Enhancing the sensitivity of optomechanical mass sensors with a laser in a squeezed state

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The non-Hermitian system has been widely studied recently in various fields from quantum physics to condensed matter, in which the "exceptional point" (EP) as an essential feature of the systems can be used to design sensors, for example, a mass sensor to detect the mass of nano-objects. Inspired by the LIGO gravitational wave detector by using squeezed states of light, we here aim to enhance the sensitivity of the mass sensor by a nonlinear laser drive. The system consists of two optomechanical cavities that are mechanically coupled and driven nonlinearly by detuned lasers (squeezed lasers). Compared to the case of linear drive, our results are more sensitive to the mass, and the split width of the eigenvalues at EP can be further increased by using the squeezed lasers. The sensitivity enhancement factor and optical damping of resonators are also calculated and discussed, and a great improvement is found consequently. This work would provide a wider view for the new quantum sensors in order to be applied in the fields of nanoparticle detection, precision measurement, and quantum metrology.

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I. INTRODUCTION

Owing to the rapid progress in the micro/nanoengineering of mechanical resonators, mass sensors based on an optomechanical system come to play an important role in the field of ultrasensitive detection, for example, the mass detection of biomolecules or viruses and the detection of nanoscale substances or particles [1,2]. The advantage of a sensor based on an optomechanical system is its simplicity and high sensitivity as well as that the optical field serves both as an actuator and a probe for the precise monitoring of mechanical frequencies.

In addition to these sensors based on Hermitian optomechanical systems, mass sensors working with non-Hermitian systems have also been proposed, aimed at reaching new levels of sensitivity and to break through the limitation of frequency restriction [3,4]. Non-Hermitian systems have attracted great attention since 1998 [5], where the effective Hamiltonian of the system is no longer Hermitian and has complex eigenvalues and peculiar topological structures in the complex space—an exceptional point (EP) [6–8]. At this point, the eigenvalues and the eigenvectors of the system coalesce. Once the system obtains a slight perturbation, the eigenvalues and the eigenvectors at the exceptional point would split rapidly, the scales of the splitting width being proportional to the square root of the perturbation. Based on this feature, the sensitivity of the optomechanical mass sensor is greatly enhanced. This is different from a conventional mass sensor system, where the linewidth and the shift and split of the frequency is in proportion to the perturbation [9,10].

The squeezing of quantum observables is a central strategy to improve measurement sensitivities beyond classical limits and has thus become a key concept in quantum metrology [11], leading to major theoretical and experimental advancements in many fields [12–17]. Especially, in Ref. [18], the performance of one of the detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) beyond the quantum noise limit has been improved by injecting a squeezed state of light. With the injection of squeezed states, this LIGO detector demonstrated the best broadband sensitivity to gravitational waves achieved to date.

Inspired by the LIGO experiment and in order to further enhance the sensitivity, in this paper we consider driving the system by a laser in squeezed states. In quantum optics [19], squeezed states significantly affect the sensitivity of laser interferometers even under the effect of quantum noise. A squeezed laser can be generated by four-wave mixing in an optical fiber and by degenerate parametric down-conversion (PDC) in a second-order nonlinear crystal placed in an optical cavity.

The advantages of the present scheme are twofold. First, as aforementioned, our model, based on a non-Hermitian system with EP, has a unique topological structure compared to other optical mass sensors or single-cavity optomechanical systems. Owing to the absence of $\mathcal{PT}$ symmetry [20], the eigenvalues at the EPs rapidly split due to mass deposition. The splitting scales as the square root of the perturbation, leading to an enhancement of the sensitivity of the sensor. Second, a squeezed-laser drive can further enhance the scales of the split of the eigenvalues, leading to a more robust sensitivity enhancement factor related closely to the average photon numbers.

The remainder of this paper is organized as follows. In Sec. II, we introduce our model and obtain the equation of motion for the system. In Sec. III, we will discuss the sensitivity of the sensor, and finally in Sec. IV, we conclude and discuss our results.

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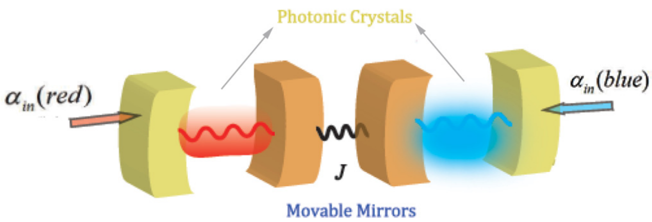

FIG. 1. Schematic illustration. Two optomechanical cavities, one with gain and the other of loss, are mechanically coupled. Both sides are driven by a red- (blue-) detuned squeezed laser.

II. OPTOMECHANICAL MODEL AND EQUATIONS

The schematic setup of our proposal is illustrated in Fig. 1. The setup consists of two optomechanical cavities that are mechanically coupled via a movable mechanical resonator. The coupling constant is denoted by J [3,4]. The tunable coupling of two mechanical resonators can be achieved through the piezoelectric effect or the photothermal effect [21]. Instead of driving both cavities linearly, for the left cavity, we use a red-detuned squeezed laser to drive, while for the right cavity, we use a blue-detuned squeezed laser. By driving this system symmetrically, we can effectively control the mechanical gain and loss, and make this system work at the EP. When the cavity is driven below resonance, the mechanical resonator can be cooled to its ground state; instead, if the cavity is driven above resonance, then the mechanical resonator is in states of self-sustaining oscillation. In the rotating frame, setting the driving frequency Ω_p, $\hbar = 1$, we can write the Hamiltonian of the system as follows,

$$H = H_{\text{optM}} + H_{\text{cp}} + H_{\text{drive}}, \tag{1}$$

with

$$\begin{aligned} H_{\text{optM}} &= \sum_{j=1,2} [\omega_j b_j^\dagger b_j - \Delta_j a_j^\dagger a_j - g a_j^\dagger a_j (b_j + b_j^\dagger)], \\ H_{\text{cp}} &= -J(b_1 b_2^\dagger + b_1^\dagger b_2), \\ H_{\text{drive}} &= \sum_{j=1,2} E\left(a_j^{\dagger 2} + a_j{}^2\right), \end{aligned} \tag{2}$$

where H_{optM} is the Hamiltonian of the optomechanical cavities, H_{cp} denotes the Hamiltonian of these two coupled systems, and H_{drive} describes the Hamiltonian of the driving mode. In quantum optics, a squeezed-laser drive can be obtained by means of parametric down-conversion. In this Hamiltonian, a_j and b_j are the annihilation bosonic field operators describing the optical and mechanical modes, respectively. The mechanical frequency is ω_j and $j = 1, 2$. The optical detuning between the driving frequency and the cavity (Ω_{cav}^j) is defined as Δ_j. J denotes the mechanical coupling strength between the two mechanical resonators, which are assumed to be tunable in the following discussion—it is defined differently in many experiments. By adjusting the laser power, one can effectively control the mechanical coupling between the two cantilevers [22–26]. For generality, in the following discussion we will not specify our theoretical model to any particular system. The linear optomechanical coupling is g, which defined as $g = g_0\sqrt{\bar{n}_{\text{cav}}}$, where g_0 is the optomechanical single-photon coupling strength, and $\bar{n}_{\text{cav}}$ indicates the photon number circulating inside the cavity.

The evolution of the system is described by the standard master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + L_m\rho + L_c\rho, \tag{3}$$

where the coupling of the mechanical resonators to their thermal surroundings at temperature T is described by

$$\begin{aligned} L_m\rho = &-\frac{\chi_m}{2}(\bar{n}+1)(b^\dagger b\rho + \rho b^\dagger b - 2b\rho b^\dagger) \\ &-\frac{\chi_m}{2}\bar{n}(bb^\dagger\rho + \rho bb^\dagger - 2b^\dagger\rho b), \end{aligned} \tag{4}$$

with χ_m the mechanical damping rate and $\bar{n} = [\exp(\hbar\omega_j/k_B T) - 1]^{-1}$. The cavity is described by

$$L_c\rho = -\frac{\gamma}{2}(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger), \tag{5}$$

where γ denotes the decay rate, and here we assume $\hbar\Omega_p \gg k_B T$ so that we can neglect thermal fluctuations while Ω_p is the frequency of the laser. According to the theory of open quantum systems [27], the strength of the driving pump can be written as $E = \sqrt{\gamma}\alpha_{\text{in}}$, where the input laser power P_{in} acts through $\alpha_{\text{in}} = \sqrt{\frac{P_{\text{in}}}{\hbar\Omega_p}}$.

We perform a Wigner transformation of the master equation, denote $\langle a_j\rangle = \alpha_j$, $\langle b_j\rangle = \beta_j$, neglecting third-order derivative terms, and use a truncated Wigner function approximation, and finally we can obtain the coupled Langevin equations,

$$\begin{aligned} \dot{\alpha_j} &= \left(i(\Delta_j + g(\beta_j^* + \beta_j)) - \frac{\gamma}{2}\right)\alpha_j - 2i\sqrt{\gamma}\alpha_{\text{in}}\alpha_j^* + \zeta_{\alpha_j}, \\ \dot{\beta_j} &= -\left(i\omega_j + \frac{\chi_m}{2}\right)\beta_j + iJ\beta_{3-j} + ig\alpha_j^*\alpha_j + \zeta_{\beta_j}. \end{aligned} \tag{6}$$

The stochastic force terms ζ_{α_j}, ζ_{β_j} have zero mean values and nonzero second-order moments $\langle\zeta_{\alpha_j^*}(t)\zeta_{\alpha_j}(t')\rangle = \delta(t-t')\gamma/2$ and $\langle\zeta_{\beta_j^*}(t)\zeta_{\beta_j}(t')\rangle = \delta(t-t')\chi_m(\bar{n}+\frac{1}{2})$. However, we seek to investigate in the classical limit where phonon numbers are assumed large in the system, and noise terms can be neglected in the analysis.

Without loss of generality, we assume that in this scheme the parameters χ_m, g, and γ are fixed for the whole system and satisfy the following condition, $\chi_m, g \ll \gamma \ll \omega_m$. From Ref. [4], we can get the overall properties of the steady state that is the solution to Eq. (6), and three regimes can be identified in the parameter's space. It is easy to find that as the driving α_{in} increases for a fixed J, the system switches from a linear regime to a nonlinear one. The EPs of the system then change and behave differently in different regimes.

In order to obtain a Hamiltonian-like description and formulate the EP as well as demonstrate the enhancement in sensitivity, we need to make some assumptions for Eq. (6) [3,4]. We assume that the solution of the mechanical oscillation takes the following form, $\beta_j(t) = \bar{\beta}_j + N_j\exp(-i\tilde{\omega}t)$, while $\bar{\beta}_j$ is the initial displacement, N_j is the amplitude that slowly changes over time, and $\tilde{\omega}$ is the mechanical degeneracy frequency when the resonator experiences a frequency locking. Bringing this hypothesis back into Eq. (6), we can get

$$\dot{\alpha_j} = \left(i(\tilde{\Delta}_j + 2g\,\mathrm{Re}(N_j)\cos\tilde{\omega}t) - \frac{\gamma}{2}\right)\alpha_j - 2i\sqrt{\gamma}\alpha_{\mathrm{in}}\alpha_j^*, \tag{7}$$

where $\tilde{\Delta}_j = \Delta_j + 2g\,\mathrm{Re}(\bar{\beta}_j)$ is nonlinear detuning, which is induced by the initial displacement of the mechanical oscillator. This equation is similar to that of a forced vibration system. Performing a Fourier expansion on α_j, we write the equation for α_j as follows,

$$\alpha_j = e^{-i\theta_j(t)}\sum_n \alpha_n^j e^{in\tilde{\omega}t}. \tag{8}$$

To simplify the expression, we defined $\theta_j(t) = -\nu_j \sin\tilde{\omega}t$, $\nu_j = \frac{2g\,\mathrm{Re}(N_j)}{\tilde{\omega}}$. Bringing all of the above formula back to Eq. (6), and then making some mathematical transformations, we can get

$$\sum_n \alpha_n^j e^{in\tilde{\omega}t} = -2i\sqrt{\gamma}\alpha_{\mathrm{in}}\alpha_j^* \frac{1}{r_n^j} e^{-2i\nu_j \sin\tilde{\omega}t}, \tag{9}$$

where $r_n^j = i(n\tilde{\omega} - \tilde{\Delta}_j) + \frac{\gamma}{2}$. Mathematically, Eq. (9) is the Jacobi-Anger expansion, which can be solved by a transformation and comparing the coefficients on both sides of the equation. After that, we get the Fourier expansion coefficient of α_j,

$$\alpha_n^j = -2i\sqrt{\gamma}\alpha_{\mathrm{in}}\alpha_j^* \frac{J_n(-2\nu_j)}{r_n^j}, \tag{10}$$

where $J_n(-2\nu_j)$ is nth Bessel function of the first kind. Therefore, we have $\alpha_j^*\alpha_j = \sum_{n,m} \alpha_m^{j\,*}\alpha_n^j e^{i(n-m)\tilde{\omega}t}$. In order to derive an expression for $\alpha_j(t)$, we set $m = n+1$. Then we arrive at

$$\dot{\beta}_1(t) = -i(\omega_{01} + \delta\omega_1)\beta_1 - \frac{\chi_m + \chi_{\mathrm{opt}}^1}{2}\beta_1 + iJ\beta_2,$$
$$\dot{\beta}_2(t) = -i(\omega_{02} + \delta\omega_2)\beta_2 - \frac{\chi_m + \chi_{\mathrm{opt}}^2}{2}\beta_2 + iJ\beta_1, \tag{11}$$

with an optical spring effect represented by

$$\delta\omega_j = -\frac{2[4\gamma(g\alpha_{\mathrm{in}})^2|\alpha_j|^2]}{\tilde{\omega}\nu_j}\mathrm{Re}\left(\sum_n \frac{J_{n+1}(-2\nu_j)J_n(-2\nu_j)}{r_{n+1}^{j*} r_n^j}\right), \tag{12}$$

and an optical damping denoted by

$$\chi_{\mathrm{opt}}^j = \frac{2[4|\alpha_j|^2(g\gamma\lambda_{\mathrm{in}})^2]}{\nu_j}\sum_n \frac{J_{n+1}(-2\nu_j)J_n(-2\nu_j)}{\left|r_{n+1}^{j*} r_n^j\right|^2}, \tag{13}$$

It is easy to see from the above expression that the optical damping term obtained by driving the cavity with the squeezed laser is changed by a factor $4|\alpha_j|^2$, which is related to the average number of photons involved. In the following discussion, we will see that this change can effectively enhance the scales of the splitting in the spectra of the mechanical system, so we can enhance the sensitivity of the sensor which is related to the average photon numbers involved.

Here, we will discuss the feature of the optical damping. For $\nu_j \ll 1$, the linear approximation is still valid and the optical damping can be rewritten accordingly. Mathematically, we have $J_n(-\nu_j) \approx \frac{1}{n!}(\frac{-\nu_j}{2})^n$ for $n \geqslant 0$ and $J_{-n}(-\nu_j) = J_n(\nu_j)$, where we take $n = -1, 0$; using these considerations in Eq. (13) yields

$$\chi_{\mathrm{opt}}^j \approx -\frac{8|\alpha_j|^2\tilde{\Delta}_j\tilde{\omega}(2g\gamma\alpha_{\mathrm{in}})^2}{\left(\tilde{\Delta}_j^2 + \frac{\gamma^2}{4}\right)\left[(\tilde{\omega} + \tilde{\Delta}_j)^2 + \frac{\gamma^2}{4}\right]\left[(\tilde{\omega} - \tilde{\Delta}_j)^2 + \frac{\gamma^2}{4}\right]}. \tag{14}$$

It is demonstrated that χ_{opt}^j is not amplitude dependent and we show the overall properties of the optical damping and their difference between two cavities in Fig. 2.

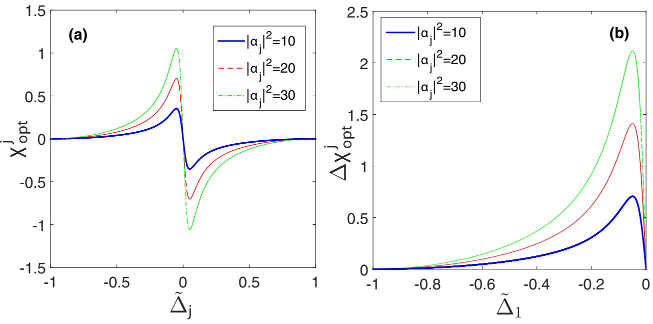

FIG. 2. (a) The optical damping vs the effective detuning in different average photon numbers in cavities. (b) The difference between the optical damping in two cavities, $\alpha_{\mathrm{in}} = 4.2 \times 10^2\sqrt{\omega_m}$, $\gamma = 10^{-1}\omega_m$, $g = 2.5 \times 10^{-4}\omega_m$, and effective detuning from -1 to 1.

III. THE EFFECTIVE HAMILTONIAN AND THE SENSITIVITY AT EP

Treating $\beta_j(t)$ $(j = 1, 2)$ as a two-component wave function, we can write an effective Hamiltonian to govern the dynamics of $\beta_j(t_0)$ according to Eq. (11). In other words, if we set $\Psi = (\beta_1, \beta_2)^T$, Eq. (11) can be rewritten as $i\frac{\partial}{\partial t}\Psi = H_{\mathrm{eff}}\Psi$, where the effective Hamiltonian H_{eff} takes

$$H_{\mathrm{eff}} = \begin{pmatrix} \omega_{\mathrm{eff}}^1 - i\frac{\chi_{\mathrm{eff}}^1}{2} & -J \\ -J & \omega_{\mathrm{eff}}^2 - i\frac{\chi_{\mathrm{eff}}^2}{2} \end{pmatrix}, \tag{15}$$

where $\omega_{\mathrm{eff}}^j = \omega_{0j} + \delta\omega_j$ and $\chi_{\mathrm{eff}}^j = \chi_m + \chi_{\mathrm{opt}}^j$ are modulated frequencies and damping rates, respectively. They depend on the average photon number of the system, and as we will show below, the sensitivity of the sensor is enhanced sharply by these modulations.

Obviously, the effective Hamiltonian is non-Hermitian, and its eigenvalues are calculated as

$$\lambda_\pm = \frac{\omega_{\mathrm{eff}}^1 + \omega_{\mathrm{eff}}^2}{2} - \frac{i}{4}\left(\chi_{\mathrm{eff}}^1 + \chi_{\mathrm{eff}}^2\right) \pm \frac{\varepsilon}{4}, \tag{16}$$

with

$$\varepsilon = \sqrt{16J^2 + \left[2\left(\omega_{\mathrm{eff}}^1 - \omega_{\mathrm{eff}}^2\right) + i\left(\chi_{\mathrm{eff}}^2 - \chi_{\mathrm{eff}}^1\right)\right]^2}, \tag{17}$$

where the eigenfrequencies and eigendampings of the system are defined as $\omega_\pm = \mathrm{Re}(\lambda_\pm)$, which denotes the mechanical frequencies, and $\chi_\pm = \mathrm{Im}(\lambda_\pm)$, which denotes the spectral linewidths. So the eigenvalues of this non-Hermitian system can be written in the form of $\lambda_\pm = \omega_\pm + i\chi_\pm$. In general, the dissipation terms are not zero. Now we deposit a tiny

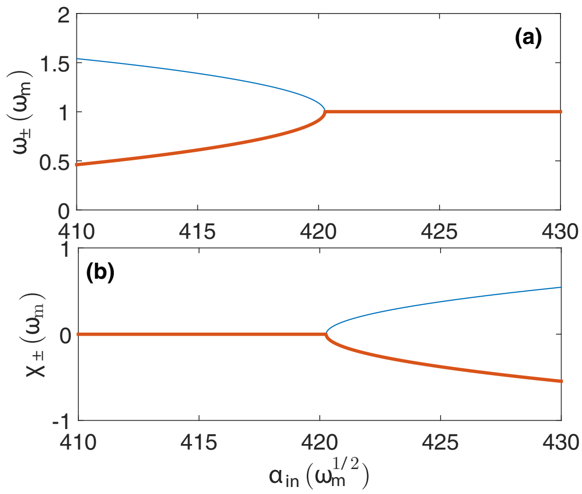

FIG. 3. (a) Real part of the eigenvalues vs the driving strength $\alpha_{\rm in}$, and $J = 1.760\omega_m$ was chosen for this plot. With increasing driving strength, the exceptional point appears. (b) Imaginary part of the eigenvalues vs the driving strength $\alpha_{\rm in}$.

mass on the optomechanical mass sensor, and the well-known relationship between the frequency shift and the deposition mass will be used in the discussion,

$$\delta m = k^{-1}\delta\omega, \tag{18}$$

with $k = \frac{\omega_m}{2m}$. This states that the frequency shift is proportional to the intensity of the perturbation. At the exceptional point, the eigenfrequency ($\omega_\pm$) and the eigendamping ($\chi_\pm$) are coalesced, and any external perturbation will cause these eigenvalues to split with a high sensitivity of the perturbation. We assume that the mass is deposited in the mechanical resonator driven by the blue-detuned squeezed laser. When the resonators are coupled, this local perturbation will affect the entire system, causing the splitting of eigenvalues at the exceptional point. Considering the two resonators to be degenerate (cavities of equal frequency) and $\delta\omega_j \ll \omega_m$, Eq. (16) reduces to

$$\lambda_\pm \approx \omega_m - \frac{i}{4}\left(\chi^1_{\rm eff} + \chi^2_{\rm eff}\right) \pm \sqrt{J^2 - \left(\frac{\Delta\chi_{\rm eff}}{4}\right)^2}, \tag{19}$$

while $\Delta\chi_{\rm eff} = \chi^2_{\rm opt} - \chi^1_{\rm opt}$. From Eq. (19), we can know that if $4J > \Delta\chi_{\rm eff}$, the real parts of the eigenvalues can be written as $\omega_m \pm J\cos\kappa$ with $\sin\kappa = \frac{\Delta\chi_{\rm eff}}{4J}$. When $4J < \Delta\chi_{\rm eff}$, the imaginary parts of the eigenvalues can be written as $-\frac{\chi^1_{\rm eff}+\chi^2_{\rm eff}}{4} \pm J\sinh\kappa$, where $\cosh\kappa = \frac{\Delta\chi_{\rm eff}}{4J}$. When $4J = \Delta\chi_{\rm eff}$, a phase transition between these two regimes occurs at the exceptional point. The quantity $\Delta\chi_{\rm eff}$ is quadratic in $\alpha_{\rm in}$, and due to the factor $4|\alpha_j|^2$ in Eq. (13), we can get the same result as in Ref. [3] for a small $\alpha_{\rm in}$, where $|\alpha_j|^2$ is the intracavity photon number which can be controlled by the optical drive signal. In order to enhance the splitting width at EP in our system, we modulate the coupling constant to take $J = 1.760\omega_m$ to match $\Delta\chi_{\rm eff}$ with a mean photon number $|\alpha_j|^2 = 10$ in the numerical simulation. Figure 3 shows the dependence of the eigenvalue (both the real part and imaginary part) on the drive around the exceptional point. As the driving strength $\alpha_{\rm in}$ increases to EP, the real part of the eigenvalues begins to coalesce while the imaginary part starts to split. We can find that the EP appears at $\alpha_{\rm in} = 4.2\times 10^2\omega_m$, and $J = 1.760\omega_m$ in the parameter's space $(\alpha_{\rm in}, J)$.

When the second resonator experiences a small perturbation δm,

$$\lambda^{\delta\omega}_\pm = \frac{\sum_j \omega_j + \delta\omega^j_{\rm opt} + \delta\omega}{2} - \frac{i}{4}\left(\chi^1_{\rm eff} + \chi^2_{\rm eff}\right) \pm \frac{\varepsilon^{\delta\omega}}{4}, \tag{20}$$

with

$$\varepsilon^{\delta\omega} = \sqrt{16J^2 + [2\Gamma + i\Delta\chi_{\rm eff}]^2}, \tag{21}$$

while $\Gamma = (\omega_1 - \omega_2 - \delta\omega^1_{\rm opt} - \delta\omega^2_{\rm opt} - \delta\omega)$. Considering degenerate mechanical resonators, i.e., $\omega_j \equiv \omega_m$ and $\delta\omega^j_{\rm opt} \ll \omega_m$, Eq. (20) becomes

$$\lambda^{\delta\omega}_\pm = \omega_m + \frac{\delta\omega}{2} - \frac{i}{4}\left(\chi^1_{\rm eff} + \chi^2_{\rm eff}\right) \pm \sqrt{J^2 - \left(\frac{\Delta\chi_{\rm eff}}{4}\right)^2 + \frac{\delta\omega^2 - i\delta\omega\Delta\chi_{\rm eff}}{4}}. \tag{22}$$

When we are trying to evaluate the sensitivity of the sensor at EP, we need to know the effect of the frequency perturbations on the supermode frequency splitting near the EP, therefore, sensitivity can be measured by the difference between the unperturbed and perturbed eigenvalues,

$$\lambda^{\delta\omega}_\pm - \lambda_\pm = \frac{\delta\omega}{2} \pm \sqrt{J^2 - \left(\frac{\Delta\chi_{\rm eff}}{4}\right)^2 + \frac{\delta\omega^2 - i\delta\omega\Delta\chi_{\rm eff}}{4}} \mp \sqrt{J^2 - \left(\frac{\Delta\chi_{\rm eff}}{4}\right)^2}, \tag{23}$$

and considering the condition at the exceptional point, $4J = \Delta\chi_{\rm eff}$, we have

$$\begin{aligned}\lambda^{\rm EP}_\pm(\delta\omega) &= \lambda^{\delta\omega}_\pm - \lambda_\pm \\ &= \frac{\delta\omega}{2} \pm \sqrt{\frac{\delta\omega^2 - i\delta\omega\Delta\chi_{\rm eff}}{4}} \\ &= \frac{1}{2}\left(\delta\omega \pm \sqrt{\delta\omega^2 - i\delta\omega\Delta\chi_{\rm eff}}\right),\end{aligned} \tag{24}$$

so we write the results as real and imaginary parts, and the square root of the complex term reads

$${\rm Re}\left(\lambda^{\rm EP}_\pm\right) = \frac{1}{2}\left(\delta\omega \pm \sqrt{\frac{\delta\omega^2}{2} + \frac{\delta\omega}{2}\sqrt{\delta\omega^2 + \Delta\chi^2_{\rm eff}}}\right), \tag{25}$$

and

$${\rm Im}\left(\lambda^{\rm EP}_\pm\right) = \pm\frac{i}{2}\sqrt{\frac{\delta\omega}{2}\sqrt{\delta\omega^2 + \Delta\chi^2_{\rm eff}} - \frac{\delta\omega^2}{2}}. \tag{26}$$

The numerical results are shown in Fig. 4. We find that one branch of the eigenvalues splits as a square root at the exceptional point. This is to say, when there is mass deposition, the originally coalesced eigenvalues would split rapidly, which means the perturbation can break the steady state of

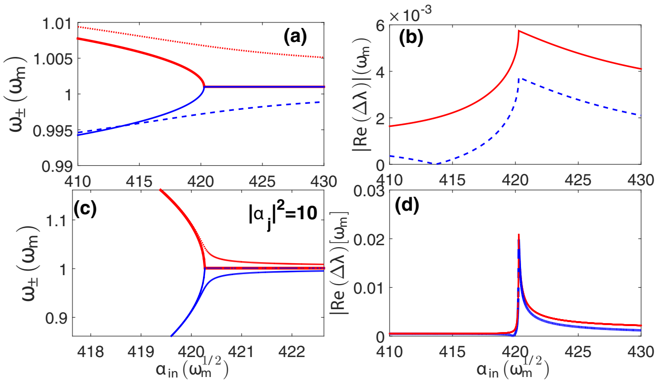

FIG. 4. (a), (c) Frequencies of the effective mechanical system without the factor $4|\alpha_j|^2$, and squeezed-laser drive with $|\alpha_j|^2 = 10$ before perturbation (solid lines) and after perturbation by a mass deposition (dashed lines). (b), (d) Gap difference between the perturbed and unperturbed vs the driving strength with $J = 0.022\omega_m$ and $1.760\omega_m$, respectively.

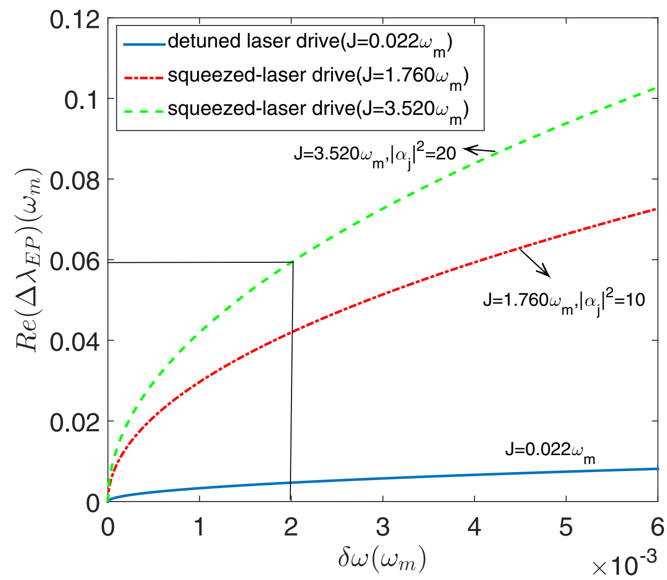

FIG. 5. Sensitivity at the exceptional point vs the strength of the perturbation $\delta\omega$ in different sets of driving fields. We set the $|\alpha_j|^2$ as 10 and 20, respectively. The squeezed-laser drive can indeed enhance the sensitivity.

the mechanical resonators. By measuring the splitting width, we can easily define the sensitivity of the mass sensor. In Refs. [3,4], their eigenvalue at EP ($J = 0.022\omega_m$, $\alpha_{\rm in}[\omega_m^{1/2}] = 420$) is $1.0057\omega_m$, while our result is increased by one order of magnitude by using a squeezed laser with the parameters $J = 1.760\omega_m$, $|\alpha_j|^2 = 10$. In Ref. [28], they provided a novel view of sensitivity on parameter estimation near the exceptional point by means of quantum Fisher information. Nevertheless, they studied the energy splitting which shows a square-root perturbation dependence by means of the difference of the positive eigenvalue and negative one in the same mechanical spectrum. In contrast, we here mainly study the split of the real parts of the eigenvalues in different spectra because $|\mathrm{Im}(\lambda_\pm^{\rm EP})| \leqslant |\mathrm{Re}(\lambda_\pm^{\rm EP})|$.

Now if we set $|\delta\omega| \ll |\Delta\chi_{\rm eff}|$, Eq. (25) reduces to $\sqrt{\frac{\delta\omega\Delta\chi_{\rm eff}}{8}}$. First, according to the special topological structure of the exceptional point, it is easy to see that the split of the eigenvalues is related to the square root of the perturbation strength ($\delta\omega$), which is different from traditional optical mass sensors that are proportional to the perturbation. Meanwhile, it is obvious that the optomechanical mass sensor working at EP has a larger splitting width after deposition, thus showing a great sensitivity [3,4]. On the other hand, the splitting width is also related to the difference of effective damping $\Delta\chi_{\rm eff}$ between the two cavities. In addition, in our model the cavities are driven by a squeezed laser. According to the above calculation and Eq. (13), this squeezed-laser driving leads to an improvement of factor $4|\alpha_j|^2$ on $\Delta\chi_{\rm eff}$. Note that $|\alpha_j|^2$ represents the average number of photons involved, and it might be manipulated by setting appropriate values of the average photon number, so we conclude that the use of lasers in squeezed states can enhance the width of the eigenvalues' splitting, thus enhancing the sensitivity. In this paper, we assume two $|\alpha_j|^2$ values, i.e., 10 and 20, to show the results (see Fig. 5). Considering the statistic properties of photon numbers and the oscillation of $\Delta\chi_{\rm eff}$ in this result, the sensitivity measured by $\mathrm{Re}(\Delta\lambda_{\rm EP})$ can be enhanced even for small perturbations $\delta\omega$ at the exceptional point.

Now, we define the sensitivity enhancement factor as [3,4]

$$\theta = \left|\frac{\mathrm{Re}(\Delta\lambda^{\rm EP})}{\delta\omega}\right| = \sqrt{\frac{\Delta\chi_{\rm eff}}{8\delta\omega}} = \sqrt{\frac{m\Delta\chi_{\rm eff}}{4\omega_m\delta m}}, \qquad (27)$$

which shows a square-root dependence of the sensitivity on the perturbation strength. In addition, we observe that the squeezed-laser drive can enhance $\Delta\chi_{\rm eff}$ by choosing an appropriate $|\alpha_j|^2$, hence it increases the sensitivity enhancement factor. From Fig. 5 we observe that the sensitivity of the sensor based on the exceptional point would end up going towards a linear relationship with the perturbation strength when the perturbation becomes large enough. Therefore, the mass sensor operating at the exceptional point can be widely used in the field of extremely small substance detection. The relationship between the sensitivity enhancement factor and the perturbation strength is shown in Fig. 6. By contrast with the earlier proposal, the enhancement factor is more prominent in the range 0–0.4 $\times 10^{-4}$ of $\delta\omega$ and it depends on the photon number involved.

Finally, we consider the influence of various noises of the system which will cause an increase in frequency uncertainty. For the mechanical resonators, the main noise is the thermomechanical fluctuations [29,30]. Mechanical resonators have resonant frequencies in the radio-frequency range or below, and thermal fluctuations would tend to mask the quantum features, so we tend to develop ways of cooling a mechanical resonator down to its ground state. In this paper, we assume $\hbar\Omega_p \ll k_B T$ so that we neglect thermal fluctuations. Here, according to the fluctuation-dissipation theorem [31,32], the frequency fluctuation induced by thermal noise can be calculated by $\delta\omega' = \sqrt{k_B T/2\pi\tau(m\omega_m\langle x_j{}^2\rangle Q)}$. Q is the mechanical quality factor, T is the effective temperature, k_B is the Boltzmann constant, and x_j is the mean amplitude of the resonator.

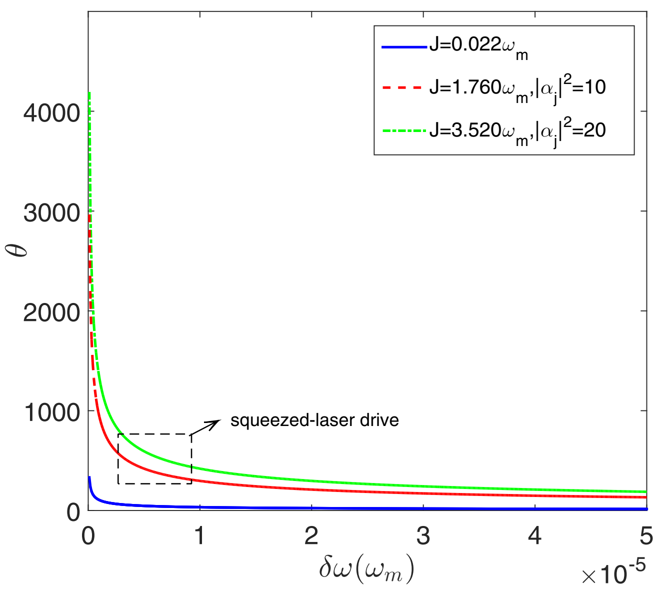

FIG. 6. Sensitivity enhancement factor θ vs $\delta\omega$ with different J and $|\alpha_j|^2$. The sensor driven by a squeezed laser performs better if the mass deposition is small enough. It ends up towards the same performance as the conventional ones as the perturbation increases.

If we take high mechanical quality and low temperature, we can reduce the thermal noise. Now we want to obtain the quantum-noise-limited sensitivity of the mass deposition, so we assume that $\delta\omega' = \sqrt{\frac{\delta\omega\Delta\chi_{\text{eff}}}{8}}$, and we take a sample time of $\tau = 1$ s,

$$\delta\omega_{\min} = \frac{k_B T_{\text{eff}}}{\pi m\omega_m \langle x_j{}^2\rangle QJ}. \tag{28}$$

If we cool down the resonators to $T_{\text{eff}} = 1$ K, take a high Q factor and appropriate $4|\alpha_j|^2$ to increase J, the sensitivity can be enhanced to a drastic level. If we take the same values of parameters in Eq. (28) without the value of J, for a linear drive we take $J = 0.022\omega_m$, and for the squeezed-laser drive we take $J = 3.520\omega_m$ (because of the different $|\alpha_j|^2$), the $\delta\omega_{\min}$ will reach $1/160$ times the linear drive, which smaller than the result of the linear drive.

IV. SUMMARY

Taking the interesting properties of non-Hermitian systems and the advantage of squeezed states into account, we propose a scheme to enhance further the optomechanical mass sensor working at the exceptional point by quadrature drives. The model system consists of two optomechanical cavities, which are mechanically coupled via a movable mechanical arm and kept in balance between the gain and loss. The cavities are driven by red- and blue-detuned squeezed lasers in each of the two cavities, respectively, and the dynamics of the system can be described by an effective Hamiltonian. At the exceptional point, all eigenvalues and eigenvectors coalesce. Any slight perturbation to the system can cause the eigenvalues at the EP to split rapidly as the square root of the perturbation. This is a unique topological structure in this system [33–36]. Therefore, the sensitivity to the perturbation can be used to detect the deposition of small masses. Moreover, it was found that driving of a squeezed laser can display more powerful sensitivity than the earlier proposal, and the present result is related to the number of photons involved that is different from the earlier one. We believe that the theoretical optimization of this model can be applied in practice, especially in the detection and development of micro- and nanotechnology and the measurement of small substances.

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