Majorana bosonic quasiparticles from twisted photons in free space

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The speed of light in vacuum, c, is a fundamental constant of nature. Photons belonging to a structured beam of finite transverse size have been observed to travel with a group velocity v_g, smaller than c, also when propagating in vacuum. The paradox of slow photons in vacuum could, in a first approach, be explained as a simple projection effect that occurs when one measures the speed of light in a diverging vortex beam along the direction of the beam propagation. This effect, instead, reveals fundamental properties of structured light beams: they form a class of quasiparticle states characterized by the geometric properties of the beams such as the beam divergence, spin angular momentum, orbital angular momentum (OAM), and the group velocity. These anomalous propagation modes with group velocities different than that of c represent a class of Majorana quasiparticles with bosonic spectrum for integer OAM modes, cast in an infinite tower of quasiparticle states similar to those expected with photons propagating in a turbulent plasma.

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I. INTRODUCTION

Structured light beams with a finite transverse size were observed to travel at a speed slower than that of light in vacuum [1–7]. Light propagating in vacuum, however, is expected to travel at the speed of light, c. This "apparent" or, better, effective observation of a subluminal propagation in free space was explained in terms of a simple projection of the effective motion of light of a diverging vortex beam, measured across the axis of propagation of the beam itself [5]. The reason why one does not measure $v_g = c$ in these experimental conditions is quite simple. Consider as an example a Laguerre-Gaussian beam propagating along the z axis [8–13]. In this case, the z axis represents the locus of points where the optical vortex is found and the amplitude and intensity of the electric field are null. Any detectable beam of light is then observed to propagate in directions different from that of the z axis, with the result that the absolute value of their speeds, projected and measured along z, obviously will be smaller than c. This behavior is equivalent to what can be found for an electromagnetic wave propagating in a cavity or in a waveguide with group velocity $v_g = \partial\omega/\partial k$, the velocity with which the envelope of an optical pulse propagates; ω and k are the angular frequency and the angular wave number, respectively. In fact, structured beams carrying orbital angular momentum (OAM) are cavity modes propagating in free space [14].

This effect has a much deeper physical meaning than that of a simple projection: vortex beams in their propagation behave as Majorana quasiparticles cast in an infinite tower of states [15,16] described by a precise relationship between the beam divergence—equivalent to a virtual mass term—and the total angular momentum $\mathbf{J}$, the conserved quantity which is the (vector) sum of the spin angular momentum (SAM), $\mathbf{\Sigma}$, and OAM, $\mathbf{L}$, that cannot always be split in these two subcomponents as they were separately conserved quantities. Additional information about Majorana particles, quasiparticles, and the Majorana tower are reported in the Appendix.

II. MAJORANA TOWER OF QUASIPARTICLES FROM OAM SLOW-VORTEX BEAMS

Majorana quasiparticles have been found mostly in terms of fermionic quasiparticle states [17], while bosonic Majorana quasiparticles were already hypothesized to be built with electromagnetic (EM) OAM carrying beams propagating in a structured plasma [18,19] and for integer-valued orbital angular momentum vortex beams propagating in free space, initially discussed in Ref. [20] and confirmed by the results presented in Ref. [21]. Like in a structured plasma, photons acquire an effective Proca mass through the Anderson-Higgs mechanism [22,23] and obey a relationship between the mass term and the total angular momentum $\mathbf{J}$ instead of the spin angular momentum (SAM) $\mathbf{\Sigma}$ present in the original mathematical formulation of the Majorana tower [15,16]. OAM acts as a term that reduces the total Proca photon mass in

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the plasma [18,19]. What actually induces a Proca mass in photons in a plasma is the breaking of spatial symmetry, and thus of spatial homogeneity, and a characteristic scale length introduced by the plasma structure at the frequencies at which the plasma is resonant. The presence of a characteristic scale length and structures in the plasma presents strong analogies with the models of space-time characterized by a modified action of the Lorentz group, such as the Magueijo-Smolin model [24], demonstrating deep analogies with the dynamics described by the Dirac equation when a characteristic scale length is present and, unavoidably, Berry phase effects are introduced [25]. The effective Proca mass falls under the general conditions provided by the Higgs mechanism. In particle physics, when a local gauge symmetry breaks up, some gauge fields may become massive. Maxwell-Proca equations fall into the affine-Higgs case studied by Stückelberg, a powerful covariance principle between free-space wavelength equations and those in a conservative force field [26], valid also for non-Abelian gauge theories [27–29].

As shown experimentally, the measured slowing down of light along the propagation axis of a structured beam in free space finds a geometrical meaning related to the proper beam divergence and its intrinsic topological properties and in the definition of group velocity. Hence, this effect is slightly different from the mechanism in which light is slowed due to the presence of matter. Structured beams of photons propagating in vacuum exhibit a different behavior than a plane wave because of the breaking of the spatial symmetry due to the field confinement induced by their finite extent and because of the structure of the beam that changes the wave vector with the result of altering the group velocity v_g, which, by definition, is measured along the propagation axis z. It is of fundamental importance to point out that the phase velocity v_p is also different for beams with different OAM modes.

Following [30,31], from the definition of group velocity, it is clear that these effects are related, especially in a helically phased beam, to the skew angle γ of the Poynting vector with respect to the direction of propagation. The heart of the matter dictating the rules of the speed of propagation of OAM eigenmodes along the z axis lies in the fact that k_z is not the only component of the wave vector that has to be taken into account in order to have a complete description of the system. If we consider a distinct z component of OAM, J_z, as a single eigenmode, characterized by its azimuthal quantum number, where its value ℓ can take any integer value ranging from $-|\ell|$ to $+|\ell|$, we also find a ϕ component of the *local* k vector that varies in time since we have a rotational degree of freedom. This additional component has implications on how to interpret the phase velocity. Moreover, the radial component of the local wave vector, k_ρ, related to the p quantum number, must also be taken into account. Thus, the effective magnitude of k is no longer k_z in the direction of propagation of the symmetry axis of the beam, but the local wave vector that identifies the propagation of photons becomes $\sqrt{(k_\rho)^2 + (\ell/\rho)^2 + (k_z)^2}$.

The consequence of this result is that for a given frequency ω, one finds a different phase velocity v_p for any different OAM quantum number ℓ as the local k vector no longer points along the z axis; instead, k varies in time and precesses along a cone having its axis coincident with the z axis. This, of course, requires averaging when measurements are made. In fact, when ℓ goes to infinity, the phase velocity v_φ goes to zero (see [32]).

To better explain this concept, consider a general electric field in cylindric coordinates $\mathbf{E}(\rho, \phi, z; t)$. In the most general case, when the field description is factorable and propagating along z,

$$\mathbf{E}(\rho, \phi, z; t) = R(\rho)\Phi(\phi)Z(z)T(t)\mathbf{E}_0, \tag{1}$$

and $\mathbf{E}_0 = E_0\hat{\mathbf{e}}$. The azimuthal function $\Phi(\phi)$ that describes the variation of $\mathbf{E}$ in the direction perpendicular to both z and ρ can most conveniently be expanded in a Fourier series with constant amplitude coefficients c_ℓ,

$$\Phi(\phi) = \sum_{\ell=-\infty}^{+\infty} c_\ell \, e^{i\ell\phi}, \tag{2}$$

which leads to an expansion of the electric field in discrete angular momentum modes $\mathbf{E}_\ell \propto e^{i\ell\phi}$. The azimuthal component of the local wave vector is $k_\phi = \ell/\rho$ and the wave vector, expanded along the given coordinates, does not coincide with the direction of propagation along the z axis.

For OAM beams, the wave vector does not necessarily have an azimuthal component; OAM beams do have a local Poynting vector with an azimuthal component, but it is not mandatory that the wave vector must have an azimuthal component such as $\frac{\ell}{\rho_0}\hat{\phi}$, for a given radius ρ_0, and thus the wave vector becomes

$$k_\ell = k_\rho\hat{\rho} + k_z\hat{z}, \tag{3}$$

where ρ_0 represents the radial mode coordinate. From this, one obtains the phase velocity v_p in vacuum,

$$v_p = \frac{\omega}{k} = \frac{\omega}{k_z\sqrt{1 + \left(\frac{k_\rho}{k_z}\right)^2}}. \tag{4}$$

Since $|k_z| \leqslant |k|$, the phase velocity projected onto the z axis will appear superluminal,

$$v_{p,z} = \frac{\omega}{k_z} \geqslant c. \tag{5}$$

For a plane wave propagating in vacuum, it is conventionally assumed that the magnitude of the wave vector k is identical to k_z (the component of the k vector along the z axis, conveniently and customarily set to be the line of sight from the source to the observer). For the group velocity v_g, the concept is the same, without assuming *a priori* constraints on the geometry of the beam [33].

We will now focus our attention on the class of circular beams carrying OAM for which a precise linear relationship between their divergence and OAM eigenvalue ℓ is present: the higher is the OAM carried by the beam, the more the beam is divergent [8–12,34–36] and more precisely to the well-known Laguerre-Gaussian (LG) beams that are solutions of Maxwell's wave equations in the paraxial approximation.

The class of Bessel beams, i.e., exact solutions of Maxwell's wave equations, present the same divergent behavior, as observed experimentally [37], without having a nonzero azimuthal component of $\mathbf{k}$ too. In this case, the scalar amplitude is $J_\ell(k_\rho\rho)e^{ik_z z}e^{i\ell\phi}e^{-i\omega t}$, which carries OAM with $\ell\hbar$ angular momentum per photon. Here one finds an azimuthal component of the Poynting's vector, but no corresponding ϕ component of the wave vector, $\omega^2 = (k_z^2 + k_\rho^2)c^2$. In any case, one will notice some differences between the behavior of Bessel beams and LG beams (or any other cylindric paraxial wave solution) only in extreme conditions such as in tightly focused beams where spin and orbital angular momenta couple to each other [38]. Consider now, for the sake of simplicity, LG beams. In this case, v_g is reduced by a precise amount that depends upon the aperture of the optical system.

This is due to a transverse spatial confinement of the field that leads to a larger modification of the axial component of the wave vector k_z, modifying the group velocity v_g, when $|\ell|$ increases. For an OAM beam propagating along the z axis, the wave vector is $k_0 = 2\pi/\lambda$ and $k_0^2 = k_x^2 + k_y^2 + k_z^2$, and the confined mode is given by the quantities $k_x,\ k_y > 0$ that imply $k_z < k_0$ with a modification in both the phase and group velocities, v_p and v_g.

When the beam is confined, the relationship $k_x, k_y =$ const implies that k_z is dispersive in free space. In polar coordinates, this relationship becomes $k_r = \sqrt{k_x^2 + k_y^2}$ and $k_z = k_0 - (k_\rho^2/2k_0^2)$.

Here, the phase velocity v_p, measured along the z direction, is

$$v_p = \frac{c}{\left(1 - \frac{k_\rho^2}{2k_0^2}\right)}, \tag{6}$$

and the group velocity along z is

$$v_g = c\left(1 - \frac{k_\rho^2}{2k_0^2}\right). \tag{7}$$

Hence, the group velocity reduction $v_g < c$ gives a measurable delay that reveals the OAM content of the beam [4,6,7,39,40]. The time delay between different beams caused by different values of the group velocity was also proposed to set up a temporal buffer for optical computing and information transfer in free space [3,41].

The group velocity can then be expressed in terms of the transverse quantum numbers p and ℓ [4],

$$v_g = \frac{c}{1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)}, \tag{8}$$

and, from this delay, the dispersion in free space of OAM beams can be characterized by a far-field beam divergence angle θ_0, which is related to the radial and azimuthal indices p and ℓ, respectively. As discussed in Ref. [4], the group velocity for Laguerre-Gaussian beams, taken as a specific example, depends on the beam divergence angle and, consequently, on the OAM value of the beam. This subluminal group velocity is only one side of what occurs in the propagation of OAM beams.

More in general, in fact, it is possible to calculate the exact group velocity of a paraxial beam at any point in space using the *wave picture* description. In the wave description, the group velocity is given by $v_g = |\partial_\omega \nabla\Phi|^{-1}$, where $\Phi(\mathbf{r})$ represents the wave phase front. For Laguerre-Gaussian modes, the group velocity along z, which has an explicit dependence

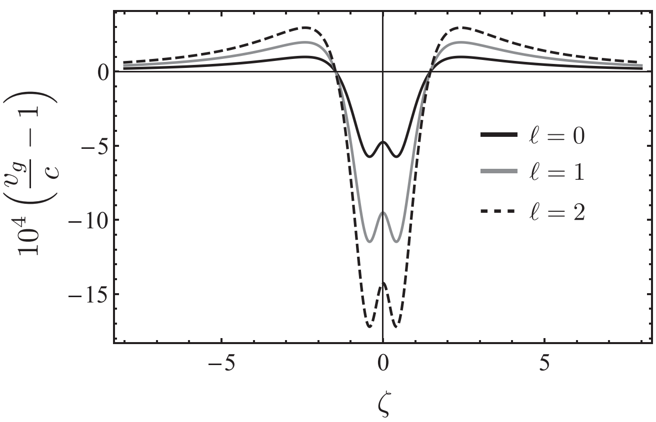

FIG. 1. Group velocity as a function of the dimensionless propagation distance $\zeta = z/z_R$ for various ℓ values, where z_R is the beam Rayleigh range. The divergence angle is set to $\theta_0 = \pi/36$.

on the propagation distance z, is given by

$$v_g = \frac{c}{1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)}, \tag{9}$$

where

$$\mathscr{F}(\zeta) = \frac{(1 + 6\zeta^2 - 3\zeta^4)}{(1 + \zeta^2)^3}, \tag{10}$$

$\zeta = z/z_\mathrm{R}$ is a dimensionless coordinate, and $z_\mathrm{R} = 8/(k\theta_0^2)$ is the Rayleigh range. The full description of the group velocity gives rise to a subluminal behavior, for $|\zeta| < \sqrt{1 + 2\sqrt{3}/3}$, that describes the observed propagation of the photons in a structured paraxial beam and a superluminal behavior for $|\zeta| > \sqrt{1 + 2\sqrt{3}/3}$. This behavior is similar to that of X-waves, i.e., nondispersive and nonmonochromatic superpositions of Bessel beams in free space that can present both superluminal group and phase velocities [42,43]. Superluminal group-velocity electromagnetic waves [44,45] are always causal solutions to Maxwell's equations that describe waves with group velocities $v_g > c$ that may naïvely appear to lead to superluminal information transmission, but they do not violate causality (see [46]).

The subluminal and superluminal behavior of the Laguerre-Gaussian modes upon propagation is shown in Fig. 1 for beams carrying OAM values of $\ell = 1$, 2, and 3.

This relationship reflects the peculiar geometry of these beams. The time delays—or, better, different group velocities—can be interpreted as the effect of a fictitious mass m_v of a quasiparticle state that characterizes the dynamics of the beam and is described by a Schrödinger-like equation propagating with the group velocity v_g,

$$v_g = \frac{1}{\hbar}\nabla_{k_z}\frac{\hbar^2 k_z^2}{2m_v} = \frac{\hbar k_z}{m_v} = \frac{p}{m_v}. \tag{11}$$

Any of these quasiparticle states shows a precise relationship between their angular momentum value and fictitious mass in vacuum that recalls the behavior of Proca photons with OAM in a plasma. This means that the process of beam confinement and structuring in phase and intensity plays a role similar to that of the Anderson-Higgs mechanism responsible to induce a Proca mass on photons that obey the rules of the Majorana tower with a precise angular momentum and mass relationship [18,19], similarly to what occurs for beams propagating in waveguides and cavities. At all effects, twisted photons observed to propagate in free space with $v_g < c$ and finite value of OAM are bosonic Majorana quasiparticle states.

The Majorana angular momentum and mass relationship is obtained by comparing the two group velocities, involving the wave vector k, the fictitious mass term m_v, and with the total angular momentum

$$\frac{\hbar k_z}{m_v} = \frac{c}{1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)}, \tag{12}$$

the fictitious mass value for this quasiparticle state, which reflects the geometrical properties of these beams, becomes

$$m_v = \frac{\hbar k_z}{c}\left[1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)\right]. \tag{13}$$

Following the calculations of Majorana in 1932 and 1937, this fictitious mass term m_v leads to a Majorana-tower mass M [15,47],

$$M = \frac{m_v}{\left[1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)\right]} = \frac{\hbar k_z}{c}, \tag{14}$$

with its corresponding energy

$$W_0 = \frac{m_v c^2}{\left[1 + \left(\frac{\theta_0}{4}\right)^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)\right]} = c\hbar k_z. \tag{15}$$

Each of these beams behaves as a particle in the low-energy limit that obeys a Schrödinger/Dirac equation with a fictitious Majorana mass M and a Majorana-tower OAM and fictitious mass relationship,

$$M = \frac{2m_v}{s^* + \frac{1}{2}}, \tag{16}$$

where $s^* = \theta_0^2 (2p + |\ell| + 1)\mathscr{F}(\zeta)$ is the angular momentum part of the virtual mass. In this case, the dynamics and structure of the beam are uniquely characterized in space and time through the rules of the Poincaré group that build up the Majorana tower. Particles with opposite OAM values differ by chirality even if they are propagating with the same group velocity. The term m_v, at these energies, is at all effects a nongravitating fictitious mass term that would turn into a real gravitating mass only when the electromagnetic and gravitational fields couple together, as in extremal astrophysical situations to explain the energy budget of extremely luminous gamma-ray bursts observed in the sky [48].

Furthermore, through the projection effect along the axis of propagation, z, one can resize and shape the beam at will to obtain different subluminal velocities for different values of OAM and spatially buffer a set of information in time obtained through the determination of the OAM value.

Mathematically, the expected apparent superluminal behavior characterized by $v_g > c$, instead, can be easily represented by fictitious tachionic solutions of the Majorana tower.

III. CONCLUSIONS

OAM beams present clear relationships between the beam divergence and the total angular momentum with the group velocity of the beam. Because of the topology of the beam, light is seen as propagating in directions different from that of the z axis, in which the latter is the locus of points where the optical vortex can be found and where the field amplitude and intensity is null. When the propagation of light is projected along the z axis, it gives rise to the subluminal group velocities that are experimentally observed. By definition, the group velocity v_g must be measured on the direction of the propagation axis z. This is what actually occurs to beams propagating into a waveguide or cavity.

We found that in these beams, the relationship between group velocity and total angular momentum or OAM can be cast in terms of a spectrum of Majorana quasiparticles with a virtual mass m_v cast as the infinite-component Majorana tower: the OAM value reduces the Majorana mass term M and also the group velocity v_g when the virtual mass m_v increases. When the OAM value is an integer, one obtains a bosonic quasiparticle. Instead, a fermionic Majorana solution, typical of the Majorana quasiparticles already known from the literature, can be obtained by applying coherent superpositions of OAM states to obtain beams carrying noninteger values of OAM [49], or to play with the component's spin angular momentum (SAM), $\mathbf{\Sigma}$, and OAM, $\mathbf{L}$, of the total angular momentum conserved quantity $\mathbf{J}$, introducing rotation operators during the propagation to obtain half-integer values of the total angular momentum J encoded also in single photons [50], which is a subject worthy of future experimental research.

Beams carrying integer values of OAM are, at all effects, an example of Majorana bosonic quasiparticles and can be used as an alternative method for the detection of the OAM value of the beam in time, without measuring the spatial phase gradient. On the other hand, this effect can be used as a precise time delay for optical communications based on the use of OAM states, as suggested in Refs. [3,41].

The mass term m_v can become a real mass term when the gravitational and EM fields couple at high energies [48]. Other solutions suggest the existence of structured waves with $v_g > c$ without violating causality, as in the well-known case of X-waves [42,43]. This phenomenology can also be applied, in principle, to other beams formed with zero-rest mass particles such as Majorana neutrinos and can be extended to beams of relativistic ultralight particles.

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APPENDIX: A BRIEF INTRODUCTION TO MAJORANA PARTICLES AND TOWERS OF PARTICLE STATES

In 1932, Majorana [15] proposed a generalization of the Dirac equation to spin values different from that of the electron by using the symmetries of the Poincaré group and introducing, in the Hamiltonian, a mass and spin angular momentum relationship.

The initial solution proposed by Majorana represents a denumerable infinite spectrum of particles that obey both the Bose-Einstein and the Fermi-Dirac statistics with a precise relationship between spin and mass due to the spin angular momentum and mass coupling in the Hamiltonian. The infinite spin spectrum of a generalized Dirac equation proposed by Majorana gives the first example of a unified spectrum of bosonic and fermionic relativistic particles, with a positive-definite or null finite squared mass, where any particle is not distinguishable from its antiparticle.

This solution, also known as the "Majorana tower" [16], does not find any correspondence with the spectrum of particles of the standard model [51], even if it finds some analogies with the pioneering work by Wess and Zumino in 1974 that describes an interacting quantum field theory in four dimensions with supersymmetry [52].

As known from the study of Majorana's unpublished works [53], he progressively came to the idea of the relativistic theory of particles with arbitrary angular momentum. He first studied the finite cases for composite systems and, in particular, a Dirac-like equation for photons, where he faced the problem by starting directly from the Maxwell classical field, using the three-dimensional complex vector $F = E \pm iH$, where E and H are the electric and magnetic fields in electrostatic units (esu), respectively [54]. This expression sets two invariants for the electromagnetic tensor, i.e., one for the real and one for the imaginary part of F, and introduces a *wave function* of the photon $\Psi = E \pm iH$, whose probabilistic interpretation is very simple and immediate and relies directly on the initial intuition of Einstein-Born: the electromagnetic energy density is proportional to the probability density of the photons. Maxwell's equations can be written in terms of this wave function; the first is the typical transverse state in quantum mechanics for the spin-1 particles, while developing the second Majorana is capable of writing the photon wave equation as a particular case of a Dirac equation, which is shown to be equivalent to quantum electrodynamics (QED) [55,56].

This was the starting point for Majorana's "theory of everything." It is important to note that the theory describes a single particle at rest and fields with variable spin, i.e., not elementary but composite systems, and applies to both bosons and fermions, anticipating the latest supermultiplet of supersymmetric (SUSY) theories. In other words, the work by Majorana consists of having extracted the most general conditions of space-temporal symmetry for a Hilbert space from a great mathematical phenomenology where all the spins can be represented by the nonhomogeneous Lorentz group. This space is infinite dimensional and can be tied to a single spin with appropriate contour conditions. Therefore, Majorana's theory is ideal for studying bound systems such as almost particle or photon systems in a structured beam, such as those we look at in this work. For theoretical developments of the theory, see, also, Refs. [57,58].

Majorana particles and quasiparticles

The modern concept of a Majorana particle or Majorana quasiparticle state such as those present in the modern literature, instead, starts from the work formulated by Majorana in 1937 [47], which was an alternative approach to the Dirac equation for relativistic particles in an attempt to avoid the problem of the negative squared mass solution emerging from the Dirac equation that instead were due to antielectrons, experimentally discovered by Anderson [59]. Majorana particles are solutions to the Dirac equation that are not distinguishable from their antiparticles.

Since any particle and antiparticle must have opposite electric charge, this type of solution is valid only for known standard-model neutral particles such as the particular class of fermions, the so-called Majorana neutrinos [60–62]. Clearly, known elementary charged particles such as electrons and positrons in the framework of the standard model cannot be Majorana particles.

The Majorana particles derived from the work written in 1937 have spin $s = 1/2$ and represent a subset of the more general spectrum of spin states described by the Majorana tower in which, instead, are present both fermionic and bosonic spin solutions.

Following this line of thought and extending the modern concept of the fermionic Majorana particle to the more general formulation found in the "tower," Majorana particles can also be bosons such as gravitons, photons, and axions [63], for which particles coincide with their own antiparticles.

"Majorana-like particles," instead, have been experimentally found up to now only in terms of fermionic quasiparticle states, in agreement with the work written in 1937 only. Examples of fermionic Majorana quasiparticles can be found in different scenarios such as condensed-matter physics [17], where composite states of elementary particles, emerging as the resulting product of electromagnetic interactions between electrons, photons, and the atomic structures that are always present in a condensed-matter scenario, behave collectively like Majorana fermions [64]. These quasiparticles can acquire mass from a self-interaction mechanism that recalls the Anderson-Higgs mechanism [22,23]. Of course, (quasi-) particle and antiparticle states coincide.

Other examples of fermionic Majorana quasiparticle excitations have been observed in topological superconductors [65,66], characterized by carrying null electric charge and energy [67]. Majorana zero modes—fermionic quasiparticles bound to a defect at null energy—were instead observed in Josephson junctions and in other solid-state systems [68] and hold promise for interesting future applications in information processing [69–71] and photonic systems [72]. For a deeper insight, see Ref. [73].

[1] D. Giovannini, J. Romero, S. Potoček *et al.*, Spatially structured photons that travel in free space slower than the speed of light, Science **347**, 857 (2015).

[2] F. Bouchard, J. Harris, H. Mand, R. W. Boyd, and E. Karimi, Observation of subluminal twisted light in vacuum, Optica **3**, 351 (2016).

[3] R. R. Alfano and D. A. Nolan, Slowing of Bessel light beam group velocity, Opt. Commun. **361**, 25 (2016).

[4] N. D. Bareza and N. Hermosa, Subluminal group velocity and dispersion of Laguerre Gauss beams in free space, Sci. Rep. **6**, 26842 (2016).

[5] Z. L. Horváth and B. Major, Comment on spatially structured photons that travel in free space slower than the speed of light, arXiv:1504.06059.

[6] P. Saari, Reexamination of group velocities of structured light pulses, Phys. Rev. A **97**, 063824 (2018).

[7] P. Saari, Observation of subluminal twisted light in vacuum: Comment, Optica **4**, 204 (2017).

[8] E. Karimi, G. Zito, B. Piccirillo, L. Marrucci, and E. Santamato, Hypergeometric-Gaussian modes, Opt. Lett. **32**, 3053 (2007).

[9] Z. S. Sacks, D. Rozas, and G. A. Swartzlander, Jr., Holographic formation of optical-vortex filaments, J. Opt. Soc. Am. B **15**, 2226 (1998).

[10] A. Y. Bekshaev and A. I. Karamoch, Spatial characteristics of vortex light beams produced by diffraction gratings with embedded phase singularity, Opt. Commun. **281**, 1366 (2008).

[11] G. Anzolin, F. Tamburini, A. Bianchini, and C. Barbieri, Method to measure off-axis displacements based on the analysis of the intensity distribution of a vortex beam, Phys. Rev. A **79**, 033845 (2009).

[12] G. Vallone, G. Parisi, F. Spinello, E. Mari, F. Tamburini, and P. Villoresi, A general theorem on the divergence of vortex beams, Phys. Rev. A **94**, 023802 (2016).

[13] N. I. Petrov, Speed of structured light pulses in free space, Sci. Rep. **9**, 18332 (2019).

[14] L. W. Davis, Theory of electromagnetic beams, Phys. Rev. A **19**, 1177 (1979).

[15] E. Majorana, Relativistic theory of particles with intrinsic arbitrary momentum, Il Nuovo Cimento **9**, 335 (1932).

[16] J. Magueijo and A. C. E. Slaughter, A brilliant darkness: The extraordinary life and mysterious disappearance of Ettore Majorana, the troubled genius of the nuclear age, Am. J. Phys. **78**, 446 (2010).

[17] F. Wilczek, Majorana and condensed matter physics, in *The Physics of Ettore Majorana: Theoretical, Mathematical, and Phenomenological*, edited by S. Esposito (Cambridge University Press, Cambridge, 2014).

[18] F. Tamburini, A. Sponselli, B. Thidé, and J. T. Mendonça, Photon orbital angular momentum and mass in a plasma vortex, Europhys. Lett. **90**, 45001 (2010).

[19] F. Tamburini and B. Thidé, Storming Majorana's tower with OAM states of light in a plasma, Europhys. Lett. **96**, 64005 (2011).

[20] F. Tamburini and B. Thidé, Majorana states for apparently slow vortex photons in vacuum, arXiv:1707.07160v1.

[21] S. Mamani Reyes, D. A. Nolan, L. Shi, and R. R. Alfano, Special classes of optical vector vortex beams are Majorana-like photons, Opt. Commun. **464**, 125425 (2020).

[22] P. W. Higgs, Broken symmetries and the masses of gauge bosons, Phys. Rev. Lett. **13**, 508 (1964).

[23] P. W. Anderson, Plasmons, gauge invariance, and mass, Phys. Rev. **130**, 439 (1963).

[24] J. Magueijo and L. Smolin, Lorentz Invariance with an Invariant Energy Scale, Phys. Rev. Lett. **88**, 190403 (2002).

[25] P. Gosselin, A. Berard, H. Mohrbach, and S. Ghosh, Phys. Lett. B **660**, 267 (2008).

[26] E. Stückelberg, The interaction forces in electrodynamics and in the field theory of nuclear forces (II), Helv. Phys. Acta **11**, 299 (1938).

[27] H. Ruegg and M. Ruiz-Altaba, The Stueckelberg field, Int. J. Mod. Phys. A **19**, 3265 (2004).

[28] V. Aldaya, M. Calixto, and F. López-Ruiz, Symmetry group of massive Yang-Mills theories without Higgs and their quantization, J. Phys. A: Math. Theor. **44**, 305405 (2011).

[29] H. Nishino and S. Rajipoot, Variant $N = 1$ supersymmetric non-Abelian Proca-Stueckelberg formalism in four dimensions, Nucl. Phys. B **872**, 213 (2013).

[30] L. Allen and M. Padgett, The Poynting vector in Laguerre-Gaussian beams and the interpretation of their angular momentum density, Opt. Commun. **184**, 67 (2000).

[31] J. Leach, S. Keen, M. Padgett *et al.*, Direct measurement of the skew angle of the Poynting vector in a helically phased beam, Opt. Exp. **14**, 11919 (2006).

[32] In the interpretation of the role of the instantaneous propagation vector $k(t)$, its (temporal or cycle) average in this scenario, one should also consider the role of its root-mean-square (RMS) deviation $\Delta k = \sqrt{\langle k\rangle^2 - \langle k^2\rangle}$, since this quantity enters in the uncertainty relations [74]. Interpreting the azimuthal component of the wave vector as the cycle average of the azimuthal component of the instantaneous propagation vector $k(t)$, it, of course, vanishes as it should on physical grounds. Even so, its RMS deviation does not vanish. So, in a physical (Heisenberg) sense, different azimuthal OAM modes give zero azimuthal components of the wave vector (vanishing momentum p_ϕ), but different RMS deviations of the OAM, corresponding to different OAM states and hence different phase delay vectors (which is a more stringent way of describing it, rather than different phase speeds; see, e.g., Ref. [75]). That wave modes can propagate with different delays and speeds is a well-known thing in waveguide physics. This is, of course, a property of the EM field, not of the (sheet metal making up the) waveguide itself. The waveguide technique is only one way of engineering EM modes that propagate with different delays and speeds. Modes with the same properties can, of course, be engineered in several other ways, such as specially designed antennas, phase masks, and so on.

[33] B. Thidé, F. Tamburini, H. Then, C. G. Someda, and R. A. Ravanelli, The physics of angular momentum radio, arXiv:1410.4268.

[34] *Twisted Photons: Applications of Light with Orbital Angular Momentum*, edited by P. Torres and J. L. Torner (Wiley-VCH Verlag GmbH, Weinheim, 2011).

[35] M. Oldoni *et al.*, Space-division demultiplexing in orbital-angular-momentum-based MIMO radio systems, antennas and propagation, IEEE Trans. **63**, 4582 (2015).

[36] F. Spinello *et al.*, High-order vortex beams generation in the radio-frequency domain, IEEE Anten. Wireless Propag. Lett. **15**, 889 (2016).

[37] X. Wei, C. Liu, L. Niu, Z. Zhang, K. Wang, Z. Yang, and J. Liu, Generation of arbitrary order Bessel beams via 3D printed axicons at the terahertz frequency range, Appl. Opt. **54**, 10641 (2015).

[38] G. F. Quinteiro, C. T. Schmiegelow, D. E. Reiter, and T. Kuhn, Reexamination of Bessel beams: A generalized scheme to derive optical vortices, Phys. Rev. A **99**, 023845 (2019).

[39] R. R. Alfano and D. A. Nolan, Response to comments on slowing of Bessel light beam group velocity, Opt. Commun. **392**, 302 (2017).
[40] N. J. Bareza and N. Hermosa, Subluminal speed of higher orders of physical light beams, in *Proceedings Frontiers in Optics 2016*, OSA Technical Digest Paper No. JW4A.103 (online) (Optical Society of America, NY, 2016).
[41] M. M. Coles, An upper bound on the rate of information transfer in optical vortex beams, Laser Phys. Lett. **15**, 095202 (2018).
[42] J. Y. Lu and J. F. Greenleaf, Nondiffracting X waves-exact solutions to free-space scalar wave equation and their finite aperture realizations, IEEE Trans Ultrason. Ferroelectr. Freq. Control. **39**, 19 (1992).
[43] *Localized Waves*, edited by H. E. Hernández-Figueroa, M. Zamboni-Rached, and E. Recami (Wiley, Hoboken, NJ, 2008).
[44] A. Sommerfeld, An objection against the relativistic theory of electrodynamics and its removal, Phys. Z. **8**, 841 (1907).
[45] H. Jeong, A. M. C. Dawes, and D. J. Gauthier, Direct Observation of Optical Precursors in a Region of Anomalous Dispersion, Phys. Rev. Lett. **96**, 143901 (2006).
[46] Consider a simple example when information is sent through Gaussian light pulses; the group velocity represents the velocity of the pulses' peaks used to send information. The common error is that one identifies the peak of the pulse—even the start of it—with the position in time of the information that is transmitted. The Gaussian modulation can be detected before the arrival of the peak because a Gaussian shape exhibits a rise of intensity in the beam at an earlier time, but this rise cannot be used for information exchange at a superluminal speed. In fact, to tell the signal from the background noise, one has to reach a threshold with a precise signal-to-noise ratio required by the communication protocol. At the end, the time at which the signal can be observed and decoded is limited exactly by the value of the speed of light in vacuum, c.
[47] E. Majorana, Symmetric theory of the electron and positron, Il Nuovo Cimento **14**, 171 (1937).
[48] F. Tamburini, M. De Laurentis, L. Amati, and B. Thidé, General relativistic electromagnetic and massive vector field effects with gamma-ray burst production, Phys. Rev. D **96**, 104003 (2017).
[49] M. V. Berry, Optical vortices evolving from helicoidal integer and fractional phase steps, J. Opt. A: Pure Appl. Opt. **6**, 259 (2004).
[50] K. E. Ballantine, J. F. Donegan, and P. R. Eastham, There are many ways to spin a photon: Half-quantization of a total optical angular momentum, Sci. Adv. **2**, e1501748 (2016).
[51] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[52] J. Wess and B. Zumino, Supergauge transformations in four dimensions, Nucl. Phys. B **70**, 39 (1974).
[53] S. Esposito, *The Physics of Ettore Majorana. Theoretical, Mathematical and Phenomenological*, (Cambridge University Press, Cambridge, 2015).
[54] R. Mignani, E. Recami, and M. Baldo, About a Dirac-like equation for the photon, according to Ettore Majorana, Lett. Nuovo Cimento **11**, 568 (1974).
[55] F. Tamburini and D. Vicino, Photon wave function: A covariant formulation and equivalence with QED, Phys. Rev. A **78**, 052116 (2008).
[56] I. Bialynicki-Birula, Photon wave function, Prog. Opt. **36**, 245 (1996).
[57] I. M. Gel'fand and A. M. Yaglom, General Lorentz invariant equations and finite dimensional representations of the Lorentz group, Z. Eksp. Teor. Fiz. **18**, 703 (1948).
[58] V. V. Varlamov, Spinor structure and internal symmetries, Intl. J. Theor. Phys. **54**, 3533 (2015).
[59] C. D. Anderson, The positive electron, Phys. Rev. **43**, 491 (1933).
[60] M. Doi, T. Kotani, and E. Takasugi, Double beta decay and Majorana neutrino, Prog. Theor. Phys. Suppl. **83**, 1 (1985).
[61] R. N. Mohapatra *et al.*, Theory of neutrinos: A white paper, Rep. Prog. Phys. **70**, 1757 (2007).
[62] F. T. Avignone III, S. R. Elliott, and J. Engel, Double beta decay, Majorana neutrinos, and neutrino mass, Rev. Mod. Phys. **80**, 481 (2008).
[63] F. Tamburini and I. Licata, Can the periodic spectral modulations observed in 236 Sloan Sky Survey stars be due to dark matter effects? Phys. Scr. **92**, 095001 (2017).
[64] S. Ferrara, A. Marrani, and A. Yeranyan, Majorana fermions, supersymmetry breaking, and born-infeld theory, in *53rd International School of Subnuclear Physics: The Future of our Physics Including New Frontiers* (World Scientific, Singapore, 2015).
[65] L. Fu and C. L. Kane, Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator, Phys. Rev. Lett. **100**, 096407 (2008).
[66] T. Neupert, S. Onoda, and A. Furusaki, Chain of Majorana States from Superconducting Dirac Fermions at a Magnetic Domain Wall, Phys. Rev. Lett. **105**, 206404 (2010).
[67] T. Kawakami and X. Hu, Evolution of Density of States and a Spin-Resolved Checkerboard-Type Pattern Associated with the Majorana Bound State, Phys. Rev. Lett. **115**, 177001 (2015).
[68] C. Ohm and F. Hassler, Majorana fermions coupled to electromagnetic radiation, New J. Phys. **16**, 015009 (2014).
[69] S. Das Sarma, M. Freedman, and C. Nayak, Majorana zero modes and topological quantum computation, npj Quantum Inf. **1**, 15001 (2015).
[70] L. Borsten and M. J. Duff, Majorana fermions in particle physics, solid state and quantum information, in *Proceedings of the 53rd International School of Subnuclear Physics Erice, Celebration of the Triumph of Ettore Majorana* (World Scientific, Singapore, 2015).
[71] A. Yu. Kitaev, Unpaired Majorana fermions in quantum wires, Phys. Usp. **44**, 131 (2001).
[72] J.-S. Xu, K. Sun, Y.-J. Han *et al.*, Simulating the exchange of Majorana zero modes with a photonic system, Nat. Commun. **7**, 13194 (2016).
[73] S. R. Elliott and M. Franz, Colloquium: Majorana fermions in nuclear, particle and solid-state physics, Rev. Mod. Phys. **87**, 137 (2015).
[74] A. Messiah, *Quantum Mechanics*, Vol. 1 (Elsevier, Amsterdam, 1981).
[75] M. Born, E. Wolf, A. Bhatia *et al.*, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 7th ed. (Cambridge University Press, Cambridge, 1999).