Scalable W-type entanglement resource in neutral-atom arrays with Rydberg-dressed resonant dipole-dipole interaction

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While the Rydberg-blockade regime provides the natural setting for creating W-type entanglement with cold neutral atoms, it is demonstrated here that a scalable entanglement resource of this type can even be obtained under completely different physical circumstances. To be more precise, a special instance of twisted W states—namely, π-twisted ones—can be engineered in one-dimensional arrays of cold neutral atoms with Rydberg-dressed resonant dipole-dipole interaction. In particular, it is shown here that this is possible even when a (dressed) Rydberg excitation is coupled to the motional degrees of freedom of atoms in their respective, nearly harmonic optical-dipole microtraps, which are quantized into dispersionless (zero-dimensional) bosons. For a specially chosen ("sweet-spot") detuning of the off-resonant dressing lasers from the relevant internal atomic transitions, the desired π-twisted W state of Rydberg-dressed qubits is the ground state of the effective excitation—boson Hamiltonian of the system in a broad window of the relevant parameters. Being at the same time separated from the other eigenstates by a gap equal to the single-boson energy, this W state can be prepared using a Rabi-type driving protocol. The corresponding preparation times are independent of the system size and several orders of magnitude shorter than the effective lifetimes of the relevant atomic states.

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I. INTRODUCTION

The past decade has seen a surge of interest in ensembles of Rydberg atoms [1] both from the fundamental-physics [2] and technological standpoints [3]. Owing to the remarkable properties of their atomic constituents, these systems have gained prominence as analog simulators of many-body phenomena [4–6]. Large strides have simultaneously been made in the context of quantum-information processing (QIP) with this class of atomic systems [7,8], a research direction aimed at realizing a neutral-atom quantum computer [9,10]. In particular, the scalability milestones recently achieved with arrays of laser-cooled atoms trapped in optical microtraps (tweezers) [11–16] have rekindled interest in quantum-state engineering in such systems [17–24]. Owing to the possibility of integrating multiqubit storage, readout, and transport [25] in these systems as well as their inherent capability for the coherent control of spin and motional states of trapped atoms [26], tweezer arrays have acquired their present status of the most powerful platform for QIP with neutral atoms.

Rydberg blockade (RB) [27–29]—a phenomenon whereby the van der Waals (vdW) interaction prevents Rydberg excitation of more than one atom within a certain radius—established itself as the enabling physical mechanism for QIP with neutral atoms [9]. Although RB also lies at the heart of many other phenomena [30,31], perhaps its most important implication is that it gives rise to a conditional logic that enables the realization of entangling two-qubit gates [9,32], such as controlled-NOT [33–35]. Another important facet of RB is that it leads to the creation of coherent-superposition "superatom" states with a single Rydberg excitation being shared among all atoms in an ensemble [34]. Such entangled states [36] belong to a special "twisted" type of W states [37]. The latter represent one of the two most important classes of maximally entangled multiqubit states; the other class, inequivalent with respect to local operations and classical communication [38], is furnished by Greenberger-Horne-Zeilinger (GHZ) states [39]. W states are known to be the most robust ones to particle losses among all N-qubit states [40] and have proven useful in many QIP protocols [41,42]. This prompted proposals for their preparation in various physical systems [43–50].

While RB naturally engenders a W-type entanglement in ensembles of cold neutral atoms, the present paper aims to show that the same type of entanglement can also be engineered in such systems under quite different physical circumstances. More precisely, this paper describes a scheme for a fast deterministic preparation of π-twisted W states in one-dimensional (1D) arrays of cold neutral atoms with Rydberg-dressed resonant dipole-dipole interaction [51]. These arrays of atoms are assumed to be trapped in optical tweezers [11–15].

Generally speaking, Rydberg dressing entails an off-resonant laser coupling between the ground states and the Rydberg states of an atom such that a small Rydberg component is admixed to the ground state [52,53]. This allows the dipole-dipole interaction to be felt even among atoms that almost reside in their ground states. In particular, the use of Rydberg dressing in the realm of QIP arose from the desire to strengthen qubit-qubit interactions for Rydberg qubits encoded in two long-lived low-lying atomic states (typically two hyperfine sublevels of an electronic ground state [8]). This, in turn, naturally led to the concept of Rydberg-dressed qubits [54–57].

The essential ingredient of the present paper is that it explicitly takes into account the coupling of an itinerant dressed Rydberg excitation to the motional degrees of freedom of trapped atoms. Upon quantizing the latter into dispersionless (zero-dimensional) bosons, the ensuing system dynamics are governed by a nonlocal excitation-boson (e-b) interaction [58–60]. In particular, for a specially chosen ["sweet-spot" (ss)] detuning of the dressing lasers from the relevant internal atomic transitions, the ground state of the effective e-b Hamiltonian is the bare-excitation Bloch state with the quasimomentum $k = \pi$ (measured in units of the inverse period of the underlying lattice), which in the system at hand coincides with the π-twisted W state of Rydberg-dressed qubits.

In addition to being the ground state of the system in a broad window of the relevant parameters, the sought-after W state is separated from the other eigenstates by a gap equal to the single-boson energy. This last circumstance—a generic feature of systems in which an itinerant excitation is coupled to gapped bosons—represents a key protection mechanism for the desired W states and facilitates their preparation using a simple Rabi-type driving protocol. Importantly, the resulting state-preparation times are independent of the system size (i.e., the number of Rydberg-dressed qubits), thus, the proposed system provides a scalable W-type entanglement resource. Another favorable property of the envisioned protocol is that these last times are several orders of magnitude shorter than the relevant Rydberg-state lifetimes.

The remainder of this paper is organized as follows. In Sec. II the system under consideration is introduced, along with the necessary notation and conventions to be used throughout the paper. The effective Hamiltonian of the system, describing an itinerant (dressed) Rydberg excitation coupled to the motional degrees of freedom of atoms is discussed in Sec. III, first in its most general form (Sec. III A) and then in the special case that corresponds to the ss detuning (Sec. III B). The principal findings of the paper, which pertain to this special choice of the detuning, are presented and discussed in Sec. IV. The character of the ground states of the system in different parameter regimes is first analyzed in Sec. IV A, which also establishes the connection between the obtained quasimomentum-π bare-excitation ground states and the desired π-twisted W states. The envisioned state-preparation protocol is then presented, and its robustness discussed in Sec. IV B. Finally, the significance of the obtained results in the context of QIP with Rydberg-dressed qubits is briefly elaborated on in Sec. IV C. The paper is summarized, along with conclusions and some general remarks, in Sec. V.

II. SYSTEM

The system under consideration [for an illustration, see Fig. 1(a)] is a 1D array of N cold neutral atoms (e.g., of ^{87}Rb) with mass m, each confined in its individual, approximately harmonic optical-dipole microtrap. In particular, the distance a between the minima of adjacent microtraps represents the period of the underlying 1D lattice. Importantly, the quantized displacement of the nth atom ($n = 1, \ldots, N$) from its equilibrium position is given by $u_n \equiv (\hbar/2m\omega_b)^{1/2}(b_n + b_n^\dagger)$,

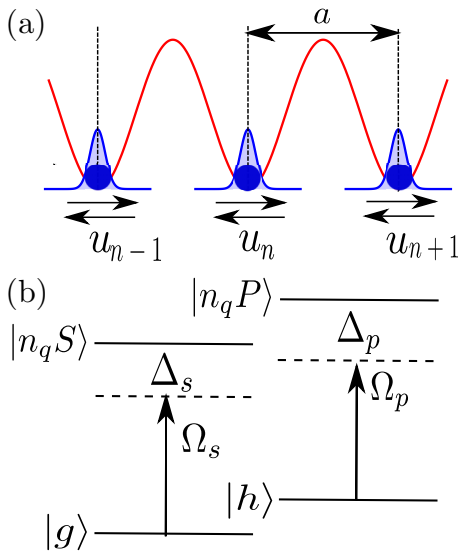

FIG. 1. (a) Illustration of the system under consideration: cold neutral atoms are confined in individual, nearly harmonic, optical-dipole microtraps whose minima are separated by distance a. (b) Schematic level diagram of an atom with ground-states $|g\rangle$ and $|h\rangle$, which are off-resonantly laser coupled to highly excited Rydberg states $|n_q S\rangle$ and $|n_q P\rangle$, respectively.

where ω_b is the longitudinal trap frequency and $b_n^\dagger$ (b_n) creates (destroys) a boson with energy $\hbar\omega_b$ in the respective microtrap. It should be stressed that an effectively 1D system of this kind is realized in practice by choosing the transverse trapping frequencies to be an order of magnitude larger than the longitudinal frequency ω_b.

Unlike the vdW case, dressing resonant dipole-dipole interactions necessitates the use of two laser couplings and four electronic states (two ground states and two Rydberg states) [51]. Thus, an off-resonant coherent coupling of two ground states (i.e., two different levels in the hyperfine manifold of the alkali-atom electronic ground state)—here denoted by $|g\rangle$ and $|h\rangle$—to a pair of highly excited Rydberg states $|n_q S\rangle$ and $|n_q P\rangle$ (where n_q is the principal quantum number) is envisaged here [for an illustration, see Fig. 1(b)]. The last two states correspond to the angular momentum quantum numbers $l = 0, 1$. In addition, all atoms are hereafter assumed to be prepared in states corresponding to the value $m_l = 0$ of the azimuthal quantum number, never acquiring $m_l \neq 0$. Along with the previously assumed geometric confinement of atoms, this ensures that the effective dressing-induced interaction potential for a pair of atoms will have no angular dependence.

As a result of Rydberg dressing, an atom that initially resided in the ground-state $|g\rangle$ ($|h\rangle$) finds itself in the dressed state $|0\rangle \approx |g\rangle + \alpha_s |n_q S\rangle$ ($|1\rangle \approx |h\rangle + \alpha_p |n_q P\rangle$), where $\alpha_{s,p} \equiv \Omega_{s,p}/(2\Delta_{s,p})$ are the effective (dimensionless) dressing parameters, fixed by the respective total Rabi frequencies $\Omega_{s,p}$ of the driving fields and the total laser detunings $\Delta_{s,p}$ [cf. Fig. 1(b)] [51]. [Note that because the coherent coupling between the ground and the Rydberg states is in practice realized through two-photon (or multiphoton) transitions, the Rabi frequencies $\Omega_{s,p}$ and the detunings $\Delta_{s,p}$ have to be considered as effective quantities.] These parameters represent a

quantitative measure for how far-detuned the coherent laser coupling is. For the sake of simplicity, it is hereafter assumed that $\alpha_s = \alpha_p \equiv \alpha$ and $\Delta_s = \Delta_p \equiv \Delta/2$, where $\Delta \equiv \Delta_s + \Delta_p$. These assumptions imply that $\Omega_s = \Omega_p \equiv \Omega = \Delta\alpha$.

The physical mechanism behind the coupling of the internal states of atoms in this system to their motional degrees of freedom represents a twofold generalization of the conventional excitation transport enabled by the resonant dipole-dipole interaction [61]; the latter scales as the inverse third power of the interatomic distance ($V_{\rm dd} = C_3/R^3$). First, in the usual setting for a pair of atoms prepared in two different Rydberg states $|s\rangle$ and $|p\rangle$ (the latter state being higher in energy—the "excited state") the resonant dipole-dipole interaction gives rise to the hopping of a p excitation between the two atoms. Thus, this interaction is accompanied by the electronic-state transfer $|s, p\rangle \rightleftarrows |p, s\rangle$ between the two atoms [62–64]. On the other hand, here the role of the ordinary Rydberg states is played by the dressed ones (denoted above by $|0\rangle$ and $|1\rangle$) [51], which represent the logical qubit states. Second, due to the vibrational motion of atoms interatomic distances in the system under consideration are not fixed but instead dynamically fluctuate so that, e.g., the distance between atoms n and $n+1$ is given by $a + u_{n+1} - u_n$. This leads to an effective dependence of both the excitation's on-site energy and its hopping amplitude on the boson degrees of freedom, two e-b coupling mechanisms reminiscent of those encountered in solid-state systems [65–69].

III. EFFECTIVE EXCITATION-BOSON HAMILTONIAN

In what follows, an effective e-b Hamiltonian of the system at hand is first presented (Sec. III A). The form of this Hamiltonian follows from the effective interaction potential for a pair of Rydberg-dressed atoms (in the resonant dipole-dipole configuration of their internal states), which was derived using van Vleck perturbation theory in Ref. [51] (with the dressing parameters α_s and α_p serving as small parameters for the perturbative expansion). In the present paper this last result is used as the point of departure for the treatment of an ensemble of atoms with the additional assumption that the interatomic distances dynamically fluctuate (recall the discussion in Sec. II). Following the discussion of the most general e-b Hamiltonian of the system, further considerations are devoted to a special case of relevance for the engineering of the sought-after W states (Sec. III B).

A. General case

The system Hamiltonian, describing an itinerant dressed Rydberg excitation coupled to dispersionless bosons, can succinctly be written as

$$H = \sum_n \varepsilon_n(\mathbf{u}) c_n^\dagger c_n + \sum_n t_{n,n+1}(\mathbf{u})(c_{n+1}^\dagger c_n + \text{H.c.}) + \hbar\omega_{\rm b} \sum_n b_n^\dagger b_n, \tag{1}$$

where $\mathbf{u} \equiv \{u_n | \, n = 1, \ldots, N\}$ is shorthand for the set of the atom displacements and $c_n^\dagger$ (c_n) creates (destroys) an excitation at site n, with

$$\varepsilon_n(\mathbf{u}) = \frac{\alpha^4\hbar\Delta}{2}\left[\left\{1 - \left(\frac{C_3}{\hbar\Delta}\right)^2 \frac{1}{(a + u_{n+1} - u_n)^6}\right\}^{-1} + \left\{1 - \left(\frac{C_3}{\hbar\Delta}\right)^2 \frac{1}{(a + u_n - u_{n-1})^6}\right\}^{-1}\right], \tag{2}$$

being its corresponding on-site energy [51]. The latter depends on the boson displacements not only on-site n, but also on the adjacent sites $n \pm 1$. At the same time,

$$t_{n,n+1}(\mathbf{u}) = \frac{\alpha^4 C_3}{(a + u_{n+1} - u_n)^3} \times \left\{1 - \left(\frac{C_3}{\hbar\Delta}\right)^2 \frac{1}{(a + u_{n+1} - u_n)^6}\right\}^{-1} \tag{3}$$

is the excitation hopping amplitude between sites n and $n+1$ [51], which depends on the difference $u_{n+1} - u_n$ of the respective displacements. To facilitate further analysis, it is prudent to introduce the dimensionless quantity $\zeta \equiv C_3/(\hbar\,\Delta a^3)$, the ratio of the most relevant energy scales in the system at hand.

Before embarking on further discussion, it is useful to stress that the very existence of the dressing-induced on-site term in the Hamiltonian of Eq. (1), which has no analog in the standard resonant dipole-dipole interaction case [62], is a consequence of the fact that the effective dressing-induced interaction potential for a pair of atoms has a nonzero diagonal component [51].

For small displacements ($u_n \ll a$) it is pertinent to expand the expressions on the right-hand side of Eqs. (2) and (3) to linear order in the difference of displacements using the approximation $(1 \pm x)^r \approx 1 \pm rx$ ($|x| \ll 1$). The linear dependence of $\varepsilon_n(\mathbf{u})$ on $u_{n+1} - u_{n-1}$ captures the coupling of the excitation density at site n with the boson displacements on the neighboring sites $n \pm 1$ [breathing-mode-type (B) e-b coupling]; similarly, the linear dependence of $t_{n,n+1}$ on $u_{n+1} - u_n$ describes how the excitation hopping between sites n and $n+1$ is affected by the boson displacements [Peierls-type (P) coupling] [58–60]. This lowest-order expansion reads

$$\begin{aligned} \varepsilon_n(\mathbf{u}) &= \epsilon_0 + \xi_{\rm B}(u_{n+1} - u_{n-1}), \\ t_{n,n+1}(\mathbf{u}) &= -t_e + \xi_{\rm P}(u_{n+1} - u_n), \end{aligned} \tag{4}$$

where $\xi_{\rm B}$ and $\xi_{\rm P}$ are given by

$$\begin{aligned} \xi_{\rm B} &= 3\,\frac{\alpha^4\hbar\Delta}{a}\,\frac{\zeta^2}{(1-\zeta^2)^2}, \\ \xi_{\rm P} &= 3\,\frac{\alpha^4 C_3}{a^4}\,\frac{3\zeta^2 - 1}{(1-\zeta^2)^2}, \end{aligned} \tag{5}$$

and the bare on-site energy and hopping amplitude by

$$\epsilon_0 = \frac{\alpha^4\hbar\Delta}{1-\zeta^2}, \quad t_e = -\frac{\alpha^4 C_3}{a^3(1-\zeta^2)}. \tag{6}$$

The positive sign of t_e [realized for $|\zeta| > 1$, i.e., $C_3/(\hbar|\Delta|a^3) > 1$] corresponds to the conventional situation where the bare-excitation dispersion $\epsilon_0 - 2t_e \cos k$ has its minimum at $k = 0$. However, of principal interest here is the opposite, negative sign of t_e [for $|\zeta| < 1$, i.e.,

$C_3/(\hbar|\Delta|a^3) < 1$]. This is the case with the band minimum at $k = \pi$, which corresponds to the bare-excitation Bloch state,

$$|\Psi_{k=\pi}\rangle \equiv c^\dagger_{k=\pi}|0\rangle_{\rm e} \otimes |0\rangle_{\rm b}, \tag{7}$$

where $|0\rangle_{\rm e}$ and $|0\rangle_{\rm b}$ are the excitation and boson vacuum states, respectively.

The effective system Hamiltonian has a noninteracting part that comprises free excitation and boson terms,

$$H_0 = \epsilon_0 \sum_n c^\dagger_n c_n - t_e \sum_n (c^\dagger_{n+1} c_n + \text{H.c.}) + \hbar\omega_{\rm b} \sum_n b^\dagger_n b_n. \tag{8}$$

Its interacting part is given by $H_{\text{e-b}} = H_{\rm B} + H_{\rm P}$, where

$$H_{\rm B} = g_{\rm B}\hbar\omega_{\rm b} \sum_n c^\dagger_n c_n (b^\dagger_{n+1} + b_{n+1} - b^\dagger_{n-1} - b_{n-1}),$$
$$H_{\rm P} = g_{\rm P}\hbar\omega_{\rm b} \sum_n (c^\dagger_{n+1} c_n + \text{H.c.})(b^\dagger_{n+1} + b_{n+1} - b^\dagger_n - b_n) \tag{9}$$

are the terms that correspond to the two different e-b coupling mechanisms described above [cf. Eq. (4)] with $g_{\rm B} \equiv \xi_{\rm B}/(2m\hbar\omega_{\rm b}^3)^{1/2}$ and $g_{\rm P} \equiv \xi_{\rm P}/(2m\hbar\omega_{\rm b}^3)^{1/2}$ being the dimensionless B and P coupling strengths. Importantly, owing to the discrete translational symmetry of the system, the eigenstates of $H = H_0 + H_{\text{e-b}}$ can be labeled by the eigenvalues of the total quasimomentum operator $K_{\rm tot} = \sum_k k\, c^\dagger_k c_k + \sum_q q\, b^\dagger_q b_q$. The permissible eigenvalues of this operator belong to the Brillouin zone that corresponds to the underlying 1D lattice.

The bare on-site energy ϵ_0 in Eq. (8) plays the usual role of the on-site energy in tight-binding models—that of a constant energy offset (i.e., the band-center energy) for an itinerant excitation. Although ϵ_0 is inconsequential for the physical mechanism that leads to π-twisted W states in the system at hand (competition of the B and P couplings), the fact that it depends on the dressing parameter α [cf. Eq. (6)] does have some bearing on the preparation of such states (see Sec. IV B below).

B. Special case: Sweet-spot detuning

Consider now the special case of the proposed system with equal P and B coupling strengths, i.e., $g_{\rm P} = g_{\rm B} \equiv g$. The latter physical situation corresponds to the ss value $\zeta_{\rm ss} = (1 + \sqrt{13})/6 \approx 0.77$ of ζ. Assuming that the atomic species and the principal quantum number are chosen, which fixes the interaction constant C_3, for each choice of the lattice period a this last situation is realized for the detuning $\Delta^{\rm ss} \equiv C_3/(\hbar\zeta_{\rm ss}a^3)$ and a range of values for the Rabi frequency $\Omega^{\rm ss} = \Delta^{\rm ss}\alpha$ [determined by the adopted range of values of the dressing parameter (see below)]. In this special case, the e-b coupling and the total Hamiltonians will be denoted by $H^{\rm ss}_{\text{e-b}}$ and $H^{\rm ss}$, respectively, in what follows.

To quantify the e-b coupling strength in the aforementioned special case of the system under consideration, one invokes the momentum-space form of $H^{\rm ss}_{\text{e-b}}$, which reads

$$H^{\rm ss}_{\text{e-b}} = N^{-1/2} \sum_{k,q} \gamma^{\rm ss}_{\text{e-b}}(k,q)\, c^\dagger_{k+q} c_k (b^\dagger_{-q} + b_q)\,. \tag{10}$$

The explicit form of the e-b vertex function $\gamma^{\rm ss}_{\text{e-b}}(k,q)$ in the last equation is

$$\gamma^{\rm ss}_{\text{e-b}}(k,q) = 2ig\hbar\omega_{\rm b}\,[\sin k - \sin q - \sin(k+q)]. \tag{11}$$

Consequently, the effective e-b coupling strength—generally defined as $\lambda_{\text{e-b}} = \langle|\gamma_{\text{e-b}}(k,q)|^2\rangle_{\rm BZ}/(2|t_{\rm e}|\omega_{\rm b})$ [70], where $\langle\cdots\rangle_{\rm BZ}$ stands for the Brillouin-zone average—in this special case evaluates to $\lambda^{\rm ss}_{\text{e-b}} \equiv 3g^2\,\hbar\omega_{\rm b}/|t_{\rm e}|$, i.e.,

$$\lambda^{\rm ss}_{\text{e-b}} = \frac{27}{2}\,\alpha^4\,\frac{C_3}{m\omega_{\rm b}^2 a^5}\,\frac{\left(3\zeta_{\rm ss}^2 - 1\right)^2}{\left(1 - \zeta_{\rm ss}^2\right)^3}\,. \tag{12}$$

The obtained dependence of $\lambda^{\rm ss}_{\text{e-b}}$ on $\alpha \equiv \Omega^{\rm ss}/\Delta^{\rm ss}$ implies that the Rabi frequency $\Omega^{\rm ss}$ is the main experimental knob in the system at hand. By varying $\Omega^{\rm ss}$ different characteristic regimes of this system can be explored.

To set the stage for further analysis, it is prudent to specify at this point the realistic range of values for each of the relevant system parameters. The system at hand is mostly analyzed in what follows for $n_{\rm q} = 80$ with the corresponding value of $C_3 = 2\pi\hbar \times 40\ \text{GHz}\,\mu\text{m}^3$ of the resonant dipole-dipole interaction constant for ^{87}Rb atoms. As usual for optical-tweezer arrays, the lattice period a is in the range between about 3 μm and tens of micrometers. The corresponding values of the ss detuning can vary in an extremely wide range, depending on the choice of a; for example, for $a = 4\ \mu$m one obtains $\Delta^{\rm ss} \approx 5.12$ GHz, for $a = 10\ \mu$m one finds $\Delta^{\rm ss} \approx 327.4$ MHz, whereas for $a = 15\ \mu$m one has $\Delta^{\rm ss} \approx 97$ MHz. At the same time, the typical values for the trapping frequency $\omega_{\rm b}$ are $\omega_{\rm b}/(2\pi) \sim$ (2–5) kHz. Finally, for the dressing parameter α it is worthwhile to consider values in the range of 0.01–0.1.

IV. RESULTS AND DISCUSSION

In the following, the principal findings of this paper are presented and discussed. The character of the ground states of the system at hand is first analyzed (Sec. IV A); it is explained that in a broad parameter window they coincide with the desired π-twisted W states. The W-state preparation protocol is then presented with emphasis on its robustness that stems from the specific character of the energy spectrum of the system (Sec. IV B). Finally, the significance of the obtained results for QIP with Rydberg-dressed qubits is briefly discussed in Sec. IV C.

A. Ground state and its connection to π-twisted W states

Lanczos-type exact diagonalization [70] of $H^{\rm ss} = H_0 + H^{\rm ss}_{\text{e-b}}$ is carried out here for a system with $N = 10$ sites (i.e., atoms) and the maximal number $M = 8$ of bosons in the truncated boson Hilbert space. This is performed using a well-established procedure for a controlled truncation of bosonic Hilbert spaces. This procedure entails a gradual increase in N with the concomitant increase in M, until a numerical convergence of the obtained results for the ground-state energy and other relevant quantities is achieved [70].

The performed numerical calculation shows that the ground state of $H^{\rm ss}$ undergoes a sharp level-crossing transition [70] at a certain critical value $(\lambda^{\rm ss}_{\text{e-b}})_{\rm c}$ of $\lambda^{\rm ss}_{\text{e-b}}$. For $\lambda^{\rm ss}_{\text{e-b}} <$

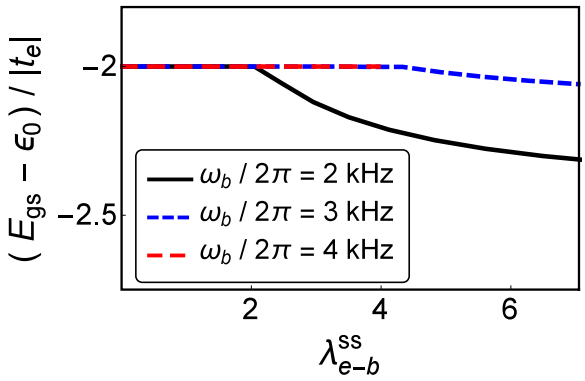

FIG. 2. Dependence of the ground-state energy of the system without the on-site-energy contribution ϵ_0, on the effective coupling strength $\lambda^{ss}_{e\text{-}b}$ for $a = 4\ \mu$m and three different values of the frequency ω_b.

$(\lambda^{ss}_{e\text{-}b})_c$ the ground state corresponds to the eigenvalue π of K_{tot} and has a peculiar character. Namely, despite being the ground state of an interacting e-b Hamiltonian, it has the form of the bare-excitation Bloch state $|\Psi_{k=\pi}\rangle$ [cf. Eq. (7)], and its energy $\epsilon_0 - 2|t_e|$ corresponds to a minimum of a 1D tight-binding dispersion. By contrast, the strongly boson-dressed ground state for $\lambda^{ss}_{e\text{-}b} \geqslant (\lambda^{ss}_{e\text{-}b})_c$ is twofold degenerate and corresponds to $K = \pm K_{gs}$, where $0 < K_{gs} < \pi$. The dependence of the ground-state energy E_{gs} (without the constant contribution ϵ_0), expressed in units of $|t_e|$ on $\lambda^{ss}_{e\text{-}b}$ is depicted in Fig. 2.

Figure 3 illustrates how the ground-state total quasimomentum K_{gs} depends on $\lambda^{ss}_{e\text{-}b}$. In particular, $K_{gs} = \pi$ in the bare-excitation ground-state $|\Psi_{k=\pi}\rangle$ has a vanishing bosonic contribution as $\langle\Psi_{k=\pi}|\sum_n b^\dagger_n b_n|\Psi_{k=\pi}\rangle = 0$. The fact that this bare-excitation Bloch state is a ground state of an interacting e-b system is a direct implication of the assumption that $g_P = g_B \equiv g$ (recall Sec. III B), i.e., it is a consequence of an effective mutual cancellation of P and B couplings for a bare excitation with this particular quasimomentum $(k = \pi)$.

It is pertinent at this point to establish the connection between the ground states of the system at hand and the desired N-qubit W states. The bare-excitation Bloch state $|\Psi_k\rangle$, recast

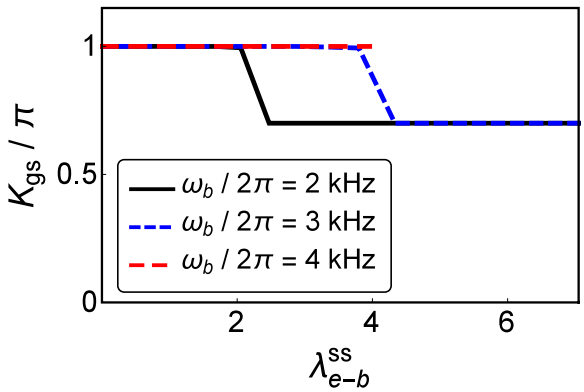

FIG. 3. Dependence of the ground-state total quasimomentum on the effective coupling strength $\lambda^{ss}_{e\text{-}b}$ for $a = 4\ \mu$m and three different values of the frequency ω_b.

in terms of the pseudospin-1/2 (qubit) degrees of freedom, coincides with the twisted W state,

$$|W_N(k)\rangle = N^{-1/2}\sum_{n=1}^{N} e^{ikn}|0\cdots 1_n\cdots 0\rangle \tag{13}$$

parametrized by the quasimomentum k from the Brillouin zone (i.e., $-\pi < k \leqslant \pi$). In particular, $|\Psi_{k=\pi}\rangle$—the ground state of the system at hand for $\lambda^{ss}_{e\text{-}b} < (\lambda^{ss}_{e\text{-}b})_c$—corresponds to the π-twisted W state $|W_N(k=\pi)\rangle$ of Rydberg-dressed qubits.

The conditions for realizing the desired states $|W_N(k=\pi)\rangle$ in the system under consideration are easily reached with realistic values of the relevant experimental parameters (a, ω_b, α). To justify that, it is worthwhile to immediately note that Figs. 2 and 3 correspond to $a = 4\ \mu$m, a relatively small lattice period which favors larger coupling strengths [cf. Eq. (12)] and allows the onset of a sharp transition. Yet, already for this choice of a, with a sufficiently large trapping frequency ($\omega_b \gtrsim 2\pi \times 3.5$ KHz) the effective coupling strength $\lambda^{ss}_{e\text{-}b}$ is always below the critical one, i.e., π-twisted W states are accessible in the entire adopted range of values (0.01–0.1) for the dressing parameter α. The fast decay of $\lambda^{ss}_{e\text{-}b}$ with a ensures that for $a \gtrsim 5\ \mu$m the sought-after W are the ground states of the system for any realistic choice of ω_b and α. For the sake of completeness, it is worthwhile mentioning that by choosing a smaller principal quantum number these conditions are even easier to satisfy because of the smaller value of C_3; for instance, for $n_q = 50$ the corresponding value of this interaction constant for ^{87}Rb is an order of magnitude smaller than for $n_q = 80$.

It is interesting to observe that—in addition to being the ground state of H^{ss} for coupling strengths below the critical one—the state $|\Psi_{k=\pi}\rangle$ is an exact eigenstate of this Hamiltonian for an arbitrary $\lambda^{ss}_{e\text{-}b}$. Namely, given that $\gamma^{ss}_{e\text{-}b}(k=\pi, q) = 0$ for an arbitrary q [cf. Eq. (11)], it is straightforward to show that $H^{ss}_{e\text{-}b}|\Psi_{k=\pi}\rangle = 0$. Thus, $|\Psi_{k=\pi}\rangle$ is an eigenstate of $H^{ss}_{e\text{-}b}$. Because this last state is an eigenstate of the free Hamiltonian H_0 as well, it follows immediately that it is also an eigenstate of the total Hamiltonian $H^{ss} = H_0 + H^{ss}_{e\text{-}b}$. To conclude, even for those parameters (i.e., values of the Rabi frequency that lead to coupling strengths above the critical one) for which $|\Psi_{k=\pi}\rangle$ does not coincide with the lowest-energy $K = \pi$ eigenstate of the system (for an illustration, see Fig. 4) this state still remains an eigenstate in the discrete (bound-state) part of the spectrum of H^{ss}.

B. W-state preparation protocol

A deterministic Rabi-type driving protocol for the preparation of π-twisted W state is discussed in what follows, assuming that the initial state of the system is $|0\rangle_{e\text{-}b} \equiv |0\rangle_e \otimes |0\rangle_b$, where $|0\rangle_e$ is the shorthand for an N-atom state with zero Rydberg-dressed excitations (i.e., all atoms occupying dressed state $|0\rangle$), whereas $|0\rangle_b$ is the collective boson vacuum, i.e., the state with all atoms being in their motional ground state. For the system to be prepared in state $|0\rangle_{e\text{-}b}$, two preliminary steps ought to be carried out. The first one is to prepare the state $|0\rangle_e$ via Rydberg-dressing starting from the state $|g\rangle_e$ with all atoms in their absolute ground-state $|g\rangle$. The other one is to

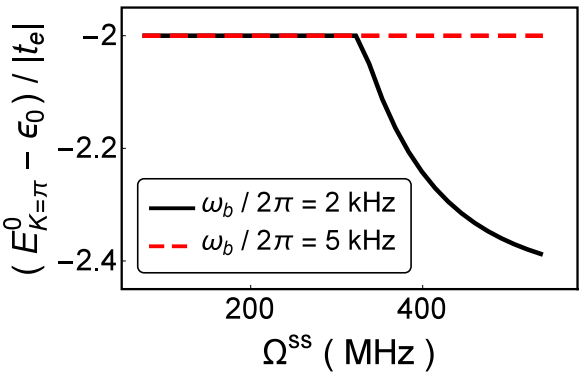

FIG. 4. Dependence of the lowest-energy $K = \pi$ eigenvalue (without the on-site-energy contribution ϵ_0), whose flat part at the energy $-2|t_e|$ corresponds to the π-twisted W state, on the Rabi frequency Ω^{ss} for $a = 4\ \mu$m and two different trapping frequencies.

prepare all atoms in their motional ground states using the established methodology for this purpose [26].

The envisioned state-preparation protocol is based on an external driving given by

$$F_{q_d}(t) = \frac{\hbar\beta(t)}{\sqrt{N}} \sum_{n=1}^{N} (\sigma_n^+ e^{-iq_d n} + \sigma_n^- e^{iq_d n}) , \qquad (14)$$

where $\beta(t)$ accounts for its time dependence and the factors $e^{\pm iq_d n}$ allow for the possibility of applying external driving to different Rydberg-atom qubits with a nontrivial phase difference. The transition matrix element of $F_{q_d}(t)$ between $|0\rangle_{\text{e-b}}$ and $|\Psi_{k=\pi}\rangle \equiv |W_N(k=\pi)\rangle \otimes |0\rangle_{\text{b}}$ is equal to $\hbar\beta(t)\,\delta_{q_d,k=\pi}$, indicating that the required phase difference between adjacent qubits is $q_d = \pi$. Furthermore, by assuming that $\beta(t) = 2\beta_p \cos(\omega_d t)$, where $\hbar\omega_d \equiv \epsilon_0 - 2|t_e|$ is the energy difference between the two relevant states, in the rotating-wave approximation these states are Rabi coupled with the Rabi frequency β_p [71]. Therefore, π-twisted W states are prepared within time $\tau_{\text{prep}} = \pi\hbar/(2\beta_p)$, which does not depend on the system size (N) at all. For example, taking $\beta_p/(2\pi\hbar)$ to be in the range of (10–100) MHz, one finds $\tau_{\text{prep}} \approx$ 3–25 ns, which is four to five orders of magnitude shorter than typical lifetimes of ordinary Rydberg states and seven to eight orders than those of Rydberg-dressed states as they are scaled by an additional factor of α^{-2}.

Apart from being the ground state of the system in a broad parameter window and remaining its eigenstate even outside of that window, the desired W state has another property that facilitates its preparation. Namely, it is separated from the other eigenstates of H^{ss} by an energy gap of $\hbar\omega_b$. Systems in which dispersionless bosons interact with a single itinerant particle [59,60] generically possess such a gap, equal to the single-boson energy, which separates their ground state from the one-boson continuum (inelastic-scattering threshold). In the weak-coupling regime ground states of such systems are typically the only bound states they have [59]. Importantly, apart from increasing the parameter window where W states can be engineered, another advantage of increasing a is a better separation of those states from other states. Namely, an increase in a leads to a decrease in $\omega_d \propto \alpha^4 C_3/a^3$ so that $\hbar\omega_b$ becomes a progressively larger fraction of the energy difference $\hbar\omega_d$. For instance, with $\alpha = 0.05$ and $a = 15\ \mu$m for the chosen range of trapping frequencies ω_b the gap energy amounts to 15–40% of this energy difference, which ensures that the above Rabi-type state-preparation protocol will not be hampered by an inadvertent population of undesired states.

The analysis of the ground-state properties of the system under consideration in Sec. IV A mostly featured the results that correspond to the relatively small lattice period $a = 4\ \mu$m. It is important to stress that this choice, which favors large effective e-b coupling strengths and a possible onset of a sharp transition [cf. Sec. IV A], was intentionally made in order to highlight the worst-case scenario as far as the realization of the desired W-type entanglement resource in the system at hand is concerned. However, from the point of view of an actual W-state preparation, for the reasons stated above it is more favorable to choose an intermediate or large lattice period, say $a \gtrsim 12\ \mu$m. Not only that this precludes an inadvertent population of undesired states in the continuum part of the spectrum of the relevant coupled e-b system, but it also leads to smaller values of the ss detuning (note that $\Delta^{ss} \propto a^{-3}$), which makes this last detuning far smaller than the typical energy spacings of Rydberg levels. (Recall that the distribution of Rydberg energy levels becomes denser as the principal quantum number n_q increases such that the energy difference ΔE of adjacent levels scales as n_q^{-3}; note also that $\Delta E \sim 1$ GHz for $n_q \sim 100$ [1]) This, in turn, prevents the possibility of inadvertently populating higher Rydberg levels in the initial Rydberg-dressing step (i.e., in the preliminary preparation of the state $|0\rangle_{\text{e}}$) of the proposed W-state preparation protocol.

C. Significance for quantum-information processing with Rydberg-dressed qubits

Owing to the rich energy-level structure of Rydberg atoms and the existing wealth of techniques for the coherent manipulation of atomic internal states, there are several possibilities for storing and manipulating quantum information, i.e., QIP with Rydberg qubits. Depending on the number of ground or Rydberg states that make up the qubit (i.e., serve as its logical $|0\rangle$ and $|1\rangle$ states), there are three main types of Rydberg qubits: (i) Those based on one weakly interacting state $|g\rangle \equiv |0\rangle$ and one strongly interacting Rydberg state $|r\rangle \equiv |1\rangle$ (gr qubits), (ii) those encoded using two different Rydberg states (rr qubits, where $|r\rangle \equiv |0\rangle$ and $|r'\rangle \equiv |1\rangle$), and, finally, (iii) those encoded in two long-lived low-lying atomic states $|g\rangle \equiv |0\rangle$ and $|h\rangle \equiv |1\rangle$ (gg qubits).

In particular, gg qubits typically involve two (usually magnetically insensitive) hyperfine sublevels of the electronic ground state—such as states $|g\rangle$ and $|h\rangle$ of the system under consideration [cf. Fig. 1(b)]. Such qubits offer the best trade-off between coherence times and switchable interactions, which makes them promising candidates for universal quantum computing. On the other hand, compared to their gr and rr counterparts, gg qubits are weakly interacting. One possible approach for inducing stronger interactions between such qubits relies on momentarily exciting and deexciting them via Rydberg states using precisely timed or shaped optical fields. An alternative approach for making gg qubits interact more strongly—of relevance for the present paper—is

to weakly admix some Rydberg-state character to the ground states using an off-resonant laser coupling thereby effectively transforming them into Rydberg-dressed qubits [54–57].

Generally speaking, creating quantum entanglement in large systems on timescales much shorter than the relevant coherence times is key to efficient QIP. In particular, it is shown here that in neutral-atom arrays π-twisted W states of Rydberg-dressed qubits can be engineered with the corresponding preparation times being independent of the system size. Importantly, those preparation times are several orders of magnitude shorter than the typical lifetimes of the relevant Rydberg states, being at the same time an even much smaller fraction of the effective lifetimes of their Rydberg-dressed counterparts.

Another favorable feature of the system at hand—and a prerequisite for universal quantum computation—stems from the XY character [72] of the effective qubit-qubit interaction in this system, which is given by the free-excitation hopping term in Eq. (8) and the Peierls-coupling term in Eq. (9). When recast in terms of the pseudospin-1/2 degrees of freedom of Rydberg-dressed qubits, the coupling between qubits n and $n+1$ is given by $J_{n,n+1}(\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$, where $J_{n,n+1}\equiv 2[-t_e+g_{\rm P}\hbar\omega_{\rm b}(b_{n+1}^\dagger+b_{n+1}-b_n^\dagger-b_n)]$ is the effective XY-coupling strength that depends dynamically on the boson degrees of freedom. Such boson-dependent coupling strengths are characteristic, for instance, of certain trapped-ion systems where collective motional modes (phonons) [73] play the role of bosons [74]. Unlike such trapped-ion systems, whose phonon spectra have a quasicontinuous character [73], the system at hand merely involves dispersionless bosons of one single frequency. This circumvents the spectral-crowding problem that poses an obstacle for QIP in large trapped-ion chains [75].

V. SUMMARY AND CONCLUSIONS

This paper proposed a scheme for a deterministic creation of a large-scale W-type entanglement in optical tweezer arrays of atoms with Rydberg-dressed resonant dipole-dipole interaction. The resulting W-state preparation times are independent of the system size, being also several orders of magnitude shorter than the effective lifetimes of the relevant atomic states. It is demonstrated here that the mechanism behind this scalable entanglement resource is robust against the unavoidable coupling of an itinerant dressed Rydberg excitation with the motional degrees of freedom of atoms. Another argument in favor of the robustness of the proposed scheme stems from the fact that π-twisted W states that it aims to realize represent ground states of the underlying system, which are at the same time separated from their other eigenstates by a sizable spectral gap. The recent advances in the manipulation, control, and readout of neutral-atom states in optical dipole traps [76] and the scalability of tweezer arrays bode well for an experimental implementation of this scheme.

In atomic physics, motional degrees of freedom [77,78] have long been perceived exclusively as sources of decoherence and dephasing [79] and have only in recent years been viewed as a quantum resource [18]. The present paper constitutes a demonstration as to how the influence of motional degrees of freedom can be suppressed for the sake of carrying out specific QIP tasks, even in systems where they may play a useful role in other tasks. This particular aspect of the present paper could be generalized to other systems, such as trapped Rydberg ions [80]. Although the latter have quite recently attracted considerable attention in the context of time-efficient gate realizations [81], quantum-state engineering in such systems is a largely unexplored subject.

The concept of Rydberg dressing has in recent years been utilized in diverse contexts [55]. In particular, the present paper underscores its usefulness in engineering maximally entangled states of Rydberg-dressed qubits. In this sense the present paper is complementary to that of Ref. [18], which discussed the preparation of various motional states of Rydberg-dressed atoms. Along with their known advantage—namely, that their effective lifetimes are significantly longer than those of ordinary Rydberg states—the capability of creating entanglement in an ensemble of atoms provides additional motivation to consider QIP with Rydberg-dressed states. Experimental realizations of the proposed W-state preparation scheme—as well as theoretical explorations of its possible generalizations—are clearly called for.

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