Unitary *p*-wave Fermi gas in one dimension

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We elucidate universal many-body properties of a one-dimensional, two-component ultracold Fermi gas near the *p*-wave Feshbach resonance. The low-energy scattering in this system can be characterized by two parameters, that is, *p*-wave scattering length and effective range. At the unitarity limit where the *p*-wave scattering length diverges and the effective range is reduced to zero without conflicting with the causality bound, the system obeys universal thermodynamics as observed in a unitary Fermi gas with contact *s*-wave interaction in three dimensions. It is in contrast to a Fermi gas with the *p*-wave resonance in three dimensions in which the effective range is inevitably finite. We present the universal equation of state in this unitary *p*-wave Fermi gas within the many-body T-matrix approach as well as the virial expansion method. Moreover, we examine the single-particle spectral function in the high-density regime where the virial expansion is no longer valid. On the basis of the Hartree-like self-energy shift at the divergent scattering length, we conjecture that the equivalence of the Bertsch parameter across spatial dimensions holds even for a one-dimensional unitary *p*-wave Fermi gas.

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I. INTRODUCTION

The concept of universality often facilitates the approach to normally complicated many-body problems. While detailed structure of the interaction potential between constituent particles generally plays a crucial role in describing the properties of a specific many-body system, the possible presence of an infinitely large length scale such as the correlation length near a critical point and the scattering length near the Feshbach resonance could be a key to understanding the universality in various systems. For example, the Bardeen-Cooper-Schrieffer (BCS) to Bose-Einstein condensate (BEC) crossover [1–4], which is experimentally realized in ultracold atoms [5–7] (for recent review, see Refs. [8–11]), not only gives a unified way of understanding Fermi and Bose superfluidity in a specific system, but also an interdisciplinary viewpoint on how to deal with different systems with different energy scales such as strongly correlated superconductors, superfluid helium, nuclear matter, and color superconductors.

Moreover, a unitary Fermi gas, which is located in the middle of the BCS-BEC crossover where the *s*-wave scattering length diverges and hence the grand-canonical thermodynamic potential includes only two energy scales, namely, the chemical potential μ and the temperature T, has been extensively investigated by theoretical [12] and experimental [13–15] approaches. In particular, the ground-state thermodynamic properties of such a gas are characterized by a single parameter called the Bertsch parameter [16,17]. These universal ground-state properties have attracted attention from the viewpoint of the similarity between an ultracold Fermi gas and dilute neutron matter [18–20] and have been quantitatively revealed in cold atom experiments [21–23]. At high temperatures, one can pin down the virial expansion coefficients [24], which could be useful for describing matter in stellar collapse [25]. In this way, there is no doubt about the importance of further investigations on these universal states of matter. Moreover, such a unitary Fermi gas is also predicted to occur in a one-dimensional system of four-component fermions with a four-body attraction [26]. Interestingly, it was reported from lattice simulations [27] that the values of the Bertsch parameter in these unitary gases with different dimensions are close to each other. In this context, other possibilities of realizing unitary gases with different kinds of interactions such as the *p*-wave resonant one are worth exploring.

While strongly interacting *s*-wave Fermi superfluids are successfully realized, the experimental realization of a superfluid Fermi gas near a *p*-wave Feshbach resonance [28–31] is still challenging. In fact, *p*-wave superfluid Fermi gases, which have been anticipated over the past few decades [32,33], suffer several difficulties such as strong three-body losses [34–38]. Fortunately, progress in experimental techniques has enabled the observation of dipolar splittings [39,40], the formation of a *p*-wave molecule [41–44], the optical control [45], characterization of the two-body relaxation [46], and the measurement of the *p*-wave Tan's contact [47]. Simultaneously, universal aspects of *p*-wave Fermi gases have attracted theoretical attention [48–54]. Also, it remains to be examined how atomic losses are suppressed in the presence of strong *p*-wave interaction.

To reach the unitarity limit, the scale of the effective range is important. In a three-dimensional system, the effective range of the *p*-wave interaction is inevitably nonzero due to

Wigner's causality bound [55]. Therefore, the unitarity limit can never be realized in a three-dimensional gas with a p-wave Feshbach resonance. In a one-dimensional system, the situation is totally different. Recently, the stabilization of p-wave Fermi gases confined in one spatial dimension [56–58] and in optical lattices [59] has been pointed out theoretically, while the three-body loss associated with a p-wave Feshbach resonance under low-dimensional confinement has experimentally been investigated [60,61]. The renormalization scheme of a contact-type p-wave interaction has also been presented in Refs. [62–67]. Note that the low-energy scattering properties are generally classified by the combination $2L + d$ of the angular momentum L and the dimension d [68,69]. In this context, a one-dimensional system with a p-wave interaction ($L = 1$ and $d = 1$) belongs to the same class as a three-dimensional system with an s-wave interaction ($L = 0$ and $d = 3$). Thus one can expect the realization of a unitary behavior in such a one-dimensional system as the unitarity limit has already been achieved in the three-dimensional counterpart. We note that this classification is valid only for the case of two-body interactions and therefore the unitarity limit may be realized in other dimensions with multibody interactions and different angular momentum of clusters as shown in Refs. [26,27].

In this work, we elucidate how the unitarity limit occurs for $L = 1$ and $d = 1$ for two-body interactions and then examine the resulting universal many-body properties. Physically, such unitarity would be realized at zero temperature because the interaction energy is of the same order as the kinetic energy with respect to the density. In one dimension, a typical momentum scale is given by the Fermi momentum $k_{\rm F}$, which is in turn proportional to the density. Note that the size of a preformed pair is given by the interparticle spacing rather than the scattering length in a many-particle system of interest here. According to the Lippmann-Schwinger equation, the effective interaction is then larger than the bare interaction by a factor of $1/(k_{\rm F} r_{\rm eff})$ with a vanishingly small effective range $r_{\rm eff}$. Since the bare interaction per fermion scales as $k_{\rm F}^3$, the effective interaction per fermion scales as $k_{\rm F}^2$, which is of the same order in $k_{\rm F}$ as the kinetic energy.

This paper is organized as follows. In Sec. II, we present our formulation based on the many-body T-matrix approach. In Sec. III, we show the numerical results for the number density and single-particle spectral weight. Finally, we conclude this paper in Sec. IV. Hereafter, we use units in which $\hbar = k_{\rm B} = 1$ and the system volume is set to unity.

II. FORMULATION

We consider a one-dimensional two-component Fermi gas near a p-wave Feshbach resonance and examine its equilibrium properties at chemical potential μ and temperature T. The corresponding two-channel Hamiltonian reads

$$H = \sum_{k,\sigma} \xi_k c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_q \xi_q^{\rm b} b_q^\dagger b_q + g \sum_{p,q} (p b_q^\dagger c_{-p+q/2,\downarrow} c_{p+q/2,\uparrow} + {\rm H.c.}), \tag{1}$$

where $\xi_k = k^2/(2m) - \mu$ and $c_{k,\sigma}^{(\dagger)}$ are the kinetic energy minus the chemical potential and annihilation (creation) operator of a Fermi atom with mass m, momentum k, and pseudospin $\sigma = \uparrow, \downarrow$, respectively. For simplicity, we consider an equal-mass mixture. The second term on the right-hand side of Eq. (1) denotes the kinetic energy term of closed channel molecules with the energy level ν, where $\xi_q^{\rm b} = q^2/(4m) - 2\mu + \nu$ and $b_q^{(\dagger)}$ are the kinetic energy minus the chemical potential and the annihilation (creation) operator of a bosonic molecule with momentum q. The last term represents the p-wave (odd parity) Feshbach coupling with coupling constant g. The p-wave scattering length a and the effective range $r_{\rm eff}$ are related to ν and g via the Lippmann-Schwinger equation as [62,63]

$$\frac{m}{2a} = -\frac{\nu_{\rm R}}{g^2}, \quad r_{\rm eff} = -\frac{4}{m^2 g^2}, \tag{2}$$

where $\nu_{\rm R} = \nu - g^2 \frac{m\Lambda}{\pi}$ is the renormalized energy level of a closed channel molecule with the momentum cutoff Λ. An explicit form of the condition for the unitarity limit ($a = \pm\infty$) reads $\nu = g^2 \frac{m\Lambda}{\pi}$. While this condition is satisfied for any finite value of g in the large-Λ limit, a dimensionless quantity $m r_{\rm eff}^2 T$ still remains to be fixed. In what follows, we shall utilize the thermal momentum scale $k_T = \sqrt{2mT}$ associated with the temperature T as well as the thermal de Broglie length $\lambda_T = \sqrt{\frac{2\pi}{mT}} \equiv 2\sqrt{\pi} k_T^{-1}$ for convenience. Also, the dimensionless range parameter $R = |r_{\rm eff} k_T|$ will be used to characterize the magnitude of $r_{\rm eff}$. In this work, we will confine ourselves to the unitary system with $a^{-1} = 0$. For the numerical calculation, we take $\Lambda = 10^2 k_T$ which is sufficiently large to avoid the cutoff effect.

In this work we focus on the equally populated case. In the presence of large spin polarization, Fermi polarons are expected to occur as has theoretically been investigated [70]. In the perfectly polarized case, an s-wave contact interaction does not work due to the Pauli principle, but a three-body interaction is inevitable [65]. This is in contrast to the present case in which an s-wave interaction is normally non-negligible, but the three-body correlation is expected to be weak. For a possible method for eliminating the s-wave interaction, see Appendix in Ref. [71]. The intracomponent p-wave interaction is not considered in this work. Such a situation is relevant for the p-wave resonance between different two hyperfine states [40,62].

We note that the present two-channel model, which involves the contact (q-independent) coupling g, reduces to the single-channel Hamiltonian with the effective p-wave interaction $H_{\rm int.} = \sum_{p,p',q} pU p' c_{p+q/2,\uparrow}^\dagger c_{-p+q/2,\downarrow}^\dagger c_{-p'+q/2,\downarrow} c_{p'+q/2,\uparrow}$, where $U = -\frac{g^2}{\nu}$ approaches zero in the large-Λ limit. This is in contrast to the s-wave case where the coupling constant is finite and therefore the Hartree shift gives a nonzero contribution [72]. Indeed, the low-energy limit of the two-body scattering T matrix $T_{\rm sc}(p, p'; \omega = 0)$ in the single-channel model

$$T_{\rm sc}(p, p'; 0) = \frac{pU p'}{1 + U \frac{m\Lambda}{\pi}} \tag{3}$$

is equivalent to the two-channel one

$$T_{\rm tc}(p,p';0)=\frac{pg^2p'}{-\nu+g^2\frac{m\Lambda}{\pi}}, \tag{4}$$

when one takes $U=-g^2/\nu$ and $g\to\infty$ [noting that $\nu=mg^2(\frac{\Lambda}{\pi}-\frac{1}{2a})\to\infty$]. Once a is determined by the two-body scattering problem in each model, the results do not depend on detailed structures of two-body interactions explicitly.

Many-body effects are incorporated into the self-energy $\Sigma_{\rm f}(p,i\omega_\ell)$ of a Fermi atom in the thermal Green's function $G(p,i\omega_\ell)=[\{G_0(p,i\omega_\ell)\}^{-1}-\Sigma_{\rm f}(p,i\omega_\ell)]^{-1}$, where $G_0(p,i\omega_\ell)=(i\omega_\ell-\xi_p)^{-1}$ is the bare propagator with the fermion Matsubara frequency $\omega_\ell=(2\ell+1)\pi T$ $(\ell\in\mathbb{Z})$. Within the many-body T-matrix approach, $\Sigma_{\rm f}(p,i\omega_\ell)$ is given by

$$\Sigma_{\rm f}(p,i\omega_\ell)=T\sum_{q,iv_n}\Gamma\Big(\frac{q}{2}-p,\frac{q}{2}-p;q,iv_n\Big)\times G_0(q-p,iv_n-i\omega_\ell), \tag{5}$$

where $v_n=2n\pi T$ is the boson Matsubara frequency $(n\in\mathbb{Z})$. The in-medium T matrix

$$\Gamma(k,k';q,iv_n)=g^2kk'D(q,iv_n) \tag{6}$$

is associated with the dressed molecular propagator $D(q,iv_n)=[iv_n-\xi_q^{\rm b}-\Sigma_{\rm b}(q,iv_n)]^{-1}$. The bosonic self-energy is given by $\Sigma_{\rm b}(q,iv_n)=g^2\Pi(q,iv_n)$, where

$$\Pi(q,iv_n)=\sum_p p^2\frac{1-f(\xi_{p+q/2})-f(\xi_{-p+q/2})}{iv_n-\xi_{p+q/2}-\xi_{-p+q/2}}, \tag{7}$$

with the Fermi-Dirac distribution function $f(x)=(e^{x/T}+1)^{-1}$, is the lowest-order particle-particle bubble.

III. RESULTS

In the two-channel model, the number density n reads $n=n_{\rm f}+n_{\rm b}$, where $n_{\rm f}=2T\sum_{p,i\omega_\ell}G(p,i\omega_\ell)$ and $n_{\rm b}=-2T\sum_{q,iv_n}D(q,iv_n)$ are the fermionic and bosonic contributions, respectively. Figure 1 shows the number density $n\lambda_T$ as a function of μ/T in the case of a negligibly small range parameter $R=0.01$. As can be seen from Fig. 1, $n_{\rm b}$ is negligibly small, which is natural because $n_{\rm b}$ reduces to zero in the limit of zero effective range $(R\to0)$. The unitary gas in this limit strictly obeys the universal thermodynamics in the sense that the grand-canonical thermodynamic potential $\Omega(\mu,T)$ has no other energy scales than μ and T [12]. In the low-density limit, $\Omega(\mu,T)$ can be obtained exactly by the virial expansion [24] as $\Omega=-2\frac{T}{\lambda_T}\sum_{j=1}b_jz^j$, where $z=e^{\mu/T}$ is the fugacity. Within the second-order virial expansion, the number density reads

$$n\lambda_T=2[b_1z+2b_2z^2+O(z^3)]. \tag{8}$$

While the first-order coefficient $b_1=1$ corresponds to the ideal classical gas contribution, the second one $b_2=b_2^{(0)}+\Delta b_2$ involves not only the noninteracting part $b_2^{(0)}=-\frac{1}{2\sqrt{2}}$ but also the interaction correction Δb_2. Δb_2 can be obtained from the low-density limit of the many-body T-matrix approach,

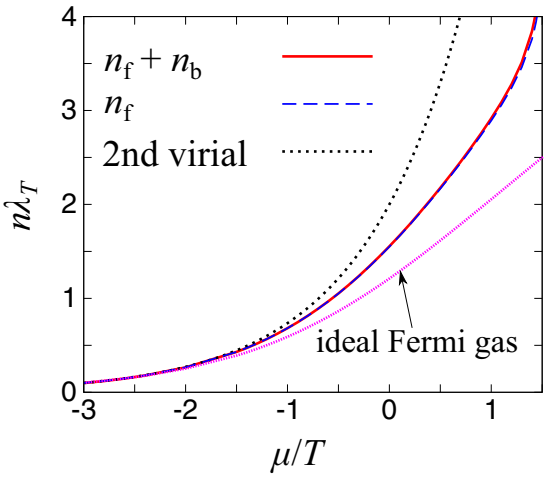

FIG. 1. Number density equation of state for a unitary p-wave Fermi gas in one dimension. The dotted curve ("2nd virial") shows the result from the second-order virial expansion. In this figure, we take $R=0.01$ which is sufficiently small to describe the universal regime. For reference, we also plot the behavior of an ideal Fermi gas.

where n is approximately given by

$$n\simeq n_0+2T\sum_{p,i\omega_\ell}[G_0(p,i\omega_\ell)]^2\Sigma_{\rm f}(p,i\omega_\ell). \tag{9}$$

The second term on the right-hand side of Eq. (11) obtained by truncating the full Green's function up to first order in $\Sigma_{\rm f}(p,i\omega_\ell)$ is equivalent to the Nozières-Schmitt-Rink (NSR) correction $\delta n_{\rm NSR}$ [11]. Using the relation $\delta n_{\rm NSR}=\frac{4}{\lambda_T}\Delta b_2z^2+O(z^3)$, we obtain $\Delta b_2=\frac{1}{2\sqrt{2}}$ and hence $b_2=0$ (see also the Appendix for the derivation of Δb_2). This indicates that the virial equation of state for a one-dimensional unitary p-wave Fermi gas up to second order in z happens to be the same as the ideal classical one $\Omega=-2\frac{Tz}{\lambda_T}$ even in the presence of strong correlations. It is in contrast to a unitary Fermi gas in three dimensions where $b_2=\frac{3}{4\sqrt{2}}$ [24]. We note that the number density of an ideal Fermi gas is smaller than the unitary gas result due to the lack of pairing fluctuations.

While the virial expansion is no longer valid for $z>1$ $(\mu>0)$, corresponding to a quantum degenerate regime of the unitary Fermi gas, the many-body T-matrix approach in this regime is still expected to give a semiquantitative description of strong-coupling effects such as the emergence of a pairing pseudogap. In Fig. 2 we present the single-particle density of states $\rho(\omega)=\sum_k A(k,\omega)$ in such a unitary p-wave Fermi gas within the many-body T-matrix approach, where the single-particle spectral weight $A(k,\omega)=-\frac{1}{\pi}{\rm Im}G(k,i\omega_\ell\to\omega+i\delta)$ is obtained by the analytic continuation of $G(k,i\omega_\ell)$ to the real frequency ω (δ is a positive infinitesimal). At zero chemical potential, $\rho(\omega)$ exhibits a single-particle peak near $\omega=0$, which is a specific behavior in one dimension, where we obtain $\rho_0(\omega)=\frac{m}{\pi\sqrt{2m\omega}}\theta(\omega)$ for the noninteracting case. It is in contrast to a square-root behavior $\rho_0(\omega)\propto\sqrt{\omega}$ of single-particle contributions in three dimensions. At larger μ, $\rho(\omega)$ shows the pseudogap opening around $\omega=0$ due to strong-pairing fluctuations. Although the emergence of a pseudogap

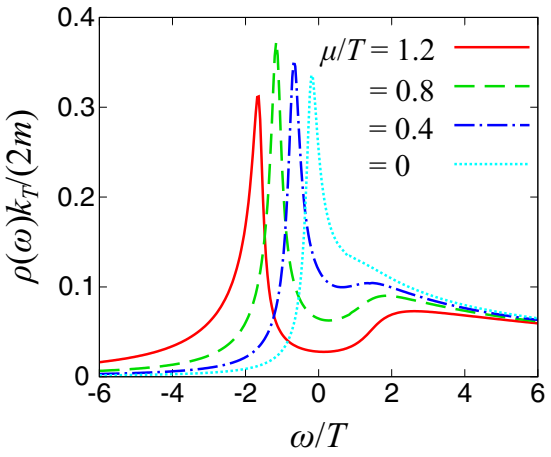

FIG. 2. Single-particle density of states $\rho(\omega)$ as a function of the single-particle energy ω in a unitary p-wave Fermi gas where $a^{-1} = 0$.

is still under debate in a three-dimensional unitary Fermi gas [73–75], the present system is expected to have a pseudogap pairing enhanced by low-dimensional fluctuations [72]. We note that a possible pseudogap in a three-dimensional Fermi gas with p-wave interaction has also been discussed in Ref. [76].

More detailed single-particle excitations can be found in $A(k, \omega)$, which is shown in Fig. 3. One can see that the single-particle branch in the low-density regime ($\mu/T = 0$) is separated into two branches due to strong-pairing fluctuations, leading to the pseudogap opening in $\rho(\omega)$ as shown in Fig. 2. A similar spectral structure can be found in the case of the s-wave interaction in one dimension [72]. While the peak position in $A(k, \omega)$ at $\mu/T = 0$ is close to the noninteracting dispersion $\omega = \xi_k$, at sufficiently large chemical potential

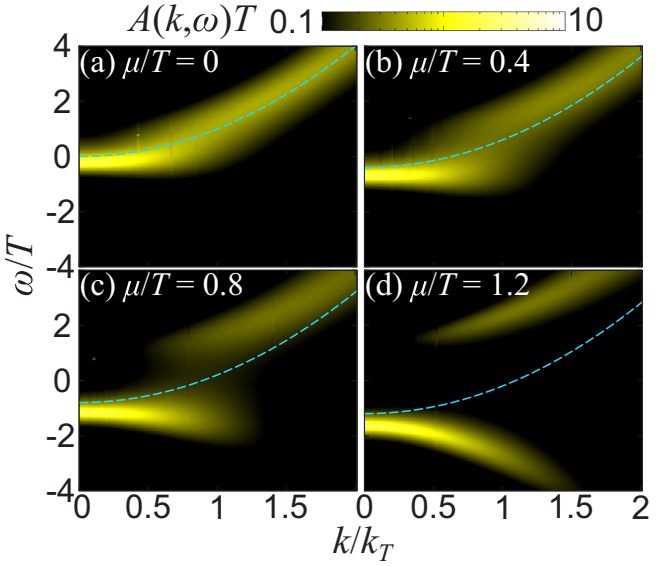

FIG. 3. Single-particle spectral weight $A(k, \omega)$ in a unitary p-wave Fermi gas in one dimension. The dashed curve represents the single-particle dispersion $\omega = k^2/(2m) - \mu$ in an ideal Fermi gas.

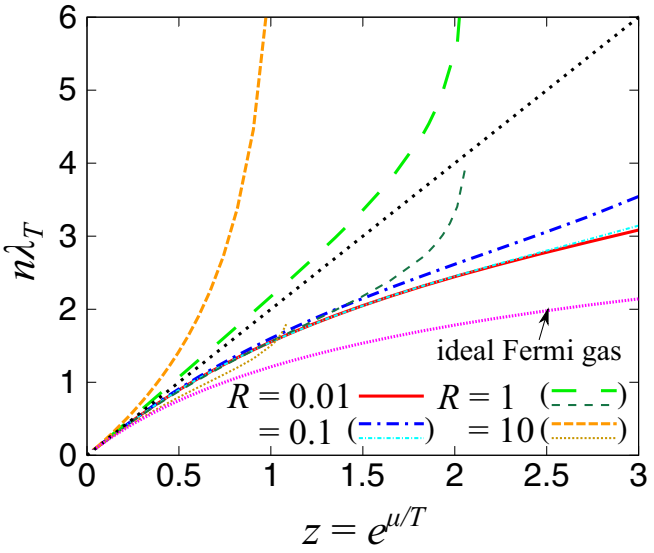

FIG. 4. Nonuniversal effect on the number density $n\lambda_T$, which can be seen by plotting $n = n_f + n_b$ (thick curves) and n_f (thin curves) at $R = 0.1$, 1, and 10. For the result at $R = 0.01$ we do not show n_f since n_b is negligibly small. The calculations are stopped at $\Delta_{TC} = 0$ [see Eq. (12) and the following text], indicating that the T-matrix approach breaks down. The diagonal dotted curve represents the second-order virial result at $R = 0$, which is the same as the behavior of an ideal classical gas, while the behavior of an ideal Fermi gas is also plotted for comparison.

where a pseudogap appears, deviation of the two branches from the noninteracting dispersion increases with μ/T.

We now turn to the nonuniversal effect of nonzero effective range on thermodynamic quantities. Figure 4 shows how $n\lambda_T$ behaves with increasing R as a function of z. For $R \gtrsim 0.1$, the fraction of molecules in the closed channel, n_b/n, is no longer negligible. Since $\Sigma_b(q, i\nu_n)$ is proportional to R^{-1}, one can qualitatively estimate $n_b \approx 2\sum_q e^{-\{\frac{q^2}{4m} - 2\mu + \nu - \Sigma_b(0,0)\}/T} \propto z^2 e^{-\frac{\alpha}{R}}$ (with the constant $\alpha > 0$), indicating that n_b becomes exponentially large when R increases. On the other hand, one can observe that n_f with finite R is close to the zero effective range result (corresponding to the solid curve with $R = 0.01$ in Fig. 4), indicating that the open-channel fraction is essential to extract the universal part of the equation of state for this system.

We note that in addition to the finite-range correction, other residual interactions such as an s-wave two-body interaction U_s cause nonuniversal effects in an experiment. In the presence of the repulsive U_s, one of possible ways to avoid such effects is to make use of an attractive interaction induced by a medium of weakly interacting third-component atoms [71]. Indeed, such a mediated interaction in the long-wavelength limit has been observed in recent experiments [77,78]. In the presence of this third-component atomic cloud interacting weakly with two pseudospins via $U_{\sigma,3}$, the total s-wave interaction is given by $V_s = U_s - U_{\uparrow,3}U_{\downarrow,3}n_3^2\kappa_3$ where n_3 and κ_3 are the number density and isothermal compressibility of the third-component atomic gas, respectively. One can realize $V_s = 0$ by adjusting $U_{\uparrow,3}$, $U_{\downarrow,3}$, and $n_3^2\kappa_3$. However, we note that this is based on the assumption that the p-wave interaction

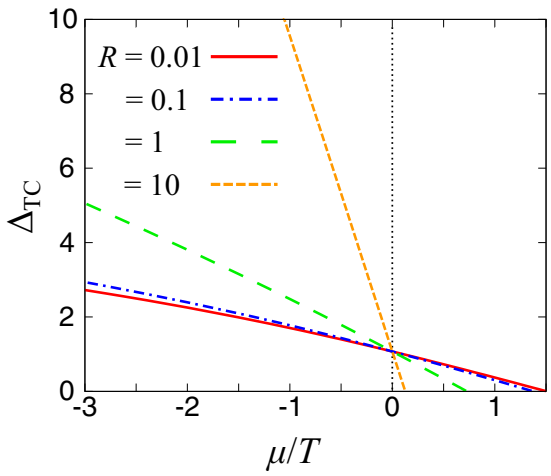

FIG. 5. Dimensionless indicator Δ_{TC} for the Thouless criterion at $R = 0.01$, 0.1, 1, and 10. In this figure, we take $a^{-1} = 0$.

is dominant and that the residual s-wave one is sufficiently weak. If the s-wave interaction is strong, the mediated interaction used for the cancellation may involve nonlocal and dynamical parts beyond the present static treatment under the local density approximation.

We mention the limitation of the present many-body T-matrix approach to this one-dimensional system. In fact, the numerical results for the number density in the high-density regime in Fig. 4 are difficult to obtain by this limitation. The many-body T-matrix approach breaks down when the dimensionless indicator of the Thouless criterion [79] defined by

$$\Delta_{TC} = \frac{kk'}{\Gamma(k, k, q = 0, i\nu_n = 0)} \frac{\pi}{mk_T} \tag{10}$$

becomes zero. Although in a three-dimensional system, an infrared divergence of the T matrix would indicate the occurrence of a superfluid phase transition, this divergence in one dimension is an artifact of theory because such a phase transition in one-dimensional systems is prohibited by the Mermin-Wagner-Hohenberg theorem [80,81]. Thus, $\Delta_{TC} = 0$ gives an artificial critical value of μ/T above which the present approach is no longer available. Figure 5 shows Δ_{TC} as a function of μ/T at $R = 0.01$, 0.1, 1, and 10. In the unitarity limit where R is negligible, corresponding to the result at $R = 0.01$ in Fig. 5, Δ_{TC} becomes zero around $\mu/T = 1.5$. Since the correction due to the effective range, which is negative as shown in Eq. (3), induces strong pairing in the two-channel model [82–84] and hence strong thermal fluctuations, however, this critical μ/T itself is doubtful. To safely avoid the above-mentioned artifact in the case of small R, therefore, one has to consider higher-order fluctuations beyond the present approach, which are left for future work.

In the large-R limit, on the other hand, $2\mu - \nu$ cannot be positive. In this limit, which is equivalent to $g \to 0$, the number density is exactly given by

$$n = 2\sum_p f(\xi_p) + 2\sum_q b\left(\xi_q^{\rm b}\right), \tag{11}$$

where $b(x) = (e^{x/T} - 1)^{-1}$ is the Bose-Einstein distribution function. Obviously, $\xi_q^{\rm b}$ cannot be negative for any q due to the Bose statistics in the second term of the right-hand side of Eq. (13), resulting in $\mu < \nu/2$ such that $\exp\left(\frac{q^2}{4mT} + \frac{\nu - 2\mu}{T}\right) - 1 > 0$ for an arbitrary q. This gives an exact upper bound for μ in the limit of $R \to \infty$, in contrast to the case of $R \to 0$ in which the condition $\Delta_{TC} = 0$ originates from the artifact of the present theoretical approach. Thus, the exact upper bound for μ at $|a| \to \infty$ is expected to be a function of R which requires $\mu < \nu/2 = 0$ at $R \to \infty$ and gives no constraints on μ at $R \to 0$. We note that the critical μ in the T-matrix approach identified by $\Delta_{TC} = 0$ is smaller than this exact upper bound at finite R.

In spite of such limitations of the present approach, the Bertsch parameter might be considered by utilizing the knowledge of a three-dimensional unitary Fermi gas. In three dimensions, one can estimate the ground-state chemical potential divided by the Fermi energy E_F, that is, the Bertsch parameter ξ_B^{3DS}, by replacing the s-wave scattering length a_s with k_F^{-1} in the Hartree shift [85,86]. Indeed, one can estimate $\xi_B^{3DS} \simeq \frac{4\pi k_F^{-1}}{mE_F} \frac{n}{2} \simeq 0.424$, which is fairly close to the experimental values $\sim$0.4 [14,15,21,22]. This implies that the characteristic length scale for the interaction is approximately given by the interparticle distance being proportional to k_F^{-1} at $|a| \to \infty$. In the present one-dimensional p-wave unitary gas, the Hartree self-energy $\Sigma_H(p, a)$ is given by

$$\Sigma_H(p, a) = \frac{2a}{m}\sum_q \left(\frac{q}{2} - p\right)^2 f(\xi_{q-p})$$
$$= \frac{ak_F^3}{6\pi m} + \frac{ak_F}{2\pi m}p^2 \quad (T \to 0). \tag{12}$$

By following the same line of argument of the three-dimensional case, we obtain the corresponding Bertsch parameter ξ_B^{1DP} as

$$\xi_B^{1DP} \simeq \frac{\Sigma_H(p = k_F, a = k_F^{-1})}{E_F}$$
$$\simeq 0.424, \tag{13}$$

where we have assumed that the self-energy shift at $p = k_F$ is relevant for our purpose. Surprisingly, it is completely equal to the value of ξ_B^{3DS} based on the same ansatz as mentioned above. This result suggests that the transdimensional equivalence of the Bertsch parameter [27] might apply even to the one-dimensional p-wave case. It is interesting to check the validity of this conjecture and examine deviation of the predicted ξ_B^{1DP} from the exact value, which will be addressed elsewhere. Also, it is useful to note that experiments for realizing the present unitary p-wave Fermi gas are expected to be more feasible than the case of a four-component unitary Fermi gas with a four-body attraction in one dimension [26,27].

IV. CONCLUSION

We have theoretically investigated universal many-body properties of a one-dimensional two-component unitary Fermi gas with a p-wave contact interaction. Thermodynamic functions in this unitary gas exhibit the universal behavior as in the case of a three-dimensional unitary Fermi gas with an

s-wave short-range interaction. We have obtained the universal equation of state in the limit of zero effective range within the many-body T-matrix approach and derived the exact result for the second-order virial expansion, which interestingly is equivalent to the ideal classical gas result. Even in the case of finite effective range, the number density of open-channel fermions is close to the universal result. Moreover, we have shown that strong-pairing fluctuations are visible as the pseudogap opening in a single-particle spectral weight. Finally, on the basis of the Hartree-like energy shift at unitarity, we have conjectured that the transdimensional equivalence of the Bertsch parameter holds even for this exotic unitary Fermi gas.

For future theoretical perspective, it is interesting to address the Bertsch parameter in this *p*-wave unitary Fermi gas in more sophisticated manner, which could be addressed by lattice simulations, the thermodynamic Bethe ansatz, variational approaches, as well as future experiments. It is worth investigating a bosonic counterpart via the Bose-Fermi mapping [87]. Indeed, a similar spin-1/2 system with intracomponent *p*-wave interactions has already been studied by using the Bose-Fermi mapping [88,89]. Moreover, a similar unitarity limit is expected to occur in a spin-polarized one-dimensional Fermi gas with *p*-wave contact interaction. We note, however, that in such a case the necessity of a three-body force for the renormalization has been pointed out in Ref. [65]. In this regard, it would be essential to investigate how three-body correlations occur simultaneously. To achieve a *p*-wave unitary Fermi gas experimentally, on the other hand, the suppression of other residual interactions would be important.

Apart from cold atomic physics, it is also interesting to consider the applications to other one-dimensional systems with *p*-wave interactions such as confined ^{3}He fluids [90,91], unconventional superconductors [92], and lattice systems [93].

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APPENDIX: SECOND-ORDER VIRIAL COEFFICIENT AT *p*-WAVE UNITARITY

In this Appendix, we present a detailed derivation of the second-order virial coefficient b_2 in the limit of zero effective range from the many-body T-matrix approach. First, the noninteracting contribution $b_2^{(0)}$ can be obtained by expanding n_0 with respect to z as

$$n_0 = 2\sum_p f(\xi_p) = 2\frac{z}{\lambda_T} - 4\frac{1}{2\sqrt{2}}\frac{z^2}{\lambda_T} + O(z^3). \tag{A1}$$

From Eq. (A1), one can find $b_2^{(0)} = -\frac{1}{2\sqrt{2}}$. The second term on the right-hand side of Eq. (11) can be rewritten as

$$\delta n = -T\sum_{q,i\nu_s}\gamma(q,i\nu_s)\frac{\partial}{\partial\mu}\gamma^{-1}(q,i\nu_s), \tag{A2}$$

where $\gamma(q,i\nu_s) = \Gamma(k,k'q,i\nu_s)/(kk')$. The summation with respect to $i\nu_s$ can be replaced with the integral along the contour C enclosing the imaginary energy axis as

$$\delta n = -\sum_q\oint_C\frac{d\zeta b(\zeta)}{2\pi i}\gamma(q,\zeta)\frac{\partial}{\partial\mu}\gamma^{-1}(q,\zeta) = -\sum_q\oint_{C'}\frac{d\zeta' b(\zeta'+\epsilon_q^{\rm b})}{2\pi i}\bar{\gamma}(q,\zeta')\frac{\partial}{\partial\mu}\bar{\gamma}^{-1}(q,\zeta'), \tag{A3}$$

where we have changed the variable $\zeta \to \zeta' + \epsilon_q^{\rm b}$ ($\epsilon_q^{\rm b} = \frac{q^2}{4m} - 2\mu$) and the contour $C \to C'$ which encloses the pole $\zeta' = i\nu_s - \epsilon_q^{\rm b}$. Also, we have used $\bar{\gamma}(q,\zeta') = \gamma(q,\zeta'+\epsilon_q^{\rm b})$. Since we are interested in the correction of $O(z^2)$ and the lowest order of $b(\zeta'+\epsilon_q^{\rm b}) \simeq z^2 e^{-\frac{\zeta'}{T}} e^{-\frac{q^2}{4mT}}$ is already $O(z^2)$, we can safely neglect the medium correction in $\bar{\gamma}(q,\zeta')$, leading to

$$\bar{\gamma}(q,\zeta') \simeq \left[\frac{m}{2a} + i\frac{m}{2}\sqrt{m\zeta'}\right]^{-1} \tag{A4}$$

and $\frac{\partial\bar{\gamma}^{-1}(q,\zeta')}{\partial\mu} = 2\frac{\partial\bar{\gamma}^{-1}(q,\zeta')}{\partial\zeta'} = \frac{m^2}{2\sqrt{m\zeta'}}i$. Using these relations at unitarity $1/a = 0$, we can obtain

$$\delta n = -z^2\sum_q e^{-\frac{q^2}{4mT}}\oint_{C'}\frac{d\zeta'}{2\pi i}\frac{e^{-\zeta'/T}}{\zeta'} + O(z^3) = 4\frac{1}{2\sqrt{2}}\frac{z^2}{\lambda_T} + O(z^3), \tag{A5}$$

which leads to $\Delta b_2 = \frac{1}{2\sqrt{2}}$. Finally, combining these results, one can obtain the vanishing second-order coefficient $b_2 = b_2^{(0)} + \Delta b_2 = 0$.

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