbianisotropic magneto-optically inactive media and bianisotropic and/or magneto-optically active dielectrics but the properties, which we need, are the same independently of whether the dielectric is only bianisotropic, or only magneto-optically active, or bianisotropic and magneto-optically active. Taking into account this fact, we will frequently call "anisotropic" a dielectric which is nonbianisotropic and magneto-optically inactive. The term "bianisotropic" will mean bianisotropic and/or magneto-optically active dielectric.

We will assume no absorption. Otherwise the impedances lose their key properties, so that the rigorous analysis of the SPP existence in the presence of losses requires a different approach. The influence of absorption on the number of SPPs is discussed in Sec. III C.

### A. Dielectrics

Electromagnetic properties of a bianisotropic material are described with the aid of the constitutive relations

$$\mathbf{D} = \hat{\boldsymbol{\varepsilon}}\mathbf{E} + \hat{\boldsymbol{\kappa}}\mathbf{H}, \quad \mathbf{B} = \hat{\boldsymbol{\kappa}}^{\dagger}\mathbf{E} + \hat{\boldsymbol{\mu}}\mathbf{H}, \tag{1}$$

where **D**, **B**, **E**, and **H** are the electric displacement, magnetic induction, and electric and magnetic fields, respectively; $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are, respectively, the dielectric permittivity and magnetic permeability; $\hat{\boldsymbol{\kappa}}$ is a complex nonsymmetric pseudotensor [41–46]. The symbol $^{\dagger}$ denotes the Hermitian conjugation. We assume $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ complex Hermitian tensors to allow for the magneto-optical activity. The components of $\hat{\boldsymbol{\varepsilon}}$, $\hat{\boldsymbol{\mu}}$, and $\hat{\boldsymbol{\kappa}}$ may depend on frequency.

In materials that are anisotropic according to our terminology, Eq. (1) simplifies to

$$\mathbf{D} = \hat{\boldsymbol{\varepsilon}}\mathbf{E}, \quad \mathbf{B} = \hat{\boldsymbol{\mu}}\mathbf{H}, \tag{2}$$

where $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are real symmetric tensors (magneto-optical activity is excluded but $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ remain frequency dependent).

Consider an electromagnetic wave

$$\tilde{\boldsymbol{\xi}}_{\alpha}(\mathbf{r}, t) = \boldsymbol{\xi}_{\alpha} e^{i[kx + p_{\alpha}z - \omega t]} \tag{3}$$

labeled by the subscript $\alpha$ and propagating in the plane $XZ$ along the axis $X$ with a frequency $\omega$ and wave number $k$. The vectors $\tilde{\boldsymbol{\xi}}_{\alpha}$ and $\boldsymbol{\xi}_{\alpha}$ have four components and these components are the $x$ and $y$ components of the electric **E** and magnetic **H** fields, $\mathbf{r} = (x\ z)^t$ is the radius vector, and the symbol $^t$ denotes the transposition. By substituting (3) in the Maxwell equations and taking into account (1) or (2) one can find that the normal wave number $p_{\alpha}$ and the vector $\boldsymbol{\xi}_{\alpha}$ are an eigenvalue and the corresponding eigenvector of a $4 \times 4$ matrix $\hat{\mathbf{N}}$. The explicit expression of $\hat{\mathbf{N}}$ depends on the order of components of **E** and **H** in $\boldsymbol{\xi}_{\alpha}$ (see, e.g., [31,47–49]). Following [32,33] we put

$$\boldsymbol{\xi}_{\alpha} = \begin{pmatrix} \mathbf{U}_{\alpha} \\ \mathbf{V}_{\alpha} \end{pmatrix}, \ \mathbf{U}_{\alpha} = \begin{pmatrix} -E_{\alpha y} \\ H_{\alpha y} \end{pmatrix}, \ \mathbf{V}_{\alpha} = \begin{pmatrix} H_{\alpha x} \\ E_{\alpha x} \end{pmatrix}. \tag{4}$$

The matrix $\hat{\mathbf{N}}$ which corresponds to the definition of vectors $\boldsymbol{\xi}_{\alpha}$ (4) is given in [32,33] and omitted in this paper because it will not be used.

Independently of whether the medium is anisotropic or bianisotropic, the four eigenvalues $p_{\alpha}$ occur either pairwise complex conjugate, $p_{\alpha} = p^*_{\alpha+2}$, $\mathrm{Im}(p_{\alpha}) = -\mathrm{Im}(p_{\alpha+2}) \neq 0$, where the symbol $^*$ denotes the complex conjugation, or pairwise real, i.e., $\mathrm{Im}(p_{\alpha}) = \mathrm{Im}(p_{\alpha+2}) = 0$. Given the geometry of propagation and the value of $k$, a limiting frequency $\omega_L$ exists such that in the interval $\omega < \omega_L$ all $p_{\alpha}$'s are complex and hence there are no bulk waves. It is this interval which is of our concern.

Assume $\mathrm{Im}(p_{\alpha}) > 0$, $\alpha = 1, 2$, at $\omega < \omega_L$. In this instance the wave fields

$$\tilde{\boldsymbol{\xi}}^{(+)} = \sum_{\alpha=1}^{2} b_{\alpha}\tilde{\boldsymbol{\xi}}_{\alpha}(\mathbf{r}, t), \quad \tilde{\boldsymbol{\xi}}^{(-)} = \sum_{\alpha=3}^{4} b_{\alpha}\tilde{\boldsymbol{\xi}}_{\alpha}(\mathbf{r}, , t), \tag{5}$$

where $b_{\alpha}$ are constants, tend to zero as $z \to +\infty$ and $-\infty$, respectively. With this in mind, we introduce two $2 \times 2$ impedance matrices $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ via the relations

$$\mathbf{V}_{\alpha} = i\hat{\mathbf{Z}}\mathbf{U}_{\alpha}, \ \mathbf{V}_{\alpha+2} = -i\hat{\mathbf{Z}}'\mathbf{U}_{\alpha+2}, \ \alpha = 1, 2. \tag{6}$$

Thus $\hat{\mathbf{Z}}$ can express the last two components of $\tilde{\boldsymbol{\xi}}^{(+)}$ in terms of its first two components at a fixed plane $z =$ const. The impedance $\hat{\mathbf{Z}}'$ relates similarly the components of $\tilde{\boldsymbol{\xi}}^{(-)}$.

The properties of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ within the interval $\omega < \omega_L$ are given in Appendix A. They are basically the same in bianisotropic and anisotropic media. The difference is that in anisotropic media

$$\hat{\mathbf{Z}}' = \hat{\mathbf{Z}}^* = \hat{\mathbf{Z}}^t. \tag{7}$$

Hence in this case

$$\mathrm{Re}(\hat{\mathbf{Z}}) = \mathrm{Re}(\hat{\mathbf{Z}})^t, \quad \mathrm{Im}(\hat{\mathbf{Z}}) = -\mathrm{Im}(\hat{\mathbf{Z}})^t. \tag{8}$$

Equality (7) holds true because $\hat{\mathbf{N}}$ is a real matrix in anisotropic media and therefore $\boldsymbol{\xi}_{\alpha} = \boldsymbol{\xi}^*_{\alpha+2}$ once $p_{\alpha} = p^*_{\alpha+2}$. In bianisotropic media $\hat{\mathbf{N}}$ is a complex matrix, so that $\boldsymbol{\xi}_{\alpha} \neq \boldsymbol{\xi}^*_{\alpha+2}$ and (7) does not hold true. In consequence, (8) may or may not hold true in such materials.

### B. Metal films

We consider that the absorption of electromagnetic waves in metal is negligibly small at frequencies higher than a critical value $\omega_a$ and that the relative dielectric permittivity of metal $\varepsilon_m(\omega)$ is isotropic and negative in the interval $\omega_a < \omega < \omega_p$, where $\omega_p$ is a bulk plasma frequency. For example, within the frame of the Drude model $\varepsilon_m(\omega) = 1 - \omega_p^2/\omega^2$.

In isotropic media there are two TE polarized modes $\alpha = 1, 3$ with $E_{\alpha x} = H_{\alpha y} = 0$ and two TM polarized modes $\alpha = 2, 4$ with $E_{\alpha y} = H_{\alpha x} = 0$. Given $k$, all the modes have the purely imaginary wave number $p_{\alpha} = -p_{\alpha+2} = ip$, where $p = \frac{1}{c}\sqrt{c^2k^2 + \omega^2|\varepsilon_m|\mu_m}$, $c$ is the light velocity in the vacuum, and $\mu_m > 0$ is the relative magnetic permeability of the metal.

We are interested in electromagnetic fields in films. By using an analogy with a general theory of acoustic waves in elastically anisotropic plates [50], we introduce the $2 \times 2$ impedances $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$ as follows. The former relates the values $H_x^{(h)}$ and $H_x^{(0)}$ of the total field $H_x$ at the edges $z = h$ and 0 of the film, respectively, with the values $E_y^{(h)}$ and $E_y^{(0)}$ of the total field $E_y$ at the same edges. The latter expresses similarly $E_x^{(h)}$ and $E_x^{(0)}$ in terms of $H_y^{(h)}$ and $H_y^{(0)}$:

$$\begin{pmatrix} H_x^{(h)} \\ -H_x^{(0)} \end{pmatrix} = i\hat{\mathbb{Z}}^{(\mathrm{TE})} \begin{pmatrix} E_y^{(h)} \\ E_y^{(0)} \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} E_x^{(h)} \\ -E_x^{(0)} \end{pmatrix} = -i\hat{\mathbb{Z}}^{(\mathrm{TM})} \begin{pmatrix} H_y^{(h)} \\ H_y^{(0)} \end{pmatrix}, \tag{10}$$

where

$$\hat{\mathbb{Z}}^{(\mathrm{TE})} = \frac{p}{\omega\mu_0\mu_m}\hat{\mathbb{Z}}, \quad \hat{\mathbb{Z}}^{(\mathrm{TM})} = -\frac{p}{\omega\varepsilon_0|\varepsilon_m|}\hat{\mathbb{Z}}, \tag{11}$$

$$\hat{\mathbb{Z}} = \begin{pmatrix} \coth(ph) & -\mathrm{csch}(ph) \\ -\mathrm{csch}(ph) & \coth(ph) \end{pmatrix}, \tag{12}$$

where $\mu_0$ and $\varepsilon_0$ are the magnetic and dielectric constants. The necessary properties of $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$ are given in Appendix B.

## III. EXISTENCE OF SURFACE PLASMON POLARITONS

Let a metal film of thickness $h$ be between half-infinite dielectric media 1 and 2 which occupy the regions $z > h$ and $z < 0$, respectively. In order to derive the dispersion equation in a suitable form we build up of $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$ a real $4 \times 4$ matrix $\hat{\mathbf{Z}}_f$ in such a way that

$$\begin{pmatrix} \mathbf{V}^{(h)} \\ -\mathbf{V}^{(0)} \end{pmatrix} = -i\hat{\mathbf{Z}}_f \begin{pmatrix} \mathbf{U}^{(h)} \\ \mathbf{U}^{(0)} \end{pmatrix}, \tag{13}$$

where $\mathbf{U}^{(h),(0)}$ and $\mathbf{V}^{(h),(0)}$ are the vectors formed of $E_{x,y}^{(h),(0)}$ and $H_{x,y}^{(h),(0)}$, respectively, similarly to $\mathbf{U}_\alpha$ and $\mathbf{V}_\alpha$ (4):

$$\hat{\mathbf{Z}}_f = \begin{pmatrix} \mathbb{Z}_{11}^{(\mathrm{TE})} & 0 & \mathbb{Z}_{12}^{(\mathrm{TE})} & 0 \\ 0 & \mathbb{Z}_{11}^{(\mathrm{TM})} & 0 & \mathbb{Z}_{12}^{(\mathrm{TM})} \\ \mathbb{Z}_{12}^{(\mathrm{TE})} & 0 & \mathbb{Z}_{11}^{(\mathrm{TE})} & 0 \\ 0 & \mathbb{Z}_{12}^{(\mathrm{TM})} & 0 & \mathbb{Z}_{11}^{(\mathrm{TM})} \end{pmatrix}, \tag{14}$$

where $\mathbb{Z}_{ij}^{(\mathrm{TE})}$ and $\mathbb{Z}_{ij}^{(\mathrm{TM})}$ are $ij$ elements of matrices (11), and the index $f$ means "film." In view of Eq. (6)

$$\mathbf{V}^{(h)} = i\hat{\mathbf{Z}}^{(1)}\mathbf{U}^{(h)}, \quad \mathbf{V}^{(0)} = -i\hat{\mathbf{Z}}^{(2)\prime}\mathbf{U}^{(0)}, \tag{15}$$

where $\hat{\mathbf{Z}}^{(J)}$ and $\hat{\mathbf{Z}}^{(J)\prime}$, with $J = 1, 2$, are, respectively, the impedances of media 1 and 2. The insertion of $\mathbf{V}^{(h)}$ and $\mathbf{V}^{(0)}$ (15) in (13) yields

$$\hat{\mathbf{Z}}_{\mathrm{st}} \begin{pmatrix} \mathbf{U}^{(h)} \\ \mathbf{U}^{(0)} \end{pmatrix} = \mathbf{0}, \tag{16}$$

where

$$\hat{\mathbf{Z}}_{\mathrm{st}} = \hat{\mathbf{Z}}_d + \hat{\mathbf{Z}}_f \tag{17}$$

and

$$\hat{\mathbf{Z}}_d = \begin{pmatrix} \hat{\mathbf{Z}}^{(1)} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{Z}}^{(2)\prime} \end{pmatrix}, \tag{18}$$

$\hat{\mathbf{0}}$ is the $2 \times 2$ zero matrix, and the indices $st$ and $d$ come from "structure" and "dielectric." As a result, the dispersion equation can be written in the form

$$\det \hat{\mathbf{Z}}_{\mathrm{st}} = 0, \tag{19}$$

wherefrom it follows that one of the eigenvalues $\lambda_\alpha$, $\alpha = 1, 2, 3, 4$, of $\hat{\mathbf{Z}}_{\mathrm{st}}$ vanishes at SPP frequencies, so below we will analyze the roots of equations

$$\lambda_\alpha(\omega) = 0, \quad \alpha = 1, 2, 3, 4, \tag{20}$$

within the frequency range $\omega_a < \omega < \Omega_L = \min(\omega_{L1}, \omega_{L2})$, where $\omega_{L1}$ and $\omega_{L2}$ are the above defined limiting frequencies of media 1 and 2, respectively. In this interval there are no bulk modes in either dielectrics or film (we assume $\omega_{L1,L2} < \omega_p$ since $\omega_p$ falls into far ultraviolet in metals).

General properties of $\lambda_\alpha$ in the interval $\omega_a < \omega < \Omega_L$ as functions of $\omega$ do not depend on whether media 1 and 2 are anisotropic or bianisotropic. These properties are derivable from properties (A1)–(A5) of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ and from properties (B1)–(B4) of $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$, viz.,

$$\mathrm{Im}(\lambda_\alpha) = 0, \quad \alpha = 1, 2, 3, 4, \tag{21}$$

$$\lambda_\alpha, \ \alpha = 1, 2, 3, 4, \ \text{monotonically decrease with increasing frequency.} \tag{22}$$

From (21) and (22) it follows that the dispersion equation (19) cannot have more than four roots because each $\lambda_\alpha$ may vanish at most once. Correspondingly, the maximum number of roots is possible when the four eigenvalues are positive at $\omega_a$, so, having in mind property (A4), we consider that

$$\lambda_\alpha > 0, \quad \alpha = 1, 2, 3, 4, \quad \text{at } \omega = \omega_a. \tag{23}$$

It turns out that there are specific restrictions on the number of vanishing $\lambda_\alpha$'s, depending on whether both dielectrics are anisotropic or at least one of them is bianisotropic, so we will discuss these two options separately. [A few additional remarks concerning Eq. (19) are given in Appendix C.]

### A. Anisotropic structure

Once medium 1 and 2 are magneto-optically inactive and nonbianisotropic, in view of Eqs. (7) and (A1)–(A5), $\mathrm{Re}(\hat{\mathbf{Z}}^{(1)})$ and $\mathrm{Re}(\hat{\mathbf{Z}}^{(2)\prime})$ are positive definite matrices in the interval $\omega < \Omega_L$. Since the matrices $\hat{\mathbf{Z}}^{(1)}$ and $\hat{\mathbf{Z}}^{(2)\prime}$ are Hermitian, their diagonal elements $Z_{11}^{(1)}$ and $Z_{11}^{(2)\prime}$ are real and therefore $Z_{11}^{(1)} > 0$ and $Z_{11}^{(2)\prime} > 0$ at $\omega < \Omega_L$, owing to the positive definiteness of $\mathrm{Re}(\hat{\mathbf{Z}}^{(1)})$ and $\mathrm{Re}(\hat{\mathbf{Z}}^{(2)\prime})$.

We represent $\hat{\mathbf{Z}}_{\mathrm{st}}$ in the form

$$\hat{\mathbf{Z}}_{\mathrm{st}} = \sum_{\alpha=1}^{4} \lambda_\alpha \mathbf{e}_\alpha \otimes \mathbf{e}_\alpha^*, \tag{24}$$

where $\mathbf{e}_\alpha$ are the orthonormalized eigenvectors of the Hermitian matrix $\hat{\mathbf{Z}}_{\mathrm{st}}$ and the symbol $\otimes$ stands for the dyadic multiplication, and assume the eigenvalue $\lambda_4$ positive at $\omega < \Omega_L$. Due to Eqs. (14), (17), (18), and (B2), by contracting $\hat{\mathbf{Z}}_{\mathrm{st}}$ with the vector $\mathbf{t} = (-e_{4,3}^* \ 0 \ e_{4,1}^* \ 0)^t$, we arrive at the inequality

$$\mathbf{t}^\dagger \hat{\mathbf{Z}}_{\mathrm{st}} \mathbf{t} = \sum_{\alpha=1}^{3} \lambda_\alpha |\mathbf{e}_\alpha^\dagger \mathbf{t}|^2 = Z_{11}^{(1)}|e_{4,3}|^2 + Z_{11}^{(2)\prime}|e_{4,1}|^2 + \mathbf{t}'^\dagger \hat{\mathbb{Z}}^{(\mathrm{TE})}\mathbf{t}' > 0, \tag{25}$$

where $\mathbf{t}' = (-e_{4,3}^* \ e_{4,1}^*)^t$. Hence, apart from $\lambda_4$, at least one of the eigenvalues $\lambda_\alpha$, $\alpha = 1, 2, 3$, has to be positive at $\omega < \Omega_L$. If it happens that $e_{4,1} = e_{4,3} = 0$, then the first and third components of at least two eigenvectors of three $\mathbf{e}_\alpha$, $\alpha = 1, 2, 3$, do not vanish, since otherwise the four $\mathbf{e}_\alpha$'s could not be linearly independent. Assuming $e_{3,1} \neq 0$ and $e_{3,3} \neq 0$, we contract $\hat{\mathbf{Z}}_{\mathrm{st}}$ with the vector $\mathbf{t} = (-e_{3,3}^* \ 0 \ e_{3,1}^* \ 0)^t$ and obtain that $\sum_{\alpha=1}^{2} \lambda_\alpha |\mathbf{e}_\alpha^\dagger \mathbf{t}|^2 > 0$, wherefrom it follows that not only

$\lambda_4 > 0$ but also either $\lambda_1 > 0$, or $\lambda_2 > 0$, or $\lambda_1 > 0$ and $\lambda_2 > 0$. Summing up, in view of (22),

given $k$, at most two nonradiative SPPs exist if both dielectrics are magneto-optically inactive and nonbianisotropic.

This statement holds true independently of the anisotropy and frequency dispersion of the dielectric permittivity and magnetic permeability. It is also independent of a particular behavior of the dielectric permittivity of metal $\varepsilon_m(\omega)$ because, as it is pointed out in Appendix B, the required properties of $\hat{\mathbf{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbf{Z}}^{(\mathrm{TM})}$ can be established without using the explicit expression of $\varepsilon_m(\omega)$.

### B. Bianisotropic structure

All the four eigenvalues of $\hat{\mathbf{Z}}_{\mathrm{st}}$ may vanish when both dielectrics are bianisotropic or/and magneto-optically active, so

given $k$, at most four nonradiative SPPs can emerge.

Let one of the dielectrics, e.g., medium 2, be magneto-optically inactive and nonbianisotropic. Due to (14), (17), and (18), the contraction of $\hat{\mathbf{Z}}_{\mathrm{st}}$ with the vector $\mathbf{q} = (0\ 0\ 1\ 0)^t$ yields

$$\mathbf{q}^t\hat{\mathbf{Z}}_{\mathrm{st}}\mathbf{q} = \sum_{\alpha=1}^{4}\lambda_\alpha|\mathbf{e}_\alpha^\dagger\mathbf{q}|^2 = Z_{11}^{(2)\prime} + Z_{11}^{(\mathrm{TE})} > 0, \tag{26}$$

because $Z_{11}^{(2)\prime} > 0$ and $Z_{11}^{(\mathrm{TE})} > 0$ [see the beginning of Sec. III A and also Eqs. (11) and (12)]. For (26) to hold true, it is necessary that at least one of $\lambda_\alpha$, $\alpha = 1, 2, 3, 4$, be positive in the interval $\omega_a < \omega < \Omega_L$. Hence,

given $k$, at most three nonradiative SPPs can emerge in a bianisotropic dielectric–metal film–anisotropic dielectric structure.

Note that the maximum number is the same if medium 2 is optically isotropic.

Additionally,

given $k$, at most three nonradiative SPPs exist if media 1 and 2 are the upper and lower halves of an infinite bianisotropic medium, respectively.

In this case $\hat{\mathbf{Z}}^{(2)\prime} = \hat{\mathbf{Z}}^{(1)\prime}$ and, by virtue of property (A3), $Z_{11}^{(1)} + Z_{11}^{(1)\prime} > 0$, where $Z_{11}^{(1)}$ and $Z_{11}^{(1)\prime}$ are diagonal elements of $\hat{\mathbf{Z}}^{(1)}$ and $\hat{\mathbf{Z}}^{(1)\prime}$, respectively. By contracting $\hat{\mathbf{Z}}_{\mathrm{st}}$ with the vector $\mathbf{t} = (1\ 0\ 1\ 0)^t$ we obtain the inequality

$$\mathbf{t}^t\hat{\mathbf{Z}}_{\mathrm{st}}\mathbf{t} = \sum_{\alpha=1}^{4}\lambda_\alpha|\mathbf{e}_\alpha^\dagger\mathbf{t}|^2 = Z_{11}^{(1)} + Z_{11}^{(1)\prime} + \mathbf{t}'^t\hat{\mathbf{Z}}^{(\mathrm{TE})}\mathbf{t}' > 0, \tag{27}$$

where $\mathbf{t}' = (1\ \ 1)^t$. Hence, not all $\lambda_\alpha$'s can turn out to be negative, so that at most three SPPs exist, which completes the proof.

### C. Absorption

It has already been mentioned that in the presence of absorption the impedances lose their properties on which our approach is based; first of all, they are not Hermitian matrices. It is worth noting that absorption changes the situation in general because in this case all partial solutions (3) of the Maxwell equations are inhomogeneous at any frequency rather than only below a certain frequency.

At the same time, some conclusions about the role of weak absorption can be made (e.g., $\varepsilon''_m/\varepsilon'_m = 0.067$ in Ag and $\varepsilon''_m/\varepsilon'_m = 0.077$ in Au at $\lambda = 1\ \mu$m [51]). In particular, weak absorption generally cannot change the number of SPPs established under assumption of no absorption. Indeed, properties (A5), (A6), and (B5) allow the demonstration via the perturbation theory that if the dispersion equation has a root in the absence of absorption then in the presence of weak absorption, e.g., due to absorption in metal, the dispersion equation has a complex root of which the imaginary part is of the sign corresponding to attenuation. Let an eigenvalue $\lambda_\alpha$ of $\hat{\mathbf{Z}}_{\mathrm{st}}$ vanish at a frequency $\omega_{\mathrm{SPP}}$ when $\varepsilon''_m = 0$. Once $\varepsilon''_m \neq 0$, the equation $\lambda_\alpha = 0$ necessarily has a root $\omega \approx \omega_{\mathrm{SPP}} + i\omega'$ of which the imaginary part $\omega'$ is of sign corresponding to attenuation:

$$\omega' = -\varepsilon''_m\frac{\mathbf{e}_\alpha^\dagger\frac{\partial\hat{\mathbf{Z}}_f}{\partial\varepsilon_m}\mathbf{e}_\alpha}{\mathbf{e}_\alpha^\dagger\frac{\partial\hat{\mathbf{Z}}_{\mathrm{st}}}{\partial\omega}\mathbf{e}_\alpha} < 0, \tag{28}$$

where $\mathbf{e}_\alpha$ is the eigenvector of $\hat{\mathbf{Z}}_{\mathrm{st}}$ associated with $\lambda_\alpha$ at $\omega = \omega_{\mathrm{SPP}}$ and $\varepsilon''_m = 0$. It is also seen that the linear correction to $\omega_{\mathrm{SPP}}$ is purely imaginary. Therefore the real correction will be of the second order, so the frequency shift due to absorption will be smaller than the SPP linewidth and the influence of the absorption on the existence of SPPs can be viewed as a second-order effect. In this connection we note that if we assume the dielectric permittivity of metal purely real below the frequency $\omega_a$ (see Sec. II B) and find an SPP at $\omega < \omega_a$ then this wave will still exist at any reasonable absorption. Correspondingly, our considerations could be extended to the interval $0 < \omega < \Omega_L$.

Weak absorption can give rise to a new attenuating SPP at $\omega < \Omega_L$ provided that one of the eigenvalues of $\hat{\mathbf{Z}}_{\mathrm{st}}$, when calculated without account for absorption, vanishes, or nearly vanishes, at $\Omega_L$ since then a small perturbation can make this eigenvalue vanish at $\omega < \Omega_L$. However, from (B5) it follows that the first derivatives of the eigenvalues $\lambda_\alpha$ of $\hat{\mathbf{Z}}_{\mathrm{st}}$ with respect to the dielectric permittivity $\varepsilon_m$ of the metal are negative:

$$\frac{\partial\lambda_\alpha}{\partial\varepsilon_m} = \mathbf{e}_\alpha^\dagger\frac{\partial\hat{\mathbf{Z}}_f}{\partial\varepsilon_m}\mathbf{e}_\alpha < 0, \quad \alpha = 1, 2, 3, 4, \tag{29}$$

so that if, e.g., in anisotropic structures $\lambda_2 < 0$ and $\lambda_3 < 0$ at $\omega < \Omega_L$ whereas $\lambda_4$ is slightly greater than zero in the vicinity of $\Omega_L$, then it would be possible to get a third SPP in a structure without losses by slightly increasing real $\varepsilon_m$. Thus a positive eigenvalue of $\hat{\mathbf{Z}}_{\mathrm{st}}$ can be small enough at $\omega = \Omega_L$ only when not more than one of the other eigenvalues is negative and hence weak absorption cannot increase the maximum number of SPPs.

## IV. CONCLUDING REMARKS

We have proved that at most two nonradiative SPPs exist in a metal film enclosed between two semi-infinite optically anisotropic but magneto-optically inactive and non-bianisotropic dielectrics. Thus the maximum number of nonradiative SPPs is the same as in the case of optically isotropic dielectrics. This maximum is universal in the sense that it is not affected either by particular material properties of dielectrics and metal, such as the frequency dependence of the dielectric permittivity and its anisotropy in dielectrics, or by the film thickness.

Bianisotropy and/or magneto-optical activity increase the admissible maximum of nonradiative SPPs in films. Three SPPs, rather than two, can emerge in a metal film placed between bianisotropic, or magneto-optically active, dielectric and nonbianisotropic magneto-optically inactive dielectrics independently of whether the latter is optically isotropic or anisotropic. Four SPPs can exist if both dielectrics are bianisotropic and different, but if the dielectrics are the upper and lower parts of a bisected bianisotropic and/or magneto-optically active dielectric then at most three SPPs emerge. For the number of SPPs in films to be greater than 2, bianisotropy and magneto-optical activity must be sufficiently strong to overcome the limit set by the real part of the dielectric permittivity and magnetic permeability. Hence, most likely, not more than two SPPs can be observed in practice.

However, more than two SPPs can be found by computations, e.g., in "model" strongly magneto-optically active materials (see Appendix D). These computations confirm the fact that the maximum numbers of SPPs established in Sec. III B are exact, that is, four or three SPPs can actually exist in relevant structures. In Appendix C it is mentioned that at most two SPPs can propagate along the bianisotropic dielectric–half-infinite metal interface whereas at most one SPP exists at the interface between a metal and a nonbianisotropic and magneto-optically inactive dielectric. Example 4 in Appendix D reveals that a pair of SPPs is also an exact maximum.

Like in the case of absorbing materials, the impedances of dielectrics lose their properties in the range $\omega > \omega_L$ because of bulk waves. In particular, they turn out to be non-Hermitian. Therefore the existence of radiative (leaky) SPPs also cannot be analyzed by the method used. We note that the same restriction on the applicability of impedances occurs when considering leaky and so-called supersonic surface acoustic waves in solids. At the same time, certain general relations between the characteristics of acoustic fields continue to hold true and they allow one to draw a number of general conclusions concerning such waves [52–54]. Similar relations hold true for electromagnetic fields but the analysis of the specific features of SPPs emerging in the frequency range of bulk waves is out of the scope of the present paper.

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## APPENDIX A

The properties of the impedances $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ of homogeneous dielectrics defined by Eq. (6) coincide with the properties of their counterparts introduced in [32–36] to analyze the existence of surface electromagnetic and acoustic waves in periodic superlattices within the so-called lowest forbidden band. The latter is an analog of the range $\omega < \omega_L$.

$$\hat{\mathbf{Z}} = \hat{\mathbf{Z}}^{\dagger}, \quad \hat{\mathbf{Z}}' = \hat{\mathbf{Z}}'^{\dagger} \text{ at } \omega < \omega_L, \tag{A1}$$

$$\hat{\mathbf{Z}} \text{ and } \hat{\mathbf{Z}}' \text{ are positive definite matrices at } \omega \to 0, \tag{A2}$$

$$\hat{\mathbf{G}} = \hat{\mathbf{Z}} + \hat{\mathbf{Z}}' \text{ is a positive definite matrix at } \omega < \omega_L, \tag{A3}$$

$$\begin{aligned}&\text{all eigenvalues of } \mathbf{Z} \text{ and } \hat{\mathbf{Z}}' \text{ are finite at } \omega < \omega_L\\ &\text{but tend to } +\infty \text{ as } \omega \to 0,\end{aligned} \tag{A4}$$

$$\frac{\partial \hat{\mathbf{Z}}}{\partial \omega} \text{ and } \frac{\partial \hat{\mathbf{Z}}'}{\partial \omega} \text{ are negative definite matrices at } \omega < \omega_L. \tag{A5}$$

The latter properties holds true irrespective of whether the material constants are frequency independent or dependent.

If the replacement of a material constant $c$ by $c + ic'$, where $c' \ll c$ is real, results in absorption, then

$$\begin{aligned}&\operatorname{sgn}(c')\frac{\partial \hat{\mathbf{Z}}}{\partial c}\bigg|_{c'=0} \text{ and } \operatorname{sgn}(c')\frac{\partial \hat{\mathbf{Z}}'}{\partial c}\bigg|_{c'=0} \text{ are}\\ &\text{negative definite matrices at } \omega < \omega_L.\end{aligned} \tag{A6}$$

## APPENDIX B

By Eqs. (11) and (12), in the interval $\omega_a < \omega < \omega_p$

$$\hat{\mathbb{Z}}^{(\mathrm{TE})} \text{ and } \hat{\mathbb{Z}}^{(\mathrm{TM})} \text{ are real symmetric matrices,} \tag{B1}$$

$$\hat{\mathbb{Z}}^{(\mathrm{TE})} \text{ is a positive definite matrix,} \tag{B2}$$

$$\hat{\mathbb{Z}}^{(\mathrm{TM})} \text{ is a negative definite matrix,} \tag{B3}$$

$$\frac{\partial \hat{\mathbb{Z}}^{(\mathrm{TE})}}{\partial \omega} \text{ and } \frac{\partial \hat{\mathbb{Z}}^{(\mathrm{TM})}}{\partial \omega} \text{ are negative definite matrices.} \tag{B4}$$

Property (B4) can be verified by taking advantage of inequality (13) from [33], i.e., to proceed similarly to the derivation of (A5). In the present case this inequality is integrated over the film thickness rather than from zero to infinity. If the Drude model is used, then (B4) can also be established just by evaluating the derivatives of $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$.

The absorption in metals is included into considerations by changing real $\varepsilon_m$ to complex $\varepsilon_m = \varepsilon_m' + i\varepsilon_m''$ [owing to the choice of the phase factor in (3), in our case $\varepsilon_m'' > 0$]. Proceeding by analogy with the derivation of (A6) in [33] and using again the integration over the film thickness we find that

$$\begin{aligned}&\frac{\partial \hat{\mathbb{Z}}^{(\mathrm{TE})}}{\partial \varepsilon_m}\bigg|_{\varepsilon_m''=0} \text{ and } \frac{\partial \hat{\mathbb{Z}}^{(\mathrm{TM})}}{\partial \varepsilon_m}\bigg|_{\varepsilon_m''=0} \text{ are}\\ &\text{negative definite matrices at } \omega_a < \omega < \omega_p.\end{aligned} \tag{B5}$$

Within the frame of the Drude model these inequalities can also be obtained just by differentiating $\hat{\mathbb{Z}}^{(\mathrm{TE})}$ and $\hat{\mathbb{Z}}^{(\mathrm{TM})}$ (11) with respect to $\varepsilon_m$.

## APPENDIX C

Equation (19) can be transformed to $\det(\hat{\mathbf{Z}}_{\mathrm{st}22}\hat{\mathbf{Z}}_{\mathrm{st}12}^{-1}\hat{\mathbf{Z}}_{\mathrm{st}11} - \hat{\mathbf{Z}}_{\mathrm{st}12}) = 0$, where $\hat{\mathbf{Z}}_{\mathrm{st}11}$ and $\hat{\mathbf{Z}}_{\mathrm{st}22}$ are the upper and lower diagonal $2\times 2$ blocks of $\hat{\mathbf{Z}}_{\mathrm{st}}$ and $\hat{\mathbf{Z}}_{\mathrm{st}12}$ is the off-diagonal $2\times 2$ block of $\hat{\mathbf{Z}}_{\mathrm{st}}$. It can be checked that in the limit $h \to 0$ this

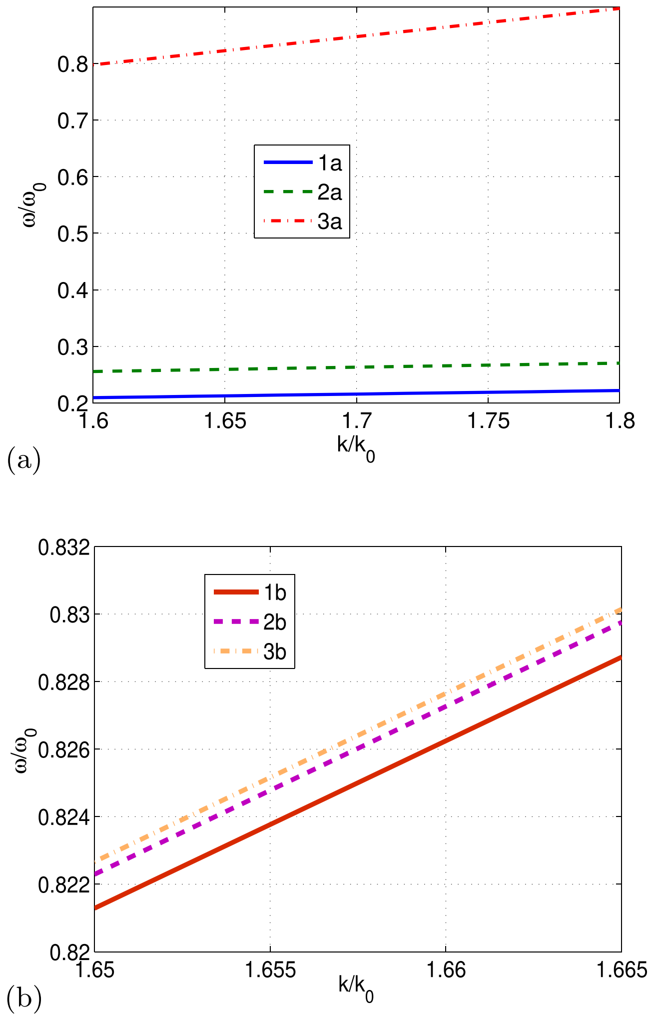


FIG. 1. Four SPPs in metallic film embedded between two different magneto-optically active dielectrics. Line 1a and line 2a are low-frequency branches 1 and 2, respectively. Line 3a represents high-frequency branches 3 and 4 as well as the dependence $\Omega_L(k)$ (these lines are hardly distinguishable). Lines 1b, 2b, and 3b are branches 3 and 4 and dependence $\Omega_L(k)$, respectively.

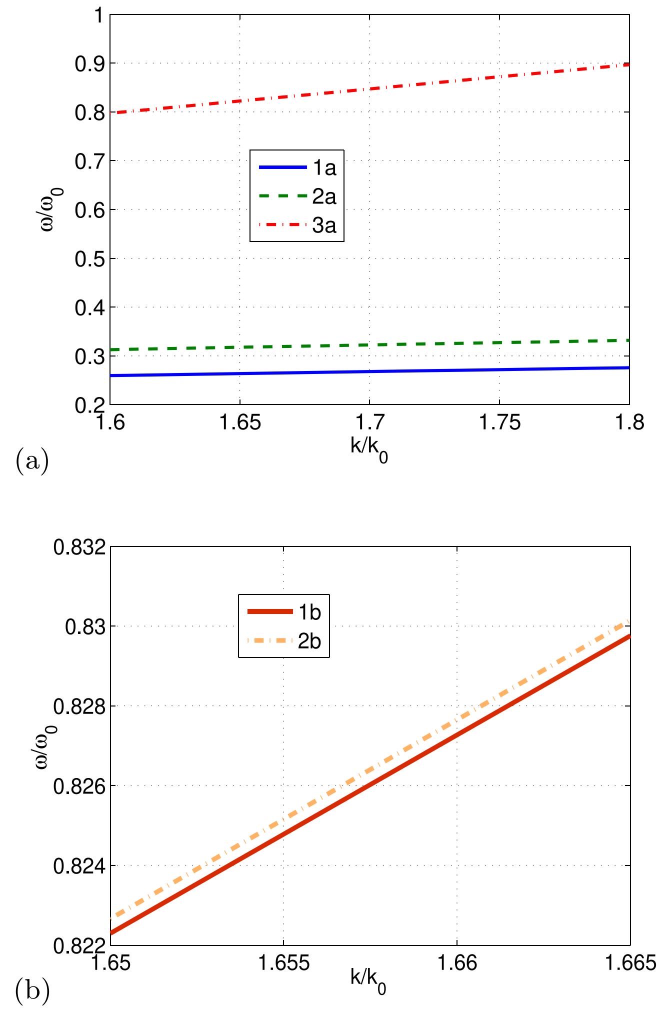


FIG. 2. Three SPPs in a metallic film immersed into a magneto-optically active dielectric. Line 1a and line 2a are low-frequency branches 1 and 2, respectively. Line 3a represents high-frequency branch 3 and the dependence $\Omega_L(k)$ (these lines are hardly distinguishable). Lines 1b and 2b are branch 3 and dependence $\Omega_L(k)$, respectively.

equation goes to the dispersion equation $\det(\hat{\mathbf{Z}}^{(1)} + \hat{\mathbf{Z}}^{(2)\prime}) = 0$ for surface electromagnetic waves at the medium 1–medium 2 interface. According to [33], at most two surface waves can exist on the interface between bianisotropic media. At most one wave can emerge if the media are anisotropic [31,33].

From Eqs. (12) and (14) it is seen that in the opposite limit $h \to \infty$ Eq. (19) splits into two independent dispersion equations for SPPs at the medium 1–infinite metal and medium 2–infinite metal interfaces. At most one SPP exists on the anisotropic media–metal interface. Two SPPs can exist if the dielectric is bianisotropic or/and magneto-optically active (an example is given in Appendix D). These conclusions follow from (A1)–(A5) and (B1)–(B4). An interesting fact is worth noting. Namely, let us bisect a bianisotropic or/and magneto-optically active dielectric and form two structures: Structure 1, where the upper part of the dielectric is on top of a metal, and structure 2, where the same metal is on top of the lower part of the dielectric. Using (A1)–(A5) and (B1)–(B4) it can be proved that if two SPPs exist in structure 1 then at most one SPP can exist in structure 2, and vice versa.

## APPENDIX D

In order to demonstrate the existence of the maximum number of SPPs established in Sec. III B we can use model materials of which the material constants secure the validity of properties (A1)–(A5) and (B1)–(B4) of the impedances. Namely, (A1)–(A5) hold true provided that the matrix $\partial(\omega\hat{\mathbf{\Gamma}})/\partial\omega$ is positive definite, where

$$\hat{\mathbf{\Gamma}} = \begin{pmatrix} \hat{\boldsymbol{\varepsilon}} & \hat{\boldsymbol{\kappa}} \\ \hat{\boldsymbol{\kappa}}^{\dagger} & \hat{\boldsymbol{\mu}} \end{pmatrix} \tag{D1}$$

is a $6 \times 6$ Hermitian matrix which involves the material constants entering Eqs. (1) and (2) [33]. Properties (B1)–(B4) hold true if a metal has isotropic real dielectric permittivity $\varepsilon_m < 0$ and magnetic permeability $\mu_m > 0$ satisfying the inequalities $\partial(\omega\varepsilon_m)/\partial\omega > 0$ and $\partial(\omega\mu_m)/\partial\omega > 0$.

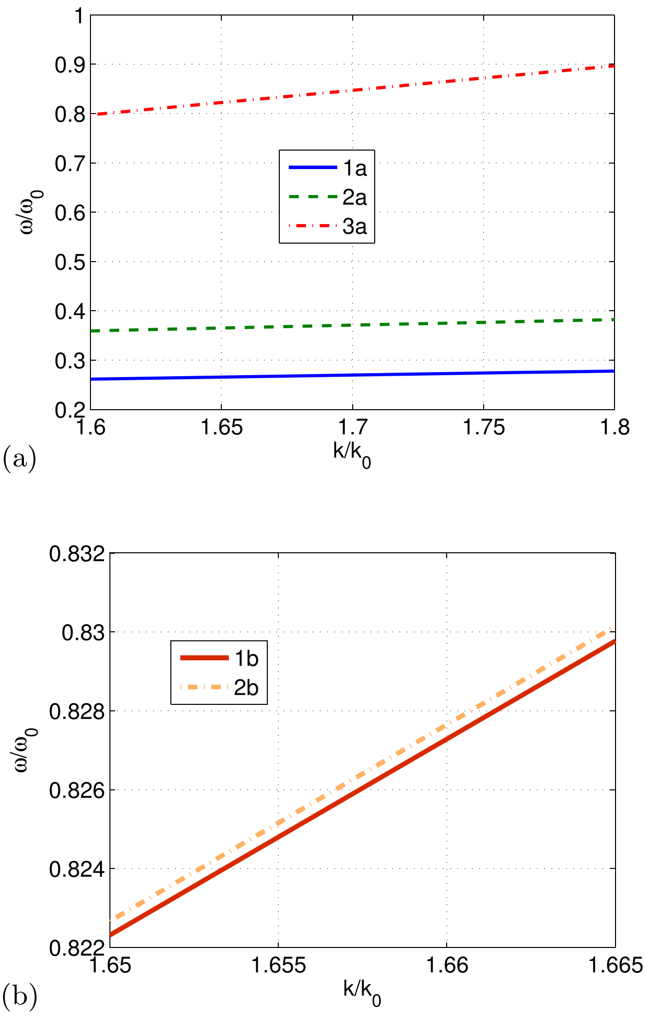


FIG. 3. Three SPPs in metallic film embedded between a magneto-optically active dielectric and a nonbianisotropic magneto-optically inactive one. Line 1a and line 2a are low-frequency branches 1 and 2, respectively. Line 3a represents high-frequency branch 3 and the dependence $\Omega_L(k)$ (these lines are hardly distinguishable). Lines 1b and 2b are branch 3 and dependence $\Omega_L(k)$, respectively.

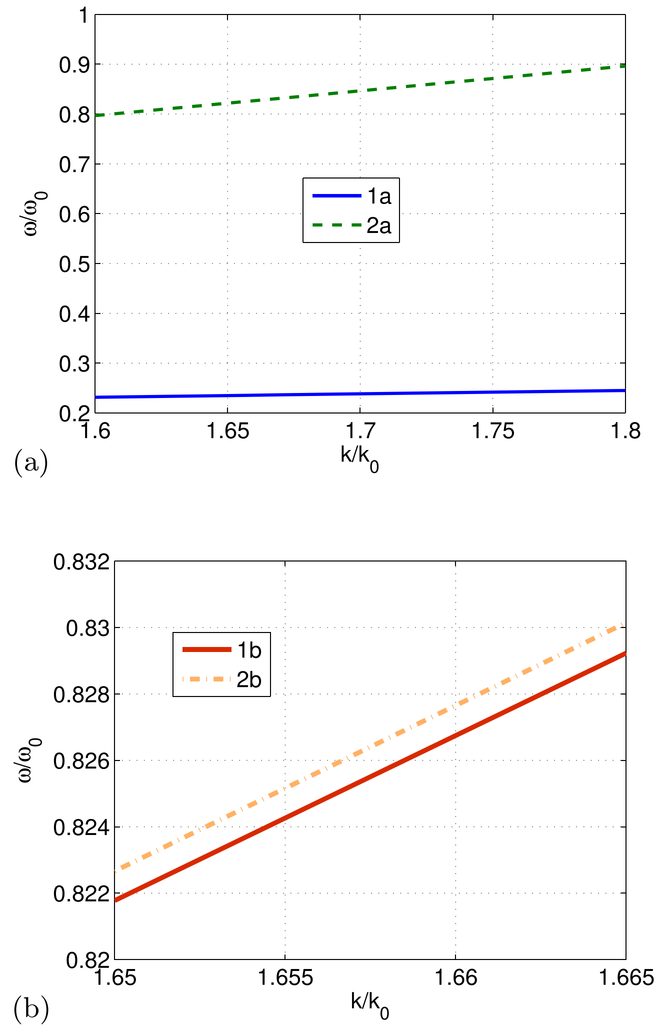


FIG. 4. Two SPPs in a magneto-optically active dielectric–half-infinite metal structure. Line 1a and line 2a are SPP branches 1 and 2, respectively. The latter practically merges with line $\Omega_L(k)$. Lines 1b and 2b are branch 2 and dependence $\Omega_L(k)$, respectively.

The number of SPPs can be greater than the maximum determined by the real parts of the dielectric permittivity and magnetic permeability of dielectrics (two SPPs in a film and one SPP in a half-infinite metal) if bianisotropy and/or magneto-optical activity are strong. In all cases given below the number of SPPs reaches a maximum thanks to strong magneto-optical activity. SPPs propagate in the positive direction of the axis $X$ of a coordinate system $XYZ$ which always remains intact. Dielectric permittivities are specified with respect to this coordinate system. Frequencies are normalized to the frequency $\omega_0 = 1.2\pi \times 10^{15}$ Hz, which corresponds to the wavelength 0.5 $\mu$m in the vacuum. Wave numbers are normalized to $k_0 = \omega_0/c$.

*Example 1.* Consider two magneto-optically active media 1 and 2 with alike dielectric permittivity:

$$\hat{\boldsymbol{\varepsilon}} = \begin{pmatrix} 5 & i4.2 & 0 \\ -i4.2 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}. \quad \text{(D2)}$$

Their magnetic permeabilities are isotropic and equal to unity. Bianisotropy is ignored, i.e., $\hat{\boldsymbol{\kappa}} = 0$. Let us rotate media 1 and 2 through the angle 20° anticlockwise and clockwise, respectively, around the axis $X$, making thereby their dielectric permittivities different in the $XYZ$ frame. Afterwards we bisect them along the plane perpendicular to the axis $Z$ and put a metallic film between the upper half of medium 1 and the lower part of medium 2. The thickness of the film is 30 nm, its dielectric permittivity is given by the Drude formula $\varepsilon_m = 1 - \omega_p^2/\omega^2$, where $\omega_p = 1.15\omega_0$, and its relative magnetic permeability equals 25. Figures 1(a) and 1(b) show four SPP branches in this structure. It is seen that four SPPs exist at a given $k$. According to Sec. III B, this is the maximum of SPPs in a film embedded between different magneto-optically active media. High-frequency branches 3 and 4 practically coincide with line $\Omega_L(k)$ of the limiting frequency of bulk electromagnetic waves.

*Example 2.* A medium having the dielectric permittivity (D2) and magnetic permeability equal to unity is rotated through the angle 20° anticlockwise around the axis $X$ and bisected along the plane perpendicular to the axis $Z$. A metal film is placed between the upper and lower parts. This film differs from the film of example 1 only by the plasmonic frequency $\omega_p = 1.5\omega_0$. Three SPP branches emerging in this structure are shown in Figs. 2(a) and 2(b). One of three branches is again very close to line $\Omega_L(k)$. Thus three SPPs exist at a given $k$. As it has been shown in Sec. III B, this is the maximum of SPPs in a film embedded between the two halves of a homogeneous magneto-optically active medium.

*Example 3.* Let us replace the lower part of the structure in example 2 by a half-infinite magneto-optically inactive medium with dielectric permittivity

$$\hat{\varepsilon} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \tag{D3}$$

and magnetic permeability equal to unity. The boundary of this medium is perpendicular to the axis $Z$. The metal film is the same as in example 2. Thus the structure involves magneto-optically active and inactive dielectrics and a metal film between them. We find three branches of SPPs (Fig. 3), so that, given $k$, three SPPs emerge, which is the allowed maximum. Notice that branches 1 (lines 1) in Figs. 2(a) and 3(a) hardly differ. Branches 3 (lines 1b) in Figs. 2(b) and 3(b) also differ only slightly.

*Example 4.* Assume that the upper part of the structures used in examples 1–3 is on top of a half-infinite metal with dielectric permittivity equal to that of the film in example 1. It has been pointed out in Appendix C that, given $k$, at most two SPPs can exist at the interface between a half-infinite magneto-optically active dielectric and a half-infinite metal. Figures 4(a) and 4(b) show two SPP branches. Again the branch of high-frequency SPPs and line $\Omega_L(k)$ nearly merge.


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