Surface electromagnetic waves in bianisotropic superlattices and homogeneous media

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This paper analyzes the existence of surface electromagnetic waves (SEWs) at the interface of bianisotropic bicrystals which are formed of half-infinite periodic superlattices or of homogeneous bianisotropic media. In either case we assume no absorption. Properties of the impedance matrices characterizing such bianisotropic media are established. On this basis, a series of statements is proved on the maximum total number of SEWs in two bicrystals composed of two superlattices or homogeneous media in such a way that the upper (lower) half of one bicrystal complements the lower (upper) part of the other to an infinite periodic superlattice or homogeneous media, respectively. It is shown that in superlattices the maximum number of SEWs at a fixed tangential wave number equals two in the lowest forbidden band (this band begins from zero frequency) and four in any upper forbidden band. It is also proved that at most two SEWs emerge in homogeneous bianisotropic media.

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I. INTRODUCTION

During the last few decades electrodynamics of bianisotropic media has attracted significant researchers' attention [1–6]. Substantial results have been obtained concerning bulk wave propagation [1–5,7–10], optical activity and circular dichroism [11–15], negative refraction [16–19], nonreciprocity effects [20–22], and the derivation of the effective constants of bianisotropic metamaterials [23–26].

The planar boundary of dielectric media, including bianisotropic ones, can support electromagnetic waves of which the amplitude decays to zero with distance in depth [6]. To some extent, such surface electromagnetic waves (SEWs) resemble surface plasmons at metal-dielectric interfaces [27] which were extensively studied in different options, in particular, in stacks of two-dimensional materials, such as metal-halide perovskites and MXenes [28,29], and widely used, e.g., in sensors [30]. Unlike surface plasmons, SEWs in dielectrics can have very small propagation losses, so they can be utilized, e.g., in communication systems and sensors where a long propagation path is necessary. The nonreciprocity of SEW propagation in bianisotropic media could also be of potential interest for applications.

SEWs in bianisotropic materials have been investigated by explicit calculations in Refs. [31–33]. TE- and TM-polarized SEWs have been found in Ref. [34] on the interface between an isotropic homogeneous half-space and a periodic system of uniaxial bianisotropic layers. In Ref. [35] the dispersion equation for SEWs in bianisotropic media has been written in a general form, but its solvability has been analyzed only for SEWs in a bicrystal formed of an isotropic medium and a bianisotropic uniaxial one with optical axis perpendicular to the interface.

The aim of the present paper is to establish how many SEWs can emerge at the interfaces between two periodic superlattices formed of bianisotropic materials of arbitrary symmetry and between two arbitrary bianisotropic homogeneous media. These problems cannot be solved either by explicit calculations or numerically, but an analytical method exists which has allowed the study of a similar problem for surface acoustic waves in phononic crystal [36–41] and later has been adapted for SEWs in nonbianisotropic magnetooptically inactive superlattices [42]. In this work we will also stick to this method, so that our analysis of the existence of SEWs will rely on general properties of the impedances of bianisotropic media. We assume no absorption. In this case impedances have a few important properties, e.g., they are Hermitian. Also, no absorption makes it possible to uniquely break the frequency range into allowed and forbidden bands. Otherwise the Bloch wave number would be complex-valued at any frequency, and therefore a rigorous account for losses proves to be a fairly intricate problem which is out of the scope of the present work.

The results obtained below apply to nonbianisotropic magnetooptically active media. Bearing this in mind, for brevity, in what follows we use the terms "bianisotropic" and "anisotropic" with the reservation that the former also implies "nonbianisotropic magnetooptically active" and the latter means "nonbianisotropic magnetooptically inactive." In other words, this paper assumes that materials, which are called anisotropic, are characterized only by the dielectric permittivity and magnetic permeability tensors, they both being purely real.

Our paper is organized as follows. The transfer matrix is discussed in Sec. II, and the necessary properties of impedances are derived in Sec. III. The statements concerning the number of SEWs in biansotropic bicrystals are given in Sec. IV. The conclusions are formulated in Sec. V. The Appendix derives important expressions which are utilized in our considerations. The Supplemental Material contains some details omitted in the main text.

II. TRANSFER MATRIX

Consider an electromagnetic wave

$$\begin{pmatrix}\mathbf{E}(\mathbf{r},t)\\ \mathbf{H}(\mathbf{r},t)\end{pmatrix}=\begin{pmatrix}\mathbb{E}(z)\\ \mathbb{H}(z)\end{pmatrix}e^{i(kx-\omega t)}, \tag{1}$$

which propagates in the plane XZ of a lossless bianisotropic medium. If its material parameters either are coordinate independent or depend only on z, then the x- and y-components of $\mathbb{E}(z)$ and $\mathbb{H}(z)$ obey a system of ordinary differential equations, *viz.*,

$$\frac{d\boldsymbol{\xi}}{dz}=i\hat{\mathbf{N}}\boldsymbol{\xi}. \tag{2}$$

The vector $\boldsymbol{\xi}(z)$ is defined differently; see, e.g., Refs. [6,35,43,44]. As in Ref. [42], we put

$$\boldsymbol{\xi}(z)=\begin{pmatrix}\mathbf{u}\\ \mathbf{v}\end{pmatrix},\quad \mathbf{u}(z)=\begin{pmatrix}-\mathbb{E}_y\\ \mathbb{H}_y\end{pmatrix},\quad \mathbf{v}(z)=\begin{pmatrix}\mathbb{H}_x\\ \mathbb{E}_x\end{pmatrix}. \tag{3}$$

The matrix $\hat{\mathbf{N}}$ in the form convenient for subsequent use is derived in the Appendix. Here we only note that

$$\hat{\mathbf{N}}=\hat{\mathbf{T}}\hat{\mathbb{N}}, \tag{4}$$

where $\hat{\mathbb{N}}$ is a Hermitian matrix (A8),

$$\hat{\mathbb{N}}^{\dagger}=\hat{\mathbb{N}}, \tag{5}$$

$$\hat{\mathbf{T}}=\begin{pmatrix}\hat{\mathbf{0}} & \hat{\mathbf{I}}\\ \hat{\mathbf{I}} & \hat{\mathbf{0}}\end{pmatrix}, \tag{6}$$

where $\hat{\mathbf{0}}$ and $\hat{\mathbf{I}}$ are 2×2 zero and identity matrices. Note that in the case of anisotropic media considered in Ref. [42], i.e., when in constitutive relations (A4) $\hat{\boldsymbol{\kappa}}=0$ and $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\mu}}$ are real, $\hat{\mathbb{N}}$ turns out to be a real symmetric matrix.

Assume a medium periodically stratified along the axis Z. The unit cell consists of n layers of thicknesses h_i, and the period is $H=\sum_{i=1}^{n}h_i$, each layer is characterized by the matrix $\hat{\mathbf{N}}_i$. We define the transfer matrix $\hat{\mathbf{M}}$ through unit cell by the relation $\boldsymbol{\xi}(H)=\hat{\mathbf{M}}\boldsymbol{\xi}(0)$, where $z=0$ and $z=H$ are the boundaries of a unit cell, so $\hat{\mathbf{M}}$ is representable as a product of the transfer matrices $\hat{\mathbf{M}}_i=\exp(ih_i\hat{\mathbf{N}}_i)$ of individual layers and, by virtue of Eqs. (4) and (5), fulfills the equality

$$\hat{\mathbf{M}}^{-1}=\hat{\mathbf{T}}\hat{\mathbf{M}}^{\dagger}\hat{\mathbf{T}}. \tag{7}$$

Due to Eq. (7), the four eigenvalues γ_α of $\hat{\mathbf{M}}$ are spilt in pairs $|\gamma_\alpha|=|\gamma_{\alpha+2}|=1$ or $\gamma_\alpha=1/\gamma_{\alpha+2}^{*}$, $|\gamma_\alpha|\neq 1$ $(\alpha=1,2)$, and, in consequence, the (ω,k)-plane is partitioned into allowed and forbidden bands. We are concerned with forbidden bands, i.e., areas of the (ω,k)-plane in which $|\gamma_\alpha|\neq 1$ for all α. Let

$$|\gamma_\alpha|<1<|\gamma_{\alpha+2}|,\ \alpha=1,2, \tag{8}$$

and $\boldsymbol{\zeta}_\alpha$, $\alpha=1,2,3,4$, be corresponding eigenvectors. In accordance with our definition of $\hat{\mathbf{M}}$, the solutions $\boldsymbol{\xi}_+(z)$ and $\boldsymbol{\xi}_-(z)$ to Eq. (2), which are specified at $z=0$ by the conditions $\boldsymbol{\xi}_+=\sum_{\alpha=1}^{2}c_\alpha\boldsymbol{\zeta}_\alpha$ and $\boldsymbol{\xi}_-=\sum_{\alpha=3}^{4}c_\alpha\boldsymbol{\zeta}_\alpha$, where c_α are constants, decay to zero as $z\to+\infty$ and $z\to-\infty$, respectively. This fact is taken into account in the definition of impedances.

III. IMPEDANCES AND THEIR PROPERTIES

Let us define the 2×2 impedance matrices $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ by relations

$$\mathbf{V}_\alpha=i\hat{\mathbf{Z}}\mathbf{U}_\alpha,\quad \mathbf{V}_{\alpha+2}=-i\hat{\mathbf{Z}}'\mathbf{U}_{\alpha+2},\quad \alpha=1,2, \tag{9}$$

where the vectors $\mathbf{U}_\alpha$ and $\mathbf{V}_\alpha$ are the first two and second two components of $\boldsymbol{\zeta}_\alpha=(\mathbf{U}_\alpha\ \mathbf{V}_\alpha)^t$. Hence, at $z=0$, $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ relate the components $\mathbf{u}$ and $\mathbf{v}$ of those wave fields which decay, respectively, in half-spaces $z>0$ and $z<0$.

The impedances $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ have the following properties in forbidden bands. First, since the z-component of the averaged energy flux is to vanish,

$$\hat{\mathbf{Z}}=\hat{\mathbf{Z}}^{\dagger},\quad \hat{\mathbf{Z}}'=\hat{\mathbf{Z}}'^{\dagger}. \tag{10}$$

Second,

$$\hat{\mathbf{Z}}\text{ and }\hat{\mathbf{Z}}'\text{ are positive definite matrices at }\omega\to 0. \tag{11}$$

The proof is similar to that given in Ref. [42]; see also the Supplemental Material [45].

A third important property is that

$$\frac{\partial\hat{\mathbf{Z}}}{\partial\omega}\text{ and }\frac{\partial\hat{\mathbf{Z}}'}{\partial\omega}\text{ are negative definite matrices in forbidden bands.} \tag{12}$$

It follows from the inequality

$$-i\frac{d}{dz}\left(\boldsymbol{\xi}^{\dagger}\hat{\mathbf{T}}\frac{\partial\boldsymbol{\xi}}{\partial\omega}\right)>0, \tag{13}$$

where $\boldsymbol{\xi}(z)$ is a solution to system (2); see the Supplemental Material [45]. In order to prove (13), we use a formula which generalizes to bianisotropic media the Brillouin formula for the average energy density W of electromagnetic fields in anisotropic media with dispersion [8,46,47]:

$$W=\frac{1}{4}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}^{\dagger}\frac{\partial(\omega\hat{\boldsymbol{\Gamma}})}{\partial\omega}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}, \tag{14}$$

where $\hat{\boldsymbol{\Gamma}}$ is matrix (A4). Let us write (14) in the form

$$W=\frac{1}{4}\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\phi}\end{pmatrix}^{\dagger}\frac{\partial(\omega\hat{\boldsymbol{\Omega}})}{\partial\omega}\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\phi}\end{pmatrix}, \tag{15}$$

where $(\boldsymbol{\xi}\ \boldsymbol{\phi})^t$ is the vector on the right-hand side of (A5) and $\hat{\boldsymbol{\Omega}}$ is matrix (A6). Further, we expand (15) and substitute (A7) for $\boldsymbol{\phi}$. This yields

$$W=\frac{1}{4}\boldsymbol{\xi}^{\dagger}\left[\frac{\partial(\omega\hat{\boldsymbol{\Omega}}_1)}{\partial\omega}-\hat{\mathbf{P}}_1-\hat{\mathbf{P}}_1^{\dagger}+\hat{\mathbf{P}}_2\right]\boldsymbol{\xi}, \tag{16}$$

where

$$\hat{\mathbf{P}}_1=\frac{\partial(\omega\hat{\boldsymbol{\Omega}}_2)}{\partial\omega}\hat{\boldsymbol{\Omega}}_4^{-1}\left(\hat{\boldsymbol{\Omega}}_2^{\dagger}+\frac{k}{\omega}\hat{\mathbf{J}}^t\right), \tag{17}$$

$$\hat{\mathbf{P}}_2=\left(\hat{\boldsymbol{\Omega}}_2+\frac{k}{\omega}\hat{\mathbf{J}}\right)\hat{\boldsymbol{\Omega}}_4^{-1}\frac{\partial(\omega\hat{\boldsymbol{\Omega}}_4)}{\partial\omega}\hat{\boldsymbol{\Omega}}_4^{-1}\left(\hat{\boldsymbol{\Omega}}_2^{\dagger}+\frac{k}{\omega}\hat{\mathbf{J}}^t\right), \tag{18}$$

and, by regrouping the terms in (16) with due account for Eqs. (A8)–(A11), we obtain that

$$W=\frac{1}{4}\boldsymbol{\xi}^{\dagger}\frac{\partial\hat{\mathbb{N}}}{\partial\omega}\boldsymbol{\xi}. \tag{19}$$

Owing to Eqs. (2), (4), and (5),

$$-i\frac{d}{dz}\left(\boldsymbol{\xi}^{\dagger}\hat{\mathbf{T}}\frac{\partial\boldsymbol{\xi}}{\partial\omega}\right)=\boldsymbol{\xi}^{\dagger}\frac{\partial\hat{\mathbb{N}}}{\partial\omega}\boldsymbol{\xi}, \tag{20}$$

so positiveness of W proofs inequality (13) and hence statement (12) holds true.

Let the replacement of a material constant c by $c+i\Delta c$, where Δc takes on a small real value, result in absorption. Denote $\text{sgn}(\Delta c)=s$ and then

$$s\frac{\partial\hat{\mathbf{Z}}}{\partial c} \text{ and } s\frac{\partial\hat{\mathbf{Z}}'}{\partial c} \text{ are negative definite matrices in forbidden bands.} \tag{21}$$

This property stems from an expression of the time-averaged heat power density Q released because of absorption of a harmonic wave [46,47],

$$Q=\frac{1}{2}\text{Re}\left(\mathbf{E}^{\dagger}\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}^{\dagger}\frac{\partial\mathbf{B}}{\partial t}\right), \tag{22}$$

which, due to (A4), transforms to

$$\begin{aligned}Q&=\frac{i\omega}{4}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}^{\dagger}\{\hat{\Gamma}^{\dagger}-\hat{\Gamma}\}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}\\&\approx\frac{\omega\Delta c}{2}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}^{\dagger}\frac{\partial\hat{\Gamma}}{\partial c}\begin{pmatrix}\mathbb{E}\\ \mathbb{H}\end{pmatrix}\\&=\frac{\Delta c}{2}\left(\boldsymbol{\xi}^{\dagger}\frac{\partial\hat{\mathbb{N}}}{\partial c}\boldsymbol{\xi}\right)=-i\frac{\Delta c}{2}\frac{d}{dz}\left(\boldsymbol{\xi}^{\dagger}\hat{\mathbf{T}}\frac{\partial\boldsymbol{\xi}}{\partial c}\right),\end{aligned} \tag{23}$$

and the fact that $Q>0$.

Thus, we have four properties of $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$ at our disposal. These properties turn out to be the same as those of the impedances of anisotropic structures. The counterparts of properties (10)–(12) for the case of anisotropic media were used in Ref. [42]. However, we note that according to Ref. [42] the counterpart of inequality (13) holds true, and consequently $\partial\hat{\mathbf{Z}}/\partial\omega$ and $\partial\hat{\mathbf{Z}}'/\partial\omega$ are negative definite matrices in anisotropic lossless materials provided that either (a) $k=0$, (b) there is no dispersion, or (c) the stratification axis is either parallel to a symmetry axis or perpendicular to the plane of symmetry. Derivation (14)–(20) removes these three extra restrictions.

Let us discuss briefly the case of homogeneous bianisotropic media. Given k, there is a critical frequency ω_L below which all four eigenvalues of $\hat{\mathbb{N}}$ are complex-valued, and, in view of Eqs. (4) and (5), they form complex conjugate pairs. Thus the interval $0<\omega<\omega_L$ is a counterpart of the lowest forbidden band of superlattices in the sense that within it all partial solutions of Eq. (2) either decrease or increase in a half-infinite medium. The counterparts $\hat{\mathbf{Z}}_h$ and $\hat{\mathbf{Z}}'_h$ of impedances $\hat{\mathbf{Z}}$ and $\hat{\mathbf{Z}}'$, respectively, are defined in terms of the eigenvectors $\boldsymbol{\xi}_\alpha$ of $\hat{\mathbb{N}}$, and in the interval $0<\omega<\omega_L$ they possess properties (10)–(12) and (21).

IV. SURFACE ELECTROMAGNETIC WAVES

We are interested in SEWs in two bicrystals formed of the halves of two infinite periodic bianisotropic superlattices 1 and 2 cut along a plane perpendicular to the periodicity direction. The bicrystal consisting of the upper half of superlattice 1 and the lower half of superlattice 2 will be called direct [Fig. 1(a)]. By joining the upper half of superlattice 2 and the lower half of superlattice 1 we get the complementary bicrystal [Fig. 1(b)]. Such two bicrystals can be called a complementary bicrystal pair since the upper (lower) half of one of them complements the lower (upper) half of the other to an infinite periodic superlattice.

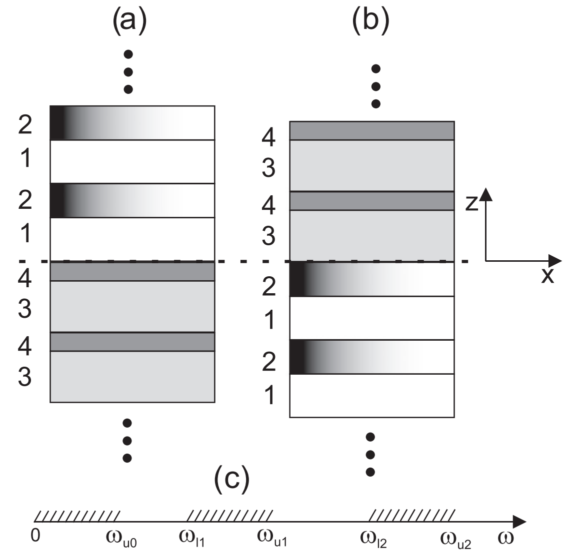

FIG. 1. Example of direct (a) and complementary (b) bicrystals composed of two-layer half-infinite superlattices $J=1$ (layers 1 and 2) and $J=2$ (layers 3 and 4) and the band structure of bicrystals at a fixed k (c). Frequency interval $(0,\omega_{u0})$ is the lowest forbidden band, and frequency intervals $(\omega_{l1},\omega_{u1})$ and $(\omega_{l2},\omega_{u2})$ are upper forbidden bands.

SEWs emerge within overlapping areas of forbidden bands of superlattices forming bicrystals. Both bicrystals have an identical set of such areas, and we will call them forbidden bands of bicrystals. The forbidden band $0<\omega<\omega_u$ which goes from 0 to a frequency ω_u will be called the lowest one. The other forbidden bands $\omega_l<\omega<\omega_u$ $(\omega_l\neq 0)$ will be called upper ones [Fig. 1(c)].

Let $\hat{\mathbf{Z}}^{(J)}$ and $\hat{\mathbf{Z}}'^{(J)}$ denote the impedances defined by (9) for superlattices $J=1$ and $J=2$. Following this definition the dispersion equations are reduced to

$$\det(\hat{\mathbf{Z}}^{(1)}+\hat{\mathbf{Z}}'^{(2)})=0 \tag{24}$$

in the direct bicrystal and

$$\det(\hat{\mathbf{Z}}^{(2)}+\hat{\mathbf{Z}}'^{(1)})=0 \tag{25}$$

in the complementary one. By utilizing properties (10)–(12) of $\hat{\mathbf{Z}}^{(J)}$ and $\hat{\mathbf{Z}}'^{(J)}$ it proves to be possible to establish the total number of roots of Eqs. (24) and (25) in a forbidden band of a complementary bicrystal pair. Insofar as these properties are the same as properties of impedances of anisotropic media [42], the conclusions regarding SEWs are derivable analogously, so we formulate only the final statements, namely,

Given k, at most two SEWs can exist in total in the lowest forbidden band of a complementary bianisotropic bicrystal pair.

Given k, at most four SEWs can exist in total in an upper forbidden band of a complementary bianisotropic bicrystals pair.

[Some details of the derivation of these statements can be found in the Supplemental Material [45].]

Numerical examples provided in Ref. [42] show that two SEWs in the lowest forbidden band and four SEWs in an upper forbidden band of a pair of anisotropic complementary bicrystals may exist. Since bianisotropy, when treated as a small perturbation, cannot remove any SEWs found in Ref. [42], we conclude that the predicted allowable numbers of SEWs in bianisotropic bicrystals are exact maxima, i.e., two SEWs and four SEWs actually may emerge in forbidden bands of a pair of bianisotropic complementary bicrystals.

Weak absorption does not remove SEWs either. For instance, assume that with no absorption a SEW exists in the direct bicrystal at a frequency $\omega = \omega_0$, i.e., an eigenvalue χ_α of $\hat{\mathbf{Z}}_B = \hat{\mathbf{Z}}^{(1)} + \hat{\mathbf{Z}}'^{(2)}$ vanishes at this frequency. Let absorption be describable via the change of a material constant c to $c + i\Delta c$. Due to Eqs. (12) and (21) the dispersion equation necessarily has a root $\omega \approx \omega_0 + i\omega'$, and the sign of ω' corresponds to attenuating wave (1),

$$\omega' = -\frac{\mathbf{e}_\alpha^\dagger \frac{\partial \hat{\mathbf{Z}}_B}{\partial c}\mathbf{e}_\alpha}{\mathbf{e}_\alpha^\dagger \frac{\partial \hat{\mathbf{Z}}_B}{\partial \omega}\mathbf{e}_\alpha}\Delta c < 0, \tag{26}$$

where $\mathbf{e}_\alpha$ is the eigenvector of $\hat{\mathbf{Z}}_B$ associated with χ_α at $\omega = \omega_0$ and no absorption.

The number of SEWs in bicrystals formed of two homogeneous bianisotropic media $J = 1$ and $J = 2$ can be established analogously. SEWs emerge in the interval $0 < \omega < \min(\omega_L^{(1)}, \omega_{L2}^{(2)})$, where $\omega_L^{(J)}$ are critical frequencies of media 1 and 2. Equations (24) and (25), after the replacement of $\hat{\mathbf{Z}}^{(J)}$ and $\hat{\mathbf{Z}}'^{(J)}$ by $\hat{\mathbf{Z}}_h^{(J)}$ and $\hat{\mathbf{Z}}_h'^{(J)}$, determine SEW frequencies. Since the interval $0 < \omega < \min(\omega_L^{(1)}, \omega_L^{(2)})$ is a counterpart of the lowest forbidden band, we conclude that

Given k, at most two SEWs can exist in total in a complementary bicrystal pair formed of homogeneous bianisotropic media.

If media are anisotropic, then, by virtue of the fact that the matrix $\hat{\mathbf{N}}$ is real in this case, $\hat{\mathbf{Z}}_h^{(J)} = \hat{\mathbf{Z}}_h'^{(J)t}$, so the dispersion equations for the direct and complementary bicrystals coincide. According to Ref. [48], at most one root exists, that is, each of the bicrystals of a complementary pair supports at most one SEW. Examples of SEWs in homogeneous anisotropic bicrystals are well known [49,50] (see also references in Refs. [6,42]). Bianisotropy leads to $\hat{\mathbf{Z}}_h^{(J)} \neq \hat{\mathbf{Z}}_h'^{(J)t}$ but small values of the pseudotensor $\hat{\boldsymbol{\kappa}}$, which enters the constitutive relations Eq. (A4), i.e., weak bianisotropy, cannot remove existing SEWs and only changes their frequencies. Hence two SEWs actually may emerge in a complementary bianisotropic bicrystal pair, so the above statement yields the exact maximum.

In particular, this statement allows the existence of two SEWs in a given bianisotropic bicrystal. Let us exemplify such a situation. Assume "model" media 1 and 2 having the same dielectric permittivity

$$\hat{\boldsymbol{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & -ig & 0 \\ ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \tag{27}$$

in the XYZ coordinate system and the same $\hat{\boldsymbol{\kappa}}$. The rotation of media 1 and 2 through the angles φ and $-\varphi$, respectively, around the axis X, makes their dielectric permittivities different in the XYZ frame. By joining media 1 and 2 along a plane perpendicular to the axis Z we get a pair of complementary bicrystals one of which can support two SEWs. For instance, if $\varepsilon_\perp = 5$, $\varepsilon_\parallel = 4$, $g = 3$, $\varphi = 20^o$ and $\hat{\boldsymbol{\kappa}} = 0$, then two SEWs exist in the positive direction of the axis X in the direct bicrystal (medium 1 is on top medium 2) at $\omega_1 = 0.485\omega_0$ and $\omega_2 = 0.494\omega_0$, where $\omega_0 = ck$, c is the light velocity in vacuum. Once two SEWs exist at $\hat{\boldsymbol{\kappa}} = 0$, both SEWs exist at least while $\hat{\boldsymbol{\kappa}}$ is small. In the negative direction of the axis X two SEWs emerge in the complementary bicrystal, whereas no SEWs exist in the direct one, and this is a manifestation of the nonreciprocity [21,22].

V. CONCLUDING REMARKS

We have analyzed the existence of SEWs in bianisotropic magnetooptically active media assuming arbitrary crystallographic symmetry and no absorption. In the presence of absorption the impedance matrices cease being Hermitian and lose other properties, so that the SEW problem in absorbing materials requires a separate investigation. Nevertheless, we have demonstrated that sufficiently weak absorption cannot decrease the number of SEWs. This work generalizes Refs. [42] and [48] where SEWs have been studied in nonbianisotropic magnetooptically inactive superlattices and homogeneous media, respectively, and, as indicated in the Introduction, we call such materials "anisotropic," while it is adopted that the term "bianisotropic" also implies "nonbianisotropic magnetooptically active."

Specifically, we have established the maximum number of SEWs which may exist in a forbidden band of a pair of complementary bicrystals constituted of two half-infinite periodic bianisotropic superlattices. It turns out that in the case of an arbitrary unit cell bianisotropy does not change the maximum number of SEWs as compared to the number of SEWs in anisotropic superlattices [42].

However, the situation is different when a unit cell consists of $2n + 1$ layers, ith and $(2n + 2 - i)$-th layers being identical (symmetric unit cell). According to Ref. [42], in anisotropic bicrystal with symmetric unit cells at most one SEW and at most two SEWs may exist in the lowest forbidden band and in an upper forbidden one, respectively. This is due to the fact that the matrix $\hat{\mathbf{N}}$ proves to be real. In bianisotropic media $\hat{\mathbf{N}}$ is complex, and it occurs that the symmetry of unit cells plays no role, so bianisotropy can increase the allowable maximum of SEWs in a bicrystal with symmetric unit cells to two and four in the lowest and upper forbidden bands, respectively.

We have also established that the maximum number of SEWs at the interface between two homogeneous half-infinite bianisotropic media is two. The example in the end of Sec. IV confirms that two SEWs may exist. Note that the same model shows that two SEWs may exist in the lowest forbidden band of a bicrystal composed of superlattices with symmetric unit cells. Indeed, it can be observed that a weak periodicity is unable to remove SEWs which exist in the bicrystal made of homogeneous media, and the interval in which two SEWs exist proves to be the lowest forbidden band.

Our results make it possible to refine some conclusions of earlier papers. As pointed out in the end of Sec. III, from the present work it follows that the absence of losses is sufficient in order for the results of Ref. [42] to hold true. Another side result is related to Ref. [48] where the existence of at most one SEW in anisotropic media has been proved assuming no dispersion. Our work shows that in such media with dispersion there also emerges at most one SEW.

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APPENDIX

We substitute (1) in the Maxwell equations and bring the resulting relations into the form

$$\frac{1}{i}\frac{d\boldsymbol{\xi}}{dz} = \hat{\mathbf{T}}(\omega\boldsymbol{\psi} + k\hat{\mathbf{J}}\boldsymbol{\phi}), \quad \text{(A1)}$$

$$-k\hat{\mathbf{J}}^t\boldsymbol{\xi} = \omega\boldsymbol{\nu}, \quad \text{(A2)}$$

where the vector $\boldsymbol{\xi}$ and the matrix $\hat{\mathbf{T}}$ are defined by Eqs. (3) and (6), respectively, and $\boldsymbol{\psi} = (-D_y\ B_y\ B_x\ D_x)^t$,

$$\boldsymbol{\phi} = \begin{pmatrix} \mathbb{H}_z \\ \mathbb{E}_z \end{pmatrix}, \quad \boldsymbol{\nu} = \begin{pmatrix} \mathbb{B}_z \\ \mathbb{D}_z \end{pmatrix}, \quad \hat{\mathbf{J}} = \begin{pmatrix} \hat{\mathbf{I}} \\ \hat{\mathbf{0}} \end{pmatrix}, \quad \text{(A3)}$$

where the symbol ^{t} stands for transposition. Let us use the bianisotropic constitutive relations

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \hat{\boldsymbol{\Gamma}}\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \quad \hat{\boldsymbol{\Gamma}} = \begin{pmatrix} \hat{\boldsymbol{\varepsilon}} & \hat{\boldsymbol{\kappa}} \\ \hat{\boldsymbol{\kappa}}^\dagger & \hat{\boldsymbol{\mu}} \end{pmatrix}, \quad \text{(A4)}$$

where the Hermitian tensors $\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{\varepsilon}}^\dagger$ and $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}^\dagger$ are, respectively, the dielectric permittivity and magnetic permeability tensors (magneto-optical activity is taken into account), and $\hat{\boldsymbol{\kappa}}$ is a complex nonsymmetric pseudotensor [4–6] and [21–24]. By regrouping Eq. (A4) we obtain

$$\begin{pmatrix} \boldsymbol{\psi} \\ \boldsymbol{\nu} \end{pmatrix} = \hat{\boldsymbol{\Omega}}\begin{pmatrix} \boldsymbol{\xi} \\ \boldsymbol{\phi} \end{pmatrix}, \quad \text{(A5)}$$

where

$$\hat{\boldsymbol{\Omega}} = \hat{\boldsymbol{\Delta}}\hat{\boldsymbol{\Gamma}}\hat{\boldsymbol{\Delta}}^t \equiv \begin{pmatrix} \hat{\boldsymbol{\Omega}}_1 & \hat{\boldsymbol{\Omega}}_2 \\ \hat{\boldsymbol{\Omega}}_2^\dagger & \hat{\boldsymbol{\Omega}}_4 \end{pmatrix} = \hat{\boldsymbol{\Omega}}^\dagger, \quad \text{(A6)}$$

$\hat{\boldsymbol{\Delta}}$ is a 6×6 permutation matrix transforming vectors involved in (A4) to those in (A5), $\hat{\boldsymbol{\Omega}}_1$ and $\hat{\boldsymbol{\Omega}}_4$ stand for the upper 4×4 and lower 2×2 diagonal blocks of $\hat{\boldsymbol{\Omega}}$, respectively, and $\hat{\boldsymbol{\Omega}}_2$ is a 4×2 matrix with elements $(\hat{\boldsymbol{\Omega}}_2)_{ij} = (\hat{\boldsymbol{\Omega}})_{ik}$, $i = 1, \ldots, 4$, $j = 1, 2$, $k = j + 4$.

The substitution of (A2) for $\boldsymbol{\nu}$ in (A5) yields

$$\boldsymbol{\phi} = -\hat{\boldsymbol{\Omega}}_4^{-1}\left(\hat{\boldsymbol{\Omega}}_2^\dagger + \frac{k}{\omega}\hat{\mathbf{J}}^t\right)\boldsymbol{\xi}. \quad \text{(A7)}$$

Afterwards we substitute (A7) for $\boldsymbol{\phi}$ in (A5), express $\boldsymbol{\psi}$ in terms of $\boldsymbol{\xi}$, insert the obtained expression of $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ (A7) in (A1), and arrive at Eq. (2) with matrix (4), where

$$\hat{\mathbf{N}} = \omega\hat{\mathbf{A}} - k\hat{\mathbf{B}} - \frac{k^2}{\omega}\hat{\mathbf{C}}, \quad \text{(A8)}$$

$$\hat{\mathbf{A}} = \hat{\boldsymbol{\Omega}}_1 - \hat{\boldsymbol{\Omega}}_2\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\boldsymbol{\Omega}}_2^\dagger, \quad \text{(A9)}$$

$$\hat{\mathbf{B}} = \hat{\boldsymbol{\Omega}}_2\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\mathbf{J}}^t + \hat{\mathbf{J}}\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\boldsymbol{\Omega}}_2^\dagger, \quad \text{(A10)}$$

$$\hat{\mathbf{C}} = \hat{\mathbf{J}}\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\mathbf{J}}^t. \quad \text{(A11)}$$

Since $\hat{\boldsymbol{\Omega}}_1$ and $\hat{\boldsymbol{\Omega}}_4$ are Hermitian matrices, so are the matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ and hence $\hat{\mathbf{N}}$.

The representation of $\hat{\mathbf{N}}$ in terms of the blocks of $\hat{\boldsymbol{\Omega}}$, rather than explicitly in terms of material constants like has been done in Ref. [42] for nonbianisotropic media, proves to be helpful in deriving relations (19) and (23).

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