


## Polarizing the medium: Fermion-mediated interactions between bosons

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We consider a homogeneous mixture of bosons and polarized fermions. We find that long-range and attractive fermion-mediated interactions between bosons have dramatic effects on the properties of the bosons. We construct the phase diagram spanned by boson-fermion mass ratio and boson-fermion scattering parameter. It consists of a stable region of mixing and unstable region toward phase separation. In the stable mixing phase, the collective long-wavelength excitations can either be well behaved with infinite lifetime or be finite in lifetime suffered from the Landau damping. We examine the effects of the induced interaction on the properties of weakly interacting bosons. It turns out that the induced interaction not only enhances the repulsion between the bosons against collapse but also enhances the stability of the superfluid state by suppressing quantum depletion.

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Ultracold atoms offer fascinating opportunities for investigating quantum many-body problems that are relevant to fields as diverse as condensed matter physics, statistical physics, quantum chemistry, and high-energy physics [1,2]. Of particular interest are Bose-Fermi mixtures [3], which allows one to explore the intriguing physics associated with the interplay between atoms of different quantum statistics. On the experimental side, tremendous progress has been achieved, which includes controlling and characterizing the interspecies interactions [4–12], realizing a mixture of Bose and Fermi superfluids [13–17], and probing physics of the phase separation state [18,19]. On the theoretical side, intense attention has been paid to study ground-state properties [20–26], nature of excitations [27–31], boson-mediated fermionic superfluidity [32–36], collective dynamics [37–39], and formation of exotic quantum phases [40–46].

Very recently, adding to the excitement are the observations of fermion-mediated long-range interactions between bosons [47–49]. The long-range nature of these mediated interactions enriches the toolbox for controlling coherent interactions [50] and opens up the possibility of correlating distant atoms and preparing quantum phases [40,51]. There have been theoretical attempts [52–54] at understanding such fermion-mediated interactions based on the linear response theory. Given current experimental relevance, thorough theoretical understanding and identifying features arising from fermion-mediated interactions becomes an urgent task. Such study will contribute to a better understanding of the emergence of induced interactions between quasiparticles [55–61].

In this work, we will carry out a comprehensive study on the fermion-mediated interactions in Bose-Fermi mixtures, with the aim of laying down a solid and tractable framework to treat such problems, fully characterizing the mediated interactions, and elucidating the effects of the induced interactions on the bosons. First, we will start from the functional representation of the partition function of the system. By tracing out the fermionic degrees of freedom, we obtain an effective action solely in terms of bosonic degrees of freedom, so that we can isolate the mediated effects of the fermions on the bosons. Second, we will examine the induced interaction at static limit in order to obtain an effective interaction potential. Third, we will construct a phase diagram by taking account of both phase stability and the Landau damping of Bogoliubov excitations arising from density response from the Fermi gases. Finally, we examine the quantum fluctuations in the presence of the effective potential on the properties of the bosons.

We consider a homogeneous mixture of Bose gases and spin-polarized Fermi gases, described by the following grand canonical Hamiltonian:

$$H = H_B + H_F + H_I, \quad (1a)$$

$$H_B = \int d\mathbf{r} \phi^\dagger(\mathbf{r}) \left( -\frac{\hbar^2 \nabla^2}{2m_B} - \mu_B \right) \phi(\mathbf{r}), \quad (1b)$$

$$H_F = \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \left( -\frac{\hbar^2 \nabla^2}{2m_F} - \mu_F \right) \psi(\mathbf{r}), \quad (1c)$$

$$H_I = \int d\mathbf{r} \left( g_I \psi^\dagger \psi \phi^\dagger \phi + \frac{g}{2} \phi^\dagger \phi^\dagger \phi \phi \right). \quad (1d)$$

For bosons,  $\phi(\mathbf{r})$  is the field operator,  $m_B$  is the mass of an atom, and  $\mu_B$  is the chemical potential. For fermions,  $\psi(\mathbf{r})$  is the field operator,  $m_F$  is the mass of an atom, and  $\mu_F$  is the chemical potential. In the interaction term  $H_I$ , the coupling

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$g_I = 2\pi\hbar^2 a_{FB}(m_F^{-1} + m_B^{-1})$  accounts for the interactions between the fermions and the bosons, and  $g = 4\pi\hbar^2 a_{BB}/m_B > 0$  accounts for the repulsive interactions between bosons, where  $a_{FB}$  and  $a_{BB}$  are the corresponding  $s$ -wave scattering lengths. For convenience, we define the Fermi momentum  $k_F = (6\pi^2 n_F)^{1/3}$  with  $n_F$  being the number density of Fermi gases, the Fermi velocity  $v_F = \hbar k_F/m_F$  and the corresponding Fermi energy  $E_F = \hbar^2 k_F^2/2m_F$ . We will take natural units by setting  $\hbar = k_B = 1$  for sake of simplicity from now on.

Within the framework of the imaginary-time field integral, the partition function of the system can be cast as  $\mathcal{Z} = \int d[\bar{\psi}, \psi] d[\phi^*, \phi] e^{-S}$  with the action given by [62]  $S = \int_0^\beta d\tau [H + \int d^3\mathbf{r}(\bar{\psi}\partial_\tau\psi + \phi^*\partial_\tau\phi)]$ , where  $\beta = 1/T$  is the inverse temperature. Carrying out the integration over the fermionic degrees of freedom, we obtain an effective action solely in terms of bosonic degrees of freedom  $S_{\text{eff}} = S_B - \text{Tr} \ln \mathcal{M}$ , where

$$\begin{aligned}
 S_B &= \int d\tau d\mathbf{r} \left[ \phi^*(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_B)\phi + \frac{g}{2}(\phi^*\phi)^2 \right], \\
 \mathcal{M} &= \partial_\tau - \frac{\nabla^2}{2m_F} - \mu_F + g_I\phi^*\phi. \quad (2)
 \end{aligned}$$

Up to this level, the formal manipulation of the partition function is exact. To distill low-energy physics, we will resort to some sort of approximations to be elaborated on.

To proceed, we may write  $\phi^*\phi = \rho_0 + \sum_{q \neq 0} \rho_q e^{iqx}$ , and we set  $\mathcal{M} = -\mathcal{G}^{-1} + \mathcal{M}_1$  where  $\mathcal{G}^{-1} = -\partial_\tau + \nabla^2/2m_F + \mu_F - g_I\rho_0$  is the inverse fermionic Green's function and  $\mathcal{M}_1 = g_I \sum_{q \neq 0} \rho_q e^{iqx}$ , with  $x$  being space-time coordinate. This allows one to write  $\text{Tr} \ln \mathcal{M} = \text{Tr} \ln(-\mathcal{G}^{-1}) + \text{Tr} \ln(1 - \mathcal{G}\mathcal{M}_1)$  and to perform series expansions as follows:

$$-\text{Tr} \ln(1 - \mathcal{G}\mathcal{M}_1) = \sum_{l=1} \frac{1}{l} \text{Tr}[(\mathcal{G}\mathcal{M}_1)^l]. \quad (3)$$

To fully exploit the translational invariance of the system, we will transform the above to momentum-frequency representation [ $q \equiv (\mathbf{q}, iw_m)$ ] resulting in

$$\text{Tr}(\mathcal{G}\mathcal{M}_1) = \mathcal{M}_1(0) \sum_{(\mathbf{k}, iw_n)} \mathcal{G}(k) = 0, \quad (4a)$$

$$\frac{1}{2} \text{Tr}[(\mathcal{G}\mathcal{M}_1)^2] = \beta V \frac{g_I^2}{2} \sum_{q \neq 0} \Pi_q \rho_q \rho_{-q}, \quad (4b)$$

$$\Pi_q = \frac{1}{\beta V} \sum_{(\mathbf{k}, iw_n)} \mathcal{G}(k)\mathcal{G}(k+q). \quad (4c)$$

Several comments are in order: For the expansion in Eq. (3), the  $l = 1$  term vanishes, as can be seen from Eq. (4a); The  $l = 2$  term corresponds to induced two-body interactions between bosons, as can be seen from Eqs. (4b) and (4c), where we have defined the so-called polarization function  $\Pi_q$ , describing the response of the Fermi gases under external density perturbation;  $V$  is the volume of the system,  $w_n = \pi(2n + 1)/\beta$  is fermionic Matsubara frequencies, while  $w_m = 2\pi n/\beta$  is the bosonic Matsubara frequencies, where  $n$ 's are integers; We will neglect  $l \geq 3$  terms, as they represent induced three-body or more than three-body interactions for bosons, which are usually irrelevant for dilute

gases. To be concrete, the effective action for the system is approximated as

$$S_{\text{eff}} = S_B - \text{Tr} \ln(-\mathcal{G}^{-1}) + \beta V \frac{g_I^2}{2} \sum_{q \neq 0} \Pi_q \rho_q \rho_{-q}. \quad (5)$$

We follow the standard Bogoliubov decomposition by splitting the bosonic field  $\phi$  into a mean-field part  $\phi_0$  and a fluctuating part  $\varphi$ :  $\phi = \phi_0 + \varphi$ . By retaining the fluctuating fields up to the quadratic order, we approximate the effective action as  $S_{\text{eff}} \approx S_0 + S_g$ , where  $S_0$  is the mean-field action and  $S_g$  is the Gaussian action with quadratic orders of the fluctuating fields  $\varphi_q^*$  and  $\varphi_q$ . Employing  $\Omega = -\ln \mathcal{Z}/\beta V$ , we obtain that the grand potential density at the mean-field level  $\Omega^{(0)} = S_0/\beta V = \frac{g|\phi_0|^4}{2} - \mu_B|\phi_0|^2 - \frac{1}{\beta V} \sum_{\mathbf{k}} \ln(1 + e^{-\beta\xi_{\mathbf{k}}})$ , where  $\xi_{\mathbf{k}} = \mathbf{k}^2/2m_F - \mu_F + g_I|\phi_0|^2$ .

Minimization of  $\Omega^{(0)}$  with respect to the condensate order parameter  $\phi_0^*$  leads to the Hugenholz-Pines theorem [63] determining the chemical potential  $\mu_B = g_I n_F + g|\phi_0|^2$ . Without loss of generality, we will take  $\phi_0 = \sqrt{n_B}$ , where  $n_B$  is the condensate density of the Bose gases. The self-consistent condition for the fermion density is determined via  $n_F = -\partial\Omega^{(0)}/\partial\mu_F$ , which gives  $\mu_F = E_F + g_I n_B$ . Therefore the ground-state energy density at mean-field level can be found as  $E_G^{(0)} = \frac{3}{5}n_F E_F + \frac{g}{2}n_B^2 + g_I n_F n_B$ .

For the system to be stable, we naturally require that the Hessian matrix  $\partial^2 E_G^{(0)}/\partial n_i \partial n_j$  ( $i, j = F, B$ ) constructed for the ground-state energy  $E_G(n_F, n_B)$  to be positively definite, which leads to an upper bound for the fermion density

$$n_F^{1/3} < \frac{g}{3m_F g_I^2} (6\pi^2)^{2/3}, \quad (6)$$

which gives the condition for mechanical stability of the system [32].

As seen from the effective action in Eq. (5), the Hamiltonian describing induced two-body interactions between bosons through coupling with fermions is given by

$$H_{\text{ind}} = \frac{g_I^2}{2} \sum_{q \neq 0} \sum_{\mathbf{k}, \mathbf{p}} \Pi_q \phi_{\mathbf{k}+\mathbf{q}}^\dagger \phi_{\mathbf{p}-\mathbf{q}}^\dagger \phi_{\mathbf{p}} \phi_{\mathbf{k}}. \quad (7)$$

Here,  $\Pi_{\mathbf{q}} \equiv \Pi_{(\mathbf{q}, 0)}$  is the polarization function evaluated at the static limit at zero temperature, which reads

$$\Pi_{\mathbf{q}} = -\frac{d(E_F)}{4} \left[ 1 + \frac{k_F^2 - q^2/4}{k_F q} \ln \left| \frac{q + 2k_F}{q - 2k_F} \right| \right], \quad (8)$$

where  $d(E_F) = m_F k_F/\pi^2$  is the density of states at the Fermi energy.

This corresponds to an induced pairwise interaction potential between two Bose atoms with relative coordinate  $\mathbf{r}$ , given by  $V_{\text{ind}}(\mathbf{r}) = \sum_{q \neq 0} g_I^2 \Pi_q e^{iq\mathbf{r}}$ , where the constant part of  $\Pi_q$  does not contribute to the  $V(\mathbf{r})$  due to the exclusion of  $q = 0$  component. As shown in Eq. (8),  $\Pi_q$  only depends on the magnitude of momentum  $q$ , so after carrying out the solid angle integration and then using the residue theorem for the

integral with respect to  $q$ , we have

$$V_{\text{ind}}(\mathbf{r}) = -\frac{d(E_F)g_I^2}{4}V_{\text{RKKY}}(r), \quad (9a)$$

$$V_{\text{RKKY}}(r) = \frac{\sin(2k_F r) - 2k_F r \cos(2k_F r)}{2\pi k_F r^4}. \quad (9b)$$

The induced attractive long-range interaction is of the RKKY type [64–66] in real space, where it decays at  $1/r^3$  and shows the Friedel oscillations at a wave vector  $2k_F$ .

The Gaussian action for the bosonic fluctuating fields can be written as  $S_g = \frac{1}{2} \sum_q \Phi_q^\dagger \mathcal{G}_B^{-1}(q) \Phi_q$  with  $\Phi_q = (\varphi_q, \varphi_{-q}^*)^T$  and the matrix  $\mathcal{G}_B^{-1}(q) = (\epsilon_q + A_q)I - iw_m \sigma_z + A_q \sigma_x$ , where  $\epsilon_q = \mathbf{q}^2/2m_B$  and  $A_q = (g + g_I^2 \Pi_q)n_B$ . The quasiparticle spectrum  $\omega(\mathbf{q})$  and the damping rate  $\gamma(\mathbf{q})$  can be obtained by seeking solutions of the secular equation  $\det \mathcal{G}_B^{-1}(\mathbf{q}, \omega - i\gamma) = 0$  with substitution of  $\Pi_q i w_m \rightarrow \omega + i0^+$ . By analytic continuation to real frequency ( $i\omega \rightarrow \omega + i0^+$ ), one obtains the real part and the imaginary part of the polarization function, the so-called Lindhard function [67]

$$\text{Re}\Pi = -\frac{d(E_F)}{4} \left[ 1 + \sum_{s=\pm} s \frac{1 - u_s^2}{2q/k_F} \ln \left| \frac{1 + u_s}{1 - u_s} \right| \right], \quad (10a)$$

$$\text{Im}\Pi = d(E_F) \frac{\pi k_F}{8q} \left[ \sum_{s=\pm} s(1 - u_s^2) \Theta(1 - u_s^2) \right], \quad (10b)$$

where  $u_{\pm} = \omega/qv_F \pm q/2k_F$  and  $\bar{s} = -s$ . The imaginary part of the polarization function provides essential information for the damping of excitations, as it can be related to the dynamical structure factor through the fluctuation-dissipation theorem [68]. For a given  $q$ , it differs from zero only in a well-defined range of frequencies. This range is determined by the geometry of the Fermi surface and is directly related to the possible excitation energies of particle-hole pairs. We show the region in momentum-frequency space where the imaginary part of the polarization function  $\text{Im}\Pi(\mathbf{q}, \omega)$  differs from zero in Fig. 1(a). In the plot, region I is defined as  $u_-^2 < 1$  and  $u_+^2 > 1$ , while region II satisfies  $u_+^2 < 1$ . The upper (lower) bound is given by  $\omega_{\pm}/E_F = (q/k_F)^2 \pm 2q/k_F$ . In Fig. 1(b), we plot  $\text{Im}\Pi$  as a function of frequencies  $\omega$  for three typical momenta amplitude  $q/k_F = 0.5, 1.0, 1.5$ . The curve will be linear when the corresponding  $(\mathbf{q}, \omega)$  lying in region II, where both terms in Eq. (10b) contribute. Otherwise, the curve will be parabolic when the corresponding  $(\mathbf{q}, \omega)$  lying in region I.

With the information of the polarization function at hand, it is straightforward to obtain the quasiparticle spectrum  $\omega(\mathbf{q})$  and the damping rate  $\gamma(\mathbf{q})$ , shown in Fig. 2. From Eq. (10b), one obtains that the region for damping to occur is given by the inequality constraint  $(q/k_F)^2 - 2q/k_F < \omega/E_F < (q/k_F)^2 + 2q/k_F$ , as shown in the shade region on Fig. 2(a) in Fig. 2. At small momenta, the spectrum is phononlike with the sound velocity given by  $c = \sqrt{(g + g_I^2 \Pi_0)n_B/m_B}$ . The positivity of the sound velocity yields the stability constraint  $(k_F a_{FB})^2 < 2\pi k_F a_{BB} m_F m_B / (m_F + m_B)^2$ , coincident to the mechanical stability provided in Eq. (6). The mass of bosons  $m_B$  has dramatic effects on the spectrum: at low momenta, the slope is inversely proportional to  $\sqrt{m_B}$ ; while at high momenta, it gives the mass for free particle. For sufficiently small  $m_B/m_F$ , the excitations can achieve infinite lifetime.

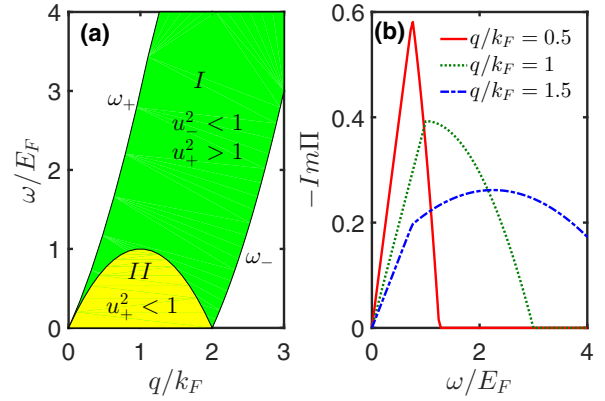


FIG. 1. (a) The shade region is the range where the imaginary part of the polarization function differs from zero, and it is referred as particle-hole continuum, since it is the region of single-particle excitations, whereby a particle below the Fermi surface is excited to above the Fermi surface. Outside this region, it is not possible to conserve energy and wave vector in a single-particle excitation process. (b) The imaginary part of the polarization function  $\text{Im}\Pi(\mathbf{q}, \omega)$  [in units of  $d(E_F)$ ] as a function of frequencies  $\omega$  at given different typical momentum amplitude  $q$ .

The damping of the excitations show sharp peak at wave vector  $q = k_F$  for all three typical boson-fermion scattering parameter  $k_F a_{FB}$ , which is associated with the largest density of states at the Fermi surface.

We are now in a position to construct a phase diagram for the system. The stability constraint marks the transition line between stable mixing phase and phase separation (PS) into fermions and bosons [20,21,23,69], shown in Fig. 3, which stays intact for different number density ratio. In the stable region, we can further classify it into quasiparticle excitations with no damping and with damping. To search for well-defined, long-lived excitations, we consider the region where  $\text{Im}\Pi(\mathbf{q}, \omega) = 0$ . This occurs when  $\omega/qv_F > 1 + q/2k_F$  [see Eq. (10b)]. At long wavelength, this becomes  $c/E_F > 2/k_F$ ,

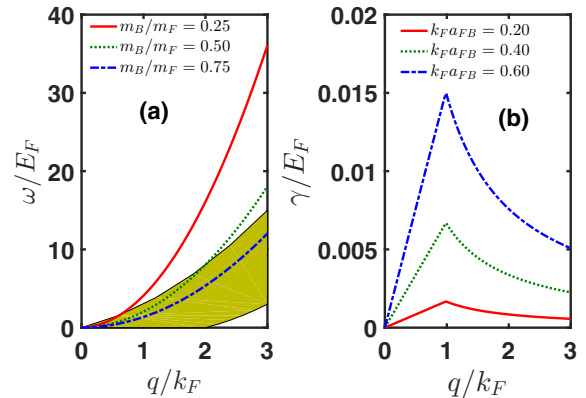


FIG. 2. Properties of the Bogoliubov quasiparticles: (a) the excitation energy  $\omega/E_F$  where  $k_F a_{FB} = 0.3$  and (b) the Landau damping rate  $\gamma/E_F$  where  $m_B/m_F = 1$ . The shade region is referred as particle-hole continuum. The quasiparticle spectrum laying outside of the shadow region is well-defined, being immune from the Landau damping. We set  $k_F a_{BB} = 0.3$  and  $n_B/n_F = 0.2$ .

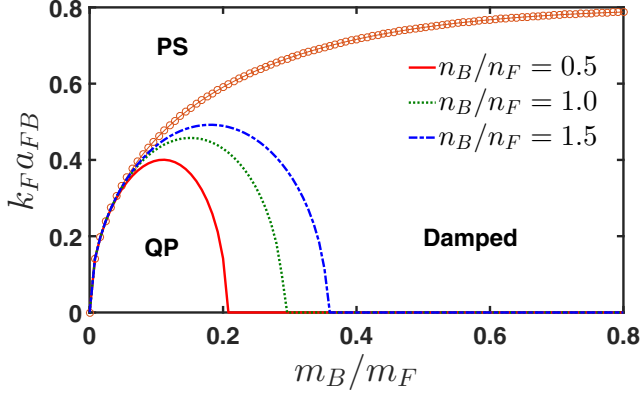


FIG. 3. Phase diagram spanned by mass ratio  $m_B/m_F$  and interspecies coupling strength  $k_F a_{FB}$ . PS stands for phase separation and QP stands for quasi-particle with infinite lifetime. Here we set  $k_F a_{BB} = 0.4$ , which sets bosons in a weakly interacting regime.

yielding

$$(k_F a_{FB})^2 < \frac{2\pi m_F m_B}{(m_F + m_B)^2} \left( k_F a_{BB} - \frac{3\pi}{2} \frac{n_F}{n_B} \frac{m_B^2}{m_F^2} \right). \quad (11)$$

The phase diagram constructed is shown in Fig. 3. The region of the quasiparticle excitations with infinite lifetime (QP) gets expanded by tuning up the number density ratio  $n_B/n_F$ . It should be pointed out that we focus on the Landau damping of the collective long-wavelength excitation, where Beliaev damping is strongly suppressed at low momenta, as it is proportional to  $q^5$  at zero temperature [70,71].

To examine the effects of the effective potential upon the Bose gases, we will evaluate the ground-state energy correction arising from quantum fluctuations. The fluctuation correction to the thermodynamic potential is given by  $\Omega_f = \frac{\beta}{2} \text{Tr} \ln \mathcal{G}_B^{-1} - \sum_{\mathbf{q}} (\epsilon_{\mathbf{q}} + A_{\mathbf{q}})$ . At zero temperature, the corresponding ground-state energy correction, becomes renormalized as  $\Delta E_G = \frac{1}{2} \sum_{\mathbf{q}} (\omega_{\mathbf{q}} - \epsilon_{\mathbf{q}} - A_{\mathbf{q}} + g^2 n_B^2 / 2\epsilon_{\mathbf{q}})$ , where  $\omega_{\mathbf{q}} = \sqrt{\epsilon_{\mathbf{q}}(\epsilon_{\mathbf{q}} + A_{\mathbf{q}})}$ , and  $A_{\mathbf{q}} = (g + g_I^2 \Pi_{\mathbf{q}}) n_B$ .

The behavior of fluctuation correction to the ground-state energy  $\Delta E_G$  is shown in Fig. 4. In Fig. 4(a), we find that as the mass ratio  $m_B/m_F$  increases the energy correction decreases, which is reasonable since the kinetic energy is inversely proportional to the mass of the bosons. Increasing the density ratio  $n_B/n_F$  contributes to the enhancement of the energy correction as  $\Delta E_G \propto n_B^{5/2}$ . Shown in Fig. 4(b), the energy correction increases monotonically with boson-fermion coupling strength  $k_F a_{FB}$ . For vanishing boson-fermion interaction, one can verify that the energy correction recovers the Lee-Huang-Yang correction [72] to the spinless weakly interacting bosons  $\Delta E_G / (g n_B^2) = 64 / (15 \sqrt{\pi}) \sqrt{n_B a_{BB}^3}$ . It is remarkable that the correction of energy increases steadily with increasing  $k_F a_{FB}$ . This raises the possibility of realizing quantum droplets states with enhanced quantum repulsion again collapse.

We turn to the depletion of the condensates due to quantum fluctuations, which provides key information about the robustness of the superfluid state. The number of excited

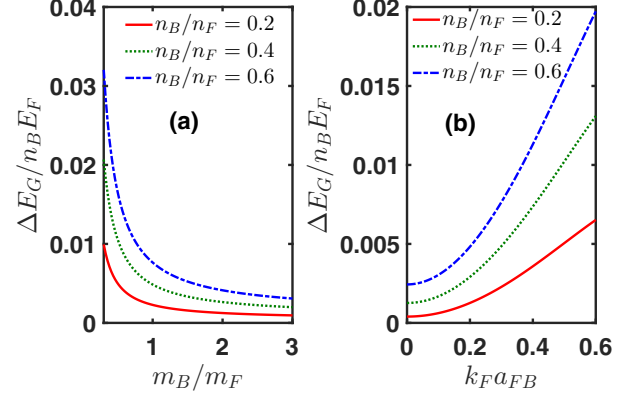


FIG. 4. Correction to the ground-state energy per density  $\Delta E_G / n_B E_F$  (a) as a function of mass ratio  $m_B/m_F$  where  $k_F a_{FB} = 0.3$  and (b) as a function of boson-fermion scattering length  $k_F a_{FB}$  where  $m_B/m_F = 1$ . We set  $k_F a_{BB} = 0.3$ .

particles is evaluated as  $n_{\text{ex}} = \sum_{\mathbf{q}} \mathcal{G}_{B11}(\mathbf{q}, i w_n)$ . The dependence of quantum depletion on tuning parameters  $m_B/m_F$  and  $k_F a_{FB}$  are shown in Fig. 5 for fixed density ratios. Aesthetically appealingly,  $n_{\text{ex}}/n_B$  develops a maximum at equal mass  $m_B = m_F$ . Remarkably, increasing the boson-fermion interaction suppresses the quantum depletion, due to attractive nature of induced interaction between bosons. At zero boson-fermion coupling, it recovers the known result [73] for spinless weakly interacting Bosons  $n_{\text{ex}} = (g m_B n_B)^{3/2} / (3\pi^2)$ . In summary, we find that the induced interaction mediated by fermions between bosons are long-range attractive interactions, tunable with fermion density as well as boson-fermion scattering length. We map out the phase boundary separating stable region of mixing phases and unstable region toward phase separation. We show that the stable region can be further classified by damping of the excitations. The predicted damping rate can be probed experimentally via two-phonon Bragg spectroscopy [74]. Extension of current work to trapped cases [3,75] will facilitate the experimental verification of our predictions. Finally, we analyze the effects

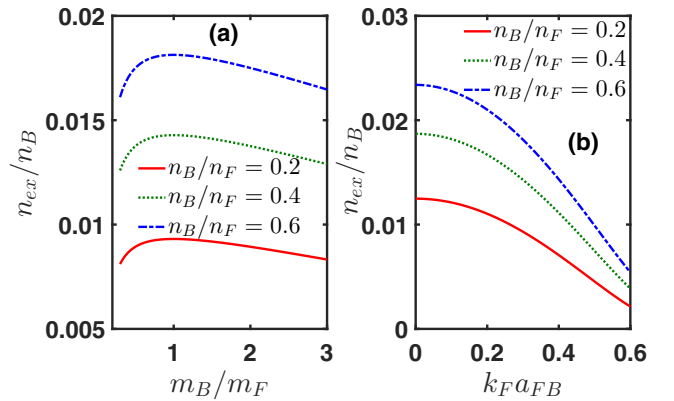


FIG. 5. Quantum depletion of the condensates  $n_{\text{ex}}/n_B$  (a) as a function of mass ratio  $m_B/m_F$  where  $k_F a_{FB} = 0.3$  and (b) as a function of boson-fermion scattering parameter  $k_F a_{FB}$  where  $m_B/m_F = 1$ . The vertical dash line intercepts with maxima of the curves at  $m_B = m_F$ . We set  $k_F a_{BB} = 0.3$ .

of the induced interactions on the ground-state energy correction and quantum depletion of the system. It suggests that by coupling to Fermi gases, weakly interacting bosons may form quantum droplet states [44,45] with enhanced stability. We expect our study to contribute to a better understanding of emergent phenomena associated with fermion-mediated interactions.

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