Letter

Three-body universality in ultracold *p*-wave resonant mixtures

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We study three-body collisions within ultracold mixtures with resonant interspecies *p*-wave interactions. Our results for the three-body effective interaction strength and decay rate are crucial towards understanding the stability and lifetime of these dilute quantum fluids. On resonance, we find that a class of universal scattering pathways emerges, regardless of the details of the short-range interactions. This gives rise quite generally to a remarkable regime where three-body effective interactions dominate over both inelastic decay and two-body effective interactions. Additionally, we find a series of mass-ratio-dependent trimer resonances further from resonance.

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Introduction. The physics of *p*-wave interactions is fundamental to many important quantum systems such as superfluid ^{3}He [1], unconventional superconductors [2], polarons [3], and halo nuclei [4,5]. This subject has received a recent surge of attention in ultracold atomic gases due to the availability of *p*-wave Feshbach resonances via which the interaction strength can be tuned in both fermionic [6–9] and mixed systems [10,11]. Recently, a powerful set of universal relations connecting thermodynamical and microscopic properties were found for ultracold Fermi gases with strong *p*-wave interactions [12–14], and such systems are predicted to display topological quantum phase transitions [15,16]. For *p*-wave resonant mixtures, an intriguing finite-momentum atomic-molecular superfluid phase is predicted [17–19]; however, these mixtures remain largely unexplored.

Determining the thermodynamics of ultracold *p*-wave resonant mixtures requires an analysis of microscopic few-body scattering processes. In the case of a mixture of weakly interacting Bose-Einstein condensates (BECs), the miscibility and stability of the system are determined by the intra- and interspecies scattering lengths which set the effective two-body interaction strengths [20–25]. However, elastic three-body scattering processes can also play a pivotal role through an effective three-body interaction, which was predicted recently to give rise to liquid quantum droplets in single-component BECs at weak interactions [26–28]. Identifying other regimes dominated by three-body effective interactions and studying the associated evolution from few- to many-body physics remains an important, open pursuit, in particular, at strong interactions, which motivates the present study.

Three-body effective interactions are typically ignored in descriptions of ultracold atomic gases due to their diluteness [29,30]. In the vicinity of an *s*-wave dimer resonance, these interactions are strong [31–35], but so are losses [33,34,36,37] and resultant heating [38–41]. We find that for *p*-wave resonant mixtures this barrier can be overcome quite generally via a set of three-body elastic scattering processes that involve *p*-wave interactions between two dissimilar particles and even occur at zero collision energy. This gives rise to an intriguing regime near a *p*-wave dimer resonance where three-body effective interactions dominate over both losses and two-body effective interactions, which opens the way to novel classes of quantum fluids.

In this Letter, we study mixed three-body systems near an interspecies *p*-wave dimer resonance. We extract the elastic transition amplitude for scattering at zero energy, which provides information on both the strength of three-body effective interactions and recombination in ultracold mixtures. We find that this transition amplitude diverges universally on resonance, depending only on a few parameters that characterize low-energy *s*- and *p*-wave two-body collisions. For two identical bosons interacting with a dissimilar particle, we also analyze how a series of trimer resonances, originating from a universal long-range three-body attraction [42,43], impacts the elastic three-body transition amplitude near the *p*-wave dimer resonance. We conclude with a discussion of the experimental and theoretical implications of our findings.

Formalism. To study three-body scattering, we start from the Alt-Grassberger-Sandhas (AGS) equations [44],

$$U_{\alpha 0}(z) = (1-\delta_{\alpha 0})G_0^{-1}(z) + \sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{3} T_\beta(z)G_0(z)U_{\beta 0}(z) \quad \text{for } \alpha = 0,1,2,3, \tag{1}$$

which define a set of transition operators $U_{\alpha 0}(z)$ for scattering of three free particles at energy z in their center-of-mass frame. The outgoing states are labeled by α and are either free-particle states ($\alpha = 0$) or a state consisting of a free particle and a dimer, in which case $\alpha = 1, 2, 3$ specifies the free particle. Here, $G_0(z)$ represents the free Green's function $(z - H_0)^{-1}$, where H_0 is the three-body kinetic energy operator in the center-of-mass frame. $T_\alpha(z)$ describes two-body scattering between particles β and γ with particle α spectating ($\alpha, \beta, \gamma = 1, 2, 3$, $\alpha \neq \beta \neq \gamma$). This means that $T_\alpha(z) =$

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$V_{\beta\gamma} + V_{\beta\gamma} G_0(z) T_\alpha(z)$, where $V_{\beta\gamma}$ indicates the pairwise potential between particles β and γ and is assumed to be spherically symmetric.

The elastic three-body transition operator $U_{00}(z)$ determines the zero-energy three-body scattering state via $|\Psi_{3b}(0)\rangle = |\mathbf{0},\mathbf{0}\rangle + G_0(0)U_{00}(0)|\mathbf{0},\mathbf{0}\rangle$, when the limit $z \to 0$ is taken from the upper half of the complex energy plane. Here, we also introduce the free-particle states $|\mathbf{p}_\alpha, \mathbf{q}_\alpha\rangle_\alpha$, where $\mathbf{p}_\alpha = \mu_{\beta\gamma}(\mathbf{P}_\beta/m_\beta - \mathbf{P}_\gamma/m_\gamma)$ and $\mathbf{q}_\alpha = \mu_{\beta\gamma,\alpha}[\mathbf{P}_\alpha/m_\alpha - (\mathbf{P}_\beta + \mathbf{P}_\gamma)/(m_\beta + m_\gamma)]$ are the Jacobi momenta describing the relative motion of the three-particle system and $\mathbf{P}_\alpha$ is the laboratory momentum of particle α. The masses m_α of particles $\alpha = 1$, 2, and 3 determine the reduced masses $\mu_{\beta\gamma} = m_\beta m_\gamma/(m_\beta + m_\gamma)$ and $\mu_{\beta\gamma,\alpha} = m_\alpha(m_\beta + m_\gamma)/(m_\alpha + m_\beta + m_\gamma)$. We normalize plane-wave states according to $\langle \mathbf{p}'|\mathbf{p}\rangle = \delta(\mathbf{p}' - \mathbf{p})$. Naturally, $|\mathbf{p}_\alpha, \mathbf{q}_\alpha\rangle_\alpha = |\mathbf{p}_\beta, \mathbf{q}_\beta\rangle_\beta$ for $\alpha, \beta = 1$, 2, or 3. The choice of $\alpha = 1$, 2, or 3 is therefore arbitrary in our definition of the elastic three-body transition amplitude ${}_\alpha\langle \mathbf{p}_\alpha, \mathbf{q}_\alpha|U_{00}(0)|\mathbf{0},\mathbf{0}\rangle$, so that we can drop the index α for notational compactness and write $\langle \mathbf{p}, \mathbf{q}|U_{00}(0)|\mathbf{0},\mathbf{0}\rangle$. This amplitude behaves as

$$\langle \mathbf{p}, \mathbf{q}|U_{00}(0)|\mathbf{0},\mathbf{0}\rangle = \sum_{\alpha=1}^{3} \Bigg\{ {}_\alpha\langle \mathbf{p}_\alpha, \mathbf{q}_\alpha|T_\alpha(0)|\mathbf{0},\mathbf{0}\rangle + \frac{A_\alpha}{q_\alpha^2} + \frac{B_\alpha}{q_\alpha} + C_\alpha \ln\left(\frac{q_\alpha \rho}{\hbar}\right) + \frac{1}{(2\pi)^6}\mathcal{U}^{(\alpha)}(\mathbf{p}_\alpha, \mathbf{q}_\alpha) \Bigg\}, \tag{2}$$

where ρ is an arbitrary length scale. The coefficients A_α, B_α, and C_α are real and depend on the masses and scattering lengths [45–48]. The functions $\mathcal{U}^{(\alpha)}(\mathbf{p}_\alpha, \mathbf{q}_\alpha)$ represent the remainder for which $\lim_{q_\alpha \to 0} \mathcal{U}^{(\alpha)}(\mathbf{0}, \mathbf{q}_\alpha)$ is finite [49]. So we define

$$\mathcal{U}_0 = \sum_{\alpha=1}^{3} \lim_{q_\alpha \to 0} \mathcal{U}^{(\alpha)}(\mathbf{0}, \mathbf{q}_\alpha). \tag{3}$$

This definition of $\mathcal{U}_0$ is closely related to the definition of the three-body scattering hypervolume considered in Refs. [26,35,50,51] for identical bosons and in Ref. [47] for dissimilar particles. In the Supplemental Material [48] we make this connection explicit. The imaginary part of $\mathcal{U}_0$ is proportional to the three-body recombination rate due to the optical theorem for three-particle scattering [48,52], whereas the real part is connected to elastic three-body scattering processes. The latter can be used to quantify the strength of an effective three-body contact interaction when modeling an ultracold quantum gas [32,33,47,53,54].

To illustrate this connection to many-body systems, we consider a dilute Bose-Bose mixture at zero temperature. A recent study [47] demonstrated that the corresponding energy density $\mathcal{E}$ can be approximated by

$$\mathcal{E} = \tfrac{1}{6}\hbar^6 \mathcal{U}_0^{(\mathrm{BBB})} n_\mathrm{B}^3 + \tfrac{1}{6}\hbar^6 \mathcal{U}_0^{(\mathrm{bbb})} n_\mathrm{b}^3 + \tfrac{1}{2}\hbar^6 \mathcal{U}_0^{(\mathrm{BBb})} n_\mathrm{B}^2 n_\mathrm{b} + \tfrac{1}{2}\hbar^6 \mathcal{U}_0^{(\mathrm{Bbb})} n_\mathrm{B} n_\mathrm{b}^2 \tag{4}$$

under the assumption that the two-body scattering lengths are negligible. Here, we have denoted the two types of bosons by B and b and the corresponding number densities by n_B and n_b, respectively. We have also added labels to $\mathcal{U}_0$ to distinguish those corresponding to different three-body systems. These amplitudes determine the stability of the mixture against collapse or phase separation [47]. The dynamics of the mixture can be studied from the corresponding Gross-Pitaevskii equations with effective three-body contact interactions whose strengths are set by the amplitudes $\mathcal{U}_0$ [47].

p-wave resonance. To see how resonant p-wave interactions influence $\mathcal{U}_0$, we expand ${}_\alpha\langle \mathbf{p}, \mathbf{q}|T_\alpha(0)|\mathbf{p}', \mathbf{q}'\rangle_\alpha$ in the Legendre polynomials $P_l(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$ as

$${}_\alpha\langle \mathbf{p}, \mathbf{q}|T_\alpha(0)|\mathbf{p}', \mathbf{q}'\rangle_\alpha = \langle \mathbf{q}|\mathbf{q}'\rangle \sum_{l=0}^{\infty} (2l+1) P_l(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') \times t_l^{(\beta\gamma)}\left(p, p', -\frac{q^2}{2\mu_{\beta\gamma,\alpha}}\right). \tag{5}$$

In contrast to identical bosons, dissimilar particles can interact via the p-wave component of the two-body transition amplitude, which behaves as

$$t_1^{(\beta\gamma)}\left(p, p', -\frac{q^2}{2\mu_{\beta\gamma,\alpha}}\right) = \frac{\frac{a_{1,\beta\gamma} p p'}{4\pi^2 \mu_{\beta\gamma}\hbar^3}}{1 - \frac{1}{2}\tilde{r}_{1,\beta\gamma} a_{1,\beta\gamma} \frac{\mu_{\beta\gamma}}{\mu_{\beta\gamma,\alpha}} \frac{q^2}{\hbar^2}} \tag{6}$$

in the limit of small p, p', and q for short-range potentials [55]. Here, $a_{1,\beta\gamma}$ is the p-wave scattering volume that diverges at the resonance, and $\tilde{r}_{1,\beta\gamma} > 0$ is the p-wave effective range [55]. For $a_{1,\beta\gamma}\tilde{r}_{1,\beta\gamma}^3 \ll -1$, the p-wave state is quasibound, whereas it is bound for $a_{1,\beta\gamma}\tilde{r}_{1,\beta\gamma}^3 \gg 1$. In the latter regime, the p-wave dimer energy is universally described by $-\hbar^2/(\mu_{\beta\gamma}\tilde{r}_{1,\beta\gamma}a_{1,\beta\gamma}^2)$. For van der Waals potentials, Eq. (6) is valid in the limit $|a_{1,\beta\gamma}| \to \infty$ [56,57], which is the exact regime we concentrate on in the following.

The p-wave component of the two-body transition amplitude contributes to $\mathcal{U}_0$ via scattering processes containing at least three T operators because the first and final T operators only contribute via their s-wave components at zero energy. The most simple scattering events containing p-wave $\beta\gamma$ interactions are thus described by $[T_\beta(0) + T_\gamma(0)]G_0(0)T_\alpha(0)G_0(0)[T_\beta(0) + T_\gamma(0)]$. A diagrammatic representation of these scattering processes is shown in Fig. 1. In the Supplemental Material [48], we demonstrate that their contributions to $\mathcal{U}_0$ near a p-wave dimer resonance scale with $\sqrt{-a_{1,\beta\gamma}}$ due to an integration over the p-wave component in Eq. (6). For positive $a_{1,\beta\gamma}$, this integration goes over a pole, resulting in the imaginary scaling $\sqrt{-a_{1,\beta\gamma}} = i\sqrt{a_{1,\beta\gamma}}$. Terms that contain more than three T operators do not contribute to the leading $\sqrt{-a_{1,\beta\gamma}}$ scaling. The dominant behavior of $\mathcal{U}_0$ close to a p-wave $\beta\gamma$ dimer resonance is thus given universally by

$$\mathcal{U}_0/\sqrt{-a_{1,\beta\gamma}} \underset{|a_{1,\beta\gamma}|\to\infty}{=} -24\sqrt{2}\pi^2\left(\frac{a_{\gamma\alpha}}{m_\gamma} - \frac{a_{\alpha\beta}}{m_\beta}\right)^2 \times \frac{\sqrt{\mu_{\beta\gamma,\alpha}\mu_{\beta\gamma}}}{\hbar^4\sqrt{\tilde{r}_{1,\beta\gamma}}}, \tag{7}$$

where the scattering lengths $a_{\alpha\beta}$ and $a_{\gamma\alpha}$ correspond to the $\alpha\beta$ and $\gamma\alpha$ interaction, respectively. This general result applies to three dissimilar particles with one resonant p-wave interaction.

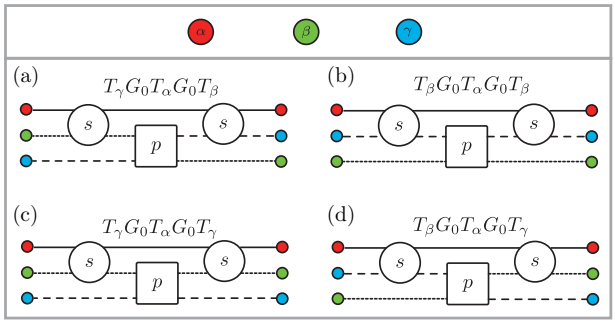

FIG. 1. Diagrammatic representation of the four distinct three-body scattering processes that result in the $\sqrt{-a_1}$ scaling of $\mathcal{U}_0$ due to one resonant p-wave interspecies interaction. In these diagrams, individual particles propagate from right to left with identities distinguished by color and line style. The vertices represent the s- (circles) or p-wave (squares) component of the two-body transition operator.

In the remainder of this Letter we focus on the BBX system, consisting of two identical bosons (B) and a distinguishable particle (X). We define m_B (m_X) as the mass of particle B (X) with mass ratio $\chi \equiv m_\mathrm{X}/m_\mathrm{B}$. In the Supplemental Material [48], we derive how the coefficients A_α, B_α, and C_α in Eq. (2) depend on χ and on the scattering lengths a_BB and a_BX corresponding to the BB and BX interaction, respectively.

For the BBX system with resonant p-wave BX interactions, the number of dominant scattering processes doubles compared with three dissimilar particles with one resonant interaction as considered in Eq. (7). This results in the universal limits

$$\mathrm{Re}(\mathcal{U}_0)/\sqrt{|a_1|} \underset{a_1\to-\infty}{=} -\frac{48\sqrt{2}\pi^2}{\sqrt{\chi(2+\chi)}}\frac{(\chi a_\mathrm{BB}-a_\mathrm{BX})^2}{m_\mathrm{X}\hbar^4\sqrt{\tilde{r}_1}} \tag{8}$$

for $a_1 < 0$ and

$$\mathrm{Im}(\mathcal{U}_0)/\sqrt{a_1} \underset{a_1\to+\infty}{=} -\frac{48\sqrt{2}\pi^2}{\sqrt{\chi(2+\chi)}}\frac{(\chi a_\mathrm{BB}-a_\mathrm{BX})^2}{m_\mathrm{X}\hbar^4\sqrt{\tilde{r}_1}} \tag{9}$$

for $a_1 > 0$ [48], where we defined $a_1 \equiv a_{1,\mathrm{BX}}$ and $\tilde{r}_1 \equiv \tilde{r}_{1,\mathrm{BX}}$ for notational convenience. Clearly, the divergent behavior of $\mathcal{U}_0$ becomes stronger for smaller mass ratios χ. Equation (9) can also be derived from the optical theorem, in which case one finds that the divergent behavior is caused only by three-body recombination into the weakly bound p-wave dimer state [48].

To study $\mathcal{U}_0$ numerically, we take a square-well potential with depth V_0 and range R to model the BX interaction. We fix the potential range R and tune the depth V_0 near $2\mu_\mathrm{BX}V_0R^2/\hbar^2 = \pi^2$, which is the point where the first p-wave dimer state gets bound. We calculate $\mathcal{U}_0$ for this BBX system by extending the method of Ref. [35], which considered three identical bosons. More specifically, starting from the AGS equations, we derive a set of integral equations for $\mathcal{U}^{(\alpha)}(\mathbf{p}_\alpha, \mathbf{q}_\alpha)$ [48], which we expand in spherical harmonics and Weinberg states [58,59] and discretize in q_α, yielding a matrix equation that can be solved numerically. For the definition of $\mathcal{U}_0$, we fix $\rho = |a_\mathrm{BX}|$ in Eq. (2). This choice of ρ is consistent with the convention of Ref. [47] when the BB interaction is set to zero as we do in our analysis presented below. This convention has, however, no effect on the universal limits in Eqs. (8) and (9).

Our numerical results for $\mathcal{U}_0$ are presented in Fig. 2 for various mass ratios and zero BB interaction. For $a_1 \to -\infty$, $\mathrm{Re}(\mathcal{U}_0)$ diverges to $-\infty$ as described by Eq. (8), whereas $\mathrm{Im}(\mathcal{U}_0)$ stays finite. Therefore elastic three-body scattering dominates over three-body recombination. For $a_1 \to +\infty$, Fig. 2 confirms the $\sqrt{a_1}$ scaling of $\mathrm{Im}(\mathcal{U}_0)$ as presented in Eq. (9), whereas $\mathrm{Re}(\mathcal{U}_0)$ diverges as $-\ln(a_1/R^3)$. The prefactor of this logarithmic behavior increases for smaller values of χ. For large mass ratios, this behavior of $\mathrm{Re}(\mathcal{U}_0)$ is very subtle for the values of a_1 considered in Fig. 2(b), since the prefactor of $-\ln(a_1/R^3)$ is very small. The inset in the lower panel of Fig. 2(b) also demonstrates that only the part of $\mathrm{Im}(\mathcal{U}_0)$ that corresponds to three-body recombination into the shallow p-wave dimer state diverges, while all other contributions stay finite at the p-wave resonance.

When $|a_1|$ decreases, $\mathcal{U}_0$ starts to behave nonuniversally. For $a_1 < 0$, Fig. 2(a) shows one trimer resonance for $\chi = 10$ near $a_1/R^3 \approx -100$ and two stronger trimer resonances for $\chi = 0.1$ near $a_1/R^3 \approx -2.17$ and -276 which result in clear peaks in $-\mathrm{Im}(\mathcal{U}_0)$. They correspond to the three-body quasibound states with zero energy and zero total angular momentum. The trimer resonances for $\chi = 0.1$ arise from a universal long-range three-body attraction that gets stronger for smaller χ [42,43], whereas the trimer resonance for $\chi = 10$ has not been predicted, and its origin is most likely nonuniversal. Figure 3 demonstrates that the trimer resonances for $\chi < 1$ constitute a series whose number increases as χ decreases. This phenomenon was predicted in Ref. [42], which investigated the trimer spectrum exactly on resonance. Our results show that the corresponding trimer resonances at the three-particle threshold are accompanied with large peaks in the three-body recombination rate. These resonances can even enhance this rate by a few orders of magnitude compared with the background value as shown in Fig. 3(a). In Fig. 3(b) we demonstrate that these trimer resonances shift towards smaller values of $|a_1|$ as χ decreases. For each trimer state there is a critical mass ratio above which the corresponding resonance has vanished. These critical mass ratios are not expected to be universal, but should depend on the details of the considered BX and BB interaction potentials.

Comparison with s-wave resonances. The universal behavior of $\mathcal{U}_0$ for the BBX system near a p-wave dimer resonance differs from the behavior near an s-wave dimer resonance (i.e., $|a_\mathrm{BX}| \to \infty$) where the Efimov effect [33,34,36,37,46,60–63] causes $\mathcal{U}_0$ to be a log-periodic function of a_BX attached to an a_BX^4 scaling. The latter is nonperturbative, while the $\sqrt{-a_{1,\mathrm{BX}}}$ scaling for resonant p-wave interactions only involves three-body collisions described by three T operators. In addition, three-body recombination into deeply bound dimer states also contributes to the leading a_BX^4 scaling on both sides of an s-wave dimer resonance, whereas such contributions are nondivergent for resonant p-wave interactions. For completeness, we present an overview of the universal behavior of $\mathcal{U}_0$ near an s-wave BX dimer resonance, of which most is already known, in the Supplemental Material [48].

Outlook. In the p-wave universal regime [Eqs. (8) and (9)], $\mathcal{U}_0$ of the BBX system diverges at a point where the s-wave

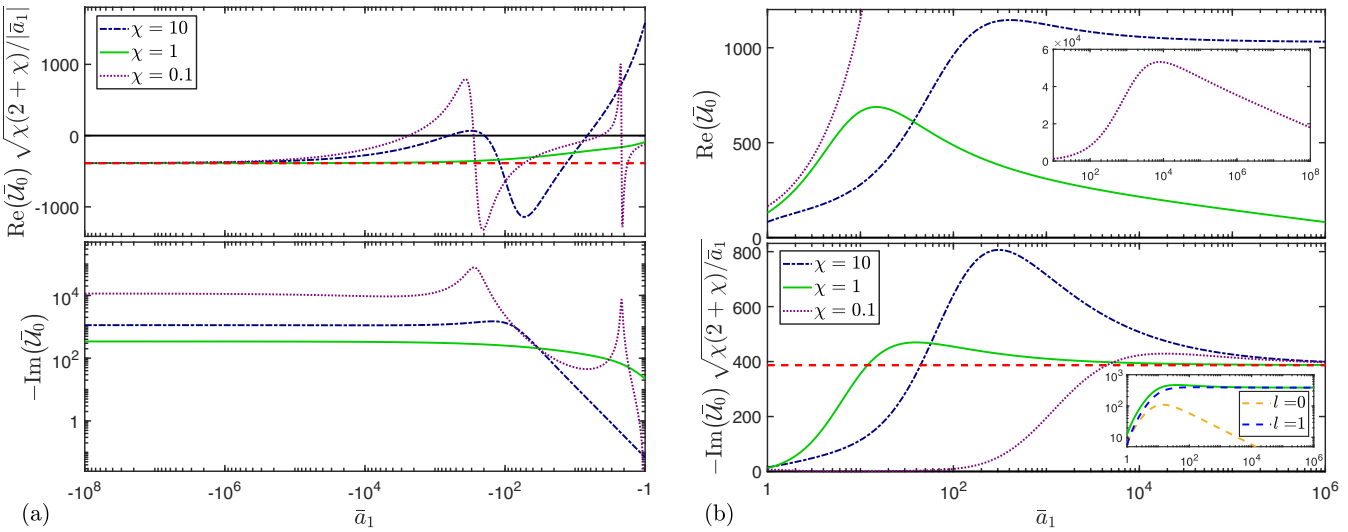

FIG. 2. $\mathcal{U}_0$ near the first p-wave BX dimer resonance of the square-well potential for various mass ratios at (a) $a_1 < 0$ and (b) $a_1 > 0$. We have defined the dimensionless quantities $\bar{\mathcal{U}}_0 \equiv \mathcal{U}_0 m_X \hbar^4/R^4$ and $\bar{a}_1 = a_1/R^3$. The BB interaction is set to zero. The red dashed lines represent Eqs. (8) and (9) with $a_{BB}/R = 0$, $a_{BX}/R = 1$, and $\tilde{r}_1 R = 3$. The parameters a_{BX} and $\tilde{r}_1$ can be regarded as constants for $|\bar{a}_1| \gtrsim 10$. For $a_1 > 0$, $\mathrm{Im}(\mathcal{U}_0)$ is determined by the three-body recombination rate into one deep s-wave dimer state ($l = 0$) and one shallow p-wave dimer state ($l = 1$). These two contributions to $\mathrm{Im}(\mathcal{U}_0)$ are presented in the inset for $\chi = 0$. The inset for $\mathrm{Re}(\mathcal{U}_0)$ at $a_1 > 0$ demonstrates its logarithmic behavior at large a_1 for $\chi = 0.1$.

scattering lengths are generally finite. This implies that three-body scattering dominates over two-body scattering at zero energy in an ultracold p-wave resonant mixture and could therefore strongly alter previous predictions for the phase diagram [17–19]. In particular, the divergent behavior of $\mathrm{Re}(\mathcal{U}_0)$ to $-\infty$ as $a_1 \to -\infty$ suggests a strong effective attraction in ultracold mixtures which could have a destabilizing effect.

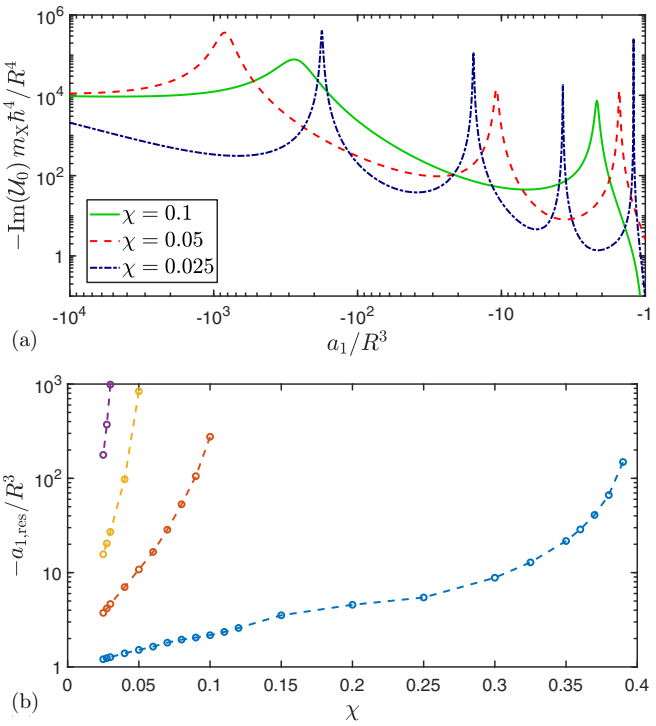

FIG. 3. (a) $-\mathrm{Im}(\mathcal{U}_0)$ near the first p-wave BX dimer resonance of the square-well potential for various mass ratios $\chi \ll 1$ at $a_1 < 0$. The BB interaction is set to zero. (b) The p-wave scattering volumes $a_{1,\mathrm{res}}$ that locate the local maxima in $-\mathrm{Im}(\mathcal{U}_0)$ for $0.025 \leqslant \chi \leqslant 0.4$.

We note that divergent behavior of $\mathrm{Re}(\mathcal{U}_0)$ can also occur when the potentials support a three-body bound state at zero energy whose total angular momentum is zero. This is only possible in the absence of dimer states to which three particles can recombine. However, our universal result in Eq. (8) applies even when deeply bound dimer states exist. This remarkable property makes the p-wave dimer resonance a promising tool to realize a divergent $\mathrm{Re}(\mathcal{U}_0)$ in atomic systems that typically support many dimer states.

On the other hand, the imaginary part of $\mathcal{U}_0$ is experimentally observable in a trapped ultracold atomic gas by measuring the atom loss from the trap as a function of time. We identify the following conditions that are required to observe the universal behavior of $\mathrm{Im}(\mathcal{U}_0)$ in Eq. (9). First, the interspecies Feshbach resonance needs to be broad enough to accurately tune a_1 up to large values. Such a broad p-wave Feshbach resonance was found in a Bose-Bose mixture of ^{85}Rb and ^{87}Rb atoms [10]. Secondly, the gas needs to be cold enough to neglect temperature effects. We expect such temperature effects to be strong due to another dominant contribution to the three-body recombination rate at positive three-body energies E and $a_1 > 0$, scaling as $E^2 a_1^{5/2}/\sqrt{\tilde{r}_1}$, which is similar for three identical fermions [64]. Therefore the thermal energy needs to be much smaller than $\hbar^2|\chi a_{BB} - a_{BX}|/(m_X a_1)$ to observe the behavior in Eq. (9). Specifically for the broad p-wave Feshbach resonance in a ^{85}Rb-^{87}Rb mixture at a magnetic field of 823.3 G [10], we find that $\hbar^2|\chi a_{BB} - a_{BX}|/(k_B m_X a_1) \approx 200$ nK for BBX = ^{85}Rb^{85}Rb^{87}Rb and 20 nK for BBX = ^{87}Rb^{87}Rb^{85}Rb [65], where we take $a_1/r_{vdW}^3 = 10^4$ according to Fig. 2(b) and r_{vdW} is the van der Waals length scale characterizing the range of the interatomic BX interaction [66]. Since the magnitude of

$\mathrm{Im}(\mathcal{U}_0)$ in Eq. (9) for BBX = ^{85}Rb ^{85}Rb ^{87}Rb is more than 100 times larger than the one for BBX = ^{87}Rb ^{87}Rb ^{85}Rb [65], the total decay rate is primarily determined by the ^{85}Rb ^{85}Rb ^{87}Rb system close to the p-wave dimer resonance. Therefore it suffices to consider temperatures that are well below 200 nK to neglect temperature effects on the total decay rate when tuning a_1/r_{vdW}^3 up to 10^4. In addition, the behavior in Eq. (9) dominates over other contributions to the total recombination rate at zero energy when a_1 is chosen large enough. Estimating these contributions generally requires accurate interaction models that account for the exact three-atom spin structure. Furthermore, it is beneficial to take $\chi \simeq 1$, since Fig. 2(b) demonstrates that the universal limit of $\mathrm{Im}(\mathcal{U}_0)$ is approached faster for $\chi = 1$ than for $\chi \ll 1$ or $\chi \gg 1$. Fortunately, a good candidate is readily available in a mixture of ^{85}Rb and ^{87}Rb. Lastly, three-body recombination into the shallow p-wave dimer state only gives rise to atom loss when the depth of the trapping potential is smaller than the binding energy of this dimer state. Tuning a_1 to large values thus provides an efficient way to create weakly bound p-wave molecules that remain trapped, since only the three-body recombination rate into the shallow dimer state diverges on resonance.

Finally, we note that the magnetic dipole-dipole interaction between the valence electrons of alkali-metal atoms splits a p-wave Feshbach resonance into two [8,9]. This splitting depends on the quantum number corresponding to the projection of the molecular orbital angular momentum onto the magnetic field axis. Therefore the universal limits in Eqs. (7)–(9) will have an additional dependence on this quantum number for these atoms. Nevertheless, we expect that the $\sqrt{-a_1}$ scaling of $\mathcal{U}_0$ is unchanged in the regime where the p-wave dimer binding energy is well described by $\hbar^2/(\mu_{\mathrm{BX}}\bar{r}_1 a_1)$.

Conclusion. We have studied zero-energy scattering for mixed three-body systems with resonant p-wave interspecies interactions. We have found a universal relation between the three-body transition amplitude $\mathcal{U}_0$ and p-wave scattering volume a_1, behaving as $\mathcal{U}_0 \propto \sqrt{-a_1}$. For $a_1 > 0$, $\mathcal{U}_0$ is dominated by three-body recombination into the weakly bound p-wave dimer state. For $a_1 < 0$, the dominant contribution comes from elastic three-body scattering processes that involve three successive two-body collisions. The limit $a_1 \to -\infty$ thus offers a special regime in which elastic three-body scattering dominates over two-body scattering and three-body recombination in ultracold mixtures. This general effect could significantly impact the phase diagram of these gases. For smaller values of $|a_1|$, $\mathcal{U}_0$ of the BBX system is influenced by a series of trimer states consisting of one light particle (X) and two heavy bosons (B). This could be relevant for nuclear systems for which other trimer states bound by strong p-wave interactions have been found [4,5].

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