Letter | Editors' Suggestion

Nonlinear Bell inequality for macroscopic measurements

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The correspondence principle suggests that quantum systems grow classical when large. Classical systems cannot violate Bell inequalities. Yet agents given substantial control can violate Bell inequalities proven for large-scale systems. We consider agents who have little control, implementing only general operations suited to macroscopic experimentalists: preparing small-scale entanglement and measuring macroscopic properties while suffering from noise. That experimentalists so restricted can violate a Bell inequality appears unlikely, in light of earlier literature. Yet we prove a Bell inequality that such an agent can violate, even if experimental errors have variances that scale as the system size. A violation implies nonclassicality, given limitations on particles' interactions. A product of singlets violates the inequality; experimental tests are feasible for photons, solid-state systems, atoms, and trapped ions. Consistently with known results, violations of our Bell inequality cannot disprove local hidden-variables theories. By rejecting the disproof goal, we show, one can certify nonclassical correlations under reasonable experimental assumptions.

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Can large systems exhibit nonclassical behaviors such as entanglement? The correspondence principle suggests not. Yet experiments are pushing the quantum-classical boundary to larger scales [1–7]: Double-slit experiments have revealed interference of organic molecules' wave functions [4]. A micron-long mechanical oscillator's quantum state has been squeezed [5]. Many-particle systems have given rise to nonlocal correlations [8–10].

Nonlocal correlations are detected with Bell tests. In a Bell test, systems are prepared, separated, and measured in each of many trials. The outcome statistics may violate a Bell inequality. If they do, they cannot be modeled with classical physics, in the absence of loopholes.

Bell inequalities have been proved for settings that involve large scales (e.g., [9–29]); see [30,31] for reviews and the Supplemental Material Note A [32] for a detailed comparison with our results. We adopt a different approach, considering which operations a macroscopic experimentalist can perform easily: preparing small-scale entanglement and measuring large-scale properties, in our model. Whether such a weak experimentalist can violate a Bell inequality, even in the absence of noise, is unclear *a priori*. Indeed, our experimentalist can violate neither Bell's 1964 inequality [33,34], nor any previously proved macroscopic Bell inequality to which our main result does not reduce [9–28].[1] Nevertheless, we prove a macroscopic Bell inequality that can be violated with these operations, even in the presence of noise. The key is the macroscopic Bell parameter's nonlinearity in the probability distributions over measurement outcomes.

Our inequality is violated by macroscopic measurements of, e.g., a product of $N > 1$ singlets. Such a state has been prepared in a wide range of platforms, including photons [35], solid-state systems [36], atoms [37,38], and trapped ions [39]. A violation of the inequality implies nonlocality if microscopic subsystems are prepared approximately independently. Similarly, independence of pairs of particles is assumed in [34,40,41], though it may be difficult to guarantee.

This independence requirement prevents violations of our inequality from disproving local hidden-variables theories (LHVTs), as no experimentalist restricted like ours can [34,40,41]. By forfeiting the goal of a disproof, we show one can certify entanglement under reasonable experimental assumptions. This certification is device independent, requiring no knowledge of the state or experimental apparatuses, apart from the aforementioned independence. Furthermore, our inequality is robust with respect to errors, including a lack of subsystem independence, whose variances scale as N. Additionally, with our strategy, similar macroscopic Bell

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[1]Navascués *et al.* prove a macroscopic Bell inequality that governs a similarly restricted experimentalist [29]. However, [29] does not address noise, with respect to which our result is robust. See the Supplemental Material Note A [32] for a detailed comparison of [29] with our result.

inequalities can be derived for macroscopic systems that satisfy different independence assumptions.

Aside from being easily testable with platforms known to produce Bell pairs, our inequality can illuminate whether poorly characterized systems harbor entanglement. Such tests pose greater challenges but offer greater potential payoffs. Possible applications include Posner molecules [42–45], tabletop experiments that simulate cosmological systems [46], and high-intensity beams.

The rest of this paper is organized as follows. We introduce the setup in Sec. I. Section II contains the main results: We present and prove the Bell inequality for macroscopic measurements, using the covariance formulation of a microscopic Bell inequality [47]. Section III contains a discussion: We compare quantum correlations and global classical correlations as resources for violating our inequality, show how to combat experimental noise, reconcile violations of the inequality with the correspondence principle [34,40,41], recast the Bell inequality as a nonlocal game, discuss a potential application to Posner molecules [42–45], and detail opportunities.

I. SETUP

Consider an experimentalist Alice who has a system A and an experimentalist Bob who has a disjoint system B. Each system consists of N microscopic subsystems, indexed with i. The ith subsystem of A can interact with the ith subsystem of B but with no other subsystems. Our setup resembles that in [34].

Alice can measure her system with settings $x = 0, 1$, and Bob can measure his system with settings $y = 0, 1$. Each measurement yields an outcome in $[0, 1]$.[2] The experimentalist observes the sum of the microscopic outcomes, the value of a *macroscopic random variable*. Measuring A with setting x yields the macroscopic random variable A_x. B_y is defined analogously.

We will often illustrate with two beams of photons. The polarization of each photon in beam A is entangled with the polarization of a photon in beam B and vice versa. Such beams can be produced through spontaneous parametric down conversion (SPDC) [48]: A laser beam strikes a nonlinear crystal. Upon absorbing a photon, the crystal emits two photons entangled in the polarization domain: $\frac{1}{\sqrt{2}}(|\mathrm{H}, \mathrm{V}\rangle + e^{i\alpha}|\mathrm{V}, \mathrm{H}\rangle)$. Horizontal and vertical polarizations are denoted by $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$. The relative phase depends on some $\alpha \in \mathbb{R}$. The photons enter different beams. Each experimentalist measures his/her beam by passing it through a polarizer, then measuring the intensity. The measurement setting (Alice's x or Bob's y) determines the polarizer's angle. A photon passing through the polarizer yields a 1 outcome. The intensity measurement counts the 1s. Supplemental Material Note B [32] addresses concerns about the feasibility of realizing our model experimentally. Supplemental Material Note C [32] details the photon example.

The randomness in the A_x's and B_y's is of three types:

(i) *Quantum randomness:* If the systems are quantum, outcomes are sampled according to the Born rule during wave-function collapse.

(ii) *Local classical randomness:* Randomness may taint the preparation of each AB pair of subsystems.

In the SPDC example, different photons enter the crystal at different locations. Suppose that the crystal's birefringence varies over short length scales. Different photon pairs will acquire different relative phases $e^{i\alpha}$ [48].

(iii) *Global classical randomness:* Global parameters that affect all the particle pairs can vary from trial to trial. In the photon example, Alice and Bob can switch on the laser; measure their post-polarizer intensities several times, performing several trials, during a time T; and then switch the laser off. The laser's intensity affects the A_x's and B_y's and may fluctuate from trial to trial.

Quantum randomness and global classical randomness can violate our macroscopic Bell inequality. Assuming a cap on the amount of global classical randomness, we conclude that violations imply nonclassicality. Local classical randomness can conceal violations achievable by quantum systems ideally. Local classical randomness also produces limited correlations, which we bound in our macroscopic Bell inequality. We quantify classical randomness with a noise variable r below.

Systems A and B satisfy two assumptions:

(a) A and B do not interact with each other while being measured. Neither system receives information about the setting with which the other system is measured.

(b) Global classical correlations are limited, as quantified in Ineq. (2).

Assumption (a) is standard across Bell inequalities. In the photon example, the beams satisfy (a) if spatially separated while passing through the polarizers and undergoing intensity measurements.

Assumption (b) is the usual assumption that parameters do not fluctuate too much between trials, due to a separation of timescales. Consider the photon example in item (ii) above. Let t denote the time required to measure the intensity, to perform one trial. The trial time must be much shorter than the time over which the global parameters drift (e.g., the laser intensity drifts): $t \gg T$. The greater the timescales' separation, the closer the system comes to satisfying assumption (b). Assumption (b) has appeared in other studies of nonclassical correlations in macroscopic systems (e.g., [34,41]).

Assumptions (a) and (b) are the conditions under which a Bell inequality is provable for the operations that a macroscopic experimentalist is expected to be able to perform: correlating small systems and measuring macroscopic observables.[3] If the experimentalist can perform different operations, different assumptions will be natural, and our macroscopic Bell test may be extended (Sec. III).

[2]In the strategies presented explicitly in this paper, every measurement outcome equals 0 or 1. But the macroscopic Bell inequality holds more generally.

[3]Why these operations? Preparing macroscopic entanglement is difficult; hence the restriction to microscopic preparation control. Given microscopic preparation control, if the experimentalist could measure microscopic observables, she/he could test the microscopic Bell inequality; a macroscopic Bell inequality would be irrelevant.

We fortify our Bell test by allowing for small global correlations and limited measurement precision. Both errors are collected in one parameter, defined as follows. In the absence of errors, A_x and B_y equal ideal random variables A'_x and B'_y. Each ideal variable equals a sum of independent random variables. We model the discrepancies between ideal and actual with random variables r, as in

$$A_x = A'_x + r_{A_x}. \tag{1}$$

Our macroscopic Bell inequality is robust with respect to errors of bounded variance:

$$\mathrm{Var}(r_{A_x}) \leqslant \epsilon N, \tag{2}$$

wherein $\epsilon > 0$. Errors r_{B_y} are defined analogously. They obey Ineq. (2) with the same ϵ. Strategies for mitigating errors are discussed in Sec. III.

Our macroscopic Bell inequality depends on the covariances of the A_x's and B_y's. The covariance of random variables X and Y is defined as

$$\mathrm{Cov}(X, Y) := \mathbb{E}([X - \mathbb{E}(X)][Y - \mathbb{E}(Y)]), \tag{3}$$

wherein $\mathbb{E}(X)$ denotes the expectation value of X. One useful combination of covariances, we define as the *macroscopic Bell parameter*:[4]

$$\mathcal{B}(A_0, A_1, B_0, B_1) := \frac{4}{N}[\mathrm{Cov}(A_0, B_0) + \mathrm{Cov}(A_0, B_1) + \mathrm{Cov}(A_1, B_0) - \mathrm{Cov}(A_1, B_1)]. \tag{4}$$

II. MAIN RESULTS

We present the nonlinear macroscopic Bell inequality and sketch the proof, detailed in Supplemental Material Note D [32]. Then, we show how to violate the inequality using quantum systems.

Theorem 1 (Nonlinear Bell inequality for macroscopic measurements). Let systems A and B, and measurement settings x and y, be as in Sec. I. Assume that the systems are classical. The macroscopic random variables satisfy the macroscopic Bell inequality

$$\mathcal{B}(A_0, A_1, B_0, B_1) \leqslant 16/7 + 16\epsilon + 32\sqrt{\epsilon}. \tag{5}$$

Proof. Here, we prove the theorem when $\epsilon = 0$, when the observed macroscopic random variables A_x and B_y equal the ideal A'_x and B'_y. The full proof is similar but requires an error analysis (Supplemental Material Note D [32]).

Let $a_x^{(i)}$ denote the value reported by the ith A particle after A is measured with setting x. A'_x and B'_y equal sums of the microscopic variables:

$$A'_x = \sum_{i=1}^{N} a_x^{(i)} \quad \text{and} \quad B'_x = \sum_{i=1}^{N} b_x^{(i)}. \tag{6}$$

Because $a_0^{(i)}$ and $b_0^{(i)}$ are independent of the other variables,

$$\mathrm{Cov}(A'_0, B'_0) = \sum_{i=1}^{N} \mathrm{Cov}\big(a_0^{(i)}, b_0^{(i)}\big). \tag{7}$$

Analogous equalities govern the other macroscopic-random-variable covariances.

Let us bound the covariances among the $a_x^{(i)}$'s and $b_y^{(i)}$'s. We use the covariance formulation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality (see [47,49] and Supplemental Material Note E [32]),[5]

$$\mathrm{Cov}\big(a_0^{(i)}, b_0^{(i)}\big) + \mathrm{Cov}\big(a_0^{(i)}, b_1^{(i)}\big) + \mathrm{Cov}\big(a_1^{(i)}, b_0^{(i)}\big) - \mathrm{Cov}\big(a_1^{(i)}, b_1^{(i)}\big) \leqslant 4/7. \tag{8}$$

Combining Eq. (7) and Ineq. (8) with the definition of $\mathcal{B}(A'_x, A'_y, B'_x, B'_y)$ [Eq. (4)] gives

$$\mathcal{B}(A'_0, A'_1, B'_0, B'_1) = \frac{4}{N}\sum_{i=1}^{N}\Big[\mathrm{Cov}\big(a_0^{(i)}, b_0^{(i)}\big) + \mathrm{Cov}\big(a_0^{(i)}, b_1^{(i)}\big) + \mathrm{Cov}\big(a_1^{(i)}, b_0^{(i)}\big) - \mathrm{Cov}\big(a_1^{(i)}, b_1^{(i)}\big)\Big] \tag{9}$$

$$\leqslant 16/7. \tag{10}$$

■

We now show that a quantum system can produce correlations that violate Ineq. (5). The system consists of singlets.

Theorem 2. There exist an N-particle quantum system and a measurement strategy, subject to the restrictions in Sec. I, whose outcome statistics violate the nonlinear Bell inequality for macroscopic measurements. The system and strategy achieve

$$\mathcal{B}(A_0, A_1, B_0, B_1) = 2\sqrt{2} \tag{11}$$

in the ideal ($\epsilon = 0$) case and

$$\mathcal{B}(A_0, A_1, B_0, B_1) \geqslant 2\sqrt{2} - 16\epsilon - 32\sqrt{\epsilon} \tag{12}$$

in the presence of noise bounded as in Ineq. (2).

Proof. As in the proof of 1, we prove the result in the ideal case here. Supplemental Material Note F [32] contains the error analysis. Let each of A and B consist of N qubits. Let the ith qubit of A and the ith qubit of B form a singlet, for all i: $|\Psi^-\rangle := \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. We denote the 1 and -1 eigenstates of the Pauli z-operator σ_z by $|0\rangle$ and $|1\rangle$. Let x and y be the measurement settings in the conventional CHSH test ([49], reviewed in Supplemental Material Note E [32]). If the measurement of a particle yields 1, the particle effectively reports 1; and if the measurement yields -1, the particle reports 0.

[4]Calculating $\mathcal{B}$ requires knowledge of N, the number of particles in each experimentalist's system. N might not be measurable precisely. But knowing N even to within $\sqrt{N}$ suffices: Taylor-approximating yields $\frac{1}{N+\sqrt{N}} = \frac{1}{N}(1 - \frac{1}{\sqrt{N}})$. The correction is of size $\frac{1}{\sqrt{N}} \ll 1$. Furthermore, uncertainty about N may be incorporated into a noise model with which a macroscopic Bell inequality can be derived (Supplemental Material Note C [32]).

[5]In the original statement of Ineq. (8), the right-hand side equals 16/7. The reason is, in [47], $a_x^{(i)}, b_y^{(i)} \in [-1, 1]$. We assume that each variable $\in [0, 1]$, so we deform the original result in two steps. First, we translate $[-1, 1]$ to $[0, 2]$. Translations preserve covariances. Second, we rescale $[0, 2]$ to $[0, 1]$. The rescaling halves each a and b, quartering products ab, the covariances, and the 16/7 in Ineq. (8). The resulting 4/7 is multiplied by 4 in Ineq. (9), returning to 16/7.

Measuring the ith particle pair yields outcomes that satisfy

$$\mathbb{E}\big(a_0^{(i)}\big) = \mathbb{E}\big(a_1^{(i)}\big) = \mathbb{E}\big(b_0^{(i)}\big) = \mathbb{E}\big(b_1^{(i)}\big) = \tfrac{1}{2}. \tag{13}$$

As shown in Supplemental Material Note F [32],

$$\mathbb{E}\big(a_0^{(i)}b_0^{(i)}\big) + \mathbb{E}\big(a_0^{(i)}b_1^{(i)}\big) + \mathbb{E}\big(a_1^{(i)}b_0^{(i)}\big) - \mathbb{E}\big(a_1^{(i)}b_1^{(i)}\big) = 2\sin^2(3\pi/8) - \tfrac{1}{2}. \tag{14}$$

Combining these two equations yields

$$\mathrm{Cov}\big(a_0^{(i)}, b_0^{(i)}\big) + \mathrm{Cov}\big(a_0^{(i)}, b_1^{(i)}\big) + \mathrm{Cov}\big(a_1^{(i)}, b_0^{(i)}\big) - \mathrm{Cov}\big(a_1^{(i)}, b_1^{(i)}\big)$$

$$= 2\sin^2(3\pi/8) - 1 \tag{15}$$

$$= 1/\sqrt{2}. \tag{16}$$

Following the proof of 1, we compute

$$\mathcal{B}(A_0', A_1', B_0', B_1') = \frac{4}{N}\sum_i \big[\mathrm{Cov}\big(a_0^{(i)}, b_0^{(i)}\big) + \mathrm{Cov}\big(a_0^{(i)}, b_1^{(i)}\big) + \mathrm{Cov}\big(a_1^{(i)}, b_0^{(i)}\big) - \mathrm{Cov}\big(a_1^{(i)}, b_1^{(i)}\big)\big] \tag{17}$$

$$= 2\sqrt{2}. \tag{18}$$

■

III. DISCUSSION

Six points merit analysis. First, we discuss the equivalence of local quantum correlations and global classical correlations as resources for violating the macroscopic Bell inequality. Second, we suggest strategies for mitigating experimental errors. Third, we reconcile our macroscopic-Bell-inequality violation with the principle of macroscopic locality, which states that macroscopic systems should behave classically [34,40,41]. Fourth, we recast our macroscopic Bell inequality in terms of a nonlocal game. Fifth, we discuss a potential application to the Posner model of quantum cognition [42–45]. Sixth, we detail opportunities engendered by this work.

Violating the macroscopic Bell inequality with classical global correlations: Violating the inequality (5) is a quantum information-processing (QI-processing) task. Entanglement fuels some QI-processing tasks equivalently to certain classical resources (e.g., [50]). In violating the macroscopic Bell inequality, entanglement within independent particle pairs serves equivalently to global classical correlations. We prove this claim in Supplemental Material Note G [32]. This result elucidates entanglement's power in QI processing.

Two strategies for mitigating experimental imperfections: Imperfections generate local classical (ii) and global classical (iii) randomness, discussed in Sec. I. Local classical randomness can conceal quantum violations of the macroscopic Bell inequality, making the macroscopic Bell parameter $\mathcal{B}$ (4) appear smaller than it should. Global classical randomness can lead classical systems to violate the inequality. These effects can be mitigated in two ways.

First, we can reduce the effects of local classical randomness on $\mathcal{B}$ by modeling noise more precisely than in Sec. I. A macroscopic Bell inequality tighter than Ineq. (5) may be derived. We illustrate in Supplemental Material Note C [32], with noise that acts on the microscopic random variables $a_x^{(i)}$ and $b_y^{(i)}$ independently. Second, we can mitigate global classical randomness by reinitializing global parameters between trials. In the photon example, the laser can be reset between measurements.

Reconciliation with the principle of macroscopic locality: Macroscopic locality has been proposed as an axiom for distinguishing quantum theory from other nonclassical probabilistic theories [34,40,41] (see [51,52] for a more restrictive proposal). Suppose that macroscopic properties of N independent quantum particles are measured with precision $\sim\sqrt{N}$. The outcomes are random variables that obey a probability distribution P. A LHVT can account for P, according to the principle of macroscopic locality.

The violation of our macroscopic Bell inequality would appear to violate the principle of macroscopic locality. But experimentalists cannot guarantee the absence of fluctuating global parameters, no matter how tightly they control the temperature, laser intensity, etc. Some unknown global parameter could underlie the Bell-inequality violation, due to the inequality's nonlinearity (Supplemental Material Note A 2 [32]). This parameter would be a classical, and so local, hidden variable. Hence violating our macroscopic Bell inequality does not disprove LHVTs. Rather, a violation signals nonlocal correlations under reasonable, if not airtight, assumptions about the experiment (Sec. I).

Nonlocal game: The macroscopic Bell inequality gives rise to a nonlocal game. Nonlocal games quantify what quantum resources can achieve that classical resources cannot. The CHSH game is based on the Bell-CHSH inequality ([49,53,54] and Supplemental Material Note E [32]): Players Alice and Bob agree on a strategy; share a resource, which might be classical or quantum; receive questions x and y from a verifier; operate on their particles locally; and reply with answers a_x and b_y. If the questions and answers satisfy $x \wedge y = a + b \pmod 2$, the players win. Players given quantum resources can win more often than classical players can.

Our macroscopic game (Supplemental Material Note H [32]) resembles the CHSH game but differs in several ways: N Alices and N Bobs play. The verifier aggregates the Alices' and Bobs' responses, but the verifier's detector has limited resolution. The aggregate responses are assessed with a criterion similar to the CHSH win condition. After many rounds of the game, the verifier scores the player's performance. The score involves no averaging over all possible question pairs xy. Players who share pairwise entanglement (such that each Alice shares entanglement with only one Bob and vice versa) can score higher than classical players.

Toy application to Posner molecules: Fisher has proposed a mechanism by which entanglement might enhance coordinated neuron firing [42]. Phosphorus nuclear spins, he argues, can retain coherence for long times when in Posner molecules $Ca_9(PO_4)_6$ [55–61]. (We call Posner molecules "Posners" for short.) He has argued that Posners might share entanglement. Fisher's work has inspired developments in quantum computation [44,62], chemistry [43,61], and many-body physics [63–65]. The experimental characterization of Posners has begun. If long-term coherence is observed, entanglement in Posners should be tested for.

How could it be? Posners tumble randomly in their room-temperature fluids. In Fisher's model, Posners can undergo the quantum-computational operations detailed in [44], not the measurements performed in conventional Bell tests. Fisher sketched an inspirational start to an entanglement test in [45]. Concretizing the test as a nonlocal game was proposed in [44]. We initiate the concretization in Supplemental Material Note I [32]. Our Posner Bell test requires microscopic control but proves that Posners can violate a Bell inequality, in principle, in Fisher's model. Observing such a violation would require more experimental effort than violating our inequality with photons. But a Posner violation would signal never-before-seen physics: entanglement among biomolecules.

Opportunities: This work establishes six avenues of research. First, violations of our inequality can be observed experimentally. Potential platforms include photons [35], solid-state systems [36], atoms [37,38], and trapped ions [39]. These systems could be conscripted relatively easily but are known to generate nonclassical correlations. More ambitiously, one could test our macroscopic Bell inequality with systems whose nonclassicality needs characterization. Examples include the cosmic microwave background (CMB) and Posner molecules. Detecting entanglement in the CMB faces difficulties: Some of the modes expected to share entanglement have such suppressed amplitudes, they cannot be measured [66]. Analogs of cosmological systems, however, can be realized in tabletop experiments [46]. Such an experiment's evolution can be paused. Consider pausing the evolution before, or engineering the evolution to avoid, the suppression. From our Bell test, one might infer about entanglement in the CMB. A Posner application would require the elimination of microscopic control from the Bell test in Supplemental Material Note I [32], opportunity two.

Third, our macroscopic Bell inequality may be generalized to systems that violate the independence requirement in Sec. I. Examples include squeezed states, as have been realized with, e.g., atomic ensembles and SPDC [67,68]. The assumptions in Sec. I would need to modified to accommodate the new setup. If an experimental system violated the new inequality while satisfying the appropriate assumptions, one could conclude that the system was nonclassical. We illustrate such a modification and violation in Supplemental Material Note C [32], with a photonic system. Tailoring our results to a high-intensity pump appears likely to enable experimentalists to witness entanglement in systems that violate a common coincidence assumption: Bell tests tend to require low intensities, so that only one particle reaches each detector per time window [69]. The coincidence of a particle's arriving at detector A and a particle's arriving at detector B implies that these particles should be analyzed jointly. High-intensity pumps violate the one-particle-per-time-window coincidence assumption. Tailoring Supplemental Material Note C [32], using Gaussian statistics, appears likely to expand Bell tests to an unexplored, high-intensity regime.

Fourth, which macroscopic Bell parameters $\mathcal{B}$ can probabilistic theories beyond quantum theory realize? Other theories can support correlations unrealizable in quantum theory [70,71]. These opportunities can help distinguish quantum theory from alternative physics while illuminating the quantum-to-classical transition.

Fifth, our macroscopic Bell parameter is nonlinear in the probabilities of possible measurements' outcomes (Supplemental Material Note A 2 [32]). We have proved that a nonlinear operation—photodetection—can violate the inequality. Can Gaussian operations [72]? The answer may illuminate the macroscopic Bell inequality's limits.

Sixth, certain Bell inequalities have applications to *self-testing* [73]. A maximal violation of such an inequality implies that the quantum state had a particular form. Whether covariance Bell inequalities can be used in self-testing merits investigation.

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