Nonlinear dynamics of a quantum cascade laser with tilted optical feedback

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Interband semiconductor lasers subject to optical feedback usually produce rich nonlinear dynamics. However, it is hard to destabilize quantum cascade lasers using common optical feedback, because of its ultrashort carrier lifetime and small linewidth broadening factor. In this work, we show that optical feedback with a tilted angle can destabilize quantum cascade lasers, which produce three types of nonlinear dynamics. We observe periodic oscillations with small tilted angles, quasiperiodic oscillations with moderate angles, and low-frequency oscillations with large angles, respectively. In contrast, this type of dynamics does not occur with well-aligned optical feedback.

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I. INTRODUCTION

Optical feedback is a simple yet powerful method to trigger semiconductor lasers for generating nonlinear dynamics. In this way, a large variety of instabilities have been observed, including periodic oscillations, quasiperiodic oscillations, low-frequency fluctuations (LFFs), regular pulse package oscillations, as well as chaos [1]. This type of dynamics is not only of high interest for fundamental physics, but also valuable for practical applications. Among these, chaos has been intensively explored for secure optical communication, for high-speed random number generation, and for Lidar applications [2–4]. Besides, period-1 oscillations are found to be a high-quality photonic microwave source for radio-over-fiber applications [5–9]. On the other hand, optical feedback in the stable regime can be used for suppressing the phase noise of both single-mode lasers and mode-locked lasers [1,10,11]. In recent years, semiconductor lasers with optical feedback operated at the edge of instability are drawing more and more attention for producing virtual neurons in the reservoir computing network, which significantly advances the hardware implementation of neural networks [12–14].

Interband semiconductor lasers are featured with a damped relaxation oscillation, and optical feedback usually destabilizes the lasers through undamping the relaxation oscillation, which yields various nonlinear dynamics [1]. However, quantum cascade lasers (QCLs) do not exhibit any relaxation oscillation because of the ultrashort carrier lifetime of intersubband transition [15].

In addition, the linewidth broadening factor of QCLs is usually less than 1.0, in comparison with 3.0–5.0 in interband semiconductor lasers [16]. Consequently, QCLs are highly stable against optical feedback [17–19]. Indeed, our recent work demonstrated that a QCL remained stable for feedback ratios up to −4.0 dB, and the spectral linewidth was narrowed by about 18 dB [20–22]. On the other hand, it is very challenging to destabilize QCLs for the generation of nonlinear dynamics. In order to trigger instabilities of QCLs with optical feedback, pulsed or modulated pump current has been introduced to perturb the lasers [23–25], and LFFs have been observed only when the QCLs were biased in the vicinity of the lasing threshold [25–27].

For common optical feedback, the external mirror is well aligned with the optical path, and hence the multiple round-trip reflections are essentially degenerate. Once the external mirror is tilted, nevertheless, the odd-order round-trip reflections from the misaligned mirror can only couple into the laser's active region by the diffraction effect [28,29]. Meanwhile, the even-order reflections still couple efficiently into the laser cavity. Therefore, the odd-order reflection beam is very sensitive to the tilted angle of the external mirror. In interband semiconductor lasers, tilted optical feedback has been found to induce regular pulsations with frequencies deviating from both the external-cavity frequency and the resonance frequency, owing to the nondegeneracy of odd and even round-trip reflections [28–31]. In addition, tilted optical feedback has been used to tailor the beam profile of broad-area semiconductor lasers [32–35]. In this work, we introduce tilted optical feedback to destabilize a QCL without any pump current perturbation. Through increasing the tilted angle of optical feedback, we observe periodic oscillations, quasiperiodic oscillations and low-frequency oscillations, respectively. In comparison, the QCL is always stable when it is subject to the well-aligned optical feedback.

II. LASER DEVICE AND EXPERIMENTAL SETUP

The QCL device under study is a Fabry-Perot laser grown on the InP substrate using solid-source molecular beam epitaxy [20,36]. The active region consists of 30 cascading gain stages, and each gain stage is formed by InGaAs/InAlAs quantum wells. It has a cavity length of 2.0 mm and a ridge width of 8.5 μm. The front cavity facet is cleaved with a

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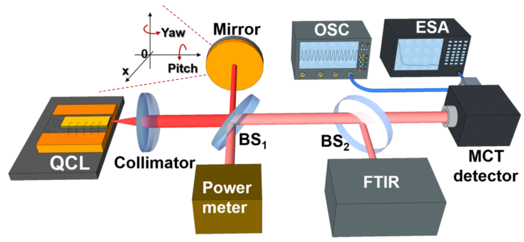

FIG. 1. Experimental setup for the QCL with tilted optical feedback. BS: beam splitter; OSC: oscilloscope; ESA: electrical spectrum analyzer. The origin of the Cartesian coordinate locates at the center of the mirror, and the *x* axis is along the optical path.

reflectivity of 27%, and the rear facet is coated with a reflectivity of 95%. Figure 1(a) shows the experimental setup for the QCL subject to the tilted optical feedback. The QCL is pumped by a continuous-wave low-noise current source (Newport, LDC-3736), and the laser operation temperature is maintained at 20 °C through using a thermoelectric cooler. The laser emission is collimated by an aspherical lens with a focal length of 6.0 mm. The optical path is then divided into two branches by a beam splitter (BS_1). One branch provides the optical feedback through a gold mirror, which locates about 97 cm away from the QCL. The tilted angle of the mirror is finely controlled in the pitch direction. The feedback power is monitored by a power meter. When the gold mirror is well aligned with the optical path (tilted angle is zero), the feedback ratio (ratio of reflected power at the laser facet to the laser emission power) reaches the maximum of about 40.4%. The other branch is used for the characterization. The optical spectrum is measured by a high resolution (0.08/cm) Fourier transform infrared spectrometer (FTIR, Bruker Vertex 80). The optical signal is converted to the electrical one through a high-bandwidth (560 MHz) HgCdTe photodetector (MCT detector, Vigo PVI-4TE-6). The electrical spectrum is recorded on an electrical spectrum analyzer (ESA, Keysight N9040B, 50 GHz bandwidth), and resolution bandwidth is set at 20 kHz. The temporal wave form is recorded on a digital oscilloscope (OSC, Keysight DSAZ594A, 59 GHz bandwidth), and the sampling rate is set at 80 GSample/s .

III. EXPERIMENTAL RESULTS

The free-running QCL exhibits a lasing threshold of $I_{th} = 287$ mA, and the well-aligned optical feedback reduces the threshold by about 10% down to 258 mA. The QCL is biased at 340 mA ($1.18 \times I_{th}$) throughout the experiment, unless stated otherwise. As shown in Fig. 2(a), the output power (dashed line) of the free-running QCL is 12.2 mW, and the well-aligned optical feedback significantly enhances it to 49.8 mW by a factor of 4.0. The output power generally decreases with increasing tilted angle in the pitch direction, due to the reduced feedback ratio. When the tilted angle is larger than 3.13°, the output power becomes comparable with the free-running one. An exceptional case occurs at 3.00°, where the output power is substantially raised. This is because one bright fringe of the diffraction coincides with the active region of the QCL, and a

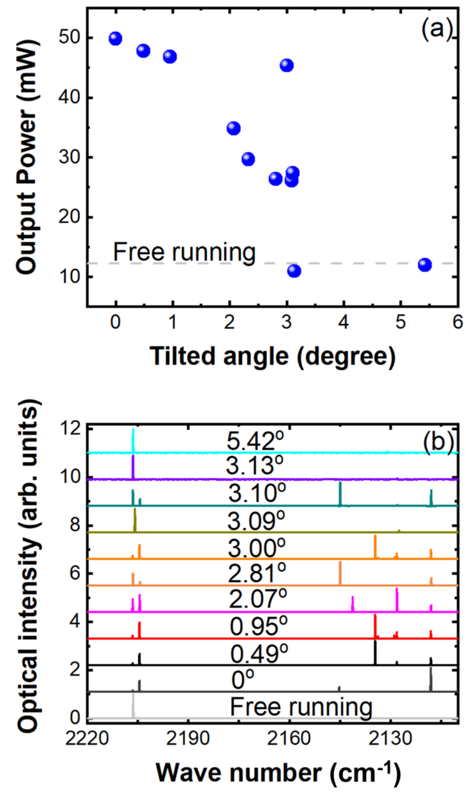

FIG. 2. (a) Laser output power as a function of the feedback tilted angle. The dashed line denotes the free-running power. (b) Optical spectra at various tilted angles.

similar effect has been observed in interband semiconductor lasers [28]. Figure 2(b) shows that the free-running QCL emits on a single mode at 2206.3 cm. However, optical feedback evokes multimode emission in the QCL. One mode locates close to the free-running mode around 2206.4 cm, while another mode arises around 2204.4 cm. The spacing of this two-mode is about 2.5 times the free spectral range (0.78 cm). In addition, several modes arise in the range of 2145–2117 cm, depending on the tilted angle. However, the optical spectrum provides little information on the laser dynamics, due to the limited resolution bandwidth (0.08 cm).

When the QCL is subject to the well-aligned feedback (0 ° tilted angle), the electrical spectrum in Fig. 3(a) is almost flat, and the temporal wave form in Fig. 3(b) is continuous wave. This suggests that the QCL remains stable for the feedback ratio of 40.4%. It is also confirmed that the QCL is stable for weaker feedback ratios, as long as the reflection mirror is well aligned. The high stability is in agreement with our previous work in [20,22]. However, when the reflection mirror is tilted by a small angle ($<2.5°$) in the pitch direction, periodic oscillations arise at certain tilted angles, and four examples are illustrated in Fig. 3. In the electrical spectrum of Fig. 3(a), a sharp peak appears at the fundamental frequency of 154.5 MHz, and the peak amplitude is more than 50 dB higher than the background noise (−83.9 dB). The fundamental frequency is exactly the same as the external cavity frequency, which is determined by the external cavity length of 97 cm. This suggests the periodic oscillations in the QCL are due to the beating of multiple external cavity modes,

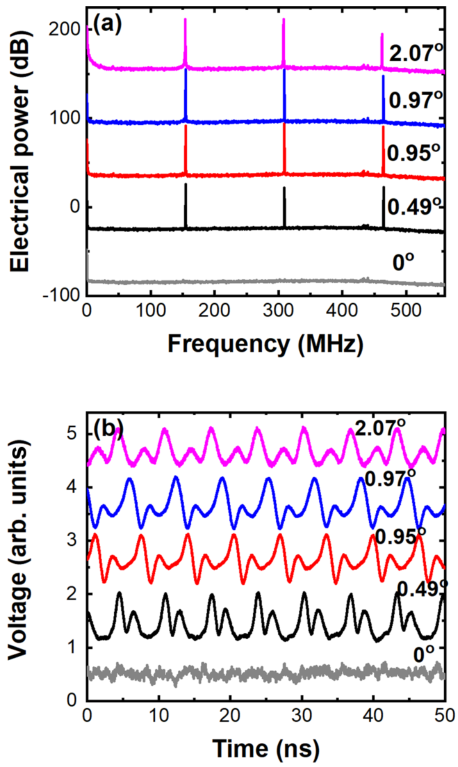

FIG. 3. Periodic oscillations at small tilted angles. (a) Electrical spectrum. The background noise level is around −83.9 dBm including both the intrinsic laser noise and the technical noise. (b) Temporal wave form. The recorded time span is 5.0 μs.

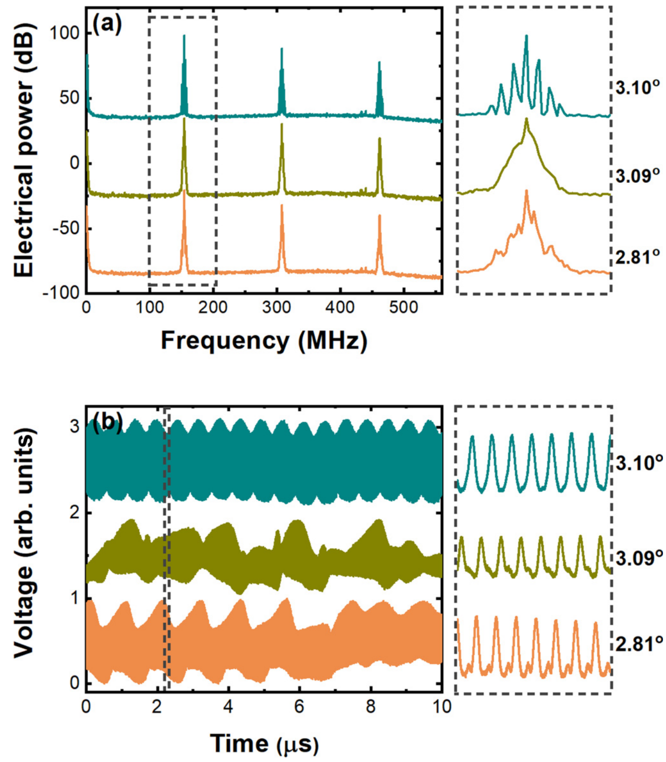

FIG. 4. Quasiperiodic oscillations at moderate tilted angles. (a) Electrical spectrum. (b) Temporal wave form. The dashed boxes on the right present the zoom-in spectrum and wave form. The recorded time span is 10 μs.

as theoretically predicted in [18]. This is different than the periodic oscillations of interband semiconductor lasers, where the fundamental frequency is governed by the relaxation resonance frequency [1]. The strong second-order and third-order harmonics in the electrical spectrum indicate that the periodic oscillation signals are highly distorted from the sinusoidal wave form, which are confirmed in Fig. 3(b).

When the tilted angle of the reflection mirror is increased to the range of 2.5–3.1°, the QCL exhibits quasiperiodic oscillations as shown in Fig. 4. The fundamental oscillation frequency remains 154.5 MHz as that in Fig. 3. However, the peaks in the electrical spectrum [Fig. 4(a)] at 2.81° and 3.09° become noisy and do not show clear sidebands, due to the irregular envelope of the temporal wave forms in Fig. 4(b). On the other hand, the electrical spectrum at 3.10° shows clear sidebands with a separation of 1.67 MHz. The sideband generation may arise from the nondegeneracy of the odd- and even-order feedbacks, which leads to two local gain minima of similar threshold, as a function of the lasing frequency [29]. Therefore, the sideband separation is determined by the frequency distance of the two local gain minima, and in turn by the tilted angle of the reflection mirror.

Further increasing the tilted angle of the reflection mirror above 3.1°, the oscillation peaks disappear in the electrical spectrum. Instead, the tilted feedback raises the low-frequency noise for frequencies below 200 MHz in Fig. 5(a). Meanwhile, the temporal wave forms in Fig. 5(b) oscillate in the time scale of microsecond, which are two orders of magnitude slower than those in Figs. 3 and in 4. At 3.13°, the QCL exhibits LFF, where the power varies irregularly with a slow power drop and a sudden power recovery. This temporal behavior is similar to the reported LFFs observed in other QCLs [25–27]. However, it is different from the common LFFs in interband semiconductor lasers, which are characterized by a sudden power drop followed by a gradual power recovery process [37–40]. The recorded time span of the LFF is 50 μs. However, a much longer span is preferred to reconstruct the phase space for the analysis of Lyapunov exponents [41,42].

When the reflection mirror is tilted at 3.34°, two states coexist in the time trace, where one state fluctuates weakly while the other one oscillates with a large magnitude. Both states switch irregularly in the time traces. The two-state coexistence phenomenon has been observed in interband semiconductor lasers with optical feedback, including the stable state and LFF [43], laminar state and chaos [44], as well as the stable state and periodic state [45]. For tilted angles of 4.16° and 5.42°, the QCL produces square-wave-like oscillations, with periods of 2.1 and 1.2 μs, respectively. Similar square waves have been observed in QCLs subject to polarization-rotated feedback in [46]. However, the physical origin remains unclear, which might be due to the optothermal effect with a characteristic time scale of microsecond [47–49]. This low-frequency square wave is different from those triggered by counterdirectional feedback in semiconductor ring lasers [50,51], or by polarization-rotated feedback in edge-emitting lasers and in vertical cavity surface emitting lasers [52,53]. In the latter cases, the fundamental period of the square wave

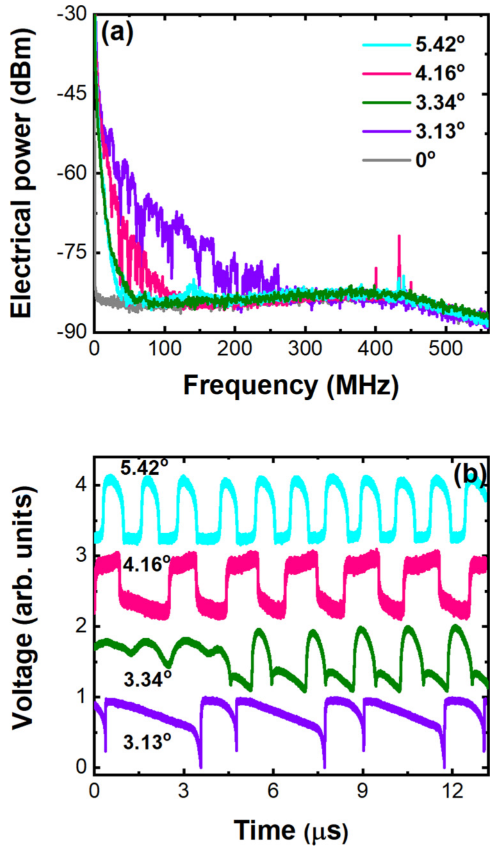

FIG. 5. Low-frequency oscillations at large tilted angles. (a) Electrical spectrum. (b) Temporal wave form. The recorded time span is 50 μs.

is twice the feedback delay time, that is, about two orders of magnitude faster than the ones in Fig. 5(b). Further increasing the tilted angle, the feedback strength becomes very weak, and the QCL is restabilized to be a continuous-wave emission.

It is worthwhile to point out that the pulse oscillations in Figs. 3–5 are recorded either by increasing or by decreasing the tilted angle of the reflection mirror. During the measurement, multistability between pulse oscillations and steady state is observed, which is a common phenomenon occurring in semiconductor lasers subject to optical feedback [54–56]. Particularly, multistability has been theoretically demonstrated in QCLs subject to optical injection as well [57,58]. The properties of hysteresis and coexisting attractors will be discussed elsewhere.

In a word, the QCL mainly exhibits periodic oscillations at small tilted angles (<2.5° in Fig. 3), quasiperiodic oscillations at moderate tilted angles (2.5–3.1° in Fig. 4), and low-frequency oscillations at large tilted angles (>3.1° in Fig. 5). However, it is also possible to observe other kinds of dynamics in each regime although it is indeed rare. For instance, we observe continuous-wave emission at a small tilted angle (2.33°), and periodic oscillations at moderate angles (2.90° and 3.00°). We also find that it is easier to observe the above nonlinear dynamics at a high bias current than at a low one. When the bias current is decreased from $1.18 \times I_{\text{th}}$ to $1.11 \times I_{\text{th}}$, we only observe periodic oscillations and quasiperiodic oscillations. At $1.04 \times I_{\text{th}}$, the QCL exhibits only periodic oscillations. This is likely due to the large linewidth broadening factor of the QCL at a high bias current, which was measured to be 1.1 at $1.04 \times I_{\text{th}}$, 1.7 at $1.11 \times I_{\text{th}}$, and 1.9 at $1.18 \times I_{\text{th}}$, respectively [22]. In addition, the generation of nonlinear dynamics is not limited to the mirror tilting in the pitch direction, and we also observe similar dynamics when the external mirror is tilted in the yaw direction. In order to trigger the nonlinear dynamics, the tilted angles in the yaw direction are much smaller (<0.2°) than the ones in the pitch direction, which is due to the much larger width of the active region than the height [30].

IV. DISCUSSION

In order to provide an intuitive image on the nondegeneracy of even- and odd-order reflections, we describe the rate equation of the photon number $S(t)$ of QCLs with tilted optical feedback as follows [28,29]:

$$\frac{dS}{dt} = \left(mG_0\Delta N - \frac{1}{\tau_p}\right)S + m\beta\frac{N_3}{\tau_{\text{sp}}} + 2k_c\sqrt{r_{\text{ext}}S(t)} \times\left[\sum_{n=1}^{\infty}\left(-\sqrt{Rr_{\text{ext}}}\right)^{n-1}\sqrt{S(t-n\tau_{\text{ext}})}\cos(\Delta\phi_n)\right] \times\begin{cases}\frac{\sin(\theta\pi)}{\theta\pi} & \text{for } n = 1, 3, 5\ldots\\ 1 & \text{for } n = 2, 4, 6\ldots\end{cases}, \tag{1}$$

where m is the number of cascading gain stage, G_0 is the gain coefficient, ΔN is the inversion population, τ_p is the photon lifetime in the cavity, τ_{sp} is the spontaneous emission lifetime, and β is the spontaneous emission factor. r_{ext} is the feedback ratio, τ_{ext} is the round-trip feedback delay time, k_c is the feedback coupling coefficient to the laser cavity, and R is the reflectivity of the laser facet. The phase difference is given by $\Delta\phi_n = n\phi_0 + \phi(t) - \phi(t - n\tau_{\text{ext}})$, with ϕ_0 being the initial feedback phase, and n being the order of round-trip reflection. θ is the tilted angle of the reflection mirror, which is normalized with respect to the tilt angle at which the first diffractive coupling minimum occurs. The odd-order reflections ($n = 1, 3, 5\ldots$) are sensitive to the tilted angle, and the feedback strength is varied by a factor of $\sin(\theta\pi)/\theta\pi$, which is a sinc function determined by the power distribution of the diffraction pattern. However, the even-order reflections ($n = 2, 4, 6\ldots$) are insensitive to the tilted angle. In addition, the phase dynamics of the electrical field is nondegenerate as well. The tilted optical feedback alters both the threshold gain and the lasing frequency. The relation between the change of the threshold gain Δg and the change of the lasing angular frequency $\Delta\omega$ is given by

$$\Delta g = \frac{1}{\alpha_H}[\Delta\omega\tau_{\text{in}} + \arg(r_{\text{eff}})] \tag{2}$$

with α_H being the linewidth broadening factor and τ_{in} being the round-trip time in the laser cavity. The complex effective field amplitude reflectivity of the tilted feedback is given by

$$r_{\text{eff}} = \sqrt{R} - \frac{r_{21}\exp(j\omega\tau_{\text{ext}}) - r_{22}\exp(j2\omega\tau_{\text{ext}})}{1 - Rr_{\text{ext}}(j2\omega\tau_{\text{ext}})} \tag{3}$$

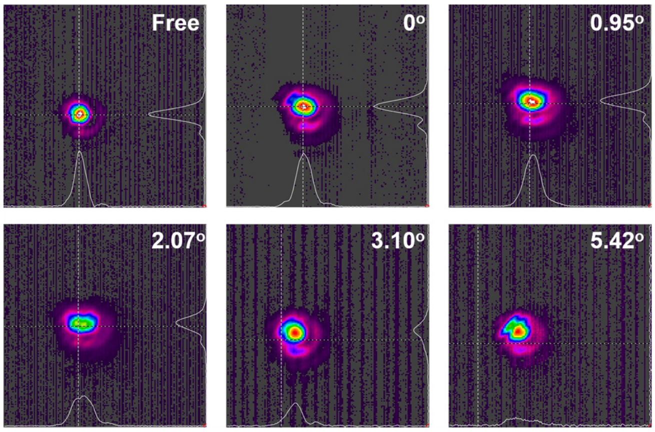

FIG. 6. Laser beam profiles at different tilted angles.

with

$$r_{21} = k_c \tau_{\text{in}} \sqrt{R r_{\text{ext}}} \frac{\sin(\theta\pi)}{\theta\pi}; \quad r_{22} = k_c \tau_{\text{in}} R r_{\text{ext}}. \tag{4}$$

For normal optical feedback ($\theta = 0$), Δg only has one minimum as a function of $\Delta\omega$, resulting in one external cavity mode solution. Nevertheless, Δg has two local minima of similar values for tilted optical feedback ($\theta \neq 0$). Consequently, the external cavity mode is split into two submodes of different frequencies, which in turn leads to the observed complex dynamics. The full rate equation model of QCLs subject to tilted optical feedback and the corresponding simulations will be reported elsewhere.

In addition, the tilted optical feedback slightly changes the far-field beam profile of the QCL, as illustrated in Fig. 6. Particularly, the beam profile at 2.07° shows the apparent effect of light diffraction from the tilted optical feedback. However, the laser remains in single transverse mode for all the measured tilted angles, and hence the observed nonlinear dynamics are not due to interaction of multiple transverse modes. On the other hand, the coupling of transverse modes in broad-area semiconductor lasers can produce nonlinear dynamics as well [59,60].

V. CONCLUSION

In summary, we demonstrate that optical feedback with a tilted angle can destabilize a QCL, which is otherwise stable when subject to well-aligned optical feedback. With increasing tilted feedback angle, the QCL exhibits periodic oscillations, quasiperiodic oscillations, and low-frequency oscillations, respectively. The rich nonlinear dynamics can be attributed to the nondegeneracy of odd-order round-trip reflections with even-order ones, which arises from the tilted optical feedback. We believe this work provides more insight on the instability of QCLs. On one hand, it is helpful for producing laser pulses without any electrical modulation. On the other hand, it suggests that an optical isolator is required for applications where high stability of the QCL emission is desired.

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