Quantum battery of interacting spins with environmental noise

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A quantum battery is a temporary energy-storage system. We construct a quantum battery model of an N-spin chain with nearest-neighbor hopping interaction and investigate the quantum battery's charging process. We obtain the maximum energy in the quantum battery charged by a coherent cavity driving field or a thermal heat bath. We confirm that for a finite-length spin chain, thermal charging results in a nonzero ergotropy, contradicting a previous result: An incoherent heat source cannot charge a single-spin quantum battery. The nearest-neighbor hopping interaction induces energy-band splitting, enhancing the energy storage and the ergotropy of the quantum battery. We find a critical point in the energy and ergotropy resulting from the ground-state quantum phase transition after which the energy significantly enhances. Finally, we also find that disorder increases the energy of the quantum battery.

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I. INTRODUCTION

A quantum battery (QB) system can potentially provide temporary energy storage. The initially proposed QB was a two-level system that stores energy from an external field [1], and the initial physical model was proposed as a single two-level spin system. Further studies verified that a N-spin chain with an external field increases the charging power of the QB [2,3]. A primary goal of QB studies is finding the maximum energy stored during the charging time and increasing the energy release after the charging process [2,4–13]. Another research focus is the entanglement and work-extraction capability of the QB [14–21]. As the energy released in a QB is usually in a thermal heat bath, a QB has commonly discussed the energy for work in terms of ergotropy [22].

In the usual case, a QB is charged by an external field. An energy-charged cavity field in an excited energy state can save such external fields [2]. Energy oscillations during the charging process necessitate accurate control of the charging time. Other charging sources include a magnetic field or a thermal heat bath [7,11,23,24]. A QB charged by a thermal bath must be discussed as an open quantum system. A single-spin QB can be charged by a thermal heat bath, but its useful energy for work is always zero [24]. Due to the decay rate and the driving field, the energy-charging process of the QB gradually stabilizes in an open quantum system [18,24–31], so controlling the charging time is not necessary.

Few previous studies have considered the hopping interaction between each spin in a QB [26,32–34]. In studies that do consider such interactions, the energy charging and release by the QB are not discussed. In a real spin-chain model, the hopping interaction is vital and cannot be ignored. The spin-chain interaction creates a ground-state quantum phase transition and influences the ground-state properties of the QB. The quantum phase transition also influences the energy charging and release of the QB. The most simple interaction in a spin chain is the nearest hopping interaction.

In this paper, we discuss a N-spin QB with nearest-neighbor hopping interaction and the quantum-phase influence of the QB. Figure 1 shows the charging protocol of our QB system. The physical model of the QB is a N-spin Dicke-spin chain with hopping interaction J. The QB is coupled to a cavity field with the decay rate of the κ-boson heat bath. Because κ decays at a certain rate, we must investigate the charging of the QB in an open quantum system. The energy charging of the QB is accomplished by an external coherent driving field or is directly provided by the thermal heat bath. We study the energy and ergotropy of the QB with or without the nearest-neighbor hopping interaction. We compare the difference between the two types of charging protocol: charging from an external coherent driving field and directly charging from a thermal heat bath. We investigate how the hopping interaction and the spin numbers influence the energy and ergotropy of the QB. Finally, we also study how disorder influences the performance of the QB.

This paper is organized as follows. In Sec. II, we introduce the QB model and the dynamic equations. We define the energy and ergotropy of the QB. In Sec. III, we report on the dynamics of QB charging without and with the hopping interaction, respectively. We also investigate the ground-state quantum phase transition (QPT) of the QB. In Sec. IV, we discuss the QPT influence on the charging time of the QB, and the disorder influence on the energy and ergotropy of the QB. The conclusion is given in Sec. V.

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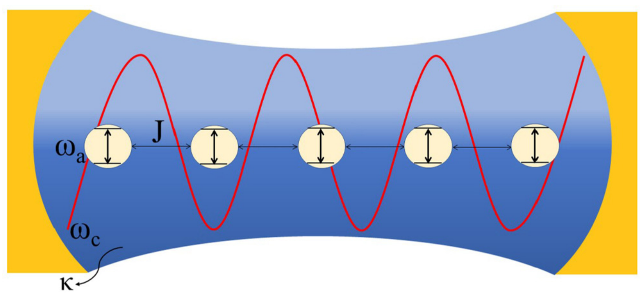

FIG. 1. A schematic of the QB charging protocol used in this paper. It includes a N-spin chain with a frequency of ω_a. The spin has a nearest-neighbor hopping interaction with a strength of J. The spin is coupled with a single-photon cavity with a frequency of ω_c and a decay rate of κ.

II. MODEL

The QB is modeled by a N-spin chain as shown in Fig. 1. The hopping interaction strength between nearest-neighbor spins is J. The spin is embedded in a microcavity with a cavity loss rate of κ. The Rabi frequency of the spin-photon interaction is g. The total Hamiltonian of this QB system is as follows:

$$H_S = H_A + H_B + H_I, \tag{1}$$

$$H_A = \omega_c c^\dagger c, \tag{2}$$

$$H_B = \omega_a \sum_{i=1}^{N} \sigma_+^i \sigma_-^i + J \sum_{i=1}^{N-1} (\sigma_+^i \sigma_-^{i+1} + \text{H.c.}), \tag{3}$$

$$H_I = \sum_{i=1}^{N} g(\sigma_+^i c + \text{H.c.}). \tag{4}$$

In the above expressions, H_A is the Hamiltonian of the cavity part with annihilation (creation) operator $c(c^\dagger)$. ω_c is the cavity frequency of the cavity field. H_B is the Hamiltonian of the QB with $\sigma_\pm^i$ as the raising or lowering spin operator for the ith spin and a spin frequency of ω_a. J is the nearest hopping interaction. H_I is the interaction term between the spin and the cavity field with spin-photon coupling constant g.

Within this setup, the cavity field is driven by an external classical field that charges the QB. The energy input from the driving field later transfers to the QB via the cavity-spin interaction. In this paper, the QB is charged in two ways: First from an external coherent driving field acting on the cavity, and second from a thermal heat bath coupled with the cavity field. We denote these two charging approaches as coherent and thermal charging, respectively.

The Hamiltonian of the coherent driving field is

$$H_d' = f(e^{-i\omega_d t} c^\dagger + e^{i\omega_d t} c). \tag{5}$$

Here, f is the driving field strength. After a unitary transformation $U = e^{i\omega_c t}$, the explicit time dependence term can be removed into [24]

$$H_d' = f(e^{-i\delta t} c^\dagger + e^{i\delta t} c), \tag{6}$$

where $\delta = \omega_d - \omega_c$. When we take $\omega_c = \omega_d$ for simplicity, the driven field Hamiltonian will reduce to

$$H_d = f(c^\dagger + c). \tag{7}$$

The dynamic process of the QB coherent charging is obtained by solving the Lindblad master equation,

$$\dot{\rho}_S(t) = -i[H_S + H_d, \rho_S(t)] + \kappa L_c[\rho_S], \tag{8}$$

where κ is the decay rate of the cavity field, and $L_c[\rho_S] = c\rho_S c^\dagger - \frac{1}{2}(c^\dagger c \rho_S + \rho_S c^\dagger c)$ is the Lindblad superoperator.

The dynamics of thermal charging are obtained by solving the Lindblad master equation,

$$\dot{\rho}_S(t) = -i[H, \rho_S(t)] + \kappa(n_B + 1)L_c[\rho_S] + \kappa n_B L_{c^\dagger}[\rho_S], \tag{9}$$

where $n_B = 1/\{\exp[\omega_c/(k_B T)] - 1\}$ is the mean occupation number of the boson heat bath.

The charging process of the QB fills the empty QB from the cavity field. We prepare an empty QB by initializing the spin in its ground-state $|g\rangle_B$. The initial state of the cavity is the vacuum state $|0\rangle_A$. Thus, the initial state of the whole system is as follows:

$$|\psi(0)\rangle = |0\rangle_A \otimes |g\rangle_B, \tag{10}$$

where $|\psi(0)\rangle$ is the initial state of the whole system and $|g\rangle_B$ is the initial state of the QB corresponding to the energy ground state of H_B. When the cavity two-level interaction g is turned on, the charging process immediately starts the energy exchange between the spin chain and the cavity field.

The energy storage in the QB at time t is given by

$$E_B(t) = \text{tr}[H_B \rho_B(t)], \tag{11}$$

where $\rho_B(t) = \text{tr}_A[\rho_S(t)]$ is the reduced density matrix of the QB at time t. The energy charged into the QB is $E_B(t) - E_B(0)$, where $E_B(0) = E_G$ is the ground-state energy of the QB. Therefore, the actual charging energy of the QB is

$$\Delta E(t) = E_B(t) - E_G. \tag{12}$$

The charging energy $\Delta E(t)$ can characterize the property of the QB. However, under the second law of thermodynamics, it cannot be transformed into work without dissipating the heat. Ergotropy characterizes the ability of the QB to generate useful work [22]. The ergotropy is defined as

$$\varepsilon_B(t) = E_B(t) - \min_U \text{tr}[H_B U \rho_B(t) U^\dagger]. \tag{13}$$

Diagonalizing H_B and $\rho_B(t)$, respectively, we obtain

$$\rho_B(t) = \sum_n r_n(t) |r_n(t)\rangle\langle r_n(t)|, \tag{14}$$

$$H_B = \sum_n e_n |e_n\rangle\langle e_n|. \tag{15}$$

The eigenvalues of $\rho_B(t)$ are arranged in descending order as $r_0 \geqslant r_1 \geqslant \cdots$, and the eigenvalues of H_B are arranged in ascending order as $e_0 \leqslant e_1 \leqslant \cdots$. The term $\min_U \text{tr}[H_B U \rho_B(t) U^\dagger]$ of Eq. (13) can be simplified as follows [22,24]:

$$\min_U \text{tr}[H U \rho(t) U^\dagger] = \sum_n r_n e_n. \tag{16}$$

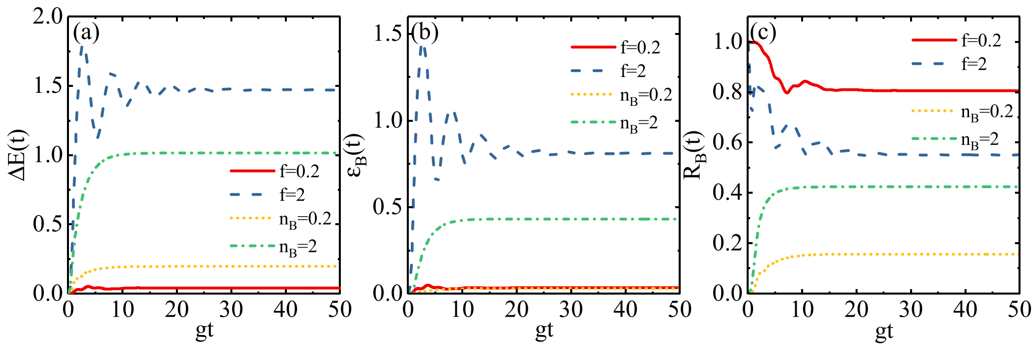

FIG. 2. The dynamic charging process of the QB. (a) Energy $\Delta E(t)$, (b) ergotropy, and (c) efficiency $R_B(t)$ as the function of gt. Other parameters are $N = 3,\ \omega_c = g = \kappa = \omega_a = 1$, and $J = 0$.

It is easily proved that $\varepsilon_B(t)$ is always non-negative and smaller than $\Delta E(t)$. Therefore, we can define the following efficiency $R_B(t)$ as the percentage of $\varepsilon_B(t)$ among the total charging energy $\Delta E(t)$:

$$R_B(t) = \frac{\varepsilon_B(t)}{\Delta E(t)}. \tag{17}$$

Using the energy ΔE, we can judge the charging energy of the QB. The ergotropy could judge ε_b is the useful energy releasing for the useful work of the QB. Furthermore, the efficiency R_B describes the useful energy release efficiency of the QB. This paper focuses on $\Delta E,\ \varepsilon_b$, and R_B during the QB charging process.

III. THE CHARGING PROPERTY OF THE QB

This section will discuss the charging properties of the QB with or without the hopping interaction. We will first discuss the QB's dynamic and find the maximum energy and the ergotropy during the charging process. Then discuss the quantum phase transition induced by the hopping interaction. Finally, we also discuss how the quantum phase influences the charging properties of the QB.

A. Charging without hopping interaction

The ground state of the QB in this case is

$$|g\rangle_B = |0\rangle^{\otimes N}, \tag{18}$$

where $|0\rangle$ is the ground state of a single spin. The ground-state energy E_G of the QB without the hopping interaction is zero.

The dynamics of the coherent and thermal charging processes are determined by Eqs. (8) and (9), respectively (see Fig. 2). Figure 2 illustrates the QB's dynamic charging process with the two charging approaches. In our QB charging model, ω_a decides the system's total energy. Meanwhile, ω_a and ω_c must be equal to ensure the maximum energy transfer. The parameter g will mainly influence the charging time. The relative value of the parameters κ will mainly influence the steady-state QB's energy and ergotropy. In all calculations, we take ω_a as a dimensionless parameter and let $\omega_a = 1$. For simplicity other parameters are taken as $\omega_c = g = \kappa = \omega_a = 1$. Owing to the decay loss, the strong and weak driving strengths differ during the charging process. To compare these two strengths, we set $f = 2$ ($n_B = 2$) and $f = 0.2$ ($n_B = 0.2$). The strong and weak driving strengths are just compared to the relative value with κ. As shown in panels (a) and (b) of Fig. 2, the energy $\Delta E(t)$ and ergotropy $\varepsilon_B(t)$ of the QB were gradually stabilized. Under both coherent and thermal chargings, the energy and ergotropy of the QB increased with driving strength, but the coherent charging oscillates during the charging. The steady-state energy at infinite time was the maximum charging energy. However, the ergotropy of the QB was maximized within the oscillatory period, indicating that the oscillations boosted the efficiency of the coherently charged QB at the beginning. In contrast, both the energy and the ergotropy of the thermally charged QB were maximized at steady state after infinite time. Besides, the QB efficiency in the thermal charging large driving strength will also correspond to a significant efficiency opposite to the thermal charging.

We could also find a nonzero ergotropy during the charging process for the thermal charging. This result differs from those of previous works on simple-spin systems in which the ergotropy ε_B is always zero [24]. A nonzero ergotropy can be obtained by increasing the spin number of the QB with the thermal charging as shown in Fig. 3. We verified the thermal charging of a single-spin ($N = 1$) QB always induces a zero ergotropy as reported in previous research [24]. However, thermal charging of a finite-length chain ($N \geqslant 2$) QB induces a nonzero ergotropy. Under strong charging conditions, the energy, ergotropy, and efficiency increase with the spin numbers increase. However, under weak charging conditions, the QB's energy and ergotropy both decrease with increasing chain length.

Above, we explained that thermal charging can induce nonzero ergotropy. If the QB exists in the thermal-state $\rho_B = \rho_B^{\text{th}} = \frac{e^{-\beta H_B}}{z}$, where $z = \text{tr}[e^{-\beta H_B}]$, it is easily verified that the ergotropy is always zero. The thermal-state density matrix ρ_B^{th} and the Hamiltonian representation of the energy, respectively, are given by

$$\rho_B^{\text{th}} = \frac{1}{z}\begin{pmatrix} e^{-\beta e_1} & 0 & \cdots & 0 \\ 0 & e^{-\beta e_1} & \cdots & 0 \\ 0 & 0 & \cdots & e^{-\beta e_n} \end{pmatrix},$$

$$H_B = \begin{pmatrix} e_1 & 0 & \cdots & 0 \\ 0 & e_2 & \cdots & 0 \\ 0 & 0 & \cdots & e_n \end{pmatrix}. \tag{19}$$

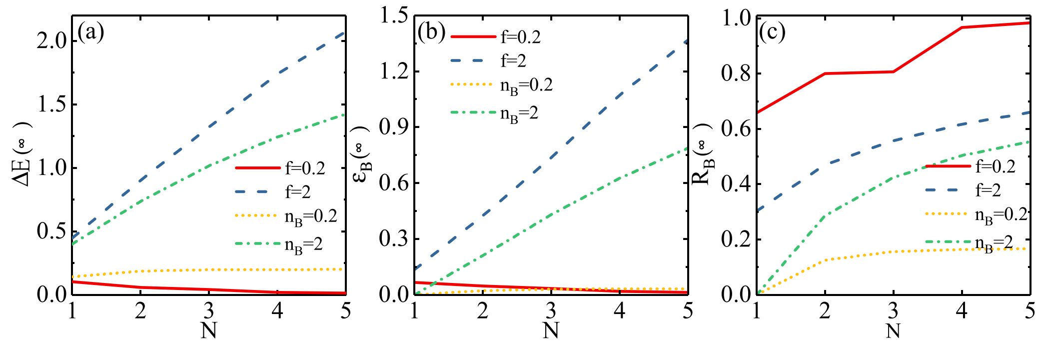

FIG. 3. The QB steady-state energy, ergotropy, and the energy release rate for the different number of spins. (a) QB energy $\Delta E(\infty)$, (b) ergotropy $\varepsilon_B(\infty)$, and (c) efficiency $R_B(\infty)$. The other parameters are the same as in Fig. 2.

Here, the energy eigenvalues are in ascending order, and the diagonal elements of the thermal-state density matrix are in descending order. Thus, the mean thermal-state energy is $E_B = \mathrm{tr}[H_B\rho^{\mathrm{th}}] = \sum_n r_n e_n$ [equaling the right hand of Eq. (13)], meaning that the ergotropy of a thermal state is always zero. When the ergotropy is nonzero, the density of the QB is reduced, and the thermal state is not attained. In this scenario (Fig. 4), the steady-state density matrix ρ_B and system density-matrix ρ of the QB differ from their corresponding density matrices ρ_B^{th} and ρ^{th} in the thermal state. As thermal driving is coupled only to photons within a β-temperature thermal bath, the total photon-spin system cannot be purified into the thermal state. The reduced density-matrix ρ_B also cannot exist in the thermal state. Thus, the ergotropy of a nonsingle-spin QB can be nonzero under thermal charging conditions.

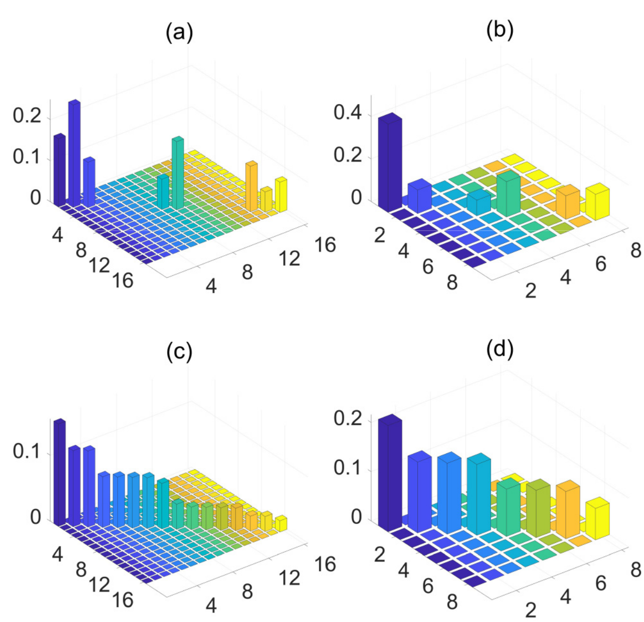

FIG. 4. Steady-state density matrices in the energy representation: Panels (a) and (b) show the steady-state densities of the whole system and the reduced density matrix of the QB under thermal charging, respectively. Panels (c) and (d) show the thermal-state density matrices of the whole system and the QB, respectively. Other parameters are $N = 3$ and $n_B = 2$.

It is worth noting that, different from the classical cyclically operating device situation and the case in Ref. [35] which Lindblad's master equations are sometimes incompatible with thermodynamic consistency. In our QB charging protocol, the thermal charging from the thermal bath cannot bring the steady state of the QB into the thermal state, which means that it cannot bring the QB system to thermal equilibrium. Besides, in our QB charging model, the initial state is a pure state, i.e., $\mathrm{tr}[\rho_B^2(0)] \equiv 1$. However, for the spin number $N \geqslant 2$ with the QB in the thermal charging, it will gradually decrease, i.e., $\mathrm{tr}[\rho_B^2(t > 0)] < 1$. Therefore, our QB extracts work from a thermal bath at the cost of spoiling the spin purity.

B. Charging with hopping interaction

In the previous subsection, we discussed the coherent and thermal charging processes of the QB without the hopping interaction. We here consider a spin-chain QB with nearest-neighbor interacting strength J in the QB. The initial state of the QB determines the energy ground state of the interacting spin chain.

When considering the hopping interaction, the charging dynamics of the QB are shown in Fig. 5. The charging dynamics of the QB is largely unaffected by the hopping interaction. But the hopping interaction increased the time in which the QB receives its maximum energy and increases the energy of the QB under coherent and thermal charging conditions. However, the hopping interaction decreases the ergotropy of the QB. Weak charging reduces the efficiency of both coherent and thermal charging. Meanwhile, the hopping interaction does not remove the oscillations from the coherent charging phase. Whether considering the hopping interaction or not, the difference between coherent and thermal charging is only the energy oscillations during the charging process.

To further discuss the influence of the hopping interaction, we calculate the steady-state energy and the ergotropy of the QB at different hopping interactions as shown in Fig. 6. At $J = 0$, the energy and ergotropy are remarkably changed by the interaction term, which breaks the symmetry of the QB. A nondifferentiable point appeases at $J = 1/\sqrt{2}$. Before this point, the QB energy is a nearly constant function of hopping interaction, but thereafter, it significantly increases with the hopping interaction.

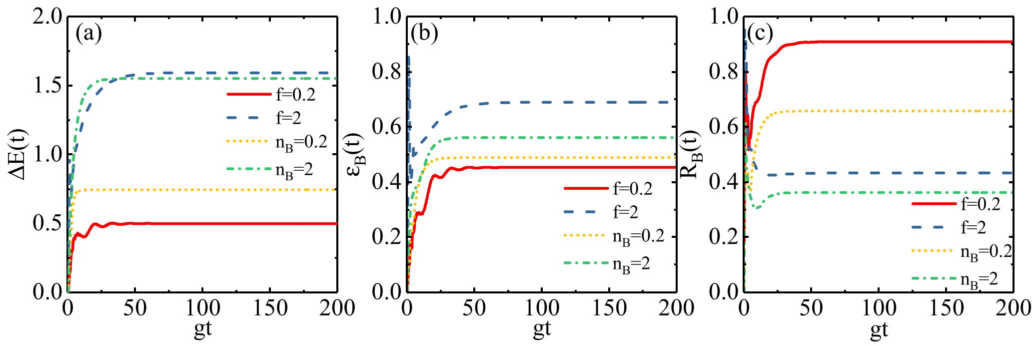

FIG. 5. The QB dynamic charging process with considering the hopping interaction. (a) Energy $\Delta E(t)$, (b) ergotropy $\varepsilon_B(t)$, and (c) efficiency R_B as the function of gt. Other parameters are $N = 3$ and $J = 1$.

We then calculate the steady-state energy and ergotropy of a thermally charged QB with different spin numbers ($N =$ 3–5) shown in panels (a)–(c), respectively, of Fig. 7. The energy versus the hopping interaction of the QB develops an external nondifferentiable point at large spin numbers, but the energy and ergotropy of the QB exhibit similar growth trends at small and large spin numbers. Whereas the energy does not significantly increase before the first nondifferentiable point, but the ergotropy was a continuously increasing function of hopping interaction.

We calculate the energy spectra to further discuss the nondifferentiable points in the QB's energy and ergotropy. The results for $N = 3$–5 are shown in panels (d)–(f) of Fig. 7, respectively. Each nondifferentiable point of the energy correspond to a crossing of the ground-energy state. Before the first ground-state energy crossing, the ground-state energy is always zero. After the first crossing point, the ground-state energy was gradually decreased by splitting of the energy bands. From Eq. (12), we find that the decreasing ground-state energy due to band splitting increases the energy by increasing the hopping interaction

The different energy behaviors of the QB's energy around the first nondifferentiable point manifest a QPT. To further discuss the QPT, we introduce the order parameter. One common order parameter is the mean magnetic-field M_z, given by

$$M_z = \frac{\langle S_z \rangle_g}{N}. \tag{20}$$

Here we define another order parameter ξ_z as

$$\xi_z = \frac{\langle S_z^2 \rangle_g}{N^2}, \tag{21}$$

where $\langle \cdots \rangle_g$ represents the average on the ground state and the total spin operator is $S_z = \sum_{i=1}^{N} \sigma_z^i$. We consider only the ground-state's ordering parameter because both the nondifferentiable points of the energy and ergotropy correspond to the ground-state energy crossing.

The calculated order parameters for $N = 3$–5 are presented in panels (g)–(i) of Fig. 7, respectively. These discontinuous points indicate a first-order QPT at this point. We have already found significant changes in the QB charging properties after the first nondifferentiable point. Before the first nondifferentiable point, the order parameter is $M_z = -1$, meaning that each spin is in the spin-down state corresponding to the ferromagnetic phase. After the first nondifferentiable point, the ordering parameter $M_z > -1$ which means the ground state is departed from the ferromagnetic phase. The ground-state quantum phase significantly influences the charging of the QB. In a noninteracting spin-chain QB, the ground state of the QB is always spin down. Accordingly, the QB exists in the ferromagnetic phase, which is unsuitable for more energy storage.

IV. CHARGING TIME AND DISORDER

In the previous section, we discussed how hopping interaction influences the energy and ergotropy of the QB. We

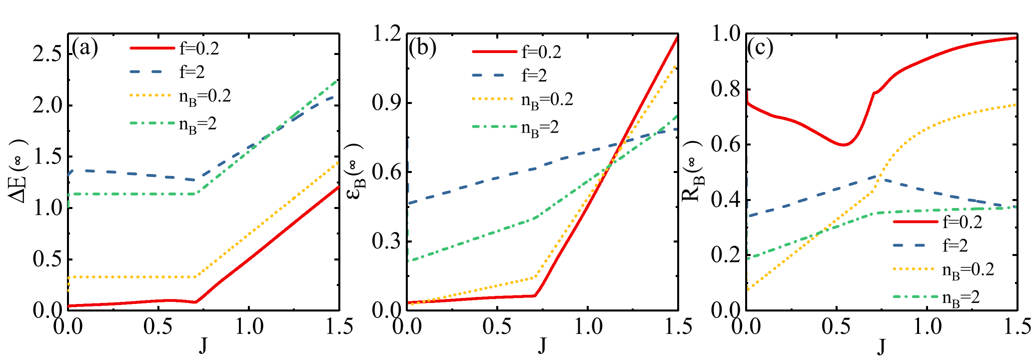

FIG. 6. The QB steady energy and ergotropy for different hopping interactions. (a) Energy $\Delta E(\infty)$, (b) ergotropy $\varepsilon_B(\infty)$, and (c) efficiency $R_B(\infty)$ as a function of J. Other parameters are the same as in Fig. 2.

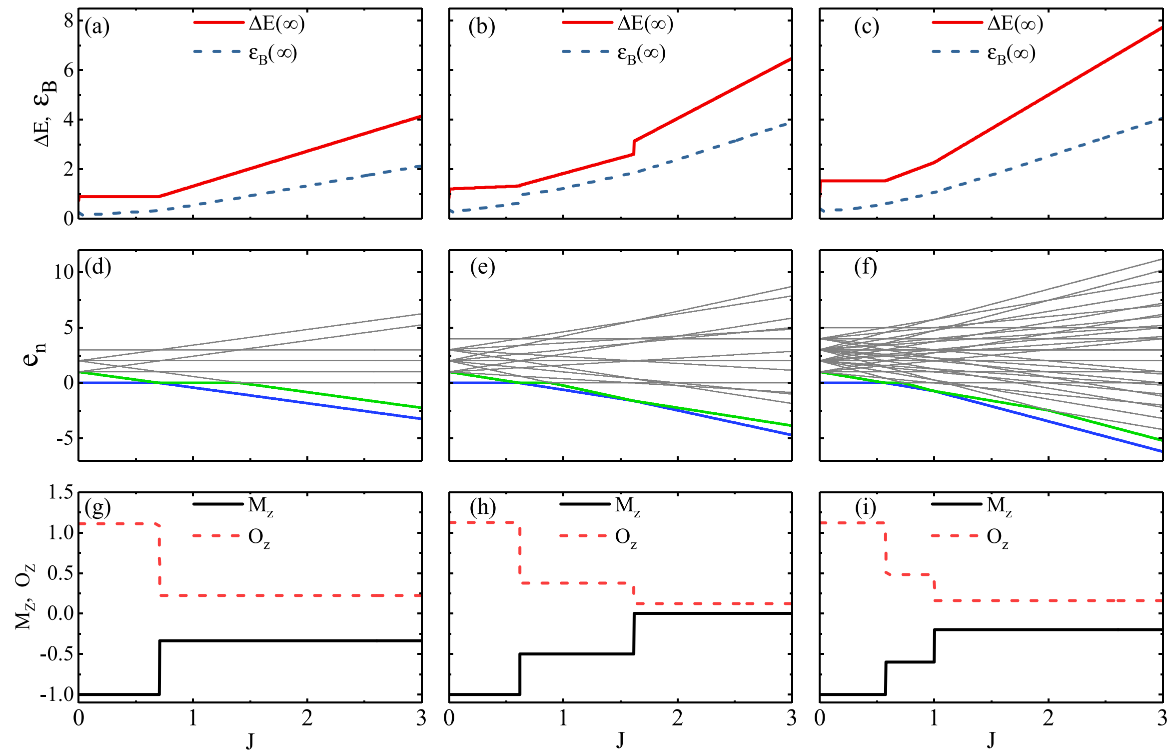

FIG. 7. (a)–(c) are the QB energy $\Delta E(\infty)$ and ergotropy $\varepsilon_B(\infty)$ as a function of the hopping interaction for the spin numbers $N = 3$–5, respectively. (d)–(f) are the respective energy spectra. (g)–(i) are the corresponding ordering parameters. Other parameters are the same as in Fig. 4.

also observe that when the hopping interaction is nonzero, the energy of the QB is enhanced after the first QPT point, and the charging speed of the QB significantly increased. This section investigates the influence of the hopping interaction on the charging speed, and the effects of on-site energy disorder.

We here focus on the steady-state energy and ergotropy of the QB during charging in an open quantum system. Theoretically, the steady state is reached only after infinite time. In reality, the charging process is ceased when the energy and ergotropy of the QB are sufficiently close to their steady-state values. To judge the charging speed, we must, therefore, define a charging time. We first define the charging power as

$$P_B(t) = \frac{\Delta E(t)}{t}. \tag{22}$$

The charging time τ_c then defines the time at which the charging power is maximized

$$\tau_c = \arg\max_t P_B(t). \tag{23}$$

Figure 8 shows the charging times τ_c at different hopping interactions. We observe that the charging time τ_c suddenly increases after the phase-transition point. Before the phase-transition point, the charging time gradually decreases with increasing hopping interaction. After the QPT point, energy, and ergotropy increase at the expense of increasing charging time.

We now discuss the influence of disorder on the energy and ergotropy of the QB. Here we consider only an on-site environmental disorder in the free-energy term, which changes the Hamiltonian of the QB as follows:

$$H_B = \omega_a \sum_{i=1}^{N}(1+\delta_i)\sigma_+^i\sigma_-^i + J\sum_{i=1}^{N-1}(\sigma_+^i\sigma_-^{i+1} + \text{H.c.}), \tag{24}$$

where $\delta_i \in [-W/2, W/2]$ is the on-site disorder and W is the disorder strength. To ensure accurate numerical calculations, the result was computed 100 times and averaged to give the result in Fig. 9. We can find that the disorders do not affect the location of the QPT point. After the phase-transition

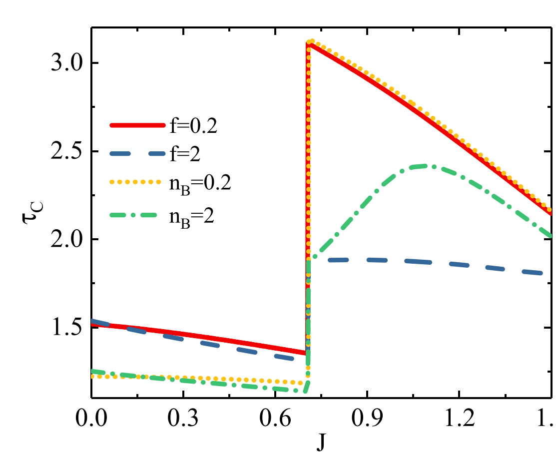

FIG. 8. The charging time τ_c as a function of the hopping interaction J. Other parameters are same as Fig. 2.

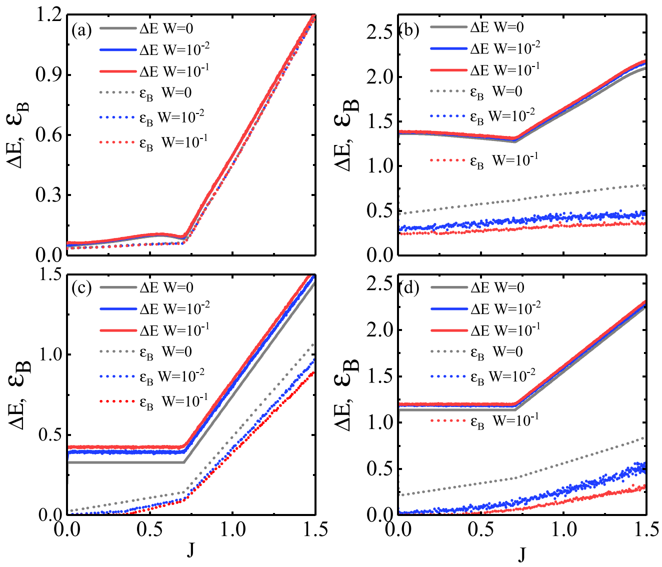

FIG. 9. The steady-state energy and ergotropy of the QB influenced by the on-site disorder W. These four figures correspond to different charging strengths with (a) $f = 0.2$, (b) $f = 2$, (c) $n_B = 0.2$, and (d) $n_B = 2$. Other parameters are same as Fig. 2.

point, the energy and ergotropy of the QB still increase with increasing hopping interaction as observed previously. The on-site disorder slightly affected the stability of the energy. However, the ergotropy is unstable in the on-site disorder. The increased disorder strength enhances the energy of the QB. The increased QB energy is similar in magnitude to the energy transfer enhanced by the disorder [36–38], although a large disorder would reduce the ergotropy.

V. CONCLUSION

We investigated QB charging in an open quantum system with coherent and thermal chargings. Without the hopping interaction, we found that for spin lengths $N \geqslant 2$, thermal charging imparts a nonzero ergotropy to the QB at the cost of spoiling the QB density-matrix purity. Under weak (strong) charging conditions, increasing the spin length decreases (increases) the ergotropy of the QB, respectively. In the system with the hopping interaction, the QB energy increases with charging time after the first ground-state energy crossing point. The ground-state QPT also affects the energy of the QB. Finally, we investigated the on-site disorders of the QB and verified that disorder increases the energy but decreases the ergotropy of the QB.

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