Bell-inequality violation and relativity of pre- and postselection

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The Bell inequalities can be violated by postselecting on the results of a measurement of the Bell states. If information about the original state preparation is available, we point out how the violation can be reproduced classically by postselecting on the basis of this information. We thus propose a variant of existing experiments that rules out such alternative explanations by having the preparation and the postselection at spacelike separation. Unlike the timelike case in which one can sharply distinguish Bell inequality violations based on pre- or postselection of a Bell state, in our scenario the distinction between these physical effects becomes foliation dependent. We call this "relativity of pre- and postselection" and conclude from it that quantum state evolution is not a fundamental process and that we should adopt an event-based Heisenbergian picture instead.

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I. INTRODUCTION

There are a number of remarkable effects of quantum postselection. Often, these effects occur due to a combination with preselection as in the three-box paradox [1]. Here we treat the case where postselection can give rise to violations of the Bell inequalities as proposed in [2,3] and realized experimentally in [4] (see also [5]). This case should be distinguished from the standard Bell inequality violations due to entanglement.

In this paper we demonstrate that these violations can be reproduced by classical means, even allowing for a saturation of the superquantum bound $S = 4$, by postselecting on information about the preparation procedure of the qubit pairs. In the quantum case, access to such information is not required. In order for this to be a genuine quantum effect, one needs to consider a setup where such information may reasonably be expected to not be available. We accordingly propose a modification of the experiment in [4].

The proposed experiment can be adapted to test simultaneously the violation of Bell inequalities due to postselection and the standard violation due to entanglement. It also adds a striking twist to the idea that entanglement is foliation dependent. We conclude that a covariant description of our proposed experiment will have to use an event-based description adopting the Heisenberg picture.

II. BELL INEQUALITIES FOR BELL STATES

We begin by summarizing a few facts about Bell inequality violations. For a pair of qubits consider the local observables

$$A_i = (P_{\alpha_i} - P_{\alpha_i}^{\perp}) \otimes \mathbb{1}, \quad B_j = \mathbb{1} \otimes (P_{\beta_j} - P_{\beta_j}^{\perp}), \tag{1}$$

with

$$P_\varphi = \begin{pmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{pmatrix}. \tag{2}$$

Next, consider the four Clauser-Horne-Shimony-Holt (CHSH) inequalities

$$\begin{aligned} S_1^\psi &= \left|E_{1,1}^\psi + E_{1,2}^\psi + E_{2,1}^\psi - E_{2,2}^\psi\right| \leqslant 2,\\ S_2^\psi &= \left|E_{1,1}^\psi + E_{1,2}^\psi - E_{2,1}^\psi + E_{2,2}^\psi\right| \leqslant 2,\\ S_3^\psi &= \left|E_{1,1}^\psi - E_{1,2}^\psi + E_{2,1}^\psi + E_{2,2}^\psi\right| \leqslant 2,\\ S_4^\psi &= \left|-E_{1,1}^\psi + E_{1,2}^\psi + E_{2,1}^\psi + E_{2,2}^\psi\right| \leqslant 2, \end{aligned} \tag{3}$$

with

$$E_{i,j}^\psi = p^\psi(A_i = B_j) - p^\psi(A_i \neq B_j). \tag{4}$$

Maximal violations of these inequalities can be obtained with each of the Bell states

$$\begin{aligned} |\Phi^+\rangle &= \tfrac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle),\\ |\Psi^+\rangle &= \tfrac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle),\\ |\Phi^-\rangle &= \tfrac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle),\\ |\Psi^-\rangle &= \tfrac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \end{aligned} \tag{5}$$

which yield the following probabilities:

$$\begin{aligned} p_{i,j}^{\Phi^+} &= \cos^2(\alpha_i - \beta_j), \quad & p_{i,j}^{\Psi^+} &= \sin^2(\alpha_i + \beta_j),\\ p_{i,j}^{\Phi^-} &= \cos^2(\alpha_i + \beta_j), \quad & p_{i,j}^{\Psi^-} &= \sin^2(\alpha_i - \beta_j), \end{aligned} \tag{6}$$

where $p_{i,j}^\psi = p^\psi(A_i = B_j)$. At most one CHSH inequality can be violated for each Bell state. For the sake of definiteness, we fix the spin directions

$$\alpha_1 = 0,\ \alpha_2 = \frac{\pi}{4},\ \beta_1 = \frac{\pi}{8},\ \beta_2 = -\frac{\pi}{8}, \tag{7}$$

which give us the violations

$$S_1^{\Phi^+} = S_2^{\Psi^+} = S_2^{\Phi^-} = S_1^{\Psi^-} = 2\sqrt{2}. \tag{8}$$

For the other combinations we have $S_i^\psi = 0$.

It follows that if Vicky prepares an equal mixture of the Bell states (5) and sends the particles to Alice and Bob to perform measurements along the directions (7), each of the four subensembles leads to a maximal violation of either S_1 or S_2, but the total ensemble exhibits no correlations whatsoever.

III. BELL INEQUALITIES WITH POSTSELECTION

A. Quantum case

Let Alice and Bob independently prepare qubits. Alice's method of preparation consists of measuring either A_1 or A_2 on a qubit in the maximally mixed state, and Bob's consists of measuring either B_1 or B_2. For this ensemble of qubit pairs we have, of course, that the outcomes of Alice's and Bob's measurements satisfy $S_i = 0$ for all i. Their qubits are then sent to Vicky in pairs (one from Alice and one from Bob).

Now Vicky performs on each pair a measurement of the Bell basis (5). Based on the outcome of this measurement Vicky constructs four subensembles of pairs of qubits. For each of these subensembles, the outcomes of Alice's and Bob's measurements do violate one of the CHSH inequalities. This violation follows simply because of the symmetry of transition probabilities; thus, violation of the Bell inequalities for this case is mathematically equivalent to the standard case in which Alice and Bob perform their measurements on preselected pairs of qubits in Bell states [2].

Although in this scenario each individual pair of qubits is in a product of eigenstates of either A_1 or A_2 and either B_1 or B_2, Vicky has no access to this information unless Alice and Bob send it classically, and it plays no role in Vicky's postselection procedure. As we shall see, however, classically, this information could be used, in principle, to define an alternative postselection procedure that also leads to violation of the Bell inequalities.

B. Classical simulation

Vicky's task is to subdivide the above totally uncorrelated ensemble into four subensembles (with the same marginals) that violate the Bell inequalities, using information about which individual states Alice and Bob have prepared. For extra vividness, our initial uncorrelated ensemble will also be purely classical. Suppose Alice chooses between flipping either a U.S. quarter dollar or a Japanese 100-yen piece, and Bob flips either a 50-euro-cent coin or a British 10 pence [6]. They will get pairs of results with the following distributions (with $=$ for two heads or two tails and $\neq$ for one head and one tail and a, b, c, d being the proportions in which the four combinations of coins are flipped):

		Coins tossed			
		\$€	\$£	¥€	¥£
Outcomes	$=$	$\frac{a}{2}$	$\frac{b}{2}$	$\frac{c}{2}$	$\frac{d}{2}$
	$\neq$	$\frac{a}{2}$	$\frac{b}{2}$	$\frac{c}{2}$	$\frac{d}{2}$

(9)

Now let Vicky take, say, the top left subensemble in (9) (\$ = €) and subdivide it at random into four subensembles in the following proportions:

$$\frac{a}{4}p^{\Phi^+}_{1,1} + \frac{a}{4}p^{\Psi^+}_{1,1} + \frac{a}{4}p^{\Phi^-}_{1,1} + \frac{a}{4}p^{\Psi^-}_{1,1} = \frac{a}{2}, \tag{10}$$

proceeding similarly with all other boxes in (9). Note that here $p^{\psi}_{i,j}$ are just theoretical numbers derived from quantum mechanics to determine the size of the subensembles. Vicky's procedure is completely classical. Vicky then collects the resulting pairs together in the following four subensembles:

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}p^{\Phi^+}_{1,1}$	$\frac{b}{4}p^{\Phi^+}_{1,2}$	$\frac{c}{4}p^{\Phi^+}_{2,1}$	$\frac{d}{4}p^{\Phi^+}_{2,2}$
$\neq$	$\frac{a}{4}q^{\Phi^+}_{1,1}$	$\frac{b}{4}q^{\Phi^+}_{1,2}$	$\frac{c}{4}q^{\Phi^+}_{2,1}$	$\frac{d}{4}q^{\Phi^+}_{2,2}$

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}p^{\Psi^+}_{1,1}$	$\frac{b}{4}p^{\Psi^+}_{1,2}$	$\frac{c}{4}p^{\Psi^+}_{2,1}$	$\frac{d}{4}p^{\Psi^+}_{2,2}$
$\neq$	$\frac{a}{4}q^{\Psi^+}_{1,1}$	$\frac{b}{4}q^{\Psi^+}_{1,2}$	$\frac{c}{4}q^{\Psi^+}_{2,1}$	$\frac{d}{4}q^{\Psi^+}_{2,2}$

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}p^{\Phi^-}_{1,1}$	$\frac{b}{4}p^{\Phi^-}_{1,2}$	$\frac{c}{4}p^{\Phi^-}_{2,1}$	$\frac{d}{4}p^{\Phi^-}_{2,2}$
$\neq$	$\frac{a}{4}q^{\Phi^-}_{1,1}$	$\frac{b}{4}q^{\Phi^-}_{1,2}$	$\frac{c}{4}q^{\Phi^-}_{2,1}$	$\frac{d}{4}q^{\Phi^-}_{2,2}$

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}p^{\Psi^-}_{1,1}$	$\frac{b}{4}p^{\Psi^-}_{1,2}$	$\frac{c}{4}p^{\Psi^-}_{2,1}$	$\frac{d}{4}p^{\Psi^-}_{2,2}$
$\neq$	$\frac{a}{4}q^{\Psi^-}_{1,1}$	$\frac{b}{4}q^{\Psi^-}_{1,2}$	$\frac{c}{4}q^{\Psi^-}_{2,1}$	$\frac{d}{4}q^{\Psi^-}_{2,2}$

where $q^{\psi}_{i,j} = 1 - p^{\psi}_{i,j}$. These postselected subensembles of pairs of classical coins reproduce *exactly the same* maximal violations of the Bell inequalities as in the quantum case above.

In fact, Vicky can do even better and can select instead subensembles of the form

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}$	$\frac{b}{4}$	$\frac{c}{4}$	0
$\neq$	0	0	0	$\frac{d}{4}$

	\$€	\$£	¥€	¥£
$=$	$\frac{a}{4}$	$\frac{b}{4}$	0	$\frac{d}{4}$
$\neq$	0	0	$\frac{c}{4}$	0

	\$€	\$£	¥€	¥£
$=$	0	0	$\frac{c}{4}$	0
$\neq$	$\frac{a}{4}$	$\frac{b}{4}$	0	$\frac{d}{4}$

	\$€	\$£	¥€	¥£
$=$	0	0	0	$\frac{d}{4}$
$\neq$	$\frac{a}{4}$	$\frac{b}{4}$	$\frac{c}{4}$	0

These subensembles now violate the same Bell inequalities with $S_i = 4$. Thus, it is possible not only to classically simulate the quantum violations using postselection but even to obtain superquantum violations.

C. Discriminating the quantum and classical cases

The structure common to the quantum and classical cases is that Vicky performs a postselection on pairs of systems in a mixture of the end products of one of two possible binary measurements. In the classical simulation the postselection protocol makes use of the fact that Vicky has full information about the eight subensembles in (9). For each coin pair, Vicky knows which coins were flipped and what the outcomes of these flips were. This information is not required in the quantum case. There Vicky uses only the outcome of the Bell measurement to construct the subensembles.

In fact, information about which measurements Alice and Bob performed and what their outcomes were is not even available in the quantum case; no quantum measurement on the qubit pair will help Vicky make a better guess as to what went on in Alice's and Bob's laboratory. This is because from Vicky's perspective the measurements performed by Alice and Bob are fiducial and the state of the qubit pair arriving at Vicky is just the maximally mixed state. If we place a similar restriction on the classical scenario, the possibility of violating the CHSH inequalities disappears. The proper analog is that instead of reporting to Vicky which coins were flipped and which outcomes were obtained, Alice and Bob just toss the coin they flipped back into the box with the coin they did not flip and then send the box of coins to Vicky. No measurement on the coins will reveal which one was flipped or what the outcome was.

What is specifically quantum is that, although any measurement on the qubit pair will be independent of both the measurements Alice and Bob performed and the outcomes they obtained, quantum measurements are not generally independent of possible correlations between the outcomes of Alice and Bob conditional on the settings. This dependence is precisely what the violations of the CHSH inequalities express.

However, classical explanations of the phenomenon are not entirely ruled out. It is conceivable that by adding hidden variables to the qubits, they do contain information about which measurements Alice and Bob performed and what their outcomes were. What is then quantum from this perspective is that there appears to be some limitation on possible measurements that prevents us from revealing these hidden variables.

One way to rule out such an explanation is by assuming *preparation noncontextuality* [7]. This assumption essentially boils down to the demand that, because whatever Alice and Bob do the end result for Vicky is the same (maximally mixed) quantum state, also at the hidden-variable level the actions of Alice and Bob leave no discernible trace in the qubit pair. Such an assumption may seem unreasonably strong though [8], and in the next section we therefore propose an alternate experimental setup to rule out classical explanations of CHSH violations using postselection.

IV. EXPERIMENTAL TEST

For our proposal we draw inspiration from a protocol that was used in the experimental violation of Bell inequalities using postselection by Ma *et al.* [4]. In Ma *et al.*'s protocol, each of the qubits in Vicky's uncorrelated pair is part of a pair

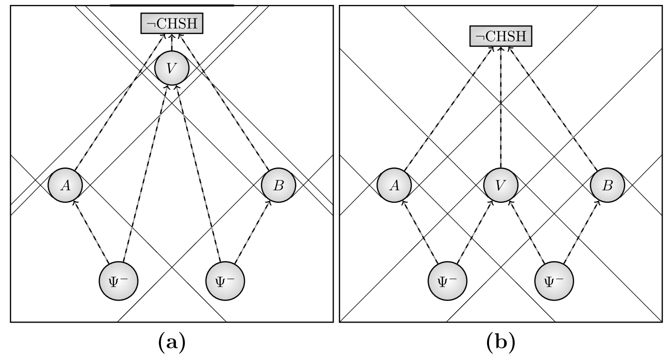

FIG. 1. Space-time diagram for (a) delayed-choice entanglement swapping and (b) for our proposed experiment.

prepared in the state $|\Psi^-\rangle$. Vicky shares one of these pairs with Alice and the other with Bob [see Fig. 1(a)]. Alice and Bob then perform their local experiments on their qubits. If we think of Alice's and Bob's measurements as collapsing the state also at Vicky's site, this procedure leads to the same mixed states for Vicky's qubits as in the previous scenario. The difference is that Alice and Bob have now prepared them from a distance.

Initially, of course, Alice's and Bob's results will be completely uncorrelated. At an arbitrary point in the future, however, Vicky can decide to perform a measurement in the Bell basis (5). This is delayed-choice entanglement swapping [3]. The outcomes of this measurement can then be used to postselect subensembles for which the measurement results of Alice and Bob become correlated and violate a CHSH inequality.

When Vicky's measurement is timelike separated from both Alice's and Bob's measurements as in Fig. 1(a), the explanation of the CHSH violation is unambiguously due to postselection. Although the experiment by Ma *et al.* [4] was accordingly set up to ensure timelike separation between Alice and Bob's and Vicky's measurements, it is precisely this feature that provides the loophole for a classical explanation of the results. In order to ensure that information about Alice's and Bob's measurements cannot reach Vicky's site and thus that the experimental violation of the Bell inequalities due to postselection is a genuine quantum effect, it is not necessary that Alice and Bob are at spacelike separation from each other, but we need to make sure that their measurements (including their choice of settings) are at spacelike separation from Vicky's. We thus propose this requirement of spacelike separation as a modification of the Ma *et al.* experiment, as in Fig. 1(b).

V. RELATIVITY OF PRE- AND POSTSELECTION

In quantum theory, spacelike separated measurements commute, which is the basis for what Shimony has called the "peaceful coexistence" of quantum theory and relativity [9,10]. But there is, of course, a tension with the idea that quantum state collapse occurs instantaneously across space. A proposal to resolve this tension is to embrace the idea that quantum states are defined on spacelike hypersurfaces and encode the probabilities for results of measurements to the

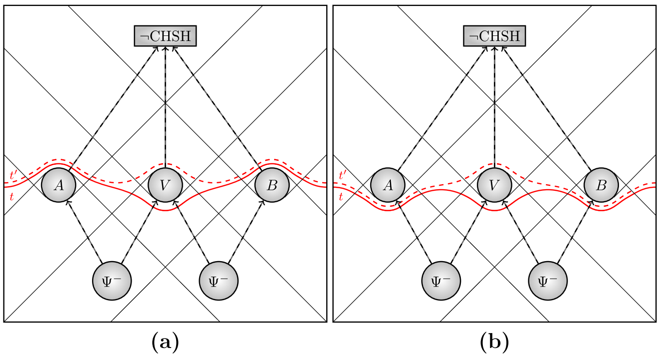

FIG. 2. Space-time diagrams with foliations with different time ordering of events. In (a) Vicky's measurement postselects subensembles, while in (b) the measurement acts as a preselection procedure.

future of the given hypersurface conditional on results of measurements to its past [11–16]. Consequently, entanglement of distant particles becomes a foliation-dependent notion: while the probabilities for Alice's and Bob's results are invariant, whether a qubit pair is entangled when Alice performs a measurement depends on the time order between their measurements. To capture this phenomenon, Myrvold [15,16] has coined the term "relativity of entanglement."

The experiment we propose adds a further dramatic touch to this idea. Because of the spacelike separation between Vicky's measurements and Alice and Bob's, the same experiment can be alternatively described in two different ways: either as Vicky performing a series of Bell measurements on maximally mixed pairs prepared by Alice and Bob [Fig. 2(a)] or as Alice and Bob performing a series of Einstein-Podolsky-Rosen (EPR) measurements on maximally entangled pairs prepared by Vicky [Fig. 2(b)]. In other words, depending on the choice of foliation, Vicky's measurement acts as a preselection or a postselection. We now have *relativity of pre- and postselection* [17,18].

This relativistic insight addresses one potential objection to our analysis in the previous section. If we imagine that Alice's and Bob's measurements actually collapse the state at a distance also at Vicky's site, then the individual pairs of qubits on which Vicky performs the Bell measurements are in definite product states. Thus, although there are no quantum-mechanical measurements Vicky can perform that will reveal the individual states of the qubits, the qubits themselves carry information that is perfectly correlated with the information about Alice's and Bob's measurements, and a hidden-variable mechanism might exploit it. However, this mechanism requires a preferred foliation in which Alice's and Bob's measurements take place before Vicky's.

Finally, the relativity of pre- and postselection suggests considering the case in which all three measurements are at spacelike separation from each other as, indeed, shown in Fig. 1(b) [19,20]. By choosing an appropriate foliation, the same three measurements can be given any arbitrary time order. Thus, in this scenario, not only does the choice of foliation affect whether Alice performs a measurement on an entangled qubit or not; it also affects with which other qubit it is entangled (Fig. 3) [21,22]. The experiment can now be seen both

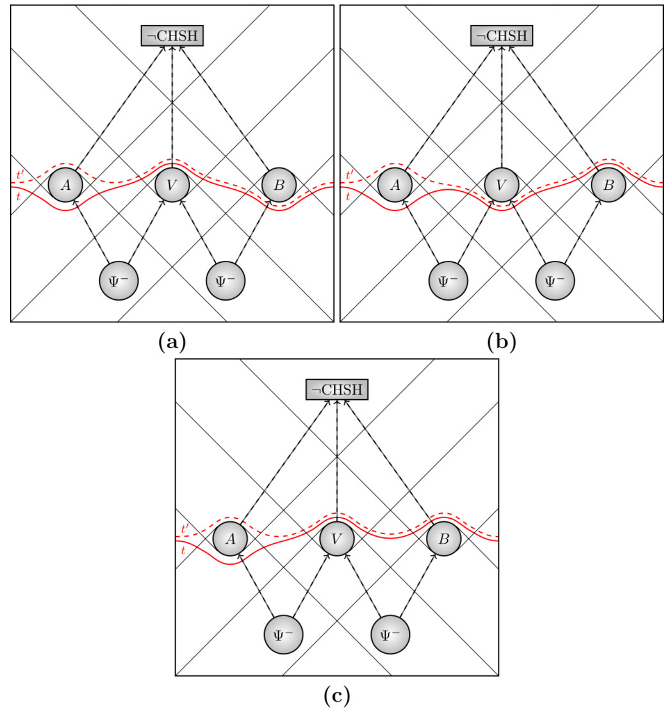

FIG. 3. Space-time diagrams with foliations with different time orderings of events. In (a) Alice performs a measurement on a qubit entangled with the qubit at B, in (b) it is entangled with the qubit in V, and in (c) it is not entangled.

as a modification of the delayed-choice entanglement swapping by Ma *et al.* and as a modification of the loophole-free Bell-EPR experiment by Hensen *et al.* [23], where Vicky's measurement is part of the preparation procedure of Alice's and Bob's qubits [24]. In this version, the experiment becomes a (loophole-free) *simultaneous test* of Bell inequality violations due to entanglement and to postselection.

VI. CONCLUSION

The quantum-mechanical predictions are invariant under change in foliation because measurements at spacelike separation commute. We have shown that, because of the relativity of pre- and postselection, the distinction between Bell inequality violations due to entanglement and those due to postselection is not invariant. What in the case of timelike separation appears to be physically different effects, in the case of spacelike separation, turns out to be one and the same physical effect.

Any dynamical explanation of our proposed experiment in terms of the evolution of the quantum state will have to be relativized to the choice of a foliation. A more fundamental frame-independent description can be given only in terms of correlations between events in the Heisenberg picture. We take it that this is also the appropriate more fundamental description when, in fact, Vicky's measurement is delayed to occur at timelike separation. After all, although in this scenario a classical dynamical explanation is, in principle, a possibility, we do not expect such a mechanism to arise simply because Vicky decided to postpone the measurement.

When in 1905 Einstein related two seemingly very different effects in the introduction to his “On the electrodynamics of moving bodies” [25], this relation led to the unification of electric and magnetic fields as one single physical object. Perhaps the relativity of pre- and postselection in violations of the Bell inequalities is trying to tell us that the very notion of a quantum state is in need of equally deep revision. This is indeed what Shimony thought about the relativity of entanglement. As he eloquently put it [10],

> The two accounts of *processes* from initial to final sets of events are in disaccord. But it is important to note that the process is a theoretical construction. . . . The thesis of peaceful coexistence presupposes a conceptually coherent reconciliation of the descriptions from the standpoints of [the frames] Σ and Σ'. Even more desirable, in the spirit of the geometrical formulation of space-time theory, would be a coordinate-free account.

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from the choice of settings at Alice's and Bob's locations to rule out a causal mechanism inducing setting-source dependence. But Alice's and Bob's measurements, although begun at space-like separation from the Bell measurement, are completed in its future light cone ([23], Fig. 2). Thus, there are space-time foliations in which the EPR measurements are performed from beginning to end in the future of the Bell measurement but no foliations in which the Bell measurement is performed in the future of the entire EPR measurements, and the violation of the Bell inequalities is unambiguously due to entanglement.

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