Analysis of necessary and sufficient conditions for quantum teleportation with non-Gaussian resources

Soumyakanti Bose [1,*] and M. Sanjay Kumar [2,†]

[1] *Indian Institute of Science Education and Research, Knowledge City, Sector 81, Sahibzada Ajit Singh Nagar, Punjab 140306, India*
[2] *S. N. Bose National Centre for Basic Sciences, Block-JD, Sector III, Salt Lake, Kolkata 700106, India*

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Recent theoretical and experimental advances have demonstrated advantages of using non-Gaussian optical resources compared to the Gaussian ones in the context of quantum teleportation (QT), an important quantum information processing task. From both theoretical and experimental points of view the question of which attributes of the resources, besides entanglement, render them useful for QT is an important one. In this paper, we examine the question of whether two well-studied attributes of optical resources, viz., squeezed vacuum affinity (SVA) and Einstein-Podolsky-Rosen (EPR) correlation are necessary and/or sufficient for QT. The specific class of non-Gaussian resources that we have considered for this purpose are the two-mode entangled states generated by mixing nonclassical inputs with vacuum at the beam splitter (BS). Our analytical results show that SVA is not always nonzero and hence it cannot be considered to be a genuine attribute. Our numerical results show that there exist some BS-generated entangled states that do not give QT in spite of being EPR correlated, implying that EPR correlation is not sufficient for QT. In conjunction with the earlier observation in the literature to the effect that EPR correlation is not necessary for QT, our results lead to the conclusion that in general, EPR correlation is *neither necessary nor sufficient* for QT. Our results leave the question open as to what attributes, in general, may be necessary and/or sufficient for QT.

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I. INTRODUCTION

Quantum teleportation (QT) [1,2] is one of the most important information processing tasks that serves as a building block for several other protocols of quantum information technology [3], such as quantum key distribution, remote state preparation, etc. Resources yielding QT could be realized in terms of discrete spin systems [4], continuous variable optical states [5], as well as hybrid systems [6]. Theoretical and experimental advances in the last few decades have also led to practical realizations of QT [7–13].

While earlier studies of QT using optical resources have dealt with two-mode Gaussian states [14], as these are easy to characterize theoretically and generate experimentally, recent studies on non-Gaussian resources [15–29] have indicated significant improvement in teleportation fidelities over the Gaussian ones. Generation of non-Gaussian resources has also become feasible as a result of recent quantum technological advances [30]. Hence in this context, the systematic characterization of non-Gaussian resources that are suitable for QT becomes important.

While entanglement is already known to be necessary for QT with optical resources [31,32], subsequent studies in the literature have examined the possibility of other attributes of the resource states that may be either crucial for QT, or aid in enhancing teleportation fidelities. In this context, Dell'Anno *et al.* [17] showed that optimized teleportation fidelities could be achieved by tuning entanglement, non-Gaussianity, and squeezed vacuum affinity (SVA) of the resource states. Later studies on other classes of resources as well as under realistic circumstances [19,23–25,28] reveal that although it is necessary, non-Gaussianity does not increase the fidelity linearly.

Besides, other works have pointed out that another attribute of the resource states, viz., Einstein-Podolsky-Rosen (EPR) correlation, is crucial for QT [18,27]. In a recent work, Hu *et al.* [29] have further made a comparison between EPR correlation and Hillery-Zubairy (HZ) correlation and concluded that EPR correlation is in fact a better witness of QT than HZ correlation. On the contrary, Lee *et al.* [21] and Wang *et al.* [26] have pointed out a counterexample, viz., the symmetrically photon added two-mode squeezed vacuum (TMSV) state, and concluded that EPR correlation is *not* necessary for QT.

In this background, we examine here the question of which attribute(s) of the resource states, besides entanglement, may be necessary and/or sufficient for QT. The specific class of non-Gaussian resource states that we have considered for this purpose are the two-mode entangled states generated by mixing nonclassical inputs with vacuum at the beam splitter (BS). The specific attributes of the resource states we consider are SVA and EPR correlation.

We find that SVA is not always nonzero. For example, SVA for the BS-generated state with input at one of the ports being an m-photon added or subtracted squeezed vacuum state (m odd) is strictly zero. Clearly hence, SVA cannot be considered to be a genuine attribute of the resource states in general. Further, our numerical results show that most of the BS-generated states that are EPR correlated do not yield QT, implying that

*soumyabose@iisermohali.ac.in
†sanjay@bose.res.in

EPR correlation is not sufficient for QT. In conjunction with the earlier observation in the literature [21,26] to the effect that EPR correlation is not necessary for QT, our results lead us to the conclusion that in general, EPR correlation is *neither necessary nor sufficient* for QT. Our results leave the question open as to what attributes, in general, may be necessary and/or sufficient for QT.

This paper is organized as follows. In Sec. II we briefly describe Braunstein-Kimble (BK) protocol for teleportation with optical resources and our analytical results on QT with BS-generated resources. Section III presents a comparative analysis of SVA and EPR correlation with the results on teleportation. Here, we provide analytical proof that SVA is identically zero for most of the resources while EPR correlation is not sufficient for QT in general. In Sec. IV we summarize and conclude our findings.

II. TELEPORTATION OF A COHERENT STATE WITH BS-GENERATED RESOURCES

A. BK protocol and QT

The most popular protocol for realizing QT with optical resources is the BK protocol [5]. In this protocol, two distant parties, Alice and Bob, transmit quantum information from one laboratory to another at the cost of their shared entanglement supplemented by classical communication [33]. In the ideal case, the output of the protocol turns out to be a perfect replica of the input state ($|\psi_{\rm in}\rangle$); however, in real practice, various technical limitations lead to a noisy output ($\rho_{\rm out}$) that differs from the input state. As a consequence, the performance of the protocol is measured in terms of the fidelity between input and the output states, $F = \langle\psi_{\rm in}|\rho_{\rm out}|\psi_{\rm in}\rangle$, known as the teleportation fidelity [1,2]. Resources are accordingly categorized as the genuine quantum ones [34] which yield F beyond the scope of classical physics.

In the BK [2] protocol, the evaluation of F becomes convenient in the characteristic function (CF) description [35,36]. The CF of an n-mode quantum optical state ρ is defined as $\chi_\rho(\{\lambda_i\}) = \mathrm{Tr}[\rho D(\{\lambda_i\})]$, where $D(\{\lambda_i\}) = \Pi_{i=1}^n \exp[\lambda_i a_i^\dagger - \lambda_i^* a_i]$, where a_i is the ith mode operator. For any two-mode state $\rho_{\rm AB}$ as a resource, the fidelity of teleportation of an unknown input state $\rho_{\rm in}$ can be expressed as [35,36]

$$F = \int \frac{d^2\lambda}{\pi}\, \chi_{\rm in}(-\lambda)\, \chi_{\rm in}(\lambda)\, \chi_{\rm AB}(\lambda, \lambda^*), \qquad (1)$$

where $\chi_{\rm in}(\lambda)$ and $\chi_{\rm AB}(\lambda, \lambda^*)$ are the CFs of $\rho_{\rm in}$ and $\rho_{\rm AB}$, respectively. In the case of a coherent state $|\alpha\rangle$ taken as the unknown input state, Eq. (1) reduces to

$$F = \int \frac{d^2\lambda}{\pi}\, e^{-\lambda^2}\, \chi_{\rm AB}(\lambda, \lambda^*). \qquad (2)$$

The maximum fidelity of teleportation of a coherent state attainable by a separable state in the BK protocol is 1/2 [31,32]. Hence, $F > 1/2$ is considered as QT. Henceforth, by teleportation fidelity we refer to Eq. (2) only.

B. QT with BS-generated resources

Here we consider the non-Gaussian entangled resources generated by mixing at the input of a passive 50:50 BS nonclassical single-mode state at one port and single-mode vacuum at the other port. Note that in this setup, BS output states are guaranteed to be entangled by virtue of the nonclassicality of the input states [37]. The specific input states that we consider here are the photon-added squeezed vacuum state (PAS), the photon-subtracted squeezed vacuum state (PSS), and the squeezed number state (SNS). Several interesting features of output entangled states, with these nonclassical states at the input of a BS, were brought out in our earlier work [38].

The PAS, PSS, and SNS are mathematically described as

$$|\psi_{\rm pas}\rangle = \frac{1}{\sqrt{N_{\rm pas}^m}} a^{\dagger m} S(r)|0\rangle$$
$$|\psi_{\rm pss}\rangle = \frac{1}{\sqrt{N_{\rm pss}^m}} a^{m} S(r)|0\rangle$$
$$|\psi_{\rm sns}\rangle = S(r)|m\rangle, \qquad (3)$$

where $S(r) = \exp[\frac{r}{2}(a^{\dagger 2} - a^2)]$ is the single-mode squeezing operator and the quantities $N_{\rm pas}^m$ and $N_{\rm pss}^m$ are defined by the relations $N_{\rm pas}^m = m!\mu^m P_m(\mu)$, $N_{\rm pss}^m = m!\nu^{2m}\sum_{k=0}^m \frac{m!}{(m-k)!k!}(\frac{-\mu}{2\nu})^k \frac{H_k^2(0)}{k!}$, $\mu = \cosh r$, and $\nu = \sinh r$. Here $P_n(x)$ and $H_n(x)$ are respectively nth-order Legendre and Hermite polynomials.

We have obtained analytic expressions of F for different input states as

$$F_{\rm pas} = \frac{m!(1+\tau)^{m+1/2}\mu^{2m}}{2^{m+1} N_{\rm pas}^m} \sum_{k=0}^m \binom{m}{k}\left(\frac{-1}{2}\right)^k \frac{H_k^2(0)}{k!}$$
$$F_{\rm pss} = \frac{m!(1+\tau)^{m+1/2}\nu^{2m}}{2^{m+1} N_{\rm pss}^m} \sum_{k=0}^m \binom{m}{k}\left(\frac{\tau-2}{2\tau}\right)^k \frac{H_k^2(0)}{k!}$$
$$F_{\rm sns} = \frac{(1+\tau)^{m+1/2}}{2^{m+1}} \sum_{k=0}^m \binom{m}{k}\left(\frac{\tau-1}{2(1+\tau)}\right)^k \frac{H_k^2(0)}{k!}, \qquad (4)$$

where $\mu = \cosh r$, $\nu = \sinh r$, $\tau = \tanh r$, $\binom{m}{k}$ is the binomial coefficient, and the subscript of F stands for the corresponding BS input state. In Fig. 1 we plot the dependence of F on the input state parameters, i.e., squeeze parameter r and the number of photon addition or subtraction, m.

As is evident from Fig. 1, in the case of all three input states, the teleportation fidelity F exhibits a rather complex, in particular a nonmonotonic, dependence on the state parameters r and m. In the case of PAS, for small r while F decreases with increase in m, as r increases the trend reverses. This might be associated with the nonmonotonic dependence of corresponding entanglement [38]. In the case of PSS, we observe an explicit distinction between the even and odd photon subtraction. While for odd photon subtraction, F decreases with m, for even photon subtraction F first increases and then decreases with m. For input SNS, however, F drops monotonically with increase in m. Nonetheless, it is noteworthy that in all three cases, F increases monotonically with r.

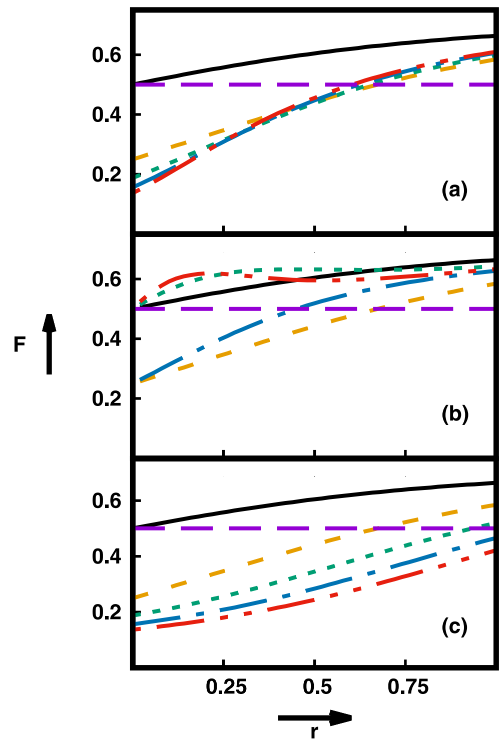

FIG. 1. Plot of F vs r for $m = 0$ (black solid line), 1 (yellow dashed line), 2 (green dotted line), 3 (blue dash-dotted line), and 4 (red dash-double-dotted line) with input states (a) PAS, (b) PSS, and (c) SNS. The violet long dashed line corresponds to the maximum "classical" limit, i.e, $F = 1/2$. Plotted quantities are dimensionless.

III. TELEPORTATION FIDELITY VERSUS SVA AND EPR CORRELATION

A. SVA is not a genuine resource

Dell'Anno *et al.* [17] showed that optimized teleportation fidelities could be achieved by tuning entanglement, non-Gaussianity, and SVA of the resource states. It turns out that in the case of the non-Gaussian resource states that Dell'Anno *et al.* have considered, viz., states obtained by addition or subtraction of an equal number of photons on the TMSV state, SVA is always nonzero. In the light of their results, while non-Gaussianity is clearly not necessary for QT, one is tempted to ask if SVA could be considered necessary for QT in general.

For any two-mode state ρ_{ab}, η (quantifying SVA) is defined as its maximal overlap with the TMSV ($|\xi_s\rangle$),

$$\eta = \max_s |\langle \xi_s|\rho_{ab}|\xi_s\rangle|, \tag{5}$$

where $|\xi_s\rangle = S_{ab}(s)|0,0\rangle$ with $S_{ab}(s) = \exp\{s(a^\dagger b^\dagger - ab\}$.

However, we notice that η becomes trivially zero for most of the resources we consider here. This could be explained in the following way. The state $|\xi_s\rangle$ has a symmetric expansion in number state basis as $|\xi_s\rangle = \frac{1}{\mu_s}\sum_k \tau_s^k |k,k\rangle$, where $\mu_s = \cosh s$ and $\tau_s = \tanh s$. Let us now consider a two-mode state $\rho = \sum_{\substack{m,n\\k,l}} C_{m,n}^{k,l}\, |m,n\rangle\langle k,l|$. The overlap between $|\xi(s)\rangle$ and ρ is given by

$$\text{overlap} = \langle \xi(s)|\rho|\xi(s)\rangle = \frac{1}{\mu_s}\sum_{\substack{m,n\\k,l}} C_{m,n}^{k,l}\, \tau_s^{m+k}\, \delta_{m,n}\, \delta_{k,l}. \tag{6}$$

Evidently, in the case of a two-mode state ρ for which the diagonal elements for all m and k vanish (e.g., $C_{m,m}^{k,k} = 0$), SVA is identically zero. In other words, for any two-mode state for which $\langle n1, n1|\rho|m1, m1\rangle = 0$, SVA vanishes identically. Hence, owing to the fact that SVA is not guaranteed to be nonzero for all resource states, it cannot be taken to be an essential attribute for QT.

B. EPR correlation is not sufficient for QT

In their seminal paper [39], Einstein, Podolsky, and Rosen proposed an ideal bipartite state, known as the EPR state [40,41], which is a common eigenstate of the relative position and total momentum of the subsystems. In view of the fact that the EPR state is a maximally correlated state, in general, for any bipartite state ρ_{ab}, one can accordingly define an EPR-type correlation parameter, known as the EPR uncertainty Δ_{EPR} [42], given by

$$\Delta_{\mathrm{EPR}} = \langle \delta^2(x_a - x_b)\rangle + \langle \delta^2(p_a + p_b)\rangle, \tag{7}$$

where $\delta^n R = (R - \langle R\rangle)^n$ and $\{x_i, p_i\}$ $(i = a, b)$ is the mode quadrature. EPR uncertainty (Δ_{EPR}) being less than 2 indicates EPR correlation while $\Delta_{\mathrm{EPR}} = 0$ signifies perfect correlation between the modes.

Recent work [18,27] has brought out the fact that EPR correlation of the resource states is crucial for QT. In this section we carry out a detailed analysis of the role of EPR correlation in the case the BS-generated resource states that we have considered with the input states described in Eq. (3).

In the case of the input state $|\psi\rangle$, EPR uncertainty for the corresponding BS output state we denote by $\Delta_{\mathrm{EPR}}(|\psi\rangle)$. Here, we derive analytic expressions of Δ_{EPR} for the BS output states as

$$\Delta_{\mathrm{EPR}}(|\psi_{\mathrm{pas}}\rangle) = 2\Bigg[\frac{N_{\mathrm{pas}}^{m+1}}{N_{\mathrm{pas}}^{m}} + \frac{\mu^{2m}(m+2)!}{N_{\mathrm{pas}}^{m}}\left(\frac{\mu\nu}{2}\right) \times \sum_{k=0}^{m}\binom{m}{k}\left(\frac{-\nu}{2\mu}\right)^k \frac{H_k(0)H_{k+2}(0)}{(k+2)!}\Bigg],$$

$$\Delta_{\mathrm{EPR}}(|\psi_{\mathrm{pss}}\rangle) = 2\Bigg[1 + \frac{N_{\mathrm{pss}}^{m+1}}{N_{\mathrm{pss}}^{m}} + \frac{\nu^{2m}(m+2)!}{N_{\mathrm{pss}}^{m}}\left(\frac{\mu\nu}{2}\right) \times \sum_{k=0}^{m}\binom{m}{k}\left(\frac{-\mu}{2\nu}\right)^k \frac{H_k(0)H_{k+2}(0)}{(k+2)!}\Bigg],$$

$$\Delta_{\mathrm{EPR}}(|\psi_{\mathrm{sns}}\rangle) = 2[1 + m(\mu-\nu)^2 - \nu(\mu-\nu)], \tag{8}$$

where $\mu = \cosh r$, $\nu = \sinh r$, and $\binom{m}{k}$ is the binomial coefficient. In Fig. 2 we plot the dependence of Δ_{EPR} for the BS-generated resource states on the corresponding input state parameters, i.e., r and m.

As is evident from Fig. 2, the dependence of Δ_{EPR} upon the state parameters r and m is not monotonic, in general. For PAS and SNS, Δ_{EPR} increases with m and decreases with r monotonically. However, in the case of PSS, we observe a

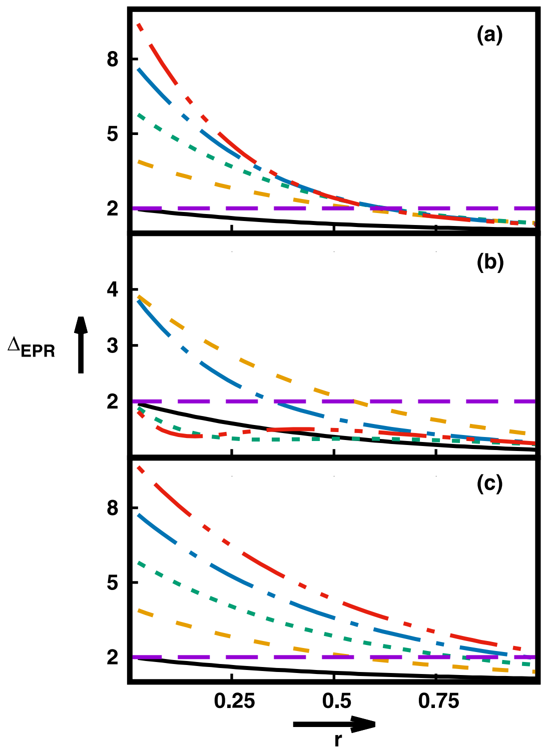

FIG. 2. Dependence of Δ_{EPR} on r for different $m = 0$ (black solid line), 1 (yellow dashed line), 2 (green dotted line), 3 (blue dash-dotted line), and 4 (red dash-double-dotted line) for input (a) PAS, (b) PSS, and (c) SNS. The long dashed violet line corresponds to $\Delta_{\mathrm{EPR}} = 2.0$. Plotted quantities are dimensionless.

clear distinction between the even and odd photon subtraction. While for odd m-PSS EPR correlation exists ($\Delta_{\mathrm{EPR}} < 2$) after a moderate r, all even m-PSS are always EPR correlated for all values of r. Nonetheless, both for the even and odd m-PSS, increase in m decreases Δ_{EPR}, contrary to the cases of PAS and SNS.

A cursory glance at the plots of teleportation fidelity (Fig. 1) and EPR correlation (Fig. 2) suggests that the latter behaves in line with the former, in the sense that whenever F increases, Δ_{EPR} decreases. However, a close inspection reveals that there exist particular parameter regions where resource states are EPR correlated ($\Delta_{\mathrm{EPR}} < 2$) yet they do not yield QT ($F > 1/2$). To bring out this point explicitly, we consider the case of input $|\psi_{\mathrm{pas}}\rangle$ with $m = 1$. It could be shown easily that

$$\Delta_{\mathrm{EPR}}(|\psi_{\mathrm{pas}}\rangle)|_{m=1} = 2(3\mu(\mu - \nu) - 1),$$
$$F_{\mathrm{pas}}|_{m=1} = \frac{(1+\tau)^{3/2}}{4}. \tag{9}$$

It follows from the above equation that the corresponding BS output state is EPR correlated, i.e., $\Delta_{\mathrm{EPR}}(|\psi_{\mathrm{pas}}\rangle)_{m=1} < 2$ for $r > 0.55$, while it yields QT, i.e., $F(|\psi_{\mathrm{pas}}\rangle)_{m=1} > 1/2$ for $r > 0.68$. This indicates that in the case of input $|\psi_{\mathrm{pas}}\rangle$ with $m = 1$, in the region $0.55 < r < 0.68$ we have EPR correlation; however, the resource state does not yield QT. Note that in the cases of input $|\psi_{\mathrm{pss}}\rangle$ with $m = 1$, and input $|\psi_{\mathrm{sns}}\rangle$ with $m = 1$, too, we can identify ranges of the squeeze parameter r over which the BS output states are EPR correlated, but do not yield QT exactly the same as in the case of $|\psi_{\mathrm{pas}}\rangle$ with $m = 1$. The reason for this is that for $m = 1$ all three states can be shown to be identical [38].

We further note that in the case of input states with higher order photon addition and subtraction, the parameter regions over which resource states are EPR correlated but do not lead to QT become much wider. This leads us to the conclusion that EPR correlation, in general, is *not sufficient* for QT. It then follows, in conjunction with the earlier observation in the literature [21,26] to the effect that EPR correlation is not necessary for QT, that in general, EPR correlation is *neither necessary nor sufficient* for QT.

IV. DISCUSSION

To summarize, we have critically examined the question of which attributes of the resource states besides entanglement may be necessary and/or sufficient for QT. In particular, we have focused on two such attributes that have been well studied in the literature, namely, SVA and EPR correlation. To this end, we have studied, both analytically and numerically, QT with a class of non-Gaussian resource states that are generated by a passive balanced BS from three different input single-mode nonclassical states, viz., photon added and subtracted squeezed vacuum states and squeezed number states.

We have found that as SVA is not nonzero for a wide range of states, it hence cannot be regarded as a genuine attribute of the resource states in general. We have given a complete characterization of the states for which SVA is trivially zero.

We have further presented numerical and analytical results on the dependence of QT upon EPR correlation. Our results lead us to the conclusion that EPR correlation is not sufficient for QT, in general. In conjunction with the earlier results [21,26], our analysis leads us to the conclusion that EPR correlation is *neither necessary nor sufficient* for QT, in general. Hence, our results leave the question open as to what attributes, in general, may be necessary and/or sufficient for QT.

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